

## Strain-induced chiral magnetic effect in Weyl semimetals

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We argue that strain applied to a time-reversal and inversion breaking Weyl semimetal in a magnetic field can induce an electric current via the chiral magnetic effect. A tight-binding model is used to show that strain generically changes the locations in the Brillouin zone but also the energies of the band touching points (tips of the Weyl cones). Since axial charge in a Weyl semimetal can relax via intervalley scattering processes, the induced current will decay with a time scale given by the lifetime of a chiral quasiparticle. We estimate the strength and lifetime of the current for typical material parameters and find that it should be experimentally observable.

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### I. INTRODUCTION

Chiral anomalies belong to the most emblematic predictions of quantum field theory [1]. In the presence of massless (chiral) fermions not all the symmetries of the classical theory are compatible with the quantum theory. The theory of anomalies has a very wide range of applications, reaching from particle physics to condensed matter physics. In recent years anomaly-induced transport phenomena, such as the chiral magnetic effect (CME), have attracted much attention in the quark-gluon plasma [2–4], and in Dirac matter [5,6]. The recently synthesized Weyl semimetals (WSMs) [7–9] are an optimum benchmark to test anomaly-related phenomena in condensed matter. These materials have chiral Weyl points separated in momentum space. The minimal model with two Weyl nodes in a continuum description is described by the action

$$S = \int d^4k \bar{\psi}_{-k} (\gamma^\mu k_\mu - m - b_\mu \gamma^\mu \gamma_5) \psi_k, \quad (1)$$

which resembles a Lorentz breaking QED action [10,11]. The WSM phase is reached when the parameters obey the condition  $-b^2 > m^2$ , in which case the separation between nodes is proportional to the four-vector  $\Delta k^\mu \equiv \lambda^\mu \sim b^\mu \sqrt{1 - \frac{m^2}{b^2}}$ .

The CME describes the generation of an electric current parallel to an applied magnetic field in chirally imbalanced matter [2–4]. In the context of Weyl semimetals it is best described by the formula [5,12]

$$\vec{j} = \frac{\mu_L - \mu_R + E_L - E_R}{4\pi^2} \vec{B} = \frac{\mu_5 - \lambda_0}{2\pi^2} \vec{B}, \quad (2)$$

where  $\mu_{L,R}$  are the chemical potentials of left- and right-handed fermions as measured from the tips of the Weyl cones whereas  $E_{L,R}$  are the energies at which the Weyl cones are located. The axial chemical potential is  $2\mu_5 = \mu_L - \mu_R$  and the difference in energy of the band touching points is  $2\lambda_0 = E_R - E_L$ .

The axial symmetry is not exact. It is broken even at the tree level by the mass term in (1). In a crystal this is unavoidable due to the compactness of the Brillouin zones. Therefore, axial charge will decay to a thermodynamic equilibrium state in which the left- and right-handed Fermi surfaces sit at the same

energy such that  $E_L + \mu_L = E_R + \mu_R$ . The CME vanishes in a Weyl semimetal in thermodynamic equilibrium [5,11].

In order to induce a nonvanishing chiral magnetic current, it is clear from (2) that there are in principle two options. First, one might modify the chemical potentials by inducing an imbalance in the occupation numbers of left- and right-handed fermions. One way to do this is to use the proper axial anomaly

$$\partial_\mu J_5^\mu = \frac{1}{2\pi^2} \vec{E} \cdot \vec{B}. \quad (3)$$

In parallel electric and magnetic fields, fermions are pumped via spectral flow from one Weyl cone to the other, thereby changing their chirality. This process leads to a dramatic enhancement of the electric conductivity along the direction of the magnetic field. The anomaly-induced negative magnetoresistivity has indeed been verified experimentally in Dirac and Weyl semimetals [13–16].

A second possibility is to change the locations of the Weyl cones  $E_{L,R}$ . We argue in the remainder of this Rapid Communication that this can indeed be achieved by applying strain, in particular, it will be induced by the time component of the axial elastic vector field that emerges when lattice deformations are included in the model. As a result, we find an elastic contribution to the chiral magnetic effect linearly proportional to the strain tensor  $u_{ij}$  and to the Grüneisen parameter of the material  $\beta$ .

