Spin-resolved orbital magnetization in Rashba two-dimensional electron gas

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We calculate orbital spin-dependent magnetization in a two-dimensional electron gas with a Rashba-type spin-orbit interaction. Such an orbital magnetization is admitted by the time-reversal symmetry of the system, and gives rise to spin currents when the system is not in thermal equilibrium. The theoretical approach is based on the linear response theory and the Matsubara Green's function formalism. To account for the spin-resolved orbital magnetization, a spin-dependent vector potential has been introduced. The spin currents which appear in thermal nonequilibrium due to spin-resolved orbital magnetization play an important role in the spin Nernst effect, and have to be included in order to correctly describe the low-temperature spin Nernst conductivity.

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I. INTRODUCTION

Heat currents coupled to electric and spin currents can be effectively used in novel spintronics devices to control not only charge and entropy/energy transport but also to control spin transport. The latter is the main goal of spin caloritronics, a new branch of spin electronics. Indeed, there is currently huge interest, both experimental and theoretical, in the thermal generation of spin currents which in turn can be used to control the magnetic state of a system. One should mention here the Seebeck and spin Seebeck effects, the Nernst and spin Nernst effects, and also others.

The theoretical description of the phenomena that occur as a system's response to a temperature gradient is generally more complex than the description of a system subject to an external electric field. To describe heat/energy transport in the framework of the Green's function formalism and the Kubo formula, an auxiliary vector potential has been introduced [1–4] instead of the Luttinger "gravitational" potential introduced earlier [5]. Such a vector potential may be considered as an analog of the vector potential in the theory of electromagnetism.

It is well known in the relevant literature that to determine the system's response (namely, the transverse electric current) to a temperature gradient, the orbital magnetization should be taken into account in order to get results that obey fundamental thermodynamics laws. In other words, electric current due to a nonzero orbital magnetization ensures the physical behavior of the off-diagonal kinetic coefficients in a zero-temperature limit, such as off-diagonal electrical conductivity due to thermal bias. This problem was studied first by Obraztsov [6], who introduced magnetization currents to the problem of off-diagonal thermal transport in order to satisfy the Onsager relations of the kinetic coefficients. Then, this problem was raised in several papers, e.g., in the context of the quantum Hall effect [7–10] or the Nernst effect in fluctuating superconductors [11,12] and graphenelike materials [13,14].

The orbital magnetization appears as a consequence of the orbital motion of electrons when the time-reversal symmetry in a system is broken [15–18]. This happens in the case of ferro- and ferrimagnets, or in nonmagnetic materials in an external magnetic field. In the presence of a spin-orbit interaction, the electron motion in a system is even more complex. Such an interaction can appear as an effective momentum-dependent magnetic field, and may lead to phenomena such as spin Hall and spin Nernst effects that require neither magnetic materials nor external magnetic fields.

An important question which arises in the context of a spin-orbit interaction concerns the behavior of off-diagonal spin-kinetic coefficients in systems with time-reversal symmetry. A typical example of such systems is the two-dimensional electron gas with a Rashba spin-orbit interaction, which appears at the interface of semiconductor heterostructures. The thermal properties of such systems have been studied recently in a couple of publications [1,19–23]. In our recent paper we have shown that to describe properly the low-temperature behavior of the spin Nernst effect in a two-dimensional electron gas in the frame of the linear response theory, one needs to introduce orbital effects as well, even though the system is symmetrical with respect to the time reversal. The usual orbital magnetization is then suppressed due to the time-reversal symmetry. Therefore, we have introduced *spin-resolved orbital magnetization* and have shown that it contributes to the spin current in thermal nonequilibrium, and therefore also to spin Nernst conductivity [22]. Thus, such a spin-resolved orbital magnetization can be considered as an additional source of thermally induced spin currents, which in turn can play a role in the thermal control of magnetic states in spintronics devices. Indeed, such control is one of the key challenges of present-day spintronics and spin caloritronics. Apart from this, spin-resolved orbital magnetization can also contribute to a proper description of the topological properties of systems with spin-orbit interactions. Finally, we note that including the spin-resolved orbital magnetization directly from the corresponding quantum-mechanical operator is a rather complex task, so we did this by introducing a spin-dependent magnetic field.

In this paper we present a detailed calculation of spin-resolved orbital magnetization. In Sec. II we describe the model and also present symmetry arguments for spin-resolved orbital magnetization. In Sec. III we introduce the spin vector potential and calculate the relevant Green's function, which is used in Sec. IV to calculate the corresponding spin-dependent orbital magnetization. Discussion and conclusions are presented in Sec. V.

II. THEORETICAL MODEL

The Hamiltonian of a two-dimensional electron gas with a Rashba spin-orbit interaction can be written as

$$H_R = \frac{\hbar^2 k^2}{2m}\sigma_0 + \alpha(k_y\sigma_x - k_x\sigma_y), \tag{1}$$

where σ_n (for $n = x,y,z$) are the Pauli matrices and σ_0 is the unit matrix. All these matrices operate in spin space. The parameter α describes the strength of the Rashba interaction, while k_x and k_y are the in-plane wave-vector components. Eigenvalues of the above Hamiltonian have the form $E_{\mathbf{k}\pm} = \varepsilon_k \pm \alpha k$, with $\varepsilon_k = \hbar^2k^2/2m$ and $k^2 = k_x^2 + k_y^2$.

