Vortex motion and flux-flow resistivity in dirty multiband superconductors

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The conductivity of vortex lattices in multiband superconductors with high concentration of impurities is calculated based on microscopic kinetic theory at temperatures significantly smaller than the critical one. Both the limits of high and low fields are considered, when the magnetic induction is close to or much smaller than the critical field strength H_{c2}, respectively. It is shown that in contrast to single-band superconductors, the resistive properties are not universal but depend on the pairing constants and ratios of diffusivities in different bands. The low-field magnetoresistance can strongly exceed the Bardeen-Stephen estimation in a quantitative agreement with experimental data for the two-band superconductor MgB_2.

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I. INTRODUCTION

Recent transport experiments reveal quite unusual behavior of the flux-flow resistive states in multiband superconductors [1–6]. The magnetic field dependencies of flux-flow resistivity $\rho_f(B)$ were found to be qualitatively different from that observed in single-band superconductors [7]. This behavior is not explained by the theories developed in previous works [8–15].

Vortex motion in conventional type-II superconductors have been investigated for several decades. Flux-flow experiments in single-band superconductors at low temperatures and magnetic fields [7] are well described by the Bardeen-Stephen (BS) theory [8]. In this regime, the flux-flow resistivity is given by the linear magnetic field dependence,

$$\rho_f/\rho_n = \beta^{-1} B/H_{c2}, \tag{1}$$

where ρ_n is the normal-state resistivity, B is an average magnetic induction, H_{c2} is the second critical field, and $\beta \approx 1$. The BS law is in qualitative agreement with the results obtained based on the microscopic theory for dirty superconductors [9] at low temperatures.

In strong magnetic fields $H_{c2} - B \ll H_{c2}$, the motion of dense vortex lattices has been extensively studied with the help of linear response theory [12–14]. In these works, the slope of flux-flow resistivity,

$$S = (H_{c2}/\rho_n)(d\rho_f/dB), \tag{2}$$

has been shown to have a universal temperature dependence in the dirty limit characterized by a monotonic increase from $S(T=0) = 1.72$ to $S(T=T_c) = 5$ [13]. This behavior was confirmed by accurate high-field measurements in Pb-In alloys [16]. Deviations observed near the critical temperature T_c were attributed to the depairing effects due to the spin-flip and electron-phonon scattering [17].

Close to T_c, the flux-flow conductivity determined by the slow diffusion mechanism of relaxation does not have a universal behavior, being parametrically larger than the BS value. As shown in Ref. [11], at high fields in the range of parameters when $1 \gg 1 - B/H_{c2} \gg 1 - T/T_c$, the flux-flow conductivity is strongly enhanced, $\sigma_f \sim (1 - T/T_c)^{-1/2}(1 - B/H_{c2})^{3/2}$. Reference [10] demonstrates that a similar tendency exists also at low fields, where $\beta \sim (1 - T/T_c)^{-1/2}$ (see the detailed review in Ref. [18]).

In contrast to the conventional behavior described above, many multiband superconductors [1–5], including MgB_2 [6], were found to have the flux-flow resistivity larger than the BS value, $\rho_f/\rho_n > B/H_{c2}$, even in the low-temperature limit. The experimentally found dependencies $\rho_f(B)$ have a steeper growth in the low-field region with an enhanced magnetoresistance characterized by $\beta < 1$ and a smaller slope $S < 1$ at $B = H_{c2}$ [6], which is not described by the single-band theory [13].

The existing theories of flux-flow states cannot be straightforwardly applied to multiband superconductors. In these systems, vortices have a composite structure consisting of multiple singularities corresponding to the order parameter phase windings in different superconducting bands. In equilibrium, an isolated composite vortex is a bound state of several co-centered fractional vortices [19]. They can split, however, under the action of fluctuations [20], interaction with other vortices and sample boundaries [21,22], or due to external drive [23]. In particular, it was shown that the moving composite vortices can split into separate fractional vortices and even dissociate in a nonlinear regime provided the interband pairing is sufficiently small [23]. It is natural to expect that vortex splitting should have a profound effect on the flux-flow resistivity, especially at high fields when the flux-flow resistivity is strongly affected by the distortions of the moving vortex lattice [12–14]. As will be shown below, the well-known solution [15] describing a moving vortex lattice is not applicable to describe multiband systems since the distortion generically splits the sublattices of fractional vortices. In the present paper, we develop a theoretical framework to take into account this effect and calculate the conductivity corrections. For that, one needs to know the Maki parameter, also known as a generalized Ginzbirg-Landau parameter κ_2 which determines, in particular, the order parameter density as a function of magnetic field near H_{c2} [12]. Recently, this parameter has been calculated for multiband superconductors [24].

To obtain a complete picture of the flux-flow conductivity behavior in multiband systems, we also consider the regime of small magnetic fields, when a picture of isolated moving vortices is an adequate description [9]. Based on the kinetic theory, we calculate the coefficient β which characterizes the initial slope of the magnetoresistance. Applying the combination of the results in two limiting cases of small and high magnetic fields, it is possible to fit the experimental curves $\rho_f(B)$ for multiband superconductors with known pairing interactions such as MgB_2.

The model of dirty-limit superconductors assumed in the present work is appropriate for a certain class of multiband materials, including MgB_2 [25,26] and iron pnictides [27]. In single crystals of MgB_2, the de Haas–van Alphen data [28] and thermal conductivity measurements [29] suggest the borderline regime when one of the superconducting bands is moderately clean and the other one is moderately dirty [26]. Scanning tunnel microscopy shows the absence of a zero-bias anomaly inside vortex cores which is typical for dirty superconductors [30]. Impurities at high concentration can be introduced in MgB_2 on demand during the preparation process, producing nontrivial magnetic properties, which have been intensively studied recently [25,31–34].

In the present work, we consider temperatures significantly lower than T_c, which is relevant to the currently known experimental data for multiband superconductors. In particular, for low fields, we assume $T \ll T_c$, which allows one to neglect the contribution of delocalized quasiparticles outside the vortex cores [9]. For high fields, we focus on the parameter interval $1 - T/T_c > 1 - B/H_{c2}$ when one can use the expansion by small order parameter amplitudes to calculate the force acting on vortices [11]. Although the opposite regimes at $T \to T_c$ are interesting to consider in multiband systems, they are beyond the scope of the present paper.

The structure of this paper is as follows. In Sec. II, we introduce the Keldysh-Usadel description of the kinetic processes in dirty multiband superconductors. Here the basic components of the kinetic theory are discussed, including kinetic equations, self-consistency equations for the order parameter and current, as well as a general expression for the viscous force acting on the moving vortices. The flux-flow conductivity at high magnetic fields is calculated in Sec. III taking into account the splitting of fractional vortex sublattices. The case of low fields is considered in Sec. IV. A quantitative comparison of theoretical results with flux-flow resistivity measurements in MgB_2 [6] is discussed in Sec. V. The work summary is given in Sec. VI.

II. KINETIC EQUATIONS AND FORCES ACTING ON THE MOVING VORTEX LINE

We consider multiband superconductors in a dirty limit when the kinetics and spectral properties are described by the Keldysh-Usadel theory. For the single-band case, the theory of vortex motion in diffusive superconductors was developed in Refs. [9,10,13,17]. Here we generalize their theory to the multiband case.

The quasiclassical Green's function (GF) in each band is defined as

$$\check{g}_k = \begin{pmatrix} \hat{g}_k^R & \hat{g}_k^K \\ 0 & \hat{g}_k^A \end{pmatrix}, \tag{3}$$

where g_k^K is the (2×2 matrix) Keldysh component, $\hat{g}_k^{R(A)}$ is the retarded (advanced) GF, and k is the band index. The GF depends on two times and a single spatial coordinate variable $\check{g}_k = \check{g}_k(t_1,t_2,\boldsymbol{r})$. We use, from the beginning, the temporal gauge where the scalar potential is zero $\Phi = 0$ with an additional constraint that in equilibrium, the vector potential is time independent and satisfies $\nabla \cdot \boldsymbol{A} = 0$.

In dirty superconductors, the matrix $\check{g}$ obeys the Usadel equation,

$$\{\tau_3 \partial_t, \check{g}_k\}_t = D_k \hat{\partial}_{\mathbf{r}}(\check{g}_k \circ \hat{\partial}_{\mathbf{r}} \check{g}_k) + [\hat{H}_k, \check{g}_k]_t - i[\check{\Sigma}^{ph}, \check{g}_k]_t. \tag{4}$$

Here, $\tau_{1,2,3}$ are Pauli matrices in Nambu space, D_k is the diffusion constant, and $\hat{H}_k(\boldsymbol{r},t) = i\hat{\Delta}_k$, where $\hat{\Delta}_k(t) = i|\Delta_k|\tau_2 e^{-i\theta_k\tau_3}$ is the gap operator in the kth band. We define the commutator operator as $[X,g]_t = X(t_1)g(t_1,t_2) - g(t_1,t_2)X(t_2)$, similarly for anticommutator $\{,\}_t$. The symbolic product operator is given by $(A \circ B)(t_1,t_2) = \int dt A(t_1,t)B(t,t_2)$. In Eq. (4), the covariant differential superoperator is defined by

$$\hat{\partial}_{\mathbf{r}} \hat{g}_k = \nabla \hat{g}_k - ie[\tau_3 \boldsymbol{A}, \hat{g}_k]_t.$$

The gap in each band is determined by the self-consistency equation,

$$\Delta_k(t,\boldsymbol{r}) = \frac{\pi}{2} \sum_j \lambda_{kj} \big(\hat{g}_j^K\big)_{12}(t,t,\boldsymbol{r}). \tag{5}$$

Here, $(\hat{g}_j^K)_{12}$ is the 12 entry of the Keldysh component in a band indexed by j and $\hat{\Lambda}$ is the coupling matrix with elements λ_{kj}. Note that symmetry relation $\nu_1\lambda_{12} = \nu_2\lambda_{21}$ is satisfied, where ν_k is the density of states. The electric current density is given by

$$\boldsymbol{j}(t,\boldsymbol{r}) = \frac{\pi e}{2} \sum_k \nu_k D_k \mathrm{Tr}(\check{g}_k \circ \hat{\partial}_{\boldsymbol{r}} \check{g}_k)^K(t,t,\boldsymbol{r}). \tag{6}$$

The Keldysh-Usadel equation (4) is complemented by the normalization condition $(\check{g}_k \circ \check{g}_k)(t_1,t_2) = \check{\delta}(t_1 - t_2)$, which allows one to introduce parametrization of the Keldysh component in terms of the distribution function,

$$\hat{g}_k^K = \hat{g}_k^R \circ \hat{f}^{(k)} - \hat{f}^{(k)} \circ \hat{g}_k^A, \tag{7}$$

$$\hat{f}^{(k)} = f_L^{(k)}\tau_0 + f_T^{(k)}\tau_3. \tag{8}$$

To proceed, we introduce the mixed representation in the time-energy domain as follows: $g_k(t_1,t_2) = \int_{-\infty}^{\infty} g_k(\varepsilon,t) e^{-i\varepsilon(t_1-t_2)} d\varepsilon/(2\pi)$, where $t = (t_1+t_2)/2$.

We employ the Larkin-Ovchinnikov theory [17,35] to calculate the force acting on the moving vortex line. In multiband superconductors, the force is given by a linear superposition of contributions from different bands [17,35],

$$\boldsymbol{F}_{env} = \sum_k \boldsymbol{F}^{(k)} + \frac{1}{c} \int d^2\boldsymbol{r} \boldsymbol{B} \times \boldsymbol{j}^{(nst)}, \tag{9}$$

$$\boldsymbol{F}^{(k)} = \nu_k \int d^2\boldsymbol{r} \int_{-\infty}^{\infty} \frac{d\varepsilon}{4} \mathrm{Tr}\big(\hat{g}_k^{nst} \hat{\partial}_{\boldsymbol{r}} \hat{\Delta}_k\big), \tag{10}$$

where the covariant differential superoperator is given by $\hat{\partial}_{\mathbf{r}} \hat{\Delta}_k = \nabla \hat{\Delta}_k - ie[\tau_3 \boldsymbol{A}, \hat{\Delta}_k]$. Equations (9) and (10) contain a nonstationary part of the electric current density $\boldsymbol{j}^{(nst)}$ and the Keldysh component of a nonstationary Green's function

$\hat{g}_k^{nst}$, which can be expressed through the gradient expansion as follows:

$$\hat{g}_k^{nst} = -\frac{i}{2}\partial_t\big(\hat{g}_k^R + \hat{g}_k^A\big)\partial_\varepsilon f_0 + \big(\hat{g}_k^R - \hat{g}_k^A\big) f_L^{(k)} + \big(\hat{g}_k^R\tau_3 - \tau_3\hat{g}_k^A\big) f_T^{(k)}. \quad (11)$$

Keeping the first-order nonequilibrium terms and neglecting the electron-phonon relaxation in Eq. (4), we obtain a system of two coupled kinetic equations that determine both the transverse $f_T^{(k)}$ and longitudinal $f_L^{(k)}$ distribution function components,

$$\nabla\big(\mathcal{D}_T^{(k)}\nabla f_T^{(k)}\big) + \boldsymbol{j}_e^{(k)}\cdot\nabla f_L^{(k)} + 2i\mathrm{Tr}\big[\big(\hat{g}_k^R + \hat{g}_k^A\big)\hat{\Delta}_k\big] f_T^{(k)} = \partial_\varepsilon f_0 \mathrm{Tr}\big[\tau_3\partial_t\hat{\Delta}_k\big(\hat{g}_k^R + \hat{g}_k^A\big)\big] - e\partial_\varepsilon f_0 \nabla\cdot\big(\mathcal{D}_T^{(k)}\boldsymbol{E}\big), \quad (12)$$

$$\nabla\big(\mathcal{D}_L^{(k)}\nabla f_L^{(k)}\big) + \boldsymbol{j}_e^{(k)}\cdot\nabla f_T^{(k)} + 2i\mathrm{Tr}\big[\tau_3\big(\hat{g}_k^R - \hat{g}_k^A\big)\hat{\Delta}_k\big] f_T^{(k)} = -\partial_\varepsilon f_0 \mathrm{Tr}\big[\partial_t\hat{\Delta}_k\big(\hat{g}_k^R - \hat{g}_k^A\big)\big] - e\partial_\varepsilon f_0 \boldsymbol{j}_e^{(k)}\cdot\boldsymbol{E}, \quad (13)$$

where $\boldsymbol{E}$ is the electric field, and the energy-dependent diffusion coefficients $\mathcal{D}_{T,L}^{(k)}$ and the spectral charge current $\boldsymbol{j}_e^{(k)}$ in each band are given by

$$\mathcal{D}_T^{(k)} = D_k\mathrm{Tr}\big(\tau_0 - \tau_3\hat{g}_k^R\tau_3\hat{g}_k^A\big), \quad (14)$$

$$\mathcal{D}_L^{(k)} = D_k\mathrm{Tr}\big(\tau_0 - \hat{g}_k^R\hat{g}_k^A\big), \quad (15)$$

$$\boldsymbol{j}_e^{(k)} = D_k\mathrm{Tr}\,\big[\tau_3\big(\hat{g}_k^R\hat{\partial}_{\mathbf{r}}\hat{g}_k^R - \hat{g}_k^A\hat{\partial}_{\mathbf{r}}\hat{g}_k^A\big)\big]. \quad (16)$$

The detailed derivation of this system is given in the Appendix. In the simplest case of weak spatial inhomogeneities, it coincides with the equations derived by Schön [36] with the substitution $f_T^{(k)} \to f_T^{(k)} + (\dot{\theta}_k/2 - e\Phi)\partial_\varepsilon f_0$.

III. LARGE MAGNETIC FIELDS

At large magnetic fields, $H_{c2} - B \ll H_{c2}$, we can use simplifying approximations related to the smallness of the order parameter $|\Delta_k| \propto \sqrt{1 - B/H_{c2}}$. From the kinetic equations (12), one can see that nonequilibrium distribution functions are of the order of magnitude $f_{L,T}^{(k)} \propto |\Delta_k|^2$. Therefore, the contribution to the force determined by g_k^{nst} is proportional to $|\Delta_k|^3$, while the second term in Eq. (9) with nonstationary current $\boldsymbol{j}^{nst}$ is proportional to $|\Delta_k|^2$, as will be shown below. Thus the term (10) can be neglected if the temperature is not very close to T_c. In the opposite case when $1 - B/H_{c2} \gg 1 - T/T_c$, this term yields a dominating contribution to the force, resulting in a strong temperature-dependent enhancement of conductivity [11].

The most efficient way to find $\boldsymbol{j}^{nst}$ is to calculate the total current and then extract a nonstationary part proportional to the vortex velocity $\boldsymbol{v}_L$. From Eq. (6), we have

$$\boldsymbol{j} = \sum_k \frac{e\nu_k}{4}\int_{-\infty}^{\infty} d\varepsilon\left(\boldsymbol{j}_e^{(k)} f_0 + e\boldsymbol{E}\mathcal{D}_T^{(k)}\frac{\partial f_0}{\partial\varepsilon}\right). \quad (17)$$

The force balance condition yields that the space-averaged net current (6) is equal to the external transport current, $\langle \boldsymbol{j}\rangle = \boldsymbol{j}_{tr}$. In equilibrium, superconducting currents circulate around stationary vortices so that the net current is zero. Under the nonequilibrium conditions created by moving vortices, both of the terms in Eq. (17) provide nonzero contributions. Hereafter, we will consider isotropic superconductors so that the average current is codirected with the electric field, $\langle \boldsymbol{j}\rangle = \sigma_f\boldsymbol{E}$. The flux-flow conductivity is given by the superposition of three terms, $\sigma_f = \sigma_n + \sigma_{fl} + \sigma_{st}$, where σ_n is a large normal-metal contribution and the last two terms are given by

$$\sigma_{fl} = \sum_k \frac{e\nu_k}{4E}\int_{-\infty}^{\infty}\big\langle \boldsymbol{j}_e^{(k)}\big\rangle f_0(\varepsilon)d\varepsilon, \quad (18)$$

$$\sigma_{st} = -\sum_k \frac{e^2\nu_k}{4}\int_{-\infty}^{\infty}\frac{\partial\big\langle\mathcal{D}_T^{(k)}\big\rangle}{\partial\varepsilon} f_0(\varepsilon)d\varepsilon. \quad (19)$$

The term σ_{fl} (18) is a conductivity correction generated by nonequilibrium distortions or fluctuations of the superconducting order parameter. The similar correction in single-band superconductors has been calculated in the pioneering works on the flux-flow conductivity at $H \approx H_{c2}(T)$ [12–14]. Besides there exists a sizable quasiparticle contribution to the current given by the second term in Eq. (17), which determines the conductivity correction σ_{st} [13,14]. As can be seen from Eq. (19), this correction is generated by nonequilibrium quasiparticles and the superconductivity-induced changes of the diffusion coefficient $\mathcal{D}_T^{(k)} - 4D_k$ as compared to the normal state which has $\mathcal{D}_T^{(k)} = 4D_k$. In contrast to σ_{fl}, the quasiparticle contribution σ_{st} can be calculated using the static order parameter distribution.

A. Conductivity correction σ_{fl}

To calculate σ_{fl} given by (18), we need to find corrections to the order parameter fields Δ_k in a moving Abrikosov lattice. In single-band superconductors, such corrections were calculated in Refs. [12,14,15]. The analogous problem in multiband superconductors cannot be approached using a straightforward generalization of the single-band solution due to the complex structure of vortices in multiband superconductors, which are composite objects consisting of several overlapping fractional vortices in different bands.

To calculate the structure of the moving vortex lattice in a two-band superconductor, let us consider the linear integral-differential system of linearized nonstationary Usadel equations together with self-consistency equation (5) for the order parameter,

$$\frac{D_k}{2}(\nabla - 2ie\boldsymbol{A})^2 f_k^{R,A} \pm i\varepsilon f_k^{R,A} = i\Delta_k, \quad (20)$$

$$\Delta_k = \sum_j \frac{\lambda_{kj}}{4}\int_{-\infty}^{\infty} d\varepsilon\left[f_j^R - f_j^A + \frac{i}{2}\frac{\partial^2}{\partial t\partial\varepsilon}\big(f_j^R + f_j^A\big)\right] \times f_0(\varepsilon). \quad (21)$$

To derive the self-consistency Eq. (21), we substituted the Keldysh component expansion $\hat{g}_k^K = (\hat{g}_k^R - \hat{g}_k^A)f_0 - \frac{i}{2}(\hat{g}_k^R + \hat{g}_k^A)\partial_\varepsilon f_0$ and integrated the second term by parts. The vector potential in Eq. (20) describes a uniform magnetic field $\boldsymbol{B} = H_{c2}\boldsymbol{z}$ and a uniform electric field in the $\boldsymbol{x}$ direction so that $\boldsymbol{A} = \boldsymbol{y}H_{c2}x - \boldsymbol{x}Et$. It is more convenient for calculations to remove a nonstationary part of the vector potential by a gauge transform introducing scalar potential $\Phi = -Ex$. Then the time derivative in Eq. (21) elongates, $\partial_t \to \partial_t + 2ie\Phi$.

A periodic vortex lattice moving with the constant velocity $\boldsymbol{v}_L = v_L \boldsymbol{y}$ is described by the following solution of Eqs. (20) and (21):

$$\Delta_k = \sum C_n e^{inp(y - v_L t)} \tilde{\Delta}_k(x - nx_0), \qquad (22)$$

$$f_k^{R,A} = \sum C_n e^{inp(y - v_L t)} \tilde{f}_k^{R,A}(x - nx_0), \qquad (23)$$

where $|C_n| = 1$, $x_0 = p/(2eH_{c2})$ and the parameter p is determined by the lattice geometry. The vortex velocity should satisfy $v_L = -E/H_{c2}$ in order for the solution to keep magnetic translation symmetry in the x direction. Substituting ansatz (22) into Eq. (20), we get

$$\frac{D_k}{2} \hat{L}_x f_k^{R,A} \pm i\varepsilon f_k^{R,A} = i\Delta_k, \qquad (24)$$

where $\hat{L}_x = \partial_x^2 - (2eH_{c2})^2 x^2$. One can see that the principal difference with a single component is due to the different diffusion constants which do not allow the solution to have a form of shifted zero Landau level (LL) eigenfunction. Instead, we should search it as a superposition of

$$\tilde{f}_k^{R,A} = a_{k0}^{R,A} \Psi_0(x) + a_{k1}^{R,A} \Psi_1(x), \qquad (25)$$

$$\tilde{\Delta}_k = b_{k0} \Psi_0(x) + b_{k1} \Psi_1(x), \qquad (26)$$

where $\Psi_0(x) = \exp(-x^2/2L_H^2)$ and $\Psi_1(x) = x\Psi_0(x)$ satisfy $\hat{L}_x \Psi_0 = -\Psi_0/L_H^2$ and $\hat{L}_x \Psi_1 = -3\Psi_1/L_H^2$. Here $L_H = 1/\sqrt{2eH_{c2}}$. Since the admixture of the first LL is proportional to a small parameter E/H_{c2}, we can determine the coefficients a_0, b_0 using a stationary equation (20),

$$a_{k0}^{R,A} = b_{k0}/(iq_k \pm \varepsilon), \qquad (27)$$

$$a_{k1}^{R,A} = b_{k1}/(3iq_k \pm \varepsilon), \qquad (28)$$

where $q_k = eH_{c2} D_k$. Substituting the relation (27) to the self-consistency Eq. (21) yields a homogeneous linear equation,

$$\hat{A} \boldsymbol{b}_0 = 0, \qquad (29)$$

$$\hat{A} = \hat{\Lambda}^{-1} - \tau_0[G_0 - \ln(t) + \psi(1/2)] + \psi(1/2 + \hat{\rho}), \qquad (30)$$

where G_0 is the minimal positive eigenvalue of the inverse coupling matrix, $t = T/T_c$, $(\hat{\rho})_{ik} = \delta_{ik}\rho_k$, and $\rho_k = q_k/2\pi T$. The solvability condition $\det\hat{A} = 0$ determines the second critical field of a multiband superconductor. The amplitudes b_{k1} of the first LL admixture are determined substituting Eq. (28) into the self-consistency Eq. (21),

$$\boldsymbol{b}_1 = \frac{ieE}{2\pi T} \hat{A}_1^{-1} \psi'(1/2 + \hat{\rho}) \boldsymbol{b}_0,$$
$$\hat{A}_1 = \hat{A} + \psi(1/2 + 3\hat{\rho}) - \psi(1/2 + \hat{\rho}). \qquad (31)$$

The above relation between components of vectors $\boldsymbol{b}_1$ and $\boldsymbol{b}_0$ characterizes the splitting of the composite vortex into separate constituents. Generally, splitting is present for any nondegenerate multiband superconductor having different diffusivities and coupling constants in the bands. By increasing the strength of the electric field, distortion of the vortex lattice becomes more evident; see Fig. 1.

The moving lattice distortions are induced by the first LL admixture in Eq. (31), which generates a finite net current perpendicular to the vortex velocity. From Eq. (18), we obtain

$$\sigma_{fl} = \sum_k \frac{\langle|\Delta_k|^2\rangle}{4\pi T e E} \frac{\sigma_k}{\rho_k} \frac{\mathrm{Im}(b_{k0}^* b_{k1})}{|b_{k0}|^2} [\psi(1/2 + 3\rho_k) - \psi_k], \qquad (32)$$

where $\psi_k = \psi(1/2 + \rho_k)$ and the average order parameter amplitude is given by $\langle|\Delta_k|^2\rangle = \sqrt{\pi}|b_{k0}|^2 L_H/x_0$.

B. Conductivity correction σ_{st}

To calculate the second-term contribution in (6) giving the conductivity correction σ_{st} (19), we need to find out how the diffusion coefficients $\mathcal{D}_T^{(k)}$ are modulated by the vortex lattice. For this, we determine spectral functions $\hat{g}_k^{R,A}$ using the linearized Usadel Eq. (20) supplemented by the normalization condition $(\hat{g}_k^{R,A})^2 = 1$:

$$\hat{g}_k^R = \left[1 + \frac{|\Delta_k|^2}{2(iq_k + \varepsilon)^2}\right]\tau_3 + \frac{i|\Delta_k|\tau_2 e^{-i\varphi_k \tau_3}}{iq_k + \varepsilon}, \qquad (33)$$

$$\hat{g}_k^A = -\left[1 + \frac{|\Delta_k|^2}{2(iq_k - \varepsilon)^2}\right]\tau_3 + \frac{i|\Delta_k|\tau_2 e^{-i\varphi_k \tau_3}}{iq_k - \varepsilon}. \qquad (34)$$

Substituting these expressions into Eq. (14), we get

$$\mathcal{D}_T^{(k)} = 2D_k\left[1 + \frac{|\Delta_k|^2}{2q_k(q_k + i\varepsilon)} - \frac{|\Delta_k|^2}{2(q_k + i\varepsilon)^2} + \text{c.c.}\right]. \qquad (35)$$

Using Eq. (35), we evaluate the conductivity correction (19) as follows:

$$\sigma_{st} = \sum_k \frac{\sigma_k \langle|\Delta_k|^2\rangle}{8\pi^2 T^2}\left(\frac{\psi_k'}{\rho_k} + \psi_k''\right), \qquad (36)$$

where $\psi_k^{(n)} = \psi^{(n)}(1/2 + \rho_k)$ and the partial conductivities are $\sigma_k = 2e^2 \nu_k D_k$. One can see that the quasiparticle current (36) is given by the superposition of two single-band contributions.

C. Slope of the flux-flow resistivity at $H = H_{c2}(T)$

We have found that both the conductivity corrections σ_{fl} and σ_{st}, Eqs. (32) and (36), respectively, are proportional to the average order parameter $\langle|\Delta_k|^2\rangle$, which should be expressed through the magnetic field. The average gap functions $\langle|\Delta_k|^2\rangle = \Delta^2 a_k^2$ have a common amplitude which have been calculated in Ref. [24],

$$\Delta = \left(\frac{eT\delta B}{2\beta_L} \frac{\sum_k \nu_k a_k^2 D_k \psi_k'}{\sum_k \nu_k a_k^4 \sigma_k D_k \psi_k'^2 \tilde{\kappa}_k^2}\right)^{1/2}, \qquad (37)$$

where $\delta B = H_{c2} - B$ and β_L is an Abrikosov parameter equal to 1.16 for a triangular lattice [37]. The parameters $\tilde{\kappa}_k$, which in the single-band case characterize the magnetization slope at $H_{c2}(T)$ [38,39], are given by

$$\tilde{\kappa}_k = \left(\frac{-\psi_k''}{16\pi\sigma_k D_k \psi_k'^2}\right)^{1/2}. \qquad (38)$$

The coefficients a_k are determined unambiguously by Eq. (29) supplemented by a normalization condition $\sum_k a_k^2 = 1$, so that $a_k = |b_{k0}|/\sqrt{\sum_k |b_{k0}|^2}$.

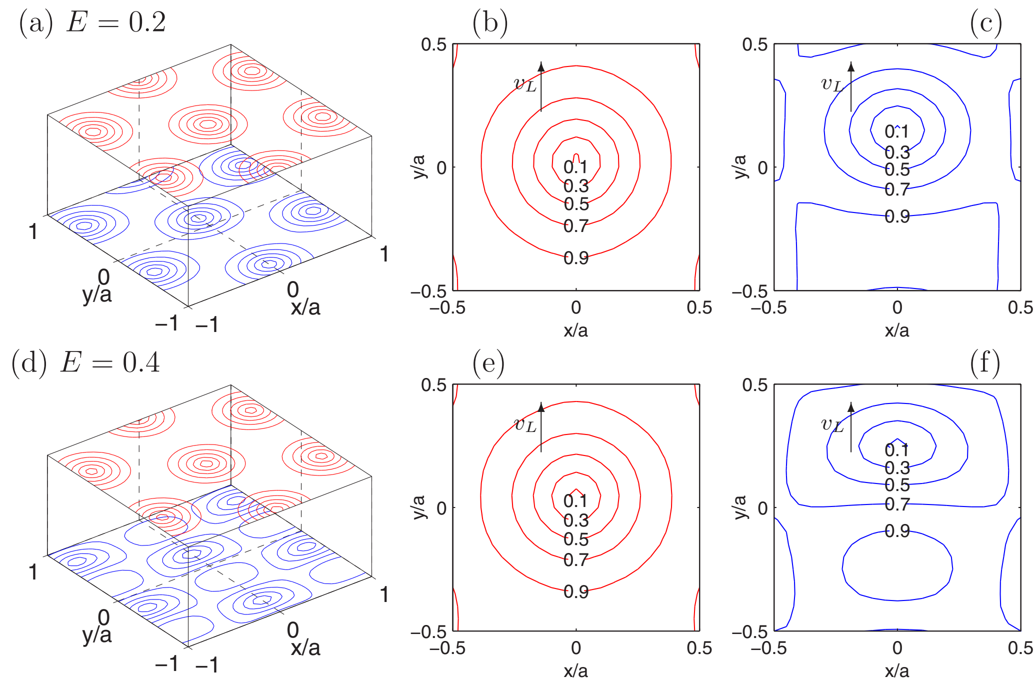

FIG. 1. Distorted lattices of fractional vortices moving along the y direction in a two-band superconductor MgB_2 with $D_2 = 0.05D_1$ and the pairing constants mentioned in the text. The temperature is $T = 0.1T_c$. The transport current is applied along x. The magnitude of the electric field is normalized by $T_c/(e\xi_1)$, where $\xi_1 = \sqrt{D_1/T_c}$. The length is normalized to the triangular lattice spacing $a = 2L_H(\pi/\sqrt{3})^{1/2}$. (a),(d) Density levels of Δ_1 (Δ_2) in upper (lower) plots for $E = 0.2,0.4$. Detailed density levels of Δ_1, Δ_2 are shown for (b),(d) $E = 0.2$ and (e),(f) $E = 0.4$. (b),(c),(e),(f) The vortex velocity direction $\boldsymbol{v}_L$ is shown by arrows.

Substituting the order parameter amplitude (37) to the expressions for conductivity corrections (18) and (19), we can find the flux-flow conductivity slope at $B = H_{c2}(T)$ (2), which can by written in the form $S = -(H_{c2}/\sigma_n)d\sigma_f/dB$. In contrast to the dirty single-band superconductors which are characterized by a universal $S = S(T)$ curve, the multiband superconductors have a significant variation of S as a function of the ratio between band diffusivities D_1/D_2. The sequences of $S(T)$ dependencies for different values of D_1/D_2 are shown in Fig. 2 for the two-band superconductor with pairing constants corresponding to the weak-coupling model of MgB_2 [40]. For reference, the universal single-band curve is shown by the dashed line in Fig. 2(a).

By applying our model at elevated temperatures, we neglect interband impurity scattering assuming that it is much smaller compared to the orbital depairing energy eD_kH_{c2}. For the same reason, we omit scattering at paramagnetic impurities and inelastic electron-phonon relaxation [17] which are known to be important near T_c but are negligible at lower temperatures.

1. Limiting values of S at temperatures close to T_c

Qualitatively, the significant deviations of $S(T)$ dependencies from the single-band case can be understood by analyzing the limiting case when $1 - T/T_c \ll 1$ but the field is sufficiently close to the critical one so that $1 - T/T_c \gg 1 - B/H_{c2}$. In this case, B is small so that $\rho_k \to 0$ and one can use the asymptotic values of functions $\psi'_k = \pi^2/2$, $\psi''_k = -14\zeta(3)$. As a result, the splitting of fractional vortex sublattices vanishes. As can be seen from Eqs. (31), to the first order by ρ_k, we have

$$\boldsymbol{b}_1 = \frac{ieE}{4\pi T}\frac{\mathrm{Tr}\hat{A}}{\sum_k A_{kk}\rho_k}\boldsymbol{b}_0. \quad (39)$$

This expression means that current-driven fractional vortices in different bands shift by the same amount.

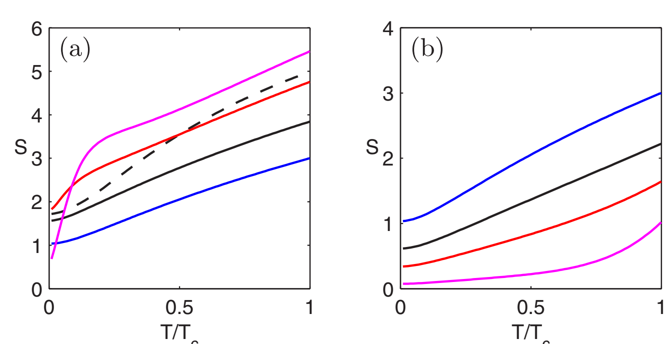

FIG. 2. Slope of the flux-flow resistivity $S = -(H/\sigma_n)d\sigma_f/dB$ at $B = H_{c2}(T)$ as a function of temperature T for different values of the ratio between band diffusivities $d = D_2/D_1$. (a) Solid curves from top to bottom: $d = 0.05, 0.2, 0.5, 1$. The dashed line shows a universal single-band behavior [13,14] obtained for the stronger-superconductivity band in the limit when interband coupling vanishes. (b) Curves from top to bottom: $d = 1, 2, 4, 20$. The pairing parameters are $\lambda_{11} = 0.101$, $\lambda_{22} = 0.045$, $\lambda_{12} = 0.034$, $\lambda_{21} = 0.026$ [40].

The conductivity corrections are given by

$$\sigma_{fl} = \frac{\mathrm{Tr}\hat{A}}{\sum_k A_{kk}\rho_k} \sum_k \frac{\sigma_k \langle |\Delta_k|^2 \rangle}{16T^2}, \quad (40)$$

$$\sigma_{st} = \sum_k \frac{\sigma_k \langle |\Delta_k|^2 \rangle}{16T^2 \rho_k}. \quad (41)$$

One can see that in contrast to the single-band case [13], these contributions are not equal if the coupling constants are not degenerate, $\lambda_{11} \neq \lambda_{22}$. From Eqs. (37) and (38), we obtain the conductivity slope at $T = T_c$,

$$S = S_c \frac{\sum_k \nu_k a_k^2 D_k}{2\sum_k \nu_k a_k^4} \sum_k \frac{\sigma_k a_k^2}{\sigma_n} \left(\frac{1}{D_k} + \frac{\mathrm{Tr}\hat{A}}{\sum_j A_{jj} D_j} \right), \quad (42)$$

where $S_c = \pi^4/(14\zeta(3)\beta_L) \approx 5$ is the universal value of $S(T = T_c)$ in the single-component case. Let us consider a two-band system and assume that $\lambda_{11} > \lambda_{22}$ and $\lambda_{12} \ll \lambda_{11} - \lambda_{12}$, which qualitatively corresponds to the pairing in MgB_2. Then, the limiting cases of Eq. (42) are as follows:

$$S = \left(1 + \frac{A_{22}}{2A_{11}}\right) S_c \quad \text{for} \quad D_1 \gg D_2, \quad (43)$$

$$S = \frac{\lambda_{21}^2 S_c}{2(\lambda_{11} - \lambda_{22})^2} \quad \text{for} \quad D_2 \gg D_1. \quad (44)$$

These expressions are in good agreement with the behavior of the curves $S(T)$ for MgB_2. As shown in Fig. 2(a), in the limiting case $D_1 \gg D_2$ (the magenta uppermost curve), the value of $S(T_c)$ is a bit larger than for the single-band case, exactly as described by Eq. (43) because in this case $A_{22}/A_{11} \approx (\lambda_{11} - \lambda_{22})^2/(\lambda_{12}\lambda_{21})$. In the opposite case $D_2 \gg D_1$, shown in Fig. 2(b) (magenta lowermost curve), the value $S(T_c) \ll S_c$ as given by Eq. (44). Quite amazingly, the deviations of $S(T)$ from the single-band case are significant even if one of the diffusivities dominates, which means that in the normal state, the current flows mostly in one of the bands. At the same time, the superconducting corrections σ_{st} and σ_{fl} are strongly renormalized by multiband effects, even in the limiting cases of strong disparity between the diffusivities.

2. *Limiting values of S: The case of decoupled bands*

To understand the qualitative features of the flux flow at high fields, it is instructive to consider the case of a superconductor with two decoupled bands characterized by different critical temperatures $T_{c1,2}$. In this case, superconductivity at high fields survives only in one of the bands which has the highest critical field, $H_{c2} = \max H_{c2}^{(k)}$. Correspondingly, the resistivity slope calculated for this particular band coincides with the universal single-band result [13]. However, even in this case, the overall S is still modified by multiband effects. Indeed, its definition (2) contains the total normal-state conductivity determined by the contribution of all bands, including nonsuperconducting ones.

Let us consider the analytically tractable low-temperature limit when the single-band critical field is given by $H_{c2}^{(k)} \propto T_{ck}/(eD_k)$ [41–43]. In this case, we obtain $S = S_0\sigma_k/(\sigma_1 + \sigma_2)$, where k is the component with larger critical field and $S_0 = 2/\beta_L \approx 1.72$ is the universal low-temperature limit

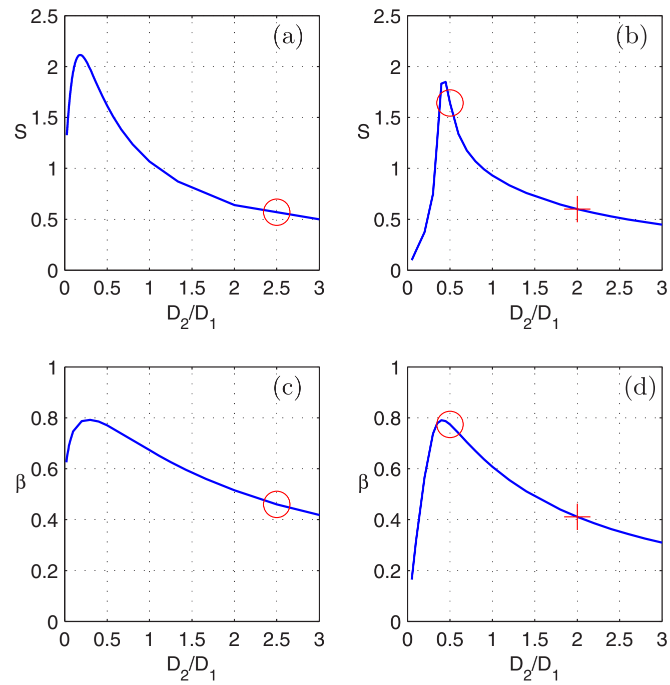

FIG. 3. The flux-flow resistivity slope S at high fields (2) and the inverse slope β at low fields (1) in a two-band superconductor as functions of the diffusivity ratio D_2/D_1. The temperature is $T = 0.05T_c$. The panels correspond to the models of (a),(c) MgB_2 [40] and (b),(d) V_3Si [45] with the pairing constants mentioned in the text. Open circles in (a),(c) mark the parameters used to fit the experimental curve for MgB_2 in Fig. 4(a). Open circles and crosses in (b),(d) show the parameters used to extrapolate magnetoresistance curves for V_3Si in Fig. 4(b).

$S(T = 0)$ in the single-component case. It is instructive to consider asymptotic behavior of S as a function of the diffusivity ratio $d = D_2/D_1$. When $d < T_{c2}/T_{c1}$, we have $S = S_0\nu_2 d/(\nu_1 + \nu_2 d)$, and we have $S = S_0\nu_1/(\nu_1 + \nu_2 d)$ in the opposite case when $d > T_{c2}/T_{c1}$. For noninteracting bands, the transition between these regimes is abrupt, resulting in the jump on $S(d)$ dependence at $d = T_{c2}/T_{c1}$. The maximal value of S which can be obtained does not exceed S_0.

The origin of a nonmonotonic $S(d)$ dependence is determined by the behavior of H_{c2} in multiband systems. At small d, the critical field is determined by the second band which has the smallest diffusivity, $H_{c2} = H_{c2}^{(2)}$. Then, with increasing d, the superconductivity changes the host band so that $H_{c2} = H_{c2}^{(1)}$. This transition is an abrupt one for noninteracting bands, but a finite interaction makes it the gradual one, washing out the cusp singularity. However, coupling does not eliminate the nonmonotonicity and the asymptotic behavior of $S(d)$ remains the same, as shown in Fig. 3.

IV. SMALL MAGNETIC FIELDS $B \ll H_{c2}$ AND LOW TEMPERATURES $T \ll T_c$

A. General formalism

In dilute vortex configurations at temperatures much below T_c, the sizable quasiparticle density exists only inside vortex cores where the superconducting order parameter is suppressed. In this regime, the deviations from equilibrium in each band are localized in vortex cores and are significant only at energies much smaller than the bulk energy gaps. Following Kopnin-Gor'kov theory [9], spectral functions $\hat{g}_k^{R,A}$ can be parametrized at $\varepsilon = 0$ as follows:

$$\hat{g}_k^R = \tau_3 \cos\theta_k + \tau_2 \sin\theta_k,$$
$$\hat{g}_k^A = -\tau_3 \cos\theta_k + \tau_2 \sin\theta_k. \quad (45)$$

Here we assume that the order parameter vortex phase is removed by gauge transformation. The distribution function can be written in the form

$$f_T^{(k)} = \tilde{f}_T^{(k)} v_L \partial_\varepsilon f_0 \sin\varphi, \quad (46)$$

where φ is a polar angle with respect to the vortex center. The amplitude $\tilde{f}_T^{(k)}$ is a function of the radial coordinate determined by the following kinetic equation:

$$\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{1}{r^2}\right)\tilde{f}_T^{(k)} = \frac{\Delta_k \sin\theta_k}{D_k}\left(2\tilde{f}_T^{(k)} - \frac{1}{r}\right), \quad (47)$$

with boundary conditions $\tilde{f}_T^{(k)}(r = 0,\infty) = 0$. The detailed derivation of Eq. (47) is given in the Appendix.

The viscous friction force acting on individual moving vortices can be written as $\boldsymbol{F}_{env} = -\eta \boldsymbol{v}_L$. The viscosity coefficient η can be calculated substituting spectral functions (45) and the distribution function (46) into the expansion (11) and the general expression for the force (10). In this way, we obtain

$$\eta = \sum_k \pi\hbar\nu_k(\alpha_k + \gamma_k),$$
$$\alpha_k = \int_0^\infty dr r \frac{\partial\Delta_k}{\partial r}\frac{\partial \sin\theta_k}{\partial r},$$
$$\gamma_k = \int_0^\infty dr \Delta_k \sin\theta_k\left(\frac{1}{r} - 2\tilde{f}_T^{(k)}\right). \quad (48)$$

To calculate the gap profiles and spectral functions, we use a stationary self-consistency equation written in the form

$$\Delta_i(\mathbf{r}) = \sum_{k=1}^N \lambda_{ik}\left[\Delta_k G_0 + 2\pi T \sum_{n=0}^\infty \left(\sin\theta_k^M - \frac{\Delta_k}{\omega_n}\right)\right], \quad (49)$$

where $G_0 = (\mathrm{Tr}\hat{\Lambda} - \sqrt{\mathrm{Tr}\hat{\Lambda}^2 - 4\mathrm{Det}\hat{\Lambda}})/(2\mathrm{Det}\hat{\Lambda}) - \ln(t)$ and $\hat{\Lambda} = \lambda_{ij}$ is the coupling matrix. In Eq. (49), the summation runs over Matsubara frequencies $\omega_n = (2n+1)\pi T$. The angle θ_k^M parametrizes imaginary-frequency Green's functions similar to Eqs. (45). It is determined by the Usadel equation,

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\theta_k^M}{dr}\right) - \frac{\sin\left(2\theta_k^M\right)}{2r^2} + \frac{2\Delta_k}{D_k}\cos\theta_k^M - \frac{2\omega}{D_k}\sin\theta_k^M = 0, \quad (50)$$

supplemented by the boundary conditions

$$\theta_k^M(r=0) = 0,$$
$$\theta_k^M(r=\infty) = \sin^{-1}\left[\Delta_k/\left(\Delta_k^2 + \omega^2\right)^{1/2}\right]. \quad (51)$$

One should put $\omega = \omega_n$ to obtain solutions at the specific Matsubara frequency. The angle θ_k parametrizing zero-energy spectral functions (45) is given by the same Eqs. (50) and (51) with $\omega = 0$.

In general, the flux-flow conductivity can be expressed through the vortex viscosity (48) as follows [9]:

$$\sigma_f = \eta/(B\phi_0), \quad (52)$$

where B is the average magnetic induction and ϕ_0 is a magnetic flux quantum. Introducing normal-state Drude conductivity $\sigma_n = \sum_k \sigma_k$, we rewrite flux-flow conductivity (52) in the form

$$\sigma_f = \beta\sigma_n H_{c2}/B, \quad (53)$$

$$\beta = \frac{1}{2eH_{c2}}\frac{\sum_k \nu_k(\alpha_k + \gamma_k)}{\sum_k \nu_k D_k}. \quad (54)$$

In Sec. V, we analyze parameter β for several known multiband superconducting compounds.

B. Limiting values of β: The case of decoupled bands

In multiband superconductors, the coefficient β can change a lot depending on the ratio of the diffusion constants and pairing potentials in different bands. Below we investigate the maximal accessible values and the asymptotic behavior of β in superconductors with decoupled bands characterized by different critical temperatures T_{ck}. In this case, one can adopt single-band value β_0 calculated in Ref. [9] to obtain

$$\beta = \beta_0 \min\left(\frac{D_k}{T_{ck}}\right)\frac{\nu_1 T_{c1} + \nu_2 T_{c2}}{\nu_1 D_1 + \nu_2 D_2}. \quad (55)$$

Here we have used the same single-band expression for $H_{c2} = \max H_{c2}^{(k)}$ as in Sec. III C 2.

It is instructive to consider the asymptotic behavior of β as a function of the diffusivity ratio $d = D_2/D_1$. When $d < T_{c2}/T_{c1}$, we have $\beta = \beta_0 d(\nu_2 + \nu_1 T_{c1}/T_{c2})/(\nu_1 + \nu_2 d)$, and $\beta = \beta_0(\nu_1 + \nu_2 T_{c2}/T_{c1})/(\nu_1 + \nu_2 d)$ in the opposite case when $d > T_{c2}/T_{c1}$. For noninteracting bands, the transition between these regimes is abrupt, resulting in the sharp maximum $\beta = \beta_0$ with a cusp at $d = T_{c2}/T_{c1}$. The transition results from switching of the superconductivity at H_{c2} between different bands. If there is a finite interband coupling, the cusp in the behavior of β changes to a smooth maximum, but the maximal value cannot be remarkably enhanced. As a result, parameter β in the two-band scenario appears to be always limited by its single-band value.

V. EXAMPLES AND COMPARISON WITH EXPERIMENTS

Having in hand general results, we can calculate the flux-flow resistivity in particular for multiband superconducting compounds. For that we choose MgB_2 and V_3Si, which have been described by the two-band weak-coupling models [25,40,44,45]. Moreover, these compounds can have a rather large impurity scattering rate to fit the dirty-limit conditions [25,44].

Basically, the only input parameters needed to calculate the flux-flow resistivity are the pairing constants which we choose as follows: (i) MgB_2 with $\lambda_{11} = 0.101$, $\lambda_{22} = 0.045$, $\lambda_{12} = 0.034$, $\lambda_{21} = 0.026$ [40] and (ii) V_3Si with $\lambda_{11} = 0.26$, $\lambda_{22} = 0.205$, $\lambda_{12} = \lambda_{21} = 0.0088$ [45]. These values are consistent with the weak-coupling superconductivity model. They were obtained by fitting the temperature dependencies of superfluid density and specific heat for MgB_2 [40] and the microwave response for V_3Si [45]. Note that the parameters of V_3Si correspond to the case of weakly interacting superconducting bands since the interband pairing is much weaker than the intraband one, $\lambda_{12} \ll \lambda_{11},\lambda_{22}$. In that sense, it is drastically different from the model of MgB_2, where λ_{12} has the same order of magnitude as λ_{11} and λ_{22}.

For such parameters, we apply the results of Secs. III C and IV to find the dependencies $S(d)$ and $\beta(d)$, where $d = D_2/D_1$ is the ratio of diffusivities in the two bands. The results are shown in Fig. 3. On can see that the dependencies are qualitatively similar for the two sets of pairing constants. The nonmonotonicity and asymptotic behavior of both S and β were explained in Secs. III C 2 and IV B using a model of noninteracting bands. As was discussed above, the origin of a nonmonotonic behavior is determined by the multiband effects in the near-H_{c2} physics. In that regime, by increasing the ratio D_2/D_1, one makes the superconductivity change the host band. This directly affects the resistive states at high fields (i.e., the S parameter), but also indirectly the low-field parameter β (1) because there the magnetic field dependence is normalized by H_{c2}.

With the help of calculated parameters S and β, we can reconstruct by extrapolation the flux-flow resistivity curve for the entire range of magnetic field and compare it with the experimental results. In Fig. 4(a), we show by dashed lines the slopes that give the best fit of the approximated flux-flow resistivity curve for MgB_2 at low temperatures $T \ll T_c$, adopted from Ref. [6]. The slopes were calculated using the two-band model for MgB_2 described above. The fitting parameter was the ratio of diffusivities chosen to be $D_2/D_1 = 2.5$, marked in Figs. 3(a) and 3(c) by open circles.

To understand the possible variations in the shape of the curve $\rho_f(B)$, we consider the model corresponding to V_3Si and consider two characteristic values of $D_2/D_1 = 2$ and 0.5. For such parameters, the values of S and β are shown

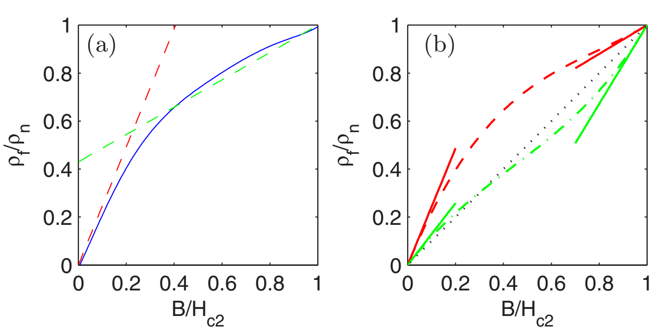

FIG. 4. (a) Solid line: the approximate flux-flow resistivity as a function of magnetic induction B adopted from the experimental curve for MgB_2 [6]. Dashed lines show theoretical slopes $\beta = 0.406$ and $S = 0.57$ corresponding to $D_2/D_1 = 2.5$ for the same model as in Figs. 3(a) and 3(c), where these slopes are shown by circles. (b) The slopes of resistivity corresponding to $D_2/D_2 = 0.5$ (dash-dotted line) and $D_2/D_1 = 2$ (dashed line) for the same model of V_3Si as in Figs. 3(b) and 3(d). The solid curves show calculated slopes. The dashed and dash-dotted lines show cubic interpolation between the low- and high-field regimes. The temperature is $T = 0.05T_c$ in both panels.

by open circles and crosses in Figs. 3(b) and 3(d). One can see that one of these points is in the regime qualitatively similar to the one considered above for MgB_2. Indeed, the cubic extrapolation of the $\rho_f(B)$ dependence shown by a red dashed curve is qualitatively similar to the approximated experimental curve for MgB_2; see Fig. 4. On the other hand, the point $D_2/D_1 = 0.5$ belongs to the region where $S > 1$, which results in a different behavior, shown in Fig. 4(b) with green color. Experimental data for V_3Si [46] demonstrate the flux-flow resistivity curve between the two considered cases; however, more measurements are needed to cover the whole range of magnetic fields. Note that the green dash-dotted curve in Fig. 4(b) is quite close to the usual BS linear dependence (black dotted curve), although it deflects slightly, changing its shape from the concave at small fields to the convex one at large fields. By slightly varying the ratio D_2/D_1 around 0.5, one can achieve a better approach to the BS line. At the same time, since β does not much exceed its single-band limit value, it is impossible to get a convex curve already at small fields since that would require $\beta > 1$ which we did not obtain for the models considered in the present work.

VI. SUMMARY

To summarize, we have developed a theoretical framework to study nonequilibrium processes in multiband superconductors and applied it to calculate flux-flow resistivity of such systems in the dirty limit with a high concentration of nonmagnetic impurities. We have considered both the regions of high and low magnetic fields. To calculate the conductivity in the former case, we have derived the solution characterizing moving vortex lattices, which reveals the effect of splitting into sublattices of fractional vortices. The maximal value of the flux-flow resistivity slope S at high fields is shown to be close to the universal single-band limit. At the same time, the minimal value can be arbitrarily small, proportional to the disparity of diffusivities in different bands $S \propto \min(D_{1,2})/\max(D_{1,2})$.

We calculated the parameter β which is the inverse slope of the flux-flow magnetoresistance curve at low magnetic field. For different models of multiband superconductors, we have found that the maximal possible value of β is close to the universal single-band constant β_0 found by Kopnin and Gor'kov [47]. For large disparity of diffusivities, it has the asymptotic behavior $\beta \propto \min(D_{1,2})/\max(D_{1,2})$, which is similar to that of parameter S.

We demonstrated that multiband superconductors exhibit an unconventional generic regime which is characterized by small values of parameters β and S and corresponds to the concave flux-flow magnetoresistance curves $\rho_f(B)$. Several recent experiments [1–6] confirm that behavior. For MgB_2, we have obtained a quantitative agreement with experimental results [6], choosing the ratio of diffusivities in two bands, $D_2/D_1 = 2.5$. At the same time, we have shown that by varying the parameter D_2/D_1, it is possible to obtain regimes when the curve $\rho_f(B)$ is quite close to the single-band Bardeen-Stephen law. Therefore, the suggested theory naturally explains quite diverse experimental data on the flux-flow resistivity in different multiband superconducting compounds.

ACKNOWLEDGMENTS

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APPENDIX : DERIVATION OF KINETIC EQUATIONS AND FORCES ACTING ON THE MOVING VORTEX LINE

1. General formalism

The quasiclassical GF matrix in band k is defined as

$$\check{g}_k = \begin{pmatrix} \hat{g}_k^R & \hat{g}_k^K \\ 0 & \hat{g}_k^A \end{pmatrix}, \tag{A1}$$

where g_k^K is the $(2\times 2$ matrix) Keldysh component and $\hat{g}_k^{R(A)}$ is the retarded (advanced) GF. In a diffusive superconducting wire with band diffusion constants D_k, the matrix $\check{g}_k$ obeys the Usadel equation,

$$\{\tau_3\partial_t,\check{g}_k\}_t = D_k\hat{\partial}_{\mathbf{r}}(\check{g}_k\circ\hat{\partial}_{\mathbf{r}}\check{g}_k) + [\hat{H}_k,\check{g}_k]_t - i\big[\check{\Sigma}_k^{ph},\check{g}_k\big]_t, \tag{A2}$$

where $\hat{H}_k(\boldsymbol{r},t) = i\hat{\Delta}_k$ and $\hat{\Delta}_k(t) = |\Delta_k|\sigma_3\tau_3\tau_1 e^{-i\varphi_k\tau_3}$ is the gap operator and φ_k is the gap phase. It is convenient to remove the spin dependence of the gap by transformation $\check{g} = \check{U}\check{g}^{\rm new}\check{U}^+$, where

$$\check{U} = \exp[i\pi(\sigma_3\tau_3 - \sigma_0\tau_3 - \sigma_3\tau_0)/4], \tag{A3}$$

which leads to

$$\check{\Delta}_k^{\rm new} = \check{U}^+\check{\Delta}_k\check{U} = i|\Delta_k|\tau_2 e^{-i\varphi_k\tau_3}, \tag{A4}$$

so that $\hat{H}_k(\boldsymbol{r},t) = i\hat{\Delta}_k^{\rm new}$. Note that we use, from the beginning, the temporal gauge where the scalar potential is zero $\Phi = 0$, with an additional constraint that in equilibrium the vector potential is time independent and satisfies $\nabla\cdot\boldsymbol{A} = 0$. Throughout the derivation, we assume $k_B = \hbar = c = 1$.

The covariant differential superoperator in Eq. (A2) is given by

$$\hat{\partial}_{\mathbf{r}}\hat{g}_k = \nabla\hat{g}_k - ie[\tau_3\boldsymbol{A}.\hat{g}_k]_t.$$

Here the commutator operator is defined as $[X,g]_t = X(t_1)g(t_1,t_2) - g(t_1,t_2)X(t_2)$, similarly for anticommutator $\{X,g\}_t$. We also introduce the symbolic product operator $(A\circ B)(t_1,t_2) = \int dt A(t_1,t)B(t,t_2)$. Equation (A2) is complemented by the normalization condition $(\check{g}_k\circ\check{g}_k)(t_1,t_2) = \delta(t_1-t_2)$, which allows one to introduce parametrization of the Keldysh component in terms of the distribution function,

$$\hat{g}_k^K = \hat{g}_k^R\circ\hat{f}^{(k)} - \hat{f}^{(k)}\circ\hat{g}_k^A, \tag{A5}$$

$$\hat{f}^{(k)} = f_L^{(k)}\tau_0 + f_T^{(k)}\tau_3. \tag{A6}$$

Here we will neglect the electron-phonon relaxation given by the last term in Eq. (A2). Such approximation is valid provided the temperature is not too close to T_c. In this case, the components of the Keldysh-Usadel given by Eq. (A2) read as

$$\big\{\tau_3\partial_t,\hat{g}_k^{R(A)}\big\}_t = D_k\hat{\partial}_{\mathbf{r}}\big(\hat{g}_k^{R(A)}\circ\hat{\partial}_{\mathbf{r}}\hat{g}_k^{R(A)}\big) + \big[\hat{H}_k,\hat{g}_k^{R(A)}\big]_t,$$
$$\big\{\tau_3\partial_t,\hat{g}_k^K\big\}_t = D_k\hat{\partial}_{\mathbf{r}}\big(\hat{g}_k^R\circ\hat{\partial}_{\mathbf{r}}\hat{g}_k^K + \hat{g}_k^K\circ\hat{\partial}_{\mathbf{r}}\hat{g}_k^A\big) + \big[\hat{H}_k,\hat{g}_k^K\big]_t. \tag{A7}$$

To obtain the kinetic equation, we substitute parametrization (8) to write

$$\begin{aligned}\hat{\partial}_{\mathbf{r}}(\check{g}_k\circ\hat{\partial}_{\mathbf{r}}\check{g}_k)^K &= \hat{\partial}_{\mathbf{r}}\big(\hat{\partial}_{\mathbf{r}}\hat{f}^{(k)} - \hat{g}_k^R\circ\hat{\partial}_{\mathbf{r}}\hat{f}^{(k)}\circ\hat{g}_k^A\big)\\ &\quad+\hat{g}_k^R\circ\hat{\partial}_{\mathbf{r}}\hat{g}_k^R\circ\hat{\partial}_{\mathbf{r}}\hat{f}^{(k)} - \hat{\partial}_{\mathbf{r}}\hat{f}^{(k)}\circ\hat{g}_k^A\circ\hat{\partial}_{\mathbf{r}}\hat{g}_k^A\\ &\quad+\hat{\partial}_{\mathbf{r}}\big(\hat{g}_k^R\circ\hat{\partial}_{\mathbf{r}}\hat{g}_k^R\big)\circ\hat{f}^{(k)}\\ &\quad-\hat{f}^{(k)}\circ\hat{\partial}_{\mathbf{r}}\big(\hat{g}_k^A\circ\hat{\partial}_{\mathbf{r}}\hat{g}_k^A\big).\end{aligned} \tag{A8}$$

To derive this expression, we used the associative property of differential superoperator $\hat{\partial}_{\mathbf{r}}(g_1\circ g_2) = \hat{\partial}_{\mathbf{r}}g_1\circ g_2 + g_1\circ\hat{\partial}_{\mathbf{r}}g_2$. To get rid of the last two terms, we subtracted the spectral components of Eq. (A2) to obtain finally the equation

$$\begin{aligned}&\hat{g}_k^R\circ(\tau_3\partial_{t'}\hat{f}^{(k)} + \partial_{t_2}\hat{f}^{(k)}\tau_3) - \big(\tau_3\partial_{t_1}\hat{f}^{(k)} + \partial_{t'}\hat{f}^{(k)}\tau_3\big)\circ\hat{g}_k^A\\ &= D_k\hat{\partial}_{\mathbf{r}}\big(\hat{\partial}_{\mathbf{r}}\hat{f}^{(k)} - \hat{g}_k^R\circ\hat{\partial}_{\mathbf{r}}\hat{f}^{(k)}\circ\hat{g}_k^A\big)\\ &\quad+D_k\big(\hat{g}_k^R\circ\hat{\partial}_{\mathbf{r}}\hat{g}_k^R\circ\hat{\partial}_{\mathbf{r}}\hat{f}^{(k)} - \hat{\partial}_{\mathbf{r}}\hat{f}^{(k)}\circ\hat{g}_k^A\circ\hat{\partial}_{\mathbf{r}}\hat{g}_k^A\big)\\ &\quad+\hat{g}_k^R\circ[\hat{H}_k,\hat{f}^{(k)}]_t - \big[\hat{H}_k,\hat{f}^{(k)}\big]_t\circ\hat{g}_k^A,\end{aligned} \tag{A9}$$

where t' is the integration variable.

To proceed, we introduce the mixed representation in the time-energy domain as follows: $g(t_1,t_2) = \int_{-\infty}^{\infty} g(\varepsilon,t)e^{-i\varepsilon(t_1-t_2)}\frac{d\varepsilon}{2\pi}$, where $t = (t_1+t_2)/2$. By keeping the first-order terms in frequency, we get, for Fourier transformations,

$$[\hat{H},\hat{g}]_t = [\hat{H},\hat{g}] - \frac{i}{2}\{\partial_t\hat{H},\partial_\varepsilon\hat{g}\}, \tag{A10}$$

$$[\boldsymbol{A}\tau_3,\hat{g}]_t = \boldsymbol{A}[\tau_3,\hat{g}] - \frac{i}{2}\partial_t\boldsymbol{A}\{\tau_3,\partial_\varepsilon\hat{g}\}, \tag{A11}$$

$$\hat{\partial}_{\mathbf{r}}\hat{f}^{(k)} = \nabla\hat{f}^{(k)} + e\boldsymbol{E}\partial_\varepsilon f_0\tau_3, \tag{A12}$$

where $\boldsymbol{E} = -\partial_t\boldsymbol{A}$ is the electric field in the temporal gauge and $f_0 = \tanh\varepsilon/(2T)$ is the equilibrium distribution. To the first order in frequency and deviation from equilibrium, we also have

$$\begin{aligned}&\hat{\partial}_{\mathbf{r}}\big(\hat{\partial}_{\mathbf{r}}\hat{f}^{(k)} - \hat{g}_k^R\circ\hat{\partial}_{\mathbf{r}}\hat{f}^{(k)}\circ\hat{g}_k^A\big)\\ &= \nabla\big(\nabla f^{(k)} - \hat{g}_k^R\nabla f^{(k)}\hat{g}_k^A\big)\\ &\quad+e\partial_\varepsilon f_0\nabla\cdot\big[\boldsymbol{E}\big(\tau_3 - \hat{g}_k^R\tau_3\hat{g}_k^A\big)\big] + ie\big[\boldsymbol{A}\tau_3,\hat{g}_k^R\nabla\hat{f}^{(k)}\hat{g}_k^A\big]\\ &\quad+ie^2\partial_\varepsilon f_0(\boldsymbol{A}\cdot\boldsymbol{E})\big[\tau_3,\hat{g}_k^R\tau_3\hat{g}_k^A\big].\end{aligned}$$

The last two terms do not contribute to the kinetic equation since they are traced out.

In the mixed representation, the kinetic equation (A9) has the following gauge-invariant form:

$$\begin{aligned}&\hat{g}_k^R\tau_3\partial_t\hat{f}^{(k)} - \tau_3\partial_t\hat{f}^{(k)}\hat{g}_k^A\\ &= D_k\nabla\big(\nabla f^{(k)} - \hat{g}_k^R\nabla f^{(k)}\hat{g}_k^A\big)\\ &\quad+D_k\big(\hat{g}_k^R\hat{\partial}_{\mathbf{r}}\hat{g}_k^R\nabla\hat{f}^{(k)} - \nabla\hat{f}^{(k)}\hat{g}_k^A\hat{\partial}_{\mathbf{r}}\hat{g}_k^A\big) + \hat{g}_k^R[\hat{H}_k,\hat{f}^{(k)}]\\ &\quad-[\hat{H}_k,\hat{f}^{(k)}]\hat{g}_k^A - i\partial_\varepsilon f_0\big(\hat{g}_k^R\partial_t\hat{H}_k - \partial_t\hat{H}_k\hat{g}_k^A\big)\\ &\quad+eD_k\partial_\varepsilon f_0\nabla\cdot\big[\boldsymbol{E}\big(\tau_3 - \hat{g}_k^R\tau_3\hat{g}_k^A\big)\big]\\ &\quad+eD_k\partial_\varepsilon f_0\boldsymbol{E}\cdot\big(\hat{g}_k^R\hat{\partial}_{\mathbf{r}}\hat{g}_k^R\tau_3 - \tau_3\hat{g}_k^A\hat{\partial}_{\mathbf{r}}\hat{g}_k^A\big),\end{aligned} \tag{A13}$$

where we omit the terms which will be traced out later. The last two terms in Eq. (A13) are the sources of disequilibrium.

Multiplying by τ_3 and taking the trace, we obtain

$$\nabla\big(\mathcal{D}_T^{(k)}\nabla f_T^{(k)}\big) + \boldsymbol{j}_e^{(k)}\cdot\nabla f_L^{(k)} + 2i\mathrm{Tr}\big[\big(\hat{g}_k^R + g_k^A\big)\hat{\Delta}_k\big]f_T^{(k)} = \partial_\varepsilon f_0 \mathrm{Tr}\big[\tau_3\partial_t\hat{\Delta}_k\big(\hat{g}_k^R + \hat{g}_k^A\big)\big] - e\partial_\varepsilon f_0\nabla\cdot\big(\mathcal{D}_T^{(k)}\boldsymbol{E}\big), \quad \text{(A14)}$$

where the energy-dependent diffusion coefficients and the spectral charge currents are

$$\mathcal{D}_T^{(k)} = D_k\mathrm{Tr}\big(\tau_0 - \tau_3\hat{g}_k^R\tau_3\hat{g}_k^A\big), \quad \text{(A15)}$$

$$\boldsymbol{j}_e^{(k)} = D_k\mathrm{Tr}\,\big[\tau_3\big(\hat{g}_k^R\hat{\partial}_{\mathbf{r}}\hat{g}_k^R - \hat{g}_k^A\hat{\partial}_{\mathbf{r}}\hat{g}_k^A\big)\big]. \quad \text{(A16)}$$

Analogously, taking just the trace of Eq. (A13), we obtain

$$\nabla\big(\mathcal{D}_L^{(k)}\nabla f_L^{(k)}\big) + \boldsymbol{j}_e^{(k)}\cdot\nabla f_T^{(k)} + 2i\mathrm{Tr}\big[\tau_3\big(\hat{g}_k^R - g_k^A\big)\hat{\Delta}_k\big]f_T^{(k)} = -\partial_\varepsilon f_0\mathrm{Tr}\big[\partial_t\hat{\Delta}_k\big(\hat{g}_k^R - \hat{g}_k^A\big)\big] - e\partial_\varepsilon f_0\big(\boldsymbol{j}_e^{(k)}\cdot\boldsymbol{E}\big), \quad \text{(A17)}$$

where $\mathcal{D}_L^{(k)} = D_k\mathrm{Tr}(\tau_0 - \hat{g}_k^R\hat{g}_k^A)$. Here we took into account that $\mathrm{Tr}(\hat{g}_k^R\tau_3\hat{g}_k^A) = 0$ because of the relation $\hat{g}_k^A = -\tau_3\hat{g}_k^{R+}\tau_3$ and the general form of the equilibrium spectral function, $\hat{g}_k^R = g_3^{(k)}\tau_3 + g_2^{(k)}\tau_2 e^{-i\varphi_k\tau_3}$.

2. The low-temperature limit $T \ll T_c$

At low temperatures, the deviations from equilibrium are localized in the vortex core and are significant only at small energies. Therefore, following Kopnin-Gor'kov theory, we can use the spectral functions $\hat{g}_k^{R,A}$ calculated at $\varepsilon = 0$ when it is possible to use θ parametrization in each band,

$$\hat{g}_k^R = \tau_3\cos\theta_k + \tau_2 e^{-i\tau_3\varphi_k}\sin\theta_k, \quad \hat{g}_k^A = -\tau_3\hat{g}_k^{R+}\tau_3. \quad \text{(A18)}$$

In this case, we can simplify the kinetic equation with the help of the following identities: $\mathcal{D}_T^{(k)} = 4D_k$ and

$$2i\mathrm{Tr}\big[\big(\hat{g}_k^R + \hat{g}_k^A\big)\hat{\Delta}_k\big] = -8|\Delta_k|\sin\theta_k, \quad \text{(A19)}$$

$$\mathrm{Tr}\big[\tau_3\partial_t\hat{\Delta}_k\big(\hat{g}_k^R + \hat{g}_k^A\big)\big] = 4(\boldsymbol{v}_L\cdot\nabla\varphi_k)|\Delta_k|\sin\theta_k, \quad \text{(A20)}$$

where we took into account that for the vortex moving with constant velocity, $\partial_t\Delta_k = -\boldsymbol{v}_L\cdot\nabla\Delta_k$. Hence the kinetic equation becomes

$$D_k\nabla^2 f_T^{(k)} = \big[2f_T^{(k)} + \partial_\varepsilon f_0(\boldsymbol{v}_L\nabla\varphi_k)\big]|\Delta_k|\sin\theta_k. \quad \text{(A21)}$$

To calculate the force $\boldsymbol{F}_{env}$ (9), we use the expansion (11) substituting there the spectral functions in the form (A18) to obtain

$$\mathrm{Tr}\big(\hat{g}_k^{nst}\hat{\partial}_{\boldsymbol{r}}\hat{\Delta}_k\big) = -2\partial_\varepsilon f_0[(\boldsymbol{v}_L\nabla\sin\theta_k)]\nabla|\Delta_k| - 2\sin\theta_k|\Delta_k| \times\big[2f_T^{(k)} + \partial_\varepsilon f_0(\boldsymbol{v}_L\nabla\varphi_k)\big]\nabla\varphi_k. \quad \text{(A22)}$$

For small magnetic fields $B \ll H_{c2}$, the last term in Eq. (9) can be neglected so that the force is given by

$$\boldsymbol{F}_{env} = -\sum_k\frac{\nu_k}{2}\int d^2\boldsymbol{r}d\varepsilon\big\{\partial_\varepsilon f_0\nabla|\Delta_k|(\boldsymbol{v}_L\nabla\sin\theta_k) + |\Delta_k|\sin\theta_k\big[2f_T^{(k)} + \partial_\varepsilon f_0(\boldsymbol{v}_L\nabla\varphi_k)\big]\nabla\varphi_k\big\}. \quad \text{(A23)}$$

We can simplify the equations further by taking into account the common phase $\varphi_{1,2} = \varphi$ so that $(\boldsymbol{v}_L\nabla\varphi) = -v_L\sin\varphi/r$ and $(\boldsymbol{v}_L\nabla|\Delta_k|) = v_L\cos\varphi\partial_r|\Delta_k|$. By factorizing the angular dependence of the distribution function $f_T^{(k)} = \tilde{f}_T^{(k)}v_L\partial_\varepsilon f_0\sin\varphi$, the force becomes $\boldsymbol{F}_{env} = -\eta\boldsymbol{v}_L$, where the viscosity coefficient is given by (48).

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