### II. STRAIN-INDUCED CHIRAL IMBALANCE

The local band structure for a WSM with two Weyl points is described by the continuum model

$$S = \int d^4k \bar{\psi}_k (\gamma^\mu k_\mu - \lambda_\mu \gamma^\mu \gamma_5) \psi_k. \quad (4)$$

The schematic band structure is represented in Fig. 1. The space components of the vector  $\lambda_\mu$  break time-reversal symmetry and its magnitude sets the separation of the Weyl nodes in momentum space. Its time component  $\lambda_0$  breaks inversion symmetry and marks the separation in energy of the points. The emergence of elastic gauge fields coupling with opposite signs to the two chiralities (axial) in time-reversal-breaking Weyl semimetals was derived from a tight-binding model in

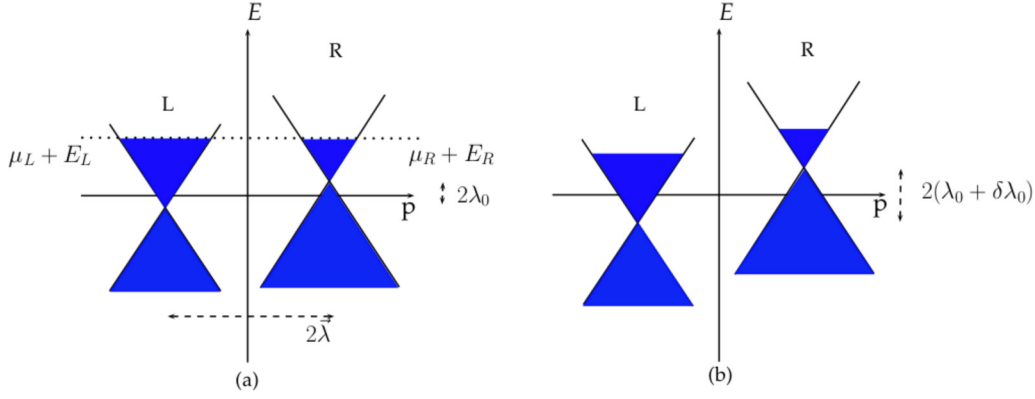


FIG. 1. Local band structure of a Weyl semimetal with two nodal points at different energies and different positions in momentum space. In (a) there are Fermi surfaces of different sizes for left- and right-handed fermions as measured by the left- and right-handed chemical potentials  $\mu_{L,R}$ . The Fermi energy of both Weyl cones is, however, the same in equilibrium, resulting in vanishing CME. In (b) the situation shortly after applying strain is depicted. The tips of the Weyl cones have shifted in energy but the Fermi surfaces have not yet had enough time to equilibrate,  $t < \tau_{\text{inter}}$ . The CME is given by  $\vec{J} = \frac{\delta\mu_5(t) - \delta b_0}{2\pi^2} \vec{B}$ . Over a time scale given by the intervalley scattering time  $\tau_{\text{inter}}$ , the axial chemical potential  $\delta\mu_5$  will build up such that for  $t \gg \tau_{\text{inter}}$  it takes the value  $\delta\mu_5 = \delta b_0$  and equilibrium with vanishing CME is reached again.

Ref. [17] (see also Ref. [18]). But, as we discussed earlier, the chiral magnetic effect requires a time component of the elastic gauge field not described before in graphene or in any other material. In what follows, we will give the main lines of the derivation of the time component for the elastic gauge field. A full detail of this derivation is provided in the Appendix.

A tight-binding model that captures the physics of a WSM containing a pair of Weyl nodes, separated both in energy and in momentum space, is described by the Hamiltonian

$$H_0 = \sum_{k,s} t \sin(k_s a) c_k^\dagger \alpha_s c_k - r \sum_{k,s} \cos(k_s a) c_k^\dagger \hat{\beta} c_k + \Delta \sum_k c_k^\dagger \hat{\beta} c_k + b_3 \sum_k c_k^\dagger \alpha_3 \gamma_5 c_k + b_0 \sum_k c_k^\dagger \gamma_5 c_k. \quad (5)$$

A detailed description of this Hamiltonian is given in the Appendix. The last term in (5) accounts for a relative energy shift of the energies of the Weyl points. The effect of the elastic deformations can be taken into account by knowing how the hopping parameters  $t$ , and  $r$  change under strain [19]:

$$t\alpha_s \rightarrow t(1 - \beta u_{ss})\alpha_s + t\beta \sum_{s' \neq s} u_{ss'}\alpha_{s'}, \quad (6a)$$

$$r \rightarrow r_s = r(1 - \beta u_{ss}). \quad (6b)$$

Inserting these changes in (5), and expanding around the Weyl nodes, we obtain the part of the Hamiltonian that depends on the strain,

$$H[u] = \sum_k \left( 2\tau b_0 \beta \frac{v\lambda_3}{m + b_3} u_{33} \right) \psi_k^\dagger \sigma_0 \psi_k + \sum_{k,s=1,2} (\tau\beta v\lambda_3 u_{3s}) \psi_k^\dagger \sigma_s \psi_k + \sum_k \left( 2\beta \frac{v^2 \lambda_3^2}{m + b_3} u_{33} - \beta r \text{Tr}[u] \right) \psi_k^\dagger \sigma_3 \psi_k, \quad (7)$$

where  $\tau = \pm 1$  labels the chirality of each Weyl node. The last two terms were derived previously [17], while the first term is the *temporal* component of the elastic vector field. The same derivation can be obtained from general symmetry arguments along the lines of Ref. [20]. We note that, from a general symmetry approach, even if the unstrained material has  $b_0 = 0$ , an effective coupling of the form  $\tau A_0(x)\sigma_0$  will be generated in the low-energy effective action since a generic strain deformation will break inversion symmetry. Moreover, since rotational invariance is broken in the material by the vector  $\mathbf{b}$ , the only symmetry remaining is rotations in the perpendicular plane. If  $\mathbf{b}$  points, say, along the  $OZ$  axis, the  $u_{33}$  component of the strain tensor is a scalar quantity that can be coupled to any term in the effective Hamiltonian.

This part of the elastic gauge field shifts the locations of the Weyl points when elastic deformations are induced in the system and induce an effective axial gauge field:

$$2(\delta\lambda_0) = (E_L - E_R)\beta u_{33}. \quad (8)$$

The strain-induced CME is proportional to the difference in energies of the Weyl cones before applying strain, to the Grüneisen parameter of the material, and to the magnitude of the strain itself. Since eventually the Fermi surfaces will equilibrate by processes such as intervalley scattering or through spectral flow through the edge states (Fermi arcs), this strain-induced CME will be observable only over a short time span set by the lifetime  $\tau_{\text{inter}}$  of a chiral quasiparticle at the corresponding Fermi surface. This time is estimated to be much larger than the intravalley scattering rate or the usual impurity-induced lifetime [21,22]. Taking anomaly and intervalley scattering into account, the time development of the axial chemical potential is given by

$$\partial_t \rho_5 = \frac{1}{2\pi^2} \vec{E} \cdot \vec{B} - \frac{1}{\tau_{\text{inter}}} (\rho_5 - \rho_5^{(\text{eq})}), \quad (9)$$

where  $\rho_5^{(\text{eq})} = \rho_5(\lambda_0 + \delta\lambda_0)$ . The relation between the axial chemical potential and the axial charge is given by the equation of state  $\rho_5 = \frac{\partial P}{\partial \mu_5}$ , with  $P$  the thermodynamic pressure for

chiral fermions. This will result in an exponentially decaying strain-induced chiral magnetic current

$$\vec{J} = \frac{\delta\lambda_0}{2\pi^2} \vec{B} e^{-t/\tau_{\text{inter}}} = \frac{(E_L - E_R)\beta u_{33}}{4\pi^2} \vec{B} e^{-t/\tau_{\text{inter}}}. \quad (10)$$

### III. PHYSICAL FEASIBILITY OF THE PROPOSAL

We can estimate the strength of the current by taking the typical separation of the Weyl nodes in energy to be of the order of 1 meV, the Grüneisen parameter  $\beta = 1$ , and the attainable amount of strain in the percentage realm (1%). The resulting current is

$$\vec{J} \approx 10^5 \cdot \vec{B} \text{ (A/m}^2\text{)}, \quad (11)$$

with the magnetic field  $B$  expressed in tesla. For a sample of a cross section of  $10 \mu\text{m}^2$ , the induced current is of the order of  $10 \mu\text{A}$  in a magnetic field of 1 T [23]. This is an observable quantity of the same order of magnitude as the one described in Refs. [24,25]. Since the considered WSM materials break inversion symmetry, they will also be piezoelectric. For comparison, the maximum current in a typical piezoelectric varies from nA to  $\mu\text{A}$  and the voltage generated in 1–100 V, depending on the size of the material. The CME signal should be easily distinguishable from the standard piezoresponse due to its linear dependence on the magnetic field. The current will decay exponentially within a typical time  $10\tau_{\text{inter}}$ .

For expression (2) to be applicable we need to assume that the strain-induced deformation acts on a time scale which is slow compared to the equilibration time with a Weyl cone but faster than the equilibration time for the axial charge, i.e., the intervalley scattering time. This can be fulfilled in WSM where the intervalley relaxation time is estimated to be at least two orders of magnitude larger than the standard lifetime [21].

A potential difficulty to measure the effect lies in the fact that the intervalley relaxation time (estimated to be of the order of  $10^{-9}$  s at best [22]) is of the same order of magnitude as the time needed for the elastic perturbation to propagate along the sample (assumed of micron size) which might attenuate the signal. Still, the magnitude of the induced current will be within the observable range.

Time-reversal symmetry-breaking Weyl semimetals (both type I and type II) have been experimentally reported recently [26,27], while others have been theoretically proposed [28,29]. It is thus expected that the present proposal can be tested in the near future. Anomaly-related transport phenomena in WSMs [11,30–32] and the effects of strain in WSMs [33,34] have been considered previously in various contexts, but the contribution of elastic gauge fields is potentially more relevant. Another interesting possibility is that elastic gauge fields could realize the topological Lifshitz transitions described in Refs. [35,36].

*Note added.* Recently, we became aware of two works [37,38] concerning the effect of strain on WSMs. These works rely on the elastic gauge fields deduced in Ref. [17] that only had spatial components. The chiral magnetic effect presented in this work is an entirely different contribution based on the time component of the elastic gauge field derived in the Appendix.

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### APPENDIX: TIME COMPONENT OF THE ELASTIC GAUGE FIELD

#### 1. The model with $b_0$

The minimal tight-binding model reproducing the band structure of WSM is that of  $s$ - and  $p$ -like electrons hopping in a cubic lattice and chirally coupled to an on-site constant vector field  $b_\mu$  [11,19],

$$\begin{aligned} H_0 = & \sum_i \sum_s \frac{-1}{2} c_i^+ (r\hat{\beta} + it\alpha_s) c_{i+s} \\ & - \frac{1}{2} c_{i+s}^+ (r\hat{\beta} - it\alpha_s) c_i + \sum_i \Delta c_i^+ \hat{\beta} c_i \\ & + \sum_i b_3 c_i^+ \alpha_3 \gamma_5 c_i + \sum_i b_0 c_i^+ \alpha_3 c_i, \end{aligned} \quad (A1)$$

where  $t, r, m$  are hopping parameters between the  $s$  and  $p$  states, hopping between the same kinds of states, and the difference of on-site energies between the  $s$  and  $p$  states, respectively. The parameter  $\Delta$  is  $\Delta = m + 3r$ . Without loss of generality, we choose the vector field  $\mathbf{b}$  to point along the  $OZ$  direction  $b_3$ . This spatial component breaks time-reversal  $\mathcal{T}$  and  $SO(2)$  rotational symmetry; the time component  $b_0$  shifts the Weyl cones in energy and breaks inversion symmetry. In momentum space,

$$\begin{aligned} H_0 = & \sum_{k,s} t \sin(k_s a) c_k^+ \alpha_s c_k - r \sum_{k,s} \cos(k_s a) c_k^+ \hat{\beta} c_k \\ & + \Delta \sum_k c_k^+ \hat{\beta} c_k + b_3 \sum_k c_k^+ \alpha_3 \gamma_5 c_k + b_0 \sum_k c_k^+ \gamma_5 c_k, \end{aligned} \quad (A2)$$

with  $s = 1, 2, 3$ . We will use the following set of Dirac matrices,  $\alpha_1 = \tau_0 \sigma_1$ ,  $\alpha_2 = \tau_0 \sigma_2$ ,  $\alpha_3 = \tau_1 \sigma_3$ ,  $\hat{\beta} = \tau_3 \sigma_3$ ,  $\gamma_5 = \tau_1 \sigma_0$ , so  $\alpha_3 \gamma_5 = \tau_0 \sigma_3$ .