The retarded Green's function corresponding to the Hamiltonian (1) can be written in the following form,

$$G^R_{\mathbf{k}}(\varepsilon) = G^R_{\mathbf{k}0}(\varepsilon)\,\sigma_0 + G^R_{\mathbf{k}x}(\varepsilon)\,\sigma_x + G^R_{\mathbf{k}y}(\varepsilon)\,\sigma_y, \tag{2}$$

where

$$G^R_{\mathbf{k}0}(\varepsilon) = \tfrac{1}{2}\big[G^R_{\mathbf{k}+}(\varepsilon) + G^R_{\mathbf{k}-}(\varepsilon)\big], \tag{3a}$$

$$G^R_{\mathbf{k}x}(\varepsilon) = \tfrac{1}{2}\sin(\phi)\big[G^R_{\mathbf{k}+}(\varepsilon) - G^R_{\mathbf{k}-}(\varepsilon)\big], \tag{3b}$$

$$G^R_{\mathbf{k}y}(\varepsilon) = -\tfrac{1}{2}\cos(\phi)\big[G^R_{\mathbf{k}+}(\varepsilon) - G^R_{\mathbf{k}-}(\varepsilon)\big], \tag{3c}$$

with ϕ denoting the angle between the wave vector $\mathbf{k}$ and the axis x, and $G^R_{\mathbf{k}\pm}(\varepsilon)$ defined as

$$G^R_{\mathbf{k}\pm}(\varepsilon) = \frac{1}{\varepsilon + \mu - E_{\mathbf{k}\pm} + i\Gamma}. \tag{4}$$

Here, Γ is the imaginary part of the self-energy, which is related to the appropriate relaxation time τ, $\Gamma = \hbar/2\tau$. The advanced Green's function can be written in a similar form with $\Gamma \to -\Gamma$.

The spin-orbit Rashba interaction is a consequence of a structural inversion asymmetry of the confinement potential in a quantum well. This means that whenever $E_{\mathbf{k},\pm} = E_{-\mathbf{k},\pm}$ due to time-inversion symmetry, $E_{\mathbf{k},+} \neq E_{\mathbf{k},-}$ due to the noninvariance with respect to spatial inversion. The spin-orbit interaction can be then considered as a momentum-dependent magnetic field acting on the electron spin. However, this internal magnetic field does not break time-reversal symmetry, as also follows from the form of Hamiltonian (1), which is symmetrical with respect to time reversal. As a consequence of this symmetry, orbital magnetization in the system is suppressed. However, time-reversal symmetry of the system under consideration allows for spin-dependent orbital magnetization (or equivalently for a spin-dependent magnetic field $\mathbf{B}_s$), which has an opposite orientation for the spin-up and spin-down electrons. The total orbital magnetization is then equal to zero, $\mathbf{M} = \mathbf{M}_\uparrow + \mathbf{M}_\downarrow = \mathbf{0}$, as $\mathbf{M}_\uparrow$ and $\mathbf{M}_\downarrow$ are oriented in opposite directions, but the spin-resolved orbital magnetization defined as $\mathbf{M}^s_{\rm orb} = \mathbf{M}_\uparrow - \mathbf{M}_\downarrow$ is then nonzero, $\mathbf{M}^s_{\rm orb} \neq \mathbf{0}$.

To calculate the spin-resolved orbital magnetization we introduce a spin vector potential, $\mathbf{A}_s(\mathbf{r}) = \sigma_z\mathbf{A}(\mathbf{r})$, into the Hamiltonian (1) by the substitution $-i\hbar\nabla\sigma_0 \to -i\hbar\nabla\sigma_0 - e\,\mathbf{A}_s$. This spin vector potential is related to the spin-dependent magnetic field $\mathbf{B}_s = \sigma_z\mathbf{B}$ according to the formula $\mathbf{B}_s = \mathrm{rot}\,\mathbf{A}_s$. Thus, the effective spin-dependent magnetic field affects the orbital motion of spin-up and spin-down electrons in different ways. In the case considered here, this spin magnetic field is oriented along the z axis (normal to the system's plane), $\mathbf{B} = (0,0,B)$. The resulting Hamiltonian reads

$$H_{\mathbf{A}} = \frac{\hbar^2}{2m}\Big(\mathbf{k}\sigma_0 - \frac{e}{\hbar}\mathbf{A}_s\Big)^2 + \alpha(k_y\sigma_x - k_x\sigma_y) - \alpha\frac{e}{\hbar}(A_{sy}\otimes\sigma_x - A_{sx}\otimes\sigma_y). \tag{5}$$

The main objective of the following sections is to calculate the total energy of the system in the presence of a nonzero B, and then to calculate the spