Transverse thermoelectric response as a probe for existence of quasiparticles

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The electrical Hall conductivities of any anisotropic interacting system with reflection symmetry obey $\sigma_{xy} = -\sigma_{yx}$. In contrast, we show that the analogous relation between the transverse thermoelectric Peltier coefficients, $\alpha_{xy} = -\alpha_{yx}$, does not generally hold in the same system. This fact may be traced to interaction contributions to the heat current operator and the mixed nature of the thermoelectric response functions. Remarkably, however, it appears that emergence of quasiparticles at low temperatures forces $\alpha_{xy} = -\alpha_{yx}$. This suggests that quasiparticle-free ground states (so-called non-Fermi liquids) may be detected by examining the relationship between α_{xy} and α_{yx} in the presence of reflection symmetry and microscopic anisotropy. These conclusions are based on the following results. (i) The relation between the Peltier coefficients is exact for elastically scattered noninteracting particles. (ii) It holds approximately within Boltzmann theory for interacting particles when elastic scattering dominates over inelastic processes. In a disordered Fermi liquid, the latter lead to deviations that vanish as T^3. (iii) We calculate the thermoelectric response in a model of weakly coupled spin-gapped Luttinger liquids and obtain strong breakdown of antisymmetry between the off-diagonal components of $\hat{\alpha}$. We also find that the Nernst signal in this model is enhanced by interactions and can change sign as function of magnetic field and temperature.

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I. INTRODUCTION

Typically, an electronic system sustains average charge and heat current densities, $\boldsymbol{J}^e$, $\boldsymbol{J}^h$, when subjected to a uniform temperature gradient, ∇T, and constant electric field, $\mathbf{E}$. Its linear thermoelectric response is described by

$$\begin{pmatrix} \boldsymbol{J}^e \\ \boldsymbol{J}^h \end{pmatrix} = \begin{pmatrix} \hat{\sigma} & \hat{\alpha} \\ \hat{\tilde{\alpha}} & \hat{\kappa} \end{pmatrix} \begin{pmatrix} \mathbf{E} \\ -\nabla T \end{pmatrix}, \tag{1}$$

where $\hat{\sigma}$ is the conductivity tensor, $\hat{\alpha}$ and $\hat{\tilde{\alpha}}$ are the Peltier tensors, and $\hat{\kappa}$ is the thermal conductivity tensor. In noninteracting systems, the electrical and heat-current operators are simply related to each other, giving rise to relations between $\hat{\sigma}$, $\hat{\kappa}$, and $\hat{\alpha}$. These relations continue to hold in Fermi liquids, up to asymptotically vanishing corrections. An example is the Wiedemann-Franz law, $\hat{\kappa} = (\pi^2/3e^2)T\hat{\sigma}$ (we use throughout $\hbar = k_B = 1, -e < 0$ is the electron charge), whose breakdown has been interpreted as a signature of physics beyond the Fermi liquid framework [1–4]. Another is the exact relation for noninteracting electrons [5,6] between $\hat{\alpha}$ at a given temperature T and chemical potential μ, and $\hat{\sigma}$ of the same system at zero temperature and shifted chemical potential

$$\hat{\alpha}(T,\mu) = \frac{1}{eT}\int_{-\infty}^{\infty} d\epsilon\, \epsilon \frac{\partial n_F(\epsilon)}{\partial \epsilon}\hat{\sigma}(T=0,\mu+\epsilon), \tag{2}$$

where $n_F(\epsilon)$ is the Fermi function. This formula hence implies that in the absence of interactions, $\hat{\alpha}$ shares the same symmetry properties as $\hat{\sigma}$. A similar conclusion is reached by solving the Boltzmann equation within an energy-dependent relaxation-time approximation [7,8].

Owing to the pioneering works of Onsager [9] and subsequently of Kubo [10] it is well known that various linear-response transport coefficients are related via the time reversal symmetry of microscopic dynamics. Consequently, one finds on general grounds that in the presence of a magnetic field B, $\sigma_{ij}(B) = \sigma_{ji}(-B)$ and $\tilde{\alpha}_{ij}(B) = T\alpha_{ji}(-B)$, where $i,j = x,y,z$. In turn, it is straightforward to show that even for an anisotropic system, as long as it is invariant under reflections, say with respect to the y axis, $\sigma_{xy}(B) = -\sigma_{yx}(B)$. The above discussion implies that under similar conditions one also finds $\alpha_{xy}(B) = -\alpha_{yx}(B)$, provided that the system is noninteracting or considered within approximated Boltzmann transport theory. A natural question then arises: is the relation $\alpha_{xy}(B) = -\alpha_{yx}(B)$ valid beyond the limits of these two conditions? Beside its intrinsic theoretical appeal, this issue is also important for identifying non-Fermi liquid behavior in the thermoelectric properties of correlated electronic systems.

One such property is the Nernst signal, defined by the off-diagonal elements S_{xy} and $-S_{yx}$ of the thermopower tensor $\hat{S} = \hat{\sigma}^{-1}\hat{\alpha}$. The latter relates the measured electric field to an applied temperature gradient, $\mathbf{E} = \hat{S}\nabla T$, in the presence of a magnetic field B_z and in the absence of an electrical current. The dependence of $\hat{S}$ on both the resistivity tensor $\hat{\rho} = \hat{\sigma}^{-1}$ and $\hat{\alpha}$ means that generally $S_{xy} = -S_{yx}$ only for isotropic systems. Therefore the symmetry properties of $\hat{S}$ do not carry direct information about interaction effects without independent knowledge of $\hat{\sigma}$. However, such information may be gleaned from discrepancies between the measured Nernst signal and the predictions of Boltzmann transport theory. While this theory accounts for the observed data in a number of materials [11], it underestimates the effect by orders of magnitude in several quasi-one-dimensional conductors [12–14].

The Nernst effect is also a sensitive probe of superconducting fluctuations, which contribute positively to the signal [15–21], in contrast to quasiparticles of various ordered normal states whose contribution is often of a negative sign [22,23]. A positive Nernst effect has been measured over a wide range above the critical temperature T_c in a series of superconductors including the cuprates [24–30], as well as amorphous films of $Nb_{0.15}Si_{0.85}$ and InO_x [31,32]. While the fluctuation contribution in the cuprates emerges from a high-temperature negative quasiparticle signal, the latter dominates the Nernst effect down to, and even below, T_c in other compounds such as the pnictides [33–35]. It is therefore interesting to investigate the interplay between these opposing contributions in systems which exhibit concomitant strong fluctuations towards competing orders including superconductivity.

Motivated by the aforementioned issues, we study in Sec. II the symmetry properties of $\hat{\alpha}$ within a generic model of interacting electrons. We begin by considering the thermoelectric linear response using the Kubo formula. We show that the close relation which exists between the electrical and heat current operators in the noninteracting limit naturally leads, in the presence of reflection symmetry, to $\alpha_{xy}(B) = -\alpha_{yx}(B)$. However, contrary to the corresponding relation for the Hall conductivities, the property $\alpha_{xy}(B) = -\alpha_{yx}(B)$ is not protected by reflection and time-reversal symmetries, and we demonstrate its explicit violation in the exactly solvable problem of two harmonically interacting electrons in a magnetic field. Having established this point of principle, we move on to consider the issue using Boltzmann transport theory for the interacting system. We show that $\alpha_{xy}(B) = -\alpha_{yx}(B)$ is obtained within the relaxation-time approximation of this theory, or more generally whenever inelastic processes can be neglected. Since this is the case in a disordered Fermi liquid at low temperatures, we conclude that violation of the above relation under the specified conditions is a telltale sign of interactions beyond the Fermi liquid framework.

In Sec. III, we consider a non-Fermi liquid model of weakly coupled Luttinger chains in the presence of a spin gap. We show that the antisymmetry of the off-diagonal elements of $\hat{\alpha}$ is indeed violated. Furthermore, we calculate the Nernst signal and show that interactions can lead to its substantial enhancement in such low dimensional systems. This may bare relevance to understanding the large signal observed experimentally in the quasi-one-dimensional materials. Finally, we also find that the sign of the effect in the spin gapped system changes from negative to positive as the temperature is lowered and the magnetic field increased. We interpret this behavior as being due to the stronger superconducting fluctuations induced by the spin gap. Various technical aspects of our study are relegated to the appendices.

II. THE SYMMETRY PROPERTIES OF $\hat{\alpha}$

A. $\hat{\alpha}$ within the Kubo theory

We consider interacting spinless fermions in a two-dimensional system of area A, which includes mass anisotropy and coupling to static electromagnetic potentials. The system is described by the Hamiltonian $H = \int d^2r\mathcal{H}(\mathbf{r})$, with

$$\mathcal{H}(\mathbf{r}) = \frac{1}{2m_\mu}[D_\mu\psi(\mathbf{r})]^\dagger[D_\mu\psi(\mathbf{r})] - e\phi(\mathbf{r})\rho(\mathbf{r}) + \frac{1}{2}\int d^2r' U(\mathbf{r}-\mathbf{r}')\psi^\dagger(\mathbf{r})\rho(\mathbf{r}')\psi(\mathbf{r}), \quad (3)$$

where $D_\mu = \partial_\mu + i(e/c)A_\mu(\mathbf{r})$, summation over repeated greek indices, which take the values x,y, is implied, and the interaction is assumed to obey $U(\mathbf{r}-\mathbf{r}') = U(\mathbf{r}'-\mathbf{r})$.

A route for calculating the thermoelectric coefficients was laid out by Luttinger [36], who argued that in the long-wavelength, low-frequency limit the linear response to a temperature variation $\delta T(\mathbf{r},t)$ is the same as the response to a fictitious gravitational field $g(\mathbf{r},t) = \delta T(\mathbf{r},t)/T$. An extension of Luttinger's results to the case with a magnetic field was given by Oji and Streda [37]. The gravitational field enters the calculation in two ways. First, it couples to the unperturbed density $\mathcal{K} = \mathcal{H} - \mu\rho$ of $K = H - \mu N$, such that the latter reads $K_T = K + \int d\mathbf{r} g(\mathbf{r},t)\mathcal{K}(\mathbf{r})$. Secondly, the unperturbed current density operators $\mathbf{J}^e$, $\mathbf{J}^h$ are themselves modified, with $\mathbf{J}^e$ becoming $\mathbf{J}^e + \delta\mathbf{J}^e = \mathbf{J}^e + g\mathbf{J}^e$, see Appendix A. Consequently,

$$\alpha_{ij} = \frac{1}{-\partial_j g}\frac{1}{AT}\left[\left\langle\int d^2r \mathrm{J}^e_i(\mathbf{r})\right\rangle_{K_T} + \left\langle\int d^2r\delta \mathrm{J}^e_i(\mathbf{r})\right\rangle_K\right] \equiv \alpha^{(1)}_{ij} + \alpha^{(2)}_{ij}. \quad (4)$$

Henceforth, latin indices, which take the values x,y, are not summed over, and $\langle J\rangle_K = \mathrm{Tr}(e^{-\beta K}J)/Z_K$, where $\beta = 1/T$, $Z_K = \mathrm{Tr}(e^{-\beta K})$.

The contribution $\alpha^{(2)}_{ij}$ is analogous to the diamagnetic term in the electrical conductivity. For a spatially constant temperature gradient one finds (see Appendix B)

$$\alpha^{(2)}_{ij} = -\frac{1}{AT}\left\langle\int d^2r \mathrm{J}^e_i(\mathbf{r})r_j\right\rangle_K = \frac{c}{AT}\epsilon^{ijz}M_z, \quad (5)$$

where M_z is the z component of the orbital magnetization. The importance of this contribution and its origin in the redistribution of the equilibrium magnetization currents which flow in the system, has been extensively discussed by Cooper, Halperin, and Ruzin [38]. Here, we note that its appearance is a direct consequence of the Kubo formalism.

Whereas $\alpha^{(2)}_{ij}$ is clearly antisymmetric in i and j, the other contribution (see Appendix B),

$$\alpha^{(1)}_{ij} = \lim_{\omega\to 0}\frac{A}{T}\frac{i}{\omega + i\delta}[\chi_{J^e_i,J^h_j}(\omega + i\delta) - \chi_{J^e_i,J^h_j}(i\epsilon)], \quad (6)$$

expressed in terms of the retarded correlation function $\chi_{J^e_i,J^h_j}$ of the averaged electrical and heat current densities, is generally not. The transformation properties of the correlation functions are discussed in Appendix C. Under spatial reflection, when such a transformation is a symmetry of the system, they imply that the diagonal elements of $\hat{\alpha}^{(1)}$ are even functions of the magnetic field $\mathbf{B} = B\hat{\mathbf{z}}$, while the off-diagonal elements are odd. Since also $M_z(B) = -M_z(-B)$, one finds that

$$\alpha_{ij}(B) = \begin{cases} \alpha_{ij}(-B) & i = j \\ -\alpha_{ij}(-B) & i \neq j \end{cases}, \quad (7)$$

with similar relations for $\hat{\tilde{\alpha}}$, $\hat{\sigma}$, and $\hat{\kappa}$.

Concomitantly, the transformation of $\hat{\alpha}^{(1)}$ under time reversal and the expressions for $\hat{\tilde{\alpha}}^{(1)}$ and $\hat{\tilde{\alpha}}^{(2)}$, Eqs. (B11) and (B12), lead to the conclusion

$$T\alpha_{ij}(B) = \tilde{\alpha}_{ji}(-B). \quad (8)$$

Hence, combining property (7), when applied to $\hat{\tilde{\alpha}}$, with Eq. (8) yields the relation $T\alpha_{ij}(B) = -\tilde{\alpha}_{ji}(B)$ between the off diagonal elements of the Peltier tensors. However, symmetry considerations do not imply a similar relation between the elements of $\hat{\alpha}$, *per se*. This stands in contrast to $\hat{\sigma}$ (and $\hat{\kappa}$), whose elements are related by time reversal symmetry via $\sigma_{ij}(B) = \sigma_{ji}(-B)$, thereby implying $\sigma_{xy}(B) = -\sigma_{yx}(B)$ for the Hall conductivity of a reflection symmetric system.

Notwithstanding, noninteracting electrons constitute an exception to the above statement. For this case, it is sufficient to consider the most general Hamiltonian H of a single particle, whose position operator we denote by $\mathbf{r}_0$. In first quantization, $\mathcal{H}(\mathbf{r}) = \{H,\delta(\mathbf{r}-\mathbf{r}_0)\}/2$, where the curly brackets denote the anticommutator. Using the continuity equation, $-\nabla\cdot\mathbf{J}^E = \partial_t\mathcal{H} = \{H,\partial_t\delta(\mathbf{r}-\mathbf{r}_0)\}/2 = \nabla\cdot\{H,\mathbf{J}^e\}/2e$, to identify the energy current density $\mathbf{J}^E$, one finds for $\mathbf{J}^h = \mathbf{J}^E + (\mu/e)\mathbf{J}^e$,

$$\mathbf{J}^h = -\frac{1}{2e}\{H-\mu,\mathbf{J}^e\}. \tag{9}$$

As a result, the correlation functions appearing in $\alpha_{ij}^{(1)}$ transform in the same way as the $\langle \mathrm{J}_i^e\mathrm{J}_j^e\rangle$ correlation functions determining $\hat{\sigma}$. Specifically, $\langle \mathrm{J}_i^e(t)\mathrm{J}_j^h(0)\rangle_K = \langle \mathrm{J}_i^e(t)\{H-\mu,\mathrm{J}_j^e(0)\}\rangle_K/2e = \langle\{H-\mu,\mathrm{J}_i^e(t)\}\mathrm{J}_j^e(0)\rangle_K/2e = \langle \mathrm{J}_i^h(t)\mathrm{J}_j^e(0)\rangle_K$, implying together with Eq. (8) that $\alpha_{ij}(B) = \alpha_{ji}(-B)$. This, in turn, when combined with reflection symmetry, gives $\alpha_{xy}(B) = -\alpha_{yx}(B)$. However, we reiterate that such a behavior is not guaranteed in the presence of interactions.

Let us note in passing that when $B=0$ the above discussion implies that for a generic interacting system with no reflection symmetry $\alpha_{ij} \neq \alpha_{ji}$ [39]. In this case, it is impossible to make $\hat{\alpha}$ purely diagonal by choosing suitably aligned principle axes. Such an "anomalous" Peltier effect is different from the Hall conductivity under the same conditions, which can always be made to vanish, and is necessarily a consequence of interactions, since in their absence $\alpha_{ij} = \alpha_{ji}$. We now proceed to demonstrate the explicit violation of $\alpha_{xy}(B) = -\alpha_{yx}(B)$ in an exactly solvable example.

B. Two interacting particles in a magnetic field

Consider two interacting particles in a magnetic field, whose Hamiltonian

$$\begin{aligned} H &= H_0 + U(\mathbf{r}_1-\mathbf{r}_2) \\ &= \frac{1}{2}\sum_{i=1,2}\left(m_x v_{i,x}^2 + m_y v_{i,y}^2\right) + \frac{1}{8}m_x\omega_x^2(x_1-x_2)^2 \\ &\quad + \frac{1}{8}m_y\omega_y^2(y_1-y_2)^2, \end{aligned} \tag{10}$$

is reflection symmetric, but anisotropic due to the rotation asymmetry of the mass tensor and harmonic interaction. The latter is characterized by the frequencies $\omega_{x,y}$, which together with the cyclotron frequency, $\omega_c = eB/\sqrt{m_x m_y}c$, set the energy scales of the problem. We work in the symmetric gauge for which the velocity operators take the form

$$v_x = \frac{1}{m_x}\left(p_x - \frac{eB}{2c}y\right), \tag{11}$$

$$v_y = \frac{1}{m_y}\left(p_y + \frac{eB}{2c}x\right). \tag{12}$$

The above Hamiltonian does not include a boundary potential, which is responsible for generating equilibrium edge currents and magnetization. However, in a system much larger than the magnetic lengths $l_{x,y} = 1/\sqrt{m_{x,y}\omega_c}$ it has a negligible effect on the current correlation functions in the bulk, which are our main point of interest.

Transforming to the center of mass coordinates, $\mathbf{R} = (\mathbf{r}_1+\mathbf{r}_2)/\sqrt{2}$, and relative coordinates $\mathbf{r} = (\mathbf{r}_1-\mathbf{r}_2)/\sqrt{2}$, separates the Hamiltonian into two commuting sectors $H = H_{\mathrm{CM}} + H_r$, with

$$\begin{aligned} H_{\mathrm{CM}} &= \frac{\omega_c}{2}\left[\left(-il_x\frac{\partial}{\partial X} - \frac{Y}{2l_y}\right)^2 + \left(-il_y\frac{\partial}{\partial Y} + \frac{X}{2l_x}\right)^2\right], \\ H_r &= \frac{\omega_c}{2}\left[\left(-il_x\frac{\partial}{\partial x} - \frac{y}{2l_y}\right)^2 + \left(-il_y\frac{\partial}{\partial y} + \frac{x}{2l_x}\right)^2\right] \\ &\quad + \frac{1}{4\omega_c}\left[\omega_x^2\left(\frac{x}{l_x}\right)^2 + \omega_y^2\left(\frac{y}{l_y}\right)^2\right]. \end{aligned} \tag{13}$$

Defining the complex coordinate $Z = X/l_x + iY/l_y$ and the operators

$$a_1 = \frac{Z^*}{2^{3/2}} + 2^{1/2}\frac{\partial}{\partial Z}, \tag{14}$$

$$a_2 = \frac{Z}{2^{3/2}} + 2^{1/2}\frac{\partial}{\partial Z^*}, \tag{15}$$

satisfying $[a_1,a_1^\dagger] = [a_2,a_2^\dagger] = 1$ and $[a_1,a_2] = [a_1,a_2^\dagger] = 0$, leads to the familiar diagonalized form of H_{CM},

$$H_{\mathrm{CM}} = \omega_c\left(a_1^\dagger a_1 + \tfrac{1}{2}\right). \tag{16}$$

The relative Hamiltonian can be diagonalized via a series of canonical transformations that are detailed in Appendix D. The result is

$$H_r = \omega_1\left(d_1^\dagger d_1 + \tfrac{1}{2}\right) + \omega_2\left(d_2^\dagger d_2 + \tfrac{1}{2}\right), \tag{17}$$

where $[d_1,d_1^\dagger] = [d_2,d_2^\dagger] = 1$ and $[d_1,d_2] = [d_1,d_2^\dagger] = 0$, and the frequencies $\omega_{1,2}$ are given in Eq. (D13). The energy eigenstates $|N,n\rangle \equiv |N_1,N_2,n_1,n_2\rangle$ are therefore characterized by the eigenvalues of $a_1^\dagger a_1$, $a_2^\dagger a_2$, $d_1^\dagger d_1$, and $d_2^\dagger d_2$, respectively, with energies $E_{N,n} = \omega_c(N_1+1/2) + \omega_1(n_1+1/2) + \omega_2(n_2+1/2)$. The fermionic statistics forces odd n_1+n_2, see Appendix D.

The first quantized form of Eq. (A2), $\mathbf{J}^e(\mathbf{r}) = -(e/2)\sum_{i=1,2}\{\boldsymbol{v}_i,\delta_i\}$, where $\delta_i = \delta(\mathbf{r}-\mathbf{r}_i)$, leads to the averaged electrical current density $\boldsymbol{J}^e = (1/A)\int d^2r\,\mathbf{J}^e(\mathbf{r})$ with

$$J_x^e = -i\frac{e\omega_c l_x}{A}(a_1^\dagger - a_1), \tag{18}$$

$$J_y^e = -\frac{e\omega_c l_y}{A}(a_1^\dagger + a_1). \tag{19}$$

An explicit calculation then readily confirms that the electrical current correlation functions satisfy $\mathrm{Tr}[e^{-\beta H}J_x^e(t)J_y^e(0)] = -\mathrm{Tr}[e^{-\beta H}J_y^e(t)J_x^e(0)]$, as required for $\sigma_{xy} = -\sigma_{yx}$.

It follows from the results of Appendix A that the averaged energy current density takes the form

$$\begin{aligned} \boldsymbol{J}^E &= \frac{1}{4A}\sum_{i=1,2}\boldsymbol{v}_i\left[m_x v_{i,x}^2 + m_y v_{i,y}^2 + U(\mathbf{r}_1-\mathbf{r}_2)\right] \\ &\quad + \frac{1}{4e}(\mathbf{r}_1-\mathbf{r}_2)\left[\boldsymbol{J}^e\cdot\frac{\partial U(\mathbf{r}_1-\mathbf{r}_2)}{\partial\mathbf{r}_1}\right] + \mathrm{H.c.}, \end{aligned} \tag{20}$$

where the commutativity of $\boldsymbol{J}^e$ with $\mathbf{r}_1 - \mathbf{r}_2$ has been used. We are interested in the correlation functions

$$\mathrm{Tr}\big[e^{-\beta H} J_x^e(t) J_y^E(0)\big] = \sum_{N,N',n} e^{(it-\beta)E_{N,n} - itE_{N',n}} \langle N,n| \times J_x^e |N',n\rangle \langle N',n| J_y^E |N,n\rangle, \quad (21)$$

and $\mathrm{Tr}[e^{-\beta H} J_y^e(t) J_x^E(0)]$, relevant to α_{xy} and α_{yx}. We therefore require only the piece in $\boldsymbol{J}^E$ which is diagonal in n_1,n_2. Calculation reveals that the corresponding piece in J_x^E may be expressed as $\{I_x, J_x^e\}$, with

$$I_x = \frac{\Omega}{8e}\left[\frac{\omega_x^2}{2\Omega^2}\cos^2\phi\, e^{-2\theta_1} - \left(\cos\phi + \frac{\omega_c}{2\Omega}\sin\phi\right)^2 e^{2\theta_1}\right] \times (d_1^\dagger d_1 + d_1 d_1^\dagger) + \frac{\Omega}{8e}\left[\frac{\omega_x^2}{2\Omega^2}\sin^2\phi\, e^{2\theta_2} - \left(\sin\phi - \frac{\omega_c}{2\Omega}\cos\phi\right)^2 e^{-2\theta_2}\right](d_2^\dagger d_2 + d_2 d_2^\dagger) - \frac{1}{4e}H, \quad (22)$$

where the parameters Ω, ϕ, and $\theta_{1,2}$ are given in Appendix D. At the same time, the corresponding piece in J_y^E reads $\{I_y, J_y^e\}$, with

$$I_y = \frac{\Omega}{8e}\left[\frac{\omega_y^2}{2\Omega^2}\sin^2\phi\, e^{2\theta_1} - \left(\sin\phi + \frac{\omega_c}{2\Omega}\cos\phi\right)^2 e^{-2\theta_1}\right] \times (d_1^\dagger d_1 + d_1 d_1^\dagger) + \frac{\Omega}{8e}\left[\frac{\omega_y^2}{2\Omega^2}\cos^2\phi\, e^{-2\theta_2} - \left(\cos\phi - \frac{\omega_c}{2\Omega}\sin\phi\right)^2 e^{2\theta_2}\right](d_2^\dagger d_2 + d_2 d_2^\dagger) - \frac{1}{4e}H. \quad (23)$$

Since $[I_x, H] = [I_y, H] = 0$, the same argument presented following Eq. (9) would imply that $\mathrm{Tr}[e^{-\beta H} J_x^e(t) J_y^E(0)] = -\mathrm{Tr}[e^{-\beta H} J_y^e(t) J_x^E(0)]$, provided that $I_x = I_y$. However, this condition is fulfilled only when $\omega_x = \omega_y$, leading to $\cos\phi = \pm\sin\phi = 1/\sqrt{2}$ and $\theta_1 = \theta_2 = 0$. Hence we conclude that $\alpha_{xy} \neq -\alpha_{yx}$ except when the system is isotropic ($m_x = m_y$ and $\omega_x = \omega_y$), or when the anisotropy in the interaction matches the mass anisotropy ($m_x \neq m_y$ but $\omega_x = \omega_y$), in which case it may be removed by coordinate rescaling.

C. $\hat{\alpha}$ within Boltzmann transport theory

Let us next apply the Boltzmann equation to the transport of spinless electrons in a two-dimensional system subjected to a perpendicular magnetic field $\mathbf{B} = B\hat{\mathbf{z}}$. This approach is appropriate on time and length scales much larger than the corresponding scales characterizing the scattering events. Consequently, the effects of scattering are captured by a local collision integral. Close to equilibrium, the distribution function can be written as $f_{\mathbf{k}} + \delta f_{\mathbf{k}}$, with $f_{\mathbf{k}} = n_F(\varepsilon_{\mathbf{k}})$, $\delta f_{\mathbf{k}} = -(\partial f_{\mathbf{k}}/\partial\varepsilon_{\mathbf{k}}) g_{\mathbf{k}}$, and $\beta g_{\mathbf{k}} \ll f_{\mathbf{k}}$. As a result, the collision integral takes the form $-\int_{\mathbf{k}'} I_{\mathbf{k},\mathbf{k}'} g_{\mathbf{k}'}$, where the kernel $I_{\mathbf{k},\mathbf{k}'} = I_{\mathbf{k}',\mathbf{k}}$ depends on the equilibrium transition rates [7,8], and the integral $\int_{\mathbf{k}} \equiv \int d^2k/(2\pi)^2$ extends over the reciprocal unit cell spanned by the vectors $\mathbf{K}_{1,2}$, see Fig. 1. To linear order in the applied homogeneous electric field and thermal gradient, the Boltzmann equation reads [7,8]

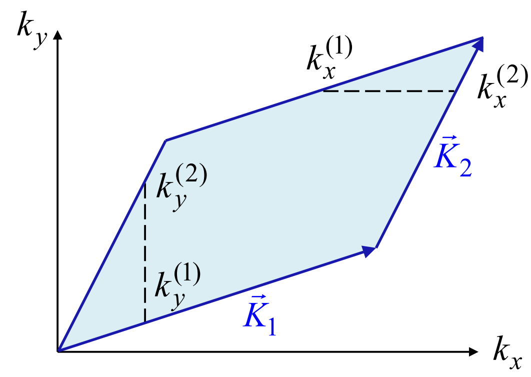

FIG. 1. The integration region in $\mathbf{k}$ space.

$$B_{\mathbf{k}} g_{\mathbf{k}} + \int_{\mathbf{k}'} I_{\mathbf{k},\mathbf{k}'} g_{\mathbf{k}'} = \mathbf{v}_{\mathbf{k}} \cdot \left[e\mathbf{E} + (\varepsilon_{\mathbf{k}} - \mu)\frac{\nabla T}{T}\right]\frac{\partial f_{\mathbf{k}}}{\partial \varepsilon_{\mathbf{k}}}, \quad (24)$$

where we have assumed that the energy spectrum consists of a single band and defined the differential operator

$$B_{\mathbf{k}} = -\frac{e}{\hbar c}\frac{\partial f_{\mathbf{k}}}{\partial\varepsilon_{\mathbf{k}}}(\mathbf{v}_{\mathbf{k}} \times \mathbf{B}) \cdot \nabla_{\mathbf{k}} = \frac{eB}{\hbar^2 c}\frac{\partial f_{\mathbf{k}}}{\partial\varepsilon_{\mathbf{k}}}\left(\frac{\partial\varepsilon_{\mathbf{k}}}{\partial k_x}\frac{\partial}{\partial k_y} - \frac{\partial\varepsilon_{\mathbf{k}}}{\partial k_y}\frac{\partial}{\partial k_x}\right). \quad (25)$$

Solving Eq. (24) yields

$$g_{\mathbf{k}} = \int_{\mathbf{k}_0} I^{-1}_{\mathbf{k},\mathbf{k}_0} \mathbf{v}_{\mathbf{k}_0} \cdot \left[e\mathbf{E} + (\varepsilon_{\mathbf{k}_0} - \mu)\frac{\nabla T}{T}\right]\frac{\partial f_{\mathbf{k}_0}}{\partial\varepsilon_{\mathbf{k}_0}} + \int_{\mathbf{k}_0} I^{-1}_{\mathbf{k},\mathbf{k}_0} \sum_{n=1}^{\infty}(-1)^n \prod_{m=1}^{n}\int_{\mathbf{k}_m} B_{\mathbf{k}_{m-1}} I^{-1}_{\mathbf{k}_{m-1},\mathbf{k}_m} \mathbf{v}_{\mathbf{k}_n} \cdot \left[e\mathbf{E} + (\varepsilon_{\mathbf{k}_n} - \mu)\frac{\nabla T}{T}\right]\frac{\partial f_{\mathbf{k}_n}}{\partial\varepsilon_{\mathbf{k}_n}}, \quad (26)$$

where $\int_{\mathbf{k}'} I_{\mathbf{k},\mathbf{k}'} I^{-1}_{\mathbf{k}',\mathbf{k}''} = (2\pi)^2\delta(\mathbf{k} - \mathbf{k}'')$. Since the electrical and heat current densities are given by $\boldsymbol{J}^e = -e\int_{\mathbf{k}} \mathbf{v}_{\mathbf{k}}\delta f_{\mathbf{k}}$, and $\boldsymbol{J}^h = \int_{\mathbf{k}} \mathbf{v}_{\mathbf{k}}(\varepsilon_{\mathbf{k}} - \mu)\delta f_{\mathbf{k}}$ it follows that

$$\begin{pmatrix} T\alpha_{ij} \\ \tilde{\alpha}_{ij} \end{pmatrix} = e\int_{\mathbf{k}}\int_{\mathbf{k}_0} \frac{\partial f_{\mathbf{k}}}{\partial\varepsilon_{\mathbf{k}}} v_{i,\mathbf{k}} I^{-1}_{\mathbf{k},\mathbf{k}_0} v_{j,\mathbf{k}_0} \frac{\partial f_{\mathbf{k}_0}}{\partial\varepsilon_{\mathbf{k}_0}} \begin{pmatrix} \varepsilon_{\mathbf{k}_0} - \mu \\ \varepsilon_{\mathbf{k}} - \mu \end{pmatrix} + e\int_{\mathbf{k}}\int_{\mathbf{k}_0} \frac{\partial f_{\mathbf{k}}}{\partial\varepsilon_{\mathbf{k}}} v_{i,\mathbf{k}} I^{-1}_{\mathbf{k},\mathbf{k}_0} \sum_{n=1}^{\infty}(-1)^n \prod_{m=1}^{n}\int_{\mathbf{k}_m} \times B_{\mathbf{k}_{m-1}} I^{-1}_{\mathbf{k}_{m-1},\mathbf{k}_m} v_{j,\mathbf{k}_n} \frac{\partial f_{\mathbf{k}_n}}{\partial\varepsilon_{\mathbf{k}_n}} \begin{pmatrix} \varepsilon_{\mathbf{k}_n} - \mu \\ \varepsilon_{\mathbf{k}} - \mu \end{pmatrix}. \quad (27)$$

The above result obeys the Onsager relation (8) at $B = 0$, as can be readily verified by using the symmetry $I_{\mathbf{k},\mathbf{k}'} = I_{\mathbf{k}',\mathbf{k}}$ and exchanging $\mathbf{k} \leftrightarrow \mathbf{k}_0$ in the first line of Eq. (27). To demonstrate that the Onsager relation continues to hold for $B > 0$ we integrate by parts the integrals in the second line, use the symmetry of the collision kernel and exchange $\mathbf{k} \leftrightarrow \mathbf{k}_n$, $\mathbf{k}_m \leftrightarrow \mathbf{k}_{n-m-1}$ for $m = 0, \cdots, \lceil(n-1)/2\rceil$. This brings the expression back to itself up to $B_{\mathbf{k}} \to -B_{\mathbf{k}}$, and $\varepsilon_{\mathbf{k}} \leftrightarrow \varepsilon_{\mathbf{k}_n}$ in the last parenthesis. Accordingly, the desired relation is established, provided that the contribution from the boundary terms, incurred during the integration by part, vanishes. On general grounds, $\varepsilon_{\mathbf{k}+\mathbf{K}} = \varepsilon_{\mathbf{k}}$ and $\mathbf{v}_{\mathbf{k}+\mathbf{K}} = \mathbf{v}_{\mathbf{k}} = (1/\hbar)\partial\varepsilon_{\mathbf{k}}/\partial\mathbf{k}$, for any reciprocal vector $\mathbf{K}$. We find that the boundary contribution vanishes if $I_{\mathbf{k},\mathbf{k}'}$ also respects the lattice periodicity, i.e., $I_{\mathbf{k}+\mathbf{K},\mathbf{k}'} = I_{\mathbf{k},\mathbf{k}'}$. Under such conditions the integrand is invariant under translation by a reciprocal wave-vector and for every contribution from an end point $[k_x,k_y^{(1)}(k_x)]$ there exists an opposite contribution from an end point at $[k_x,k_y^{(1)}(k_x)] + \mathbf{K}_2$ or $[k_x,k_y^{(1)}(k_x)] - \mathbf{K}_1$, see Fig. 1. A similar argument works for the other end points.

The preceding analysis shows that $\alpha_{ij}(B) = \alpha_{ji}(-B)$, and therefore $\alpha_{ij}(B) = -\alpha_{ji}(B)$ in reflection symmetric systems, only if $\varepsilon_{\mathbf{k}} = \varepsilon_{\mathbf{k}_n}$ in Eq. (27). This condition is fulfilled whenever $I^{-1}_{\mathbf{k},\mathbf{k}'}$ is proportional to $\delta(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}'})$, as is the case for elastic impurity scattering, or within the relaxation time approximation where $I^{-1}_{\mathbf{k},\mathbf{k}'} = \delta(\mathbf{k}-\mathbf{k}')(\partial f_{\mathbf{k}}/\partial\epsilon_{\mathbf{k}})^{-1}\tau_{\mathbf{k}}$. An important case of interest is the disordered Fermi liquid, which includes both elastic impurity scattering and inelastic processes due to electron-electron interactions. While the elastic piece in $I^{-1}_{\mathbf{k},\mathbf{k}'}$ is temperature independent, the inelastic channel contribution to $I^{-1}_{\mathbf{k},\mathbf{k}'}$ scales as T^2 in three dimensions [7,8]. Therefore, at low temperatures, the physics is dominated by the former, $\alpha_{xy} \sim T$, and the relation $\alpha_{ij}(B) = -\alpha_{ji}(B)$ holds up to corrections of order T^3. In the following section we turn our attention to the behavior of $\hat{\alpha}$ in a system which is manifestly a non-Fermi liquid.

III. THE NERNST EFFECT IN A SYSTEM OF COUPLED LUTTINGER LIQUIDS

A. The model and its $\hat{\alpha}$

We consider a model of a two-dimensional array of N_c one-dimensional chains extending along the x direction from $-L/2$ to $L/2$ and separated by a distance d in the y direction, with both $N_c, L \to \infty$. The chains are immersed in a magnetic field $\mathbf{B} = B\hat{\mathbf{z}}$, which is generated by the vector potential $A_y = Bx$. The spinfull electrons that populate the system interact via an attractive contact interaction, which opens a gap in the spin sector of each chain [40]. This gap is assumed to be much larger than any remaining energy scale in the problem, such as the temperature and inter-chain couplings. Owing to the spin gap, single-particle tunneling between the chains is irrelevant. In contrast, the superconducting and $2k_F$ charge-density wave (CDW) susceptibilities of the chains are enhanced and the interchain Josephson and CDW couplings are important [40]. In order to have a nontrivial transverse thermoelectric response one needs to include the Josephson tunneling. We will neglect the CDW coupling, whose main effect is to compete against the superconducting ordering tendency of the system, since we are interested in the case where the latter prevails. Consequently, we study the following bosonized form of $H = H_0 + H_{\mathcal{J}}$, where

$$H_0 = \frac{v}{2}\sum_{j=1}^{N_c}\int dx\left[K(\partial_x\theta_j)^2 + \frac{1}{K}(\partial_x\phi_j)^2\right], \tag{28}$$

$$H_{\mathcal{J}} = -\mathcal{J}\sum_{j=2}^{N_c}\int dx\cos[\sqrt{2\pi}(\theta_j - \theta_{j-1}) + bx]. \tag{29}$$

Here, v and $K > 1$ are the velocity and Luttinger parameter of the charge sector, respectively. $\mathcal{J}$ is the Josephson energy per unit length, and $b = 2eBd/c = 2d/l_B^2$ is a wave vector associated with the magnetic field. Equation (29) shows that the field adds an oscillatory phase to the pair hopping term, thus rendering it irrelevant in the renormalization group sense. However, a second-order term in $\mathcal{J}$ is relevant for $K > 3/2$ and induces a crossover to a strong coupling regime at $T_c \sim (v/a)(\mathcal{J}/v)^{K/(K-3/2)}$, where a is the short-distance cutoff of the theory [41]. Therefore the perturbative treatment of $\mathcal{J}$, which we employ below, is valid only for $T > T_c$.

The current density operators may be deduced from the continuity equations for the conserved quantities. For the average current densities, we obtain

$$J_x^e = -\sqrt{\frac{2}{\pi}}\frac{evK}{A}\sum_{j=1}^{N_c}\int dx\,\partial_x\theta_j, \tag{30}$$

$$J_y^e = -\frac{2e\mathcal{J}d}{A}\sum_{j=2}^{N_c}\int dx\sin[\sqrt{2\pi}(\theta_j - \theta_{j-1}) + bx], \tag{31}$$

$$J_x^h = -\frac{v^2}{2A}\sum_{j=1}^{N_c}\int dx\{\partial_x\phi_j,\partial_x\theta_j\}, \tag{32}$$

$$J_y^h = -\frac{\sqrt{2\pi}v\mathcal{J}d}{4KA}\sum_{j=2}^{N_c}\int dx\{\partial_x\phi_j + \partial_x\phi_{j-1}, \times\sin[\sqrt{2\pi}(\theta_j - \theta_{j-1}) + bx]\}, \tag{33}$$

where $A = LN_cd$. Note that in the Luttiner model (28) the energy is measured relative to the chemical potential and therefore $\mathbf{J}^h$ is calculated from the continuity equation for the Hamiltonian density.

Using the above expressions and Eq. (6), we compute $\alpha_{yx}^{(1)}$ to second order in $\mathcal{J}$, see Appendix E for details. The result

$$\alpha_{yx}^{(1)} = \lim_{\omega\to 0} -\frac{ebv^2\mathcal{J}^2}{2T}\frac{\partial^2 C(b,\omega)}{\partial\omega^2}, \tag{34}$$

is expressed in terms of the function

$$C(q,\omega) = \frac{a^2}{v}\sin\left(\frac{\pi}{K}\right)\left(\frac{l_T}{2a}\right)^{2-2/K} \times B\left[\frac{1}{2K} - \frac{i}{4}\left(\frac{\omega}{v} - q\right)l_T, 1 - \frac{1}{K}\right] \times B\left[\frac{1}{2K} - \frac{i}{4}\left(\frac{\omega}{v} + q\right)l_T, 1 - \frac{1}{K}\right], \tag{35}$$

where $B(x,y)$ is the beta function, and $l_T = v/\pi T$ is the thermal length. Appendix E also contains the computation of M_z, which, together with Eq. (5), leads to

$$\alpha_{yx}^{(2)} = -\frac{e\mathcal{J}^2}{2T}\frac{\partial C(b,0)}{\partial b}. \tag{36}$$

The final result for α_{yx} may be cast into a scaling form,

$$\alpha_{yx} = e\left(\frac{\mathcal{J}a^2}{v}\right)^2\left(\frac{l_T}{a}\right)^{4-2/K}\left[f_\alpha^{(1)}(bl_T) + f_\alpha^{(2)}(bl_T)\right], \tag{37}$$

where the functions $f_\alpha^{(1,2)}$ originate from $\alpha_{yx}^{(1,2)}$, respectively. Both $f_\alpha^{(1)}(x)$ and $f_\alpha^{(2)}(x)$ scale as x for $x \ll 1$, and decay

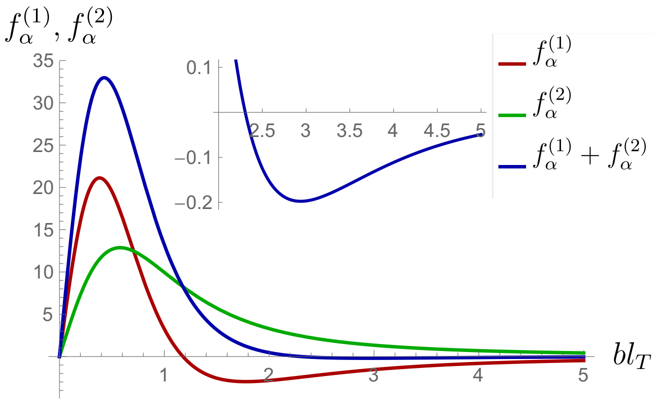

FIG. 2. The scaling functions that determine α_{yx}, shown here for $K = 2$. The inset depicts the sign change of $f_\alpha^{(1)} + f_\alpha^{(2)}$, and thus of α_{yx} for large bl_T.

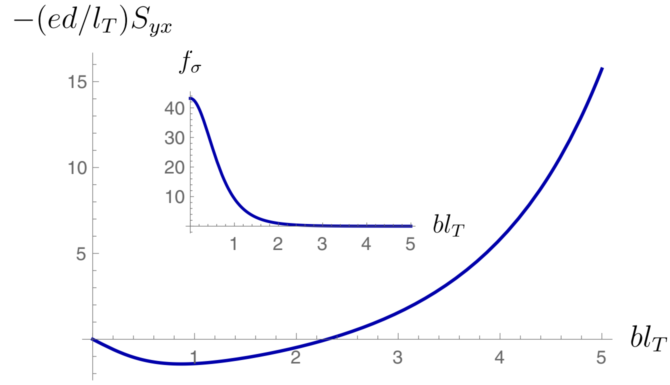

FIG. 3. The dimensionless Nernst signal $-(ed/l_T)S_{yx}$ for the case $K = 2$. The inset depicts the scaling function of σ_{yy}.

as $x^{-(3-2/K)}$ for $x \gg 1$, due to the rapid oscillations in the Josephson coupling, see Fig. 2. While $\alpha_{yx}^{(2)}$ is always positive, consistent with a diamagnetic response ($M_z < 0$), the sign of $\alpha_{yx}^{(1)}$ changes as function of bl_T. At weak fields and high temperatures, the two contributions add up, leading to a positive α_{yx}, which behaves as $B/T^{5-2/K}$. On the other hand, at large magnetic fields and low temperatures they tend to cancel each other leaving a total negative α_{yx}, which varies according to $-T/B^{5-2/K}$. The sign of α_{yx} in this regime is the one expected from superconducting fluctuations.

In contrast, we show in Appendix E that $\alpha_{xy}^{(1)}$ is smaller than $\alpha_{yx}^{(1)}$ by a factor l_T/L, and hence negligible in the thermodynamic limit. This is a consequence of the fact that in the clean model considered here $[J_x^e, H] = 0$, up to corrections from boundary terms. As a result the retarded $J_x^e J_y^h$ correlation function which determine $\alpha_{xy}^{(1)}$ vanishes identically. This demonstrates that in the inherently interacting problem studied here, $\alpha_{xy} = cM_z/AT \neq -\alpha_{yx}$. We expect that upon breaking the conservation of J_x^e, e.g., by introducing disorder into the chains, $\alpha_{xy}^{(1)}$ will no longer vanish. Nevertheless, its magnitude will be proportional to the disorder strength and will not match that of $\alpha_{yx}^{(1)}$.

Let us comment that a model for two superconducting wires, similar to $H_0 + H_{\mathcal{J}}$ defined by Eqs. (28) and (29), was considered in Ref. [42]. However, unlike the present study each wire was assumed to be in equilibrium, described by a density matrix $e^{-H_0/T}$ with a different temperature, while the Josephson coupling was turned on adiabatically. Consequently, it was found that $\alpha_{xy} = 0$. Upon including a term which breaks the linear dispersion and characterized by a dimensionless curvature $\mathcal{C}$, this result changed to $\alpha_{xy} = -cM_z/AT_0$, where $T_0 = v/(\pi a \mathcal{C})$.

B. The conductivity and the Nernst signal

For a particle-hole and reflection symmetric model, such as the one considered here, the relation between the Peltier coefficients and the thermopower is considerably simplified. Under particle-hole transformation $\boldsymbol{J}^e(B) \to -\boldsymbol{J}^e(-B)$ and $\boldsymbol{J}^h(B) \to \boldsymbol{J}^h(-B)$. Therefore, in the symmetric case, where $K(B) \to K(-B)$, we conclude that $\hat{\sigma}(B) = \hat{\sigma}(-B)$ and $\hat{\alpha}(B) = -\hat{\alpha}(-B)$. When combined with Eq. (7) due to reflection symmetry, it leads to the result $\sigma_{xy} = \sigma_{yx} = \alpha_{xx} = \alpha_{yy} = 0$. In turn, the Nernst signals become

$$S_{xy} = \frac{\alpha_{xy}}{\sigma_{xx}}, \quad -S_{yx} = -\frac{\alpha_{yx}}{\sigma_{yy}}. \tag{38}$$

For a quasi-one-dimensional system embedded in a magnetic field and possessing Galilean invariance along the chains, one finds $\sigma_{xx} \sim 1/\kappa^2$, where κ is the curvature of the free chain spectrum [43]. In our linearized model, σ_{xx} diverges and as a result $S_{xy} = 0$. To calculate σ_{yy}, we apply Eq. (B5) (with $\mathbf{J} = \mathbf{J}^e$) and obtain to second order in $\mathcal{J}$,

$$\begin{aligned}\sigma_{yy} &= \lim_{\omega\to 0} -2ide^2\mathcal{J}^2 \frac{\partial C(b,\omega)}{\partial \omega} \\ &= e^2 \frac{d}{a}\left(\frac{\mathcal{J}a^2}{v}\right)^2 \left(\frac{l_T}{a}\right)^{3-2/K} f_\sigma(bl_T).\end{aligned} \tag{39}$$

The conductivity scaling function $f_\sigma(x)$ is depicted in the inset of Fig. 3. It tends to a constant at small x and decays as $x^{-(2-2/K)}e^{-\pi x/2}$ for large x. When combined with the behavior of α_{yx} this results in a Nernst signal along the y direction that is negative and scales according to B/T^2 for low fields and large temperatures ($bl_T \ll 1$). As the field is increased and the temperature lowered the Nernst signal turns positive and eventually, when $bl_T \gg 1$, follows $(T^2/B^3)e^{(evd/c)(B/T)}$, see Fig. 3. The resulting scale for the Nernst signal, l_T/ed, is very large. For typical values relevant for the quasi-one-dimensional conductors, $v = 10^5$ ms^{-1}, $d = 1$ nm, and $T = 10$ K, the Nernst signal is of order $S_{yx} \approx 1$ mV K^{-1}, to be compared with values of order 0.1 mV K^{-1}, measured in $(TMTSF)_2ClO_4$ [13]. The Nernst coefficient, $e_N = -S_{yx}/B$, calculated for low fields where S_{yx} is linear in B, is also large. For the above parameters we find $e_N \approx 100\,\mu$V K^{-1} T^{-1}, while e_N measured in $(TMTSF)_2ClO_4$ is of order 10 μV K^{-1} T^{-1} [13]. This is in contrast to $e_N \approx 0.1\,\mu$V K^{-1} T^{-1} calculated using Boltzmann theory for a similar band structure [13].

IV. CONCLUSIONS

The transformation properties of a system under spatial reflections, time reversal, and charge conjugation relate many of its transport coefficients. Here we showed that the frequently used relation $\alpha_{xy} = -\alpha_{yx}$ does not belong to this category.

Rather, its validity requires the additional condition of no interactions between the electrons, either directly or via mediators such as phonons. Nevertheless, it becomes a good approximation whenever the interacting system can be considered to comprise of weakly and locally interacting particles, i.e., a Fermi liquid. Its violation in a reflection symmetric system is therefore a clear sign that energy is also transported via interactions between the particles, or in the extreme limit that the concept of a quasiparticle fails. In that sense, the above relation is similar to the Wiedemann-Franz law. They both reflect an underlying assumption that heat transfer is restricted to convection by motion of the charge carriers, and can be used to detect non-Fermi liquids. However, we believe that our proposal has a potential of yet larger impact since the required anisotropy may be tuned using recently developed strain techniques, and because thermoelectric properties are comparatively simpler to obtain than the thermal conductivity that figures in the Wiedemann-Franz law.

Thus it would be interesting to follow the relation between α_{xy} and α_{yx} as function of temperature. If, for example, $r = (\alpha_{xy} + \alpha_{yx})/(\alpha_{xy} - \alpha_{yx}) \ll 1$ is observed at high temperatures but approaches $r \approx 1$ below a characteristic temperature T_0, this would mean one of the following: (i) T_0 indicates a nematic transition inside a non-Fermi liquid state, i.e., a breaking of the C_4 rotation symmetry around the z axis. (ii) The system is anisotropic and breaks the reflection symmetry about the x and y directions below T_0. (iii) The system is a Fermi-liquid and breaks both reflection symmetry and C_4 rotation symmetry at low temperatures. (iv) The system is anisotropic but reflection symmetric and non-Fermi liquid behavior onsets at the temperature scale T_0. The pseudogap regime of the high-temperature superconductors, with its tendencies to develop various ordered states, seems to be a good candidate for such an experiment.

By studying the Nernst effect in an interacting quasi-one-dimensional model with strong superconducting fluctuations, we were able to demonstrate that the effect is much stronger than in two-dimensional models considered using Boltzmann transport theory. This finding points to the importance of interactions and low dimensionality in establishing a large Nernst signal, and may bare relevance to experiments done on quasi-one-dimensional materials.

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APPENDIX A: THE CURRENT DENSITY OPERATORS

Here we obtain the electrical and heat current density operators of model (3). To begin with, the continuity equation for the charge density $\rho^e = -e\rho$ in the presence of a gravitational field,

$$\partial_\mu \mathrm{J}^e_\mu = i \int d^2r' [1 + g(\mathbf{r}')][\rho^e(\mathbf{r}),\mathcal{H}(\mathbf{r}')], \tag{A1}$$

is satisfied by the electrical current density operator

$$\mathrm{J}^e_j(\mathbf{r}) = [1 + g(\mathbf{r})]\frac{ie}{2m_j}\psi^\dagger(\mathbf{r})D_j\psi(\mathbf{r}) + \text{H.c.}. \tag{A2}$$

The heat current density operator $\mathbf{J}^h = \mathbf{J}^E + (\mu/e)\mathbf{J}^e$ is related to the energy current density operator $\mathbf{J}^E$, which in turn is to be determined by the continuity equation for the energy density

$$\begin{aligned}\partial_\mu \mathrm{J}^E_\mu &= i[\mathcal{H}(\mathbf{r}),H]\\ &= \partial_\mu\left\{\frac{i}{2m_\mu}[D_\mu\psi(\mathbf{r})]^\dagger\left[-\frac{1}{2m_\nu}D_\nu^2 - e\phi(\mathbf{r})\right.\right.\\ &\quad\left.\left.+\frac{1}{2}\int d^2r' U(\mathbf{r}-\mathbf{r}')\rho(\mathbf{r}')\right]\psi(\mathbf{r})\right\}\\ &\quad+\frac{i}{4m_\mu}\int d^2r'\{[D_\mu\psi(\mathbf{r})]^\dagger[\partial_\mu U(\mathbf{r}-\mathbf{r}')]\\ &\quad\times\rho(\mathbf{r}')\psi(\mathbf{r}) + (\mathbf{r}\leftrightarrow\mathbf{r}')\} + \text{H.c.}\end{aligned} \tag{A3}$$

Here, in order to avoid a surface term which arises in the derivation, we have assumed that no charge current is flowing out of the system, i.e., $\mathbf{J}^e \cdot \mathbf{n} = 0$, with $\mathbf{n}$ the normal to the system's boundary.

To make progress, we need to integrate Eq. (A3) with the appropriate boundary conditions. To this end, we assume that the system is thermally isolated in the sense $\mathbf{J}^E \cdot \mathbf{n} = 0$. Both conditions on the currents are fulfilled if $\mathbf{D}\psi \cdot \mathbf{n} = 0$. Denoting the second term in Eq. (A3) by $F(\mathbf{r})$, we further assume that its contribution to $\mathbf{J}^E$ is irrotational and hence can be expressed as $\nabla\Phi$, where $\nabla^2\Phi(\mathbf{r}) = F(\mathbf{r})$. It follows from the divergence theorem that a solution to this equation exists only if $\int d^2r\, F(\mathbf{r}) = 0$, which holds true in our case. Consequently, we find

$$\begin{aligned}\mathrm{J}^E_j(\mathbf{r}) &= \frac{i}{2m_j}[D_j\psi(\mathbf{r})]^\dagger\left[-\frac{1}{2m_\mu}D_\mu^2 - e\phi(\mathbf{r})\right.\\ &\quad\left.+\frac{1}{2}\int d^2r' U(\mathbf{r}-\mathbf{r}')\rho(\mathbf{r}')\right]\psi(\mathbf{r})\\ &\quad+\frac{i}{4m_\mu}\int d^2r'd^2r''\partial_j[G(\mathbf{r},\mathbf{r}') - G(\mathbf{r},\mathbf{r}'')][D'_\mu\psi(\mathbf{r}')]^\dagger\\ &\quad\times[\partial'_\mu U(\mathbf{r}'-\mathbf{r}'')]\rho(\mathbf{r}'')\psi(\mathbf{r}') + \text{H.c.},\end{aligned} \tag{A4}$$

where $G(\mathbf{r},\mathbf{r}')$ is the Green's function of the Laplace equation with Neumannn boundary conditions, satisfying $\nabla^2 G(\mathbf{r},\mathbf{r}') = \delta(\mathbf{r}-\mathbf{r}') - 1/A$ and $\nabla G(\mathbf{r},\mathbf{r}')\cdot\mathbf{n} = 0$. For a rectangular domain $A = L_x \times L_y$ it is given by

$$G(\mathbf{r},\mathbf{r}') = {\sum}'^{\infty}_{m,n=0}\frac{u_{mn}(\mathbf{r})u_{mn}(\mathbf{r}')}{\lambda_{mn}}, \tag{A5}$$

where the term $m = n = 0$ is excluded from the sum and the Laplacian eigenfuncions and eigenvalues are given by

$$u_{mn}(\mathbf{r}) = \frac{c_{mn}}{\sqrt{A}}\cos\left(\frac{m\pi x}{L_x}\right)\cos\left(\frac{n\pi y}{L_y}\right), \tag{A6}$$

$$\lambda_{mn} = -\left(\frac{m\pi}{L_x}\right)^2 - \left(\frac{n\pi}{L_y}\right)^2, \tag{A7}$$

with $c_{mn} = 2^{[\mathrm{sign}(m)+\mathrm{sign}(n)]/2}$. Subsequently, it follows from

$$\int d^2r\partial_j G(\mathbf{r},\mathbf{r}') = \frac{L_j}{2} - r'_j, \tag{A8}$$

that the average current density

$$\boldsymbol{J}^E = \frac{1}{A}\int d^2r\mathbf{J}^E(\mathbf{r}), \tag{A9}$$

is readily obtained from Eq. (A4) (up to the factor $1/A$) by integrating the first line over $\mathbf{r}$ and replacing $\partial_j[G(\mathbf{r},\mathbf{r}') - G(\mathbf{r},\mathbf{r}'')]$ with $r''_j - r'_j$ in the second. Alternatively, it can also be expressed as

$$\begin{aligned} J_j^E = &-\frac{1}{2m_jA}\int d^2r[D_j\psi(\mathbf{r})]^\dagger\partial_t\psi(\mathbf{r}) \\ &-\frac{1}{4A}\int d^2rd^2r'(r_j-r'_j)U(\mathbf{r}-\mathbf{r}')\{\psi^\dagger(\mathbf{r})\rho(\mathbf{r}')\partial_t\psi(\mathbf{r}) \\ &-\psi^\dagger(\mathbf{r})\psi^\dagger(\mathbf{r}')[\partial_t\psi(\mathbf{r}')]\psi(\mathbf{r})\} + \mathrm{H.c.} \end{aligned} \tag{A10}$$

APPENDIX B: THE KUBO FORMULA FOR THE THERMOELECTRIC COEFFICIENTS

Consider a time independent K (with $[H,N]=0$), perturbed by $\delta K = \int d^2rg(\mathbf{r},t)Q(\mathbf{r})$, where $g(\mathbf{r},t) = g(\mathbf{r})e^{-i(\omega+i\delta)t}$ is an external field coupled to a conserved charge Q, satisfying $\partial_t Q + \nabla\cdot\mathbf{J} = 0$. To linear order in g, an observable $O(t) = e^{iKt}Oe^{-iKt}$, with $\langle O(t)\rangle_K = 0$, acquires the expectation value [44]

$$\langle O(t)\rangle_{K+\delta K} = \langle\delta O(t)\rangle_K - \int d^2r\nabla\varphi(\mathbf{r},t)\cdot\Pi_{\mathbf{J},O}(\mathbf{r},\omega+i\delta). \tag{B1}$$

Here we have assumed that no $\mathbf{J}$ flows out of the system. i.e., $\mathbf{J}\cdot\mathbf{n} = 0$, denoted by δO the change in the form of O in the presence of δK, and

$$\Pi_{\mathbf{J},O}(\mathbf{r},\omega) = \int_0^\infty dt\int_0^\beta d\tau e^{i\omega t}\langle\mathbf{J}(\mathbf{r},-t-i\tau)O(0)\rangle_K. \tag{B2}$$

Using a Lehmann representation in terms of K eigenstates, $K|n\rangle = \xi_n|n\rangle$, we can write the latter as

$$\begin{aligned} \Pi_{\mathbf{J},O}(\mathbf{r},\omega+i\delta) &= \frac{i}{Z_H}\sum_{m,n}e^{-\beta\xi_n}\frac{\langle n|\mathbf{J}(\mathbf{r})|m\rangle\langle m|O|n\rangle}{\xi_m-\xi_n+\omega+i\delta} \\ &\quad\times\int_0^\beta d\tau e^{(\xi_n-\xi_m)\tau} \\ &= \frac{i}{\omega+i\delta}\frac{1}{Z_H}\sum_{m,n}[e^{-\beta\xi_m} - e^{-\beta(\xi_n-i\epsilon)}] \\ &\quad\times\left[\frac{1}{\xi_m-\xi_n+\omega+i\delta} - \frac{1}{\xi_m-\xi_n+i\epsilon}\right] \\ &\quad\times\langle n|\mathbf{J}(\mathbf{r})|m\rangle\langle m|O|n\rangle, \end{aligned} \tag{B3}$$

where the limit $\epsilon\to 0$ is introduced in order to recover the correct result of the τ integration in the case $\xi_m = \xi_n$, and is to be taken first, followed by the limit $\delta\to 0$.

On the other hand, consider the imaginary-time correlation function

$$\begin{aligned} \chi_{O,\mathbf{J}}(\mathbf{r},i\omega_n) &= -\int_0^\beta d\tau e^{i\omega_n\tau}\langle O(-i\tau)\mathbf{J}(\mathbf{r},0)\rangle \\ &= \frac{1}{Z_H}\sum_{m,n}[e^{-\beta\xi_m} - e^{-\beta(\xi_n-i\epsilon)}] \\ &\quad\times\frac{\langle n|\mathbf{J}(\mathbf{r})|m\rangle\langle m|O|n\rangle}{\xi_m-\xi_n+i\omega_n+i\epsilon}, \end{aligned} \tag{B4}$$

where ω_n is a bosonic Matsubara frequency, and the limit $\epsilon\to 0$ takes care of the case $\omega_n = 0$ and $\xi_m = \xi_n$. From Eqs. (B3) and (B4), it then follows that

$$\Pi_{\mathbf{J},O}(\mathbf{r},\omega+i\delta) = \frac{i}{\omega+i\delta}[\chi_{O,\mathbf{J}}(\mathbf{r},\omega+i\delta) - \chi_{O,\mathbf{J}}(\mathbf{r},i\epsilon)], \tag{B5}$$

where χ has been analytically continued via $i\omega_n\to\omega+i\delta$ to yield the retarded correlation function.

The above results applies to the calculation of $\hat{\alpha}$ given the identification $O = \boldsymbol{J}^e = (1/A)\int d^2r\mathbf{J}^e$, $g(\mathbf{r}) = (1/T)\nabla T\cdot\mathbf{r}$, $Q = \mathcal{K}$, and $\mathbf{J} = \mathbf{J}^h$. This in turn leads, together with the definition

$$\chi_{O,\boldsymbol{J}}(\omega) = \frac{1}{A}\int d^2r\chi_{O,\mathbf{J}}(\mathbf{r},\omega), \tag{B6}$$

to Eq. (6).

From Eq. (A2) it follows that $\delta\mathbf{J}^e = g\mathbf{J}^e$, with the consequent contribution to $\hat{\alpha}$

$$\alpha_{ij}^{(2)} = -\frac{1}{AT}\left\langle\int d^2\mathrm{J}_i^e(\mathbf{r})r_j\right\rangle_K. \tag{B7}$$

To relate it to the z component of the orbital magnetization, M_z, note that

$$\begin{aligned} 0 &= \left\langle\int d^2r\partial_t\rho^e(\mathbf{r})r_ir_j\right\rangle_K = -\left\langle\int d^2r\nabla\cdot\mathbf{J}^e(\mathbf{r})r_ir_j\right\rangle_K \\ &= \int d^2r\mathrm{J}_i^e(\mathbf{r})r_j + \int d^2r\mathrm{J}_j^e(\mathbf{r})r_i, \end{aligned} \tag{B8}$$

where the first equality is a result of $\mathrm{Tr}\{e^{-\beta K}[\rho^e,H]\} = 0$, and the third a result of our assumption $\mathbf{J}^e\cdot\mathbf{n} = 0$. Eq. (B8), together with the definition

$$M_z = \frac{1}{2c}\left\langle\int d^2r\left[x\mathrm{J}_y^e(\mathbf{r}) - y\mathrm{J}_x^e(\mathbf{r})\right]\right\rangle_K, \tag{B9}$$

allow us to express $\alpha_{ij}^{(2)}$ by Eq. (5).

Next, let us discuss the calculation of $\hat{\tilde{\alpha}}$ using the Kubo formula. Since the calculation is done for finite ω, which is taken to zero only at the end, one needs to determine the appropriate form of $\mathbf{J}^E$ in the presence of a time-varying electric field. To this end, we split the scalar potential into $\phi(\mathbf{r},t) = \phi_0(\mathbf{r}) + \varphi(\mathbf{r},t)$, such that the driving electric field is given by $\mathbf{E}(\mathbf{r},t) = -\nabla\varphi(\mathbf{r},t) - (1/c)\partial_t\boldsymbol{A}(\mathbf{r},t)$, while $\phi_0(\mathbf{r})$ describes the constant background potential, due to the ions for example. We denote by $\tilde{\mathbf{J}}^E$ the current density that is given by Eq. (A4) with time-dependent electromagnetic potentials, and note that it satisfies $-\nabla\cdot\tilde{\mathbf{J}}^E(\mathbf{r},t) = i[H(t),\mathcal{H}(\mathbf{r},t)]$. Consequently, one finds

$$\begin{aligned}\partial_t[\mathcal{H}-\varphi\rho^e] &= i[H,\mathcal{H}] + \partial_t\mathcal{H} - i\varphi[H,\rho^e] - \partial_t\varphi\rho^e\\ &= -\nabla\cdot(\tilde{\mathbf{J}}^E - \varphi\mathbf{J}^e) - \varphi(\partial_t\rho^e + \nabla\cdot\mathbf{J}^e)\\ &\quad -(\nabla\varphi + \partial_t A/c)\cdot\mathbf{J}^e\\ &= -\nabla\cdot(\tilde{\mathbf{J}}^E - \varphi\mathbf{J}^e) + \mathbf{J}^e\cdot\mathbf{E},\end{aligned}\tag{B10}$$

which is to be interpreted as a continuity equation, with a source term due to Joule heating, for the energy density $\rho^E = \mathcal{H} - \varphi\rho^e$, and current $\mathbf{J}^E = \tilde{\mathbf{J}}^E - \varphi\mathbf{J}^e$ [1,2]. These ρ^E and $\mathbf{J}^E$ are also both gauge invariant, with $\boldsymbol{J}^E$ obtained from Eq. (A10) via the substitution $\partial_t \to \partial_t - ie\varphi$.

Introducing the electric field in the gauge $\varphi(\mathbf{r},t) = -\mathbf{E}\cdot\mathbf{r}e^{-i(\omega+i\delta)t}$ and applying Eq. (B1) with $O = \boldsymbol{J}^h = \boldsymbol{J}^E + (\mu/e)\boldsymbol{J}^e$, $g(\mathbf{r},t) = \varphi(\mathbf{r},t)$, $Q = \rho^e$, and $\mathbf{J} = \mathbf{J}^e$ leads to

$$\tilde{\alpha}^{(1)}_{ij} = \lim_{\omega\to 0} A\frac{i}{\omega+i\delta}\left[\chi_{J^h_i,J^e_j}(\omega+i\delta) - \chi_{J^h_i,J^e_j}(i\epsilon)\right].\tag{B11}$$

The above form of the heat current does not change in the presence of $\varphi(\mathbf{r},t)$. However, in the limit $\omega\to 0$, the system relaxes to a state which is close to local (but not global) thermodynamic equilibrium, for which $\varphi(\mathbf{r}) = -\mathbf{E}\cdot\mathbf{r}$ becomes a part of the $\phi_0(\mathbf{r})$ and $\varphi\mathbf{J}^e$ a part $\mathbf{J}^h$. As a result, an additional contribution to $\hat{\tilde{\alpha}}$ appears, and is given by

$$\tilde{\alpha}^{(2)}_{ij} = -\frac{1}{A}\left\langle\int d^2r\mathrm{J}^e_i(\mathbf{r})r_j\right\rangle_K = \frac{c}{A}\epsilon^{ijz}M_z.\tag{B12}$$

The existence of the magnetization term can be traced back to the assumed local thermodynamic equilibrium, which implies the relation $T\delta S = \delta E - \mu\delta N + M_z\delta B_z$ for an infinitesimal heat change. This, when divided by δt and combined with Faraday's law $\nabla\times\mathbf{E} = -(1/c)\partial_t\mathbf{B}$, gives Eq. (B12).

Finally, we demonstrate that $\hat{\tilde{\alpha}}^{(1)}$ is gauge invariant. To this end, we employ the conventional form of the Kubo formula [44], which for the gauge $\varphi(\mathbf{r},t) = -\mathbf{E}\cdot\mathbf{r}e^{-i(\omega+i\delta)t}$ reads

$$\begin{aligned}\left\langle\mathrm{J}^h_i(\mathbf{r},t)\right\rangle_{K+\delta K} &= i\int_{-\infty}^t dt'\int d^2r'\mathbf{E}\cdot\mathbf{r}'e^{-i(\omega+i\delta)t'}\\ &\quad\times\left\langle\left[\mathrm{J}^h_i(\mathbf{r},t),\rho^e(\mathbf{r}',t')\right]\right\rangle_K,\end{aligned}\tag{B13}$$

where we used the fact that in this gauge $\delta\boldsymbol{J}^h = 0$.

Alternatively, one can use the gauge $\mathbf{A}(\mathbf{r},t) = \mathbf{A}^{\mathrm{B}}(\mathbf{r}) + \mathbf{A}^{\mathrm{E}}(t)$, where the first piece is responsible for the magnetic field, while the electric field is introduced via $\mathbf{A}^{\mathrm{E}} = -ic\mathbf{E}e^{-i(\omega+i\delta)t}/(\omega+i\delta)$. The Kubo formula then becomes

$$\begin{aligned}\left\langle\mathrm{J}^h_i(\mathbf{r},t)\right\rangle_{K+\delta K} &= \langle\delta\mathrm{J}^h_i(\mathbf{r},t)\rangle_K + \int_{-\infty}^t dt'\int d^2r'\frac{e^{-i(\omega+i\delta)t'}}{\omega+i\delta}\mathbf{E}\\ &\quad\cdot\left\langle\left[\mathrm{J}^h_i(\mathbf{r},t),\mathbf{J}^e_\mu(\mathbf{r}',t')\right]\right\rangle_K,\end{aligned}\tag{B14}$$

where K includes $\mathbf{A}^{\mathrm{B}}$ but not $\mathbf{A}^{\mathrm{E}}$. To proceed, we note that

$$\begin{aligned}\int d^2r\mathbf{E}\cdot\mathbf{J}^e &= \int d\mathbf{n}\cdot\tilde{\mathbf{J}} - \int d^2r\,(\mathbf{E}\cdot\mathbf{r})\nabla\cdot\mathbf{J}^e\\ &= \int d^2r\,\mathbf{E}\cdot\mathbf{r}\,\partial_t\rho^e,\end{aligned}\tag{B15}$$

where in going from the first to the second line we assumed that the surface integral of $\tilde{\mathbf{J}} = (\mathbf{E}\cdot\mathbf{r})\mathbf{J}^e$ vanishes. Plugging Eq. (B15) into Eq. (B14) and integrating by parts over t' yields Eq. (B13) and a boundary term:

$$\begin{aligned}&\frac{e^{-i(\omega+i\delta)t}}{\omega+i\delta}\int d^2r'\mathbf{E}\cdot\mathbf{r}'\left\langle\left[\mathrm{J}^h_i(\mathbf{r},t),\rho^e(\mathbf{r}',t)\right]\right\rangle_K\\ &= -\frac{e}{c}A^{\mathrm{E}}_\nu(t)\Big\langle\frac{1}{2m_im_\nu}[D_i\psi]^\dagger D_\nu\psi\\ &\quad+\frac{\delta_{i,\nu}}{2m_i}\psi^\dagger\left[-\frac{1}{2m_\nu}D^2_\nu - e\phi + \frac{1}{2}\int d^2r'U(\mathbf{r}-\mathbf{r}')\rho(\mathbf{r}')\right]\psi\\ &\quad+\frac{1}{4m_\nu}\int d^2r'd^2r''\partial_i[G(\mathbf{r},\mathbf{r}') - G(\mathbf{r},\mathbf{r}'')]\\ &\quad\times\psi^\dagger(\mathbf{r}')[\partial'_\nu U(\mathbf{r}'-\mathbf{r}'')]\rho(\mathbf{r}'')\psi(\mathbf{r}') + \delta_{i,\nu}\frac{\mu}{2em_i}\rho^e\Big\rangle_K\\ &\quad+\mathrm{H.c.},\end{aligned}\tag{B16}$$

which exactly cancels $\langle\delta\mathrm{J}^h_i(\mathbf{r},t)\rangle_K$, as can be checked using Eqs. (A2) and (A4). We comment that by applying the considerations outlined in Appendix C it can be shown that $\langle\delta\mathrm{J}^h_i(\mathbf{r},t)\rangle_K$ vanishes in the presence of reflection symmetry.

APPENDIX C: BEHAVIOR OF CORRELATION FUNCTIONS UNDER REFLECTION AND TIME REVERSAL

We are interested in the case where the potential appearing in the Hamiltonian density (3) is invariant under reflection about the y axis,

$$\phi(x,y) = \phi(L_x - x,y) \equiv \phi(\tilde{x},y),\tag{C1}$$

and similarly the magnetic field satisfies $B_z(x,y) = B_z(\tilde{x},y)$. The latter condition in obeyed provided that

$$A_x(\tilde{x},y) = A_x(x,y) + \partial_x f(x,y),\tag{C2}$$

$$A_y(\tilde{x},y) = -A_y(x,y) - \partial_y f(x,y).\tag{C3}$$

It is then straightforward to check that $\pi H(B)\pi^\dagger = H(-B)$ under the reflection transformation

$$\pi\psi(x,y)\pi^\dagger = \psi(\tilde{x},y)e^{i(e/c)f(x,y)}.\tag{C4}$$

As a result, any two bosonic Hermitian operators $O_{1,2}$, transforming according to

$$\pi O_{1,2}(B)\pi^\dagger = \epsilon^\pi_{1,2}O_{1,2}(-B),\tag{C5}$$

with $\epsilon^\pi_{1,2} + \pm 1$, satisfy

$$\begin{aligned}\langle O_1(B)O_2(B)\rangle_{K(B)} &= \mathrm{Tr}[\pi e^{-\beta K}O_1O_2\pi^\dagger]/Z_{K(-B)}\\ &= \epsilon^\pi_1\epsilon^\pi_2\langle O_1(-B)O_2(-B)\rangle_{K(-B)}.\end{aligned}\tag{C6}$$

Consequently, the imaginary-time correlation function obeys

$$\begin{aligned}\chi_{O_1,O_2}(i\omega_n;B) &= -\int_0^\beta d\tau e^{i\omega_n\tau}\langle O_1(-i\tau;B)O_2(0;B)\rangle_{K(B)}\\ &= \epsilon^\pi_1\epsilon^\pi_2\chi_{O_1,O_2}(i\omega_n;-B).\end{aligned}\tag{C7}$$

Note that for both electrical and heat currents $\epsilon^\pi_{\mathrm{J}_{x,y}} = \mp 1$. Equation (C7) also holds for the disorder averaged correlation function $\overline{\langle O_1 O_2\rangle_K} = \int D\phi P(\phi)\langle O_1 O_2\rangle_K$, even when condition (C1) is not fulfilled, as long as the disorder distribution obeys $P[\phi(x,y)] = P[\phi(\tilde{x},y)]$.

Under time reversal, $\Theta H(B)\Theta^{-1} = H(-B)$. Provided that

$$\Theta O_{1,2}(B)\Theta^{-1} = \epsilon^{\Theta}_{1,2} O_{1,2}(-B), \tag{C8}$$

with $\epsilon^{\Theta}_{1,2} + \pm 1$, and using $\langle n|O|n\rangle = \langle \bar{n}|\Theta O^{\dagger}\Theta^{-1}|\bar{n}\rangle$ [45], where $|\bar{n}\rangle = \Theta|n\rangle$ is the time reversed state, one finds

$$\begin{aligned}\langle O_1(-i\tau;B)O_2(0;B)\rangle_{K(B)} \\ = \mathrm{Tr}[\Theta O_2(0;B)O_1(i\tau;B)e^{-\beta K(B)}\Theta^{-1}]/Z_{K(-B)}.\end{aligned} \tag{C9}$$

Hence

$$\chi_{O_1,O_2}(i\omega_n;B) = \epsilon^{\Theta}_1\epsilon^{\Theta}_2\chi_{O_2,O_1}(i\omega_n;-B). \tag{C10}$$

For both electrical and heat current densities, $\epsilon^{\Theta}_{J_i} = -1$.

APPENDIX D: DIAGONALIZATION OF H_r

The relative two-particle Hamiltonian in Eq. (13) is expressed in terms of the operators

$$b_1 = \frac{1}{2^{1/2}}\left(\sqrt{\frac{\Omega}{\omega_c}}\frac{x}{l_x} + \sqrt{\frac{\omega_c}{\Omega}}l_x\frac{\partial}{\partial x}\right), \tag{D1}$$

$$b_2 = \frac{1}{2^{1/2}}\left(\sqrt{\frac{\Omega}{\omega_c}}\frac{y}{l_y} + \sqrt{\frac{\omega_c}{\Omega}}l_y\frac{\partial}{\partial y}\right), \tag{D2}$$

satisfying $[b_1,b_1^{\dagger}] = [b_2,b_2^{\dagger}] = 1$, $[b_1,b_2] = [b_1,b_2^{\dagger}] = 0$, as

$$\begin{aligned}H_r &= \Omega(b_1^{\dagger}b_1 + b_2^{\dagger}b_2 + 1) - i\frac{\omega_c}{2}(b_1^{\dagger}b_2 - b_2^{\dagger}b_1) \\ &\quad - \gamma[(b_1^{\dagger}+b_1)^2 - (b_2^{\dagger}+b_2)^2],\end{aligned} \tag{D3}$$

where

$$\Omega = \tfrac{1}{2}\sqrt{\omega_c^2 + \omega_x^2 + \omega_y^2}, \tag{D4}$$

$$\gamma = \frac{\omega_y^2 - \omega_x^2}{16\Omega}. \tag{D5}$$

It may be decoupled into two independent pieces via the canonical transformation

$$\begin{pmatrix} b_1 \\ b_2\end{pmatrix} = \begin{pmatrix} i\cos\phi & \sin\phi \\ -\sin\phi & -i\cos\phi\end{pmatrix}\begin{pmatrix} c_1 \\ c_2\end{pmatrix}, \tag{D6}$$

with

$$\tan 2\phi = -\frac{\omega_c}{4\gamma}, \tag{D7}$$

which leads to

$$H_r = (c_1^{\dagger}\ c_1)\begin{pmatrix}\Omega_- & \gamma \\ \gamma & \Omega_-\end{pmatrix}\begin{pmatrix} c_1 \\ c_1^{\dagger}\end{pmatrix} + (c_2^{\dagger}\ c_2)\begin{pmatrix}\Omega_+ & -\gamma \\ -\gamma & \Omega_+\end{pmatrix}\begin{pmatrix} c_2 \\ c_2^{\dagger}\end{pmatrix}, \tag{D8}$$

where

$$\Omega_{\pm} = \frac{\Omega}{2} \pm \frac{1}{4}\sqrt{\omega_c^2 + (4\gamma)^2} \geqslant 0. \tag{D9}$$

Finally, we employ the Bogoliubov transformation

$$\begin{pmatrix} c_{1,2} \\ c_{1,2}^{\dagger}\end{pmatrix} = \begin{pmatrix}\cosh\theta_{1,2} & \sinh\theta_{1,2} \\ \sinh\theta_{1,2} & \cosh\theta_{1,2}\end{pmatrix}\begin{pmatrix} d_{1,2} \\ d_{1,2}^{\dagger}\end{pmatrix}, \tag{D10}$$

with

$$\tanh 2\theta_{1,2} = \mp\frac{\gamma}{\Omega_{\mp}}, \tag{D11}$$

to bring H_r into the diagonalized form

$$H_r = \omega_1\left(d_1^{\dagger}d_1 + \tfrac{1}{2}\right) + \omega_2\left(d_2^{\dagger}d_2 + \tfrac{1}{2}\right), \tag{D12}$$

where $[d_1,d_1^{\dagger}] = [d_2,d_2^{\dagger}] = 1$, $[d_1,d_2] = [d_1,d_2^{\dagger}] = 0$, and the eigenfrequencies are given by

$$\omega_{1,2} = 2\sqrt{\Omega_{\mp}^2 - \gamma^2}. \tag{D13}$$

The center of mass part of the two-particle eigenstate is obviously symmetric under particle exchange. To maintain antisymmetry of the full state, we require that the relative part would be antisymmetric. One can check that the wave function of the ground state, $|n_1 = 0, n_2 = 0\rangle$, of H_r is proportional to $\exp[-(ax^2 + ibxy + cy^2)]$, with a,b,c constants, and hence symmetric. From Eqs. (D1) and (D2), it follows that $b_{1,2} \to -b_{1,2}$ under particle exchange, and the linearity of the ensuing transformations means that $d_{1,2}$ share this property. Thus the allowed $|n_1,n_2\rangle \propto (d_1^{\dagger})^{n_1}(d_2^{\dagger})^{n_2}|0,0\rangle$ are those for which $n_1 + n_2$ is odd.

APPENDIX E: CALCULATING $\hat{\alpha}$ FOR THE QUASI-ONE-DIMENSIONAL MODEL

1. Calculating $\alpha^{(1)}_{yx}$

According to Eq. (6), $\alpha^{(1)}_{yx}$ is determined from the correlation function $\chi_{J^e_y,J^h_x}(i\omega_n)$, which we evaluate perturbatively in $H_{\mathcal{J}}$. One can readily verify that the zeroth-order term vanishes in the limit $L \to \infty$, and the lowest nonvanishing contribution is

$$\chi_{J^e_y,J^h_x}(i\omega_n) = \int_0^{\beta} d\tau d\tau' e^{i\omega_n\tau}\langle T_\tau J^e_y(\tau)J^h_x(0)H_{\mathcal{J}}(\tau')\rangle_0, \tag{E1}$$

where here $O(\tau) = e^{H_0\tau}Oe^{-H_0\tau}$ and T_τ is the imaginary time ordering operator. Using expressions (31) and (32) of the current densities and the averages [46]

$$\begin{aligned}F_1(x,\tau) &= K^{-1}\langle T_\tau[\phi(x,\tau) - \phi(0,0)]^2\rangle_0 \\ &= K\langle T_\tau[\theta(x,\tau) - \theta(0,0)]^2\rangle_0 \\ &= \frac{1}{2\pi}\ln\left\{\left(\frac{l_T}{a}\right)^2\left[\sinh^2\left(\frac{x}{l_T}\right) + \sin^2\left(\frac{v\tilde{\tau}}{l_T}\right)\right]\right\},\end{aligned} \tag{E2}$$

$$\begin{aligned}F_2(x,\tau) &= \langle T_\tau\phi(x,\tau)\theta(0,0)\rangle_0 \\ &= \frac{1}{4\pi}\left\{\ln\left[-i\sinh\left(\frac{x+iv\tilde{\tau}}{l_T}\right)\right]\right. \\ &\quad \left. - \ln\left[i\sinh\left(\frac{x-iv\tilde{\tau}}{l_T}\right)\right]\right\},\end{aligned} \tag{E3}$$

where $\tilde{\tau} = \tau + \text{sign}(\tau)a/v$, and $l_T = v/\pi T$, we obtain

$$\chi_{J^e_y,J^h_x}(i\omega_n) = \frac{2\pi e v^2 \mathcal{J}^2}{KA}\int_0^\beta d\tau d\tau' \int dx dx' e^{i\omega_n\tau} \times C(x-x',\tau-\tau')\sin[b(x-x')] \times [\partial_x F_1(x,\tau) - \partial_{x'}F_1(x',\tau')] \times [\partial_x F_2(x,\tau) - \partial_{x'}F_2(x',\tau')], \quad \text{(E4)}$$

with

$$C(x,\tau) = e^{-2\pi K^{-1}F_1(x,\tau)}. \quad \text{(E5)}$$

The parity of the functions F_1 and F_2 leads, after defining $r = x - x'$, to

$$\chi_{J^e_y,J^h_x}(i\omega_n) = \frac{2\pi e v^2 \mathcal{J}^2}{KA}\int_0^\beta d\tau d\tau' \int dx dr e^{i\omega_n\tau} \times C(r,\tau-\tau')\sin(br)[\partial_x F_1(x,\tau)\partial_x F_2(x-r,\tau') + \partial_x F_1(x-r,\tau')\partial_x F_2(x,\tau)]. \quad \text{(E6)}$$

For $\tau,\tau' \in [0,\beta]$, the integral over x can be evaluated with the result

$$\chi_{J^e_y,J^h_x}(i\omega_n) = \frac{iev\mathcal{J}^2}{\pi l_T A}\int_0^\beta d\tau d\tau' \int dr e^{i\omega_n\tau}\sin(br) \times [v^2(\tau-\tau')\partial_r + r\partial_{\tau'}]C(r,\tau-\tau'). \quad \text{(E7)}$$

Integrating by parts, we find that the $\partial_{\tau'}$ term vanishes. Finally, integration by parts over r and a change of variables to $\tau \pm \tau'$, gives

$$\chi_{J^e_y,J^h_x}(i\omega_n) = \frac{ev^2\mathcal{J}^2 b}{\omega_n A}[C(b,i\omega_n) - C(b,0)], \quad \text{(E8)}$$

where $\chi_{J^e_y,J^h_x}(i\omega_n = 0) = 0$, and

$$C(q,i\omega_n) = \int_{-\infty}^{\infty} dx \int_0^\beta d\tau\, e^{i(\omega_n\tau - qx)}C(x,\tau). \quad \text{(E9)}$$

We are now left with the task of calculating

$$C(q,i\omega_n) = \frac{l_T^2}{v}\left(\frac{a}{l_T}\right)^{2/K}\int_{-\infty}^{\infty} dx \int_0^\pi dy\, e^{i(\tilde{\omega}_n y - \tilde{q}x)} \times (\sinh^2 x + \sin^2 y)^{-1/K}, \quad \text{(E10)}$$

with $\tilde{\omega}_n = l_T\omega_n/v$ and $\tilde{q} = ql_T$. Clearly, this function is symmetric under $\omega_n \to -\omega_n$, so in the following we assume $\omega_n > 0$. Next, a change in the integration variable to $z = iy$ rotates the integral onto the segment C_1, see Fig. 4. Applying Cauchy's theorem, while noting the branch cuts at $\text{Re}(z) < 0$ and $\text{Im}(z) = 0,\pi$, we may trade C_1 by the contour $-C_2 - C_4$. Finally, we use the invariance of the integrand under $z \to z - i\pi$ to shift C_2 below the real axis and obtain

$$C(q,i\omega_n) = i\frac{l_T^2}{v}\left(\frac{a}{l_T}\right)^{2/K}\int_{-\infty}^{\infty} dx \int_{-\infty}^0 dz\, e^{\tilde{\omega}_n z - i\tilde{q}x} \times \{[\sinh^2 x - \sinh^2(z - i\epsilon)]^{-1/K} - [\sinh^2 x - \sinh^2(z + i\epsilon)]^{-1/K}\}, \quad \text{(E11)}$$

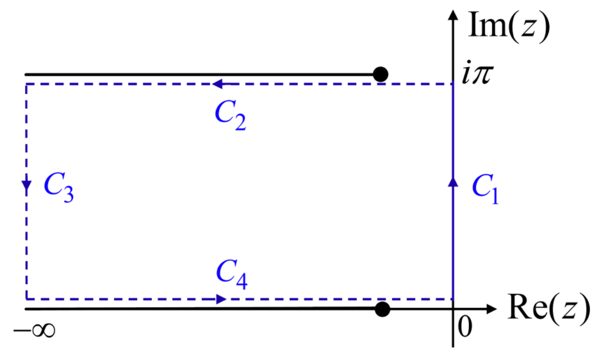

FIG. 4. The integration contour for calculating $C(q,i\omega_n)$.

where ϵ is a positive infinitesimal. Taking $z \to -z$ and using

$$[\sinh^2 x - \sinh^2(z \pm i\epsilon)]^{1/K} = \begin{cases} (\sinh^2 x - \sinh^2 z)^{1/K} & |x| > |z| \\ (\sinh^2 z - \sinh^2 x)^{1/K} e^{\mp i\pi\,\text{sign}(z)/K} & |z| > |x| \end{cases}, \quad \text{(E12)}$$

we arrive at

$$C(q,i\omega_n) = 2\frac{l_T^2}{v}\left(\frac{a}{l_T}\right)^{2/K}\sin\left(\frac{\pi}{K}\right) \times \int_0^\infty dz \int_{-z}^z dx \frac{e^{-\tilde{\omega}_n z - i\tilde{q}x}}{(\sinh^2 z - \sinh^2 x)^{1/K}}. \quad \text{(E13)}$$

By changing variables to $z \pm x$ the remaining integrals can be evaluated for $K > 1$. The result, after analytically continuing $i\omega_n \to \omega + i\delta$, is given by Eq. (35).

2. Calculating $\alpha_{xy}^{(1)}$

Within a perturbative treatment of $\mathcal{J}$ the leading contribution to $\alpha_{xy}^{(1)}$ is determined by

$$\chi_{J^e_x,J^h_y}(i\omega_n) = \int_0^\beta d\tau d\tau' e^{i\omega_n\tau}\langle T_\tau J^e_x(\tau)J^h_y(0)H_{\mathcal{J}}(\tau')\rangle_0. \quad \text{(E14)}$$

Concentrating on the spatial integrals which appear in this contribution, we find that it is proportional to

$$\frac{1}{AN_cL}\sum_{j=1}^{N_c}\sum_{j'=2}^{N_c}\int_{-L/2}^{L/2} dx dx' dx''\sin[b(x'-x'')] \times C(x'-x'',\tau'-\tau'')\{\delta_{j,j'}\partial_x^2 F_2(x-x',\tau'-\tau) + (\pi/K)(\delta_{j,j'} - \delta_{j,j'-1})\partial_{x'}F_2(x'-x'',\tau'-\tau'') \times [\partial_{x'}F_1(x'-x,\tau'-\tau) - \partial_{x''}F_1(x''-x,\tau''-\tau)]\}. \quad \text{(E15)}$$

Clearly, the sum over j' of the last two lines vanishes. The sums and x integral over the remaining part give $N_c\partial_x F_2(x - x',\tau - \tau')|_{x=-L/2}^{x=L/2}$. It follows from Eq. (E3) that this term is appreciable only for x' within a distance of order l_T from the

the edges at $\pm L/2$. Consequently, we conclude that $\alpha_{xy}^{(1)}$ is smaller by a factor l_T/L than the corresponding $\alpha_{yx}^{(1)}$.

3. Calculating M_z

The magnetization can be computed from the thermodynamic relation

$$M_z = -\left(\frac{\partial \Omega}{\partial B}\right)_{\mu,T}, \tag{E16}$$

where Ω is the grand canonical potential. To second order in $\mathcal{J}$, we obtain

$$\Omega = \Omega_0 - \frac{1}{2}\int_0^\beta d\tau \langle T_\tau H_{\mathcal{J}}(\tau) H_{\mathcal{J}}(0)\rangle_0 = \Omega_0 - \frac{\mathcal{J}^2 A}{4d}\int_0^\beta d\tau \int dx C(x,\tau)\cos(bx), \tag{E17}$$

with $\Omega_0 = -T\mathrm{Tr}[e^{-\beta H_0}]$, from which it follows, using Eqs. (E16) and (E9), that

$$M_z = -\frac{e\mathcal{J}^2 A}{2c}\frac{\partial C(b,0)}{\partial b}. \tag{E18}$$

The same result is also obtained from the definition of M_z in terms of currents, Eq. (B9). To see this, we note that Eqs. (29) and (31) imply

$$\left\langle \int d^2 r x \mathrm{J}_y^e \right\rangle_H = -2ed\left\langle \frac{\partial H_{\mathcal{J}}}{\partial b}\right\rangle_H = ed\frac{\partial}{\partial b}\int_0^\beta d\tau \langle T_\tau H_{\mathcal{J}}(\tau) H_{\mathcal{J}}(0)\rangle_0 = cM_z. \tag{E19}$$

Furthermore, explicit calculation reveals that to order $\mathcal{J}^2$,

$$\left\langle \int d^2 r y \mathrm{J}_x^e \right\rangle_H = -\frac{evd\mathcal{J}^2}{2}\sum_{j=1}^{N_c}\sum_{j'=2}^{N_c} j(\delta_{j,j'} - \delta_{j,j'-1})\int_0^\beta d\tau d\tau' \int dx dx' dx'' \sin[b(x'-x'')]C(x'-x'',\tau'-\tau'')\partial_x F_1(x-x',\tau-\tau'). \tag{E20}$$

A naive evaluation of the integral, disregarding the finite size of the system, would yield zero owing to the fact that the integrand is odd in $x - x'$ and $x' - x''$. However, a more careful analysis leads to a different conclusion. First, the sums add up to $N_c - 1$. Secondly, Eq. (E2) implies that to a good approximation $F_1(\pm L/2 - x', \tau - \tau') = [\ln(l_T/2a) + (L/2 \mp x')/l_T]/\pi$, as long as x' is situated more than l_T away from the edges. Using this, we obtain

$$\left\langle \int d^2 r y \mathrm{J}_x^e \right\rangle_H = \frac{e\mathcal{J}^2 A}{2}\int_0^\beta d\tau \int dx C(x,\tau)\, x \sin(bx) = c\frac{\partial \Omega}{\partial B} = -cM_z, \tag{E21}$$

as required.

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