We will choose to expand the Hamiltonian (A2) around the  $\mathbf{k} = \mathbf{0}$  point ( $\mathbf{k} \cdot \mathbf{p}$  theory) and compute the value of the emergent Fermi points within this approximation. When computing the effect of the strain we will start from this approximation. Around the  $\Gamma$  point,  $\sin(k_s a) \simeq k_s a$ , and

$\cos(k_s a) \simeq 1$ , so the Hamiltonian matrix reads ( $v = ta$  and  $m = \Delta - 3r$ )

$$\mathcal{H}_0(\mathbf{k}) = \begin{pmatrix} v\boldsymbol{\sigma} \cdot \mathbf{k}_\perp + (m + b_3)\sigma_3 & vk_3\sigma_3 + b_0\sigma_0 \\ vk_3\sigma_3 + b_0\sigma_0 & v\boldsymbol{\sigma} \cdot \mathbf{k}_\perp + (b_3 - m)\sigma_3 \end{pmatrix}, \quad (\text{A3})$$

acting on the spinor  $\Psi_{\mathbf{k}} = (\phi_{\mathbf{k}}, \psi_{\mathbf{k}})^T$ . The momentum  $\mathbf{k}_\perp$  is the momentum perpendicular to  $b_3$ . The main difference with respect to the original case is that now, for energies  $\omega, v|\mathbf{k}_\perp| \ll m + b_3$ , the high-energy sector represented by  $\phi_{\mathbf{k}}$  is related to  $\psi_{\mathbf{k}}$  through

$$\phi_{\mathbf{k}} \simeq -\frac{\sigma_0 vk_3 + \sigma_3 b_0}{m + b_3} \psi_{\mathbf{k}}, \quad (\text{A4})$$

and the effective two-band model is

$$H_{\text{eff}} = \sum_{\mathbf{k}} \psi_{\mathbf{k}}^+ \left( -\frac{2b_0 vk_3}{m + b_3} \sigma_0 + v\boldsymbol{\sigma} \cdot \mathbf{k}_\perp + \frac{1}{m + b_3} (b_3^2 - b_0^2 - m^2 - v^2 k_3^2) \sigma_3 \right) \psi_{\mathbf{k}}. \quad (\text{A5})$$

The presence of  $b_0$  induces two differences: First, there is a term proportional to the identity matrix  $\sigma_0$  and *linearly* dependent on  $k_3$ , breaking inversion symmetry [in the original full lattice model, it would be proportional to  $\sin(k_3 a)$ ], and second, it modifies the mass term accompanying the  $\sigma_3$  matrix.

The condition to obtain the Weyl nodes is now when the two bands intersect each other,  $E_+(\mathbf{k}) = E_-(\mathbf{k})$ , so the positions of the Weyl nodes change correspondingly to

$$\lambda_{\pm} = \pm \lambda_3 = \pm \frac{\sqrt{b_3^2 - b_0^2 - m^2}}{v}, \quad (\text{A6})$$

that is, when  $b_0 = 0$ , the two Weyl nodes appeared for values of  $b_3 > m$ . Now, the situation is a little bit more complex (and richer), since for some values of the three parameters  $m, b_3$ , and  $b_0$  it might happen that the maximum of the valence band is placed at higher energies than the minimum of the conduction band, but the two bands do not intersect at any real momentum. The energies where the new Weyl nodes are placed read

$$E_{\pm} = \mp 2b_0 \frac{\sqrt{b_3^2 - b_0^2 - m^2}}{m + b_3}. \quad (\text{A7})$$

Expanding around  $\lambda_3, k_3 = \pm \lambda_3 + \delta k_3$ , for small  $\delta k_3$  the Hamiltonian matrix around the two Fermi points takes the form ( $\tau = \pm 1$  labels the two Weyl points)

$$H_{\tau}^W(\mathbf{k}) = \tau \frac{2vb_0\lambda_3}{m + b_3} \sigma_0 + v\boldsymbol{\sigma} \cdot \mathbf{k}_\perp + \tau v_3 (k_3 + \tau \lambda_3) \sigma_3, \quad (\text{A8})$$

that is, the Hamiltonian of two Weyl modes with opposite chirality ( $v_3 = 2v \sqrt{\frac{b_3^2 - b_0^2 - m^2}{(b_3 + m)^2}}$ ).

## 2. Elastic deformations

Elastic deformations of the lattice induce the following two changes in the Hamiltonian (A2):

$$t\alpha_s \rightarrow t(1 - \beta u_{ss})\alpha_s + t\beta \sum_{s' \neq s} u_{ss'}\alpha_{s'}, \quad (\text{A9a})$$

$$r \rightarrow r_s = r(1 - \beta u_{ss}). \quad (\text{A9b})$$

The tensor  $u_{ij}$  is the strain tensor. We will make the approximation of setting all the Grüneisen parameters  $\beta$  to be equal. After expanding first around the Weyl points and applying these changes in (A2), we can split the new Hamiltonian into  $H_0 + H[u]$ :

$$H[u] = -\beta v \tau \lambda_3 \sum_{\mathbf{k}} u_{33} c_{\mathbf{k}}^+ \alpha_3 c_{\mathbf{k}} + \beta v \tau \lambda_3 \sum_{s \neq 3} u_{3s} c_{\mathbf{k}}^+ \alpha_s c_{\mathbf{k}} + r\beta \sum_{\mathbf{k}} \text{Tr}[u] c_{\mathbf{k}}^+ \hat{\beta} c_{\mathbf{k}}. \quad (\text{A10})$$

Remembering the expression (A4) relating the high-energy sector to the low-energy sector, we can evaluate the elements of the type  $c_{\mathbf{k}}^+ \alpha_s c_{\mathbf{k}}$  keeping the lowest order in a  $1/(m + b_3)$  expansion,

$$\begin{aligned} c_{\mathbf{k}}^+ \alpha_3 c_{\mathbf{k}} &\simeq \phi_{\mathbf{k}}^+ \sigma_3 \psi_{\mathbf{k}} + \psi_{\mathbf{k}}^+ \sigma_3 \phi_{\mathbf{k}} \\ &= -\frac{2}{m + b_3} \psi_{\mathbf{k}}^+ (\tau v \lambda_3 \sigma_3 + b_0 \sigma_0) \psi_{\mathbf{k}}, \end{aligned} \quad (\text{A11a})$$

$$c_{\mathbf{k}}^+ \alpha_s c_{\mathbf{k}} \simeq \psi_{\mathbf{k}}^+ \sigma_s \psi_{\mathbf{k}}, \quad (\text{A11b})$$

$$c_{\mathbf{k}}^+ \hat{\beta} c_{\mathbf{k}} \simeq -\psi_{\mathbf{k}}^+ \sigma_3 \psi_{\mathbf{k}}. \quad (\text{A11c})$$

With these expressions, we then have

$$\begin{aligned} H[u] &= \sum_{\mathbf{k}} \left( 2\tau b_0 \beta \frac{v\lambda_3}{m + b_3} u_{33} \right) \psi_{\mathbf{k}}^+ \sigma_0 \psi_{\mathbf{k}} \\ &+ \sum_{k,s=1,2} (\tau \beta v \lambda_3 u_{3s}) \psi_{\mathbf{k}}^+ \sigma_s \psi_{\mathbf{k}} \\ &+ \sum_{\mathbf{k}} \left( 2\beta \frac{v^2 \lambda_3^2}{m + b_3} u_{33} - \beta r \text{Tr}[u] \right) \psi_{\mathbf{k}}^+ \sigma_3 \psi_{\mathbf{k}}, \end{aligned} \quad (\text{A12})$$

from which we can read the components of the elastic vector field,

$$A_0^{\text{el}} = \tau b_0 \beta \frac{2v\lambda_3}{m + b_3} u_{33}, \quad (\text{A13a})$$

$$A_1^{\text{el}} = \tau \beta v \lambda_3 u_{31}, \quad (\text{A13b})$$

$$A_2^{\text{el}} = \tau \beta v \lambda_3 u_{32}, \quad (\text{A13c})$$

$$A_3^{\text{el}} = 2\beta \frac{v^2 \lambda_3^2}{m + b_3} u_{33} - \beta r \text{Tr}[u]. \quad (\text{A13d})$$

The difference now is that, because the presence of the term  $b_0$  in the original Hamiltonian, a chiral zeroth component of the elastic vector field appears.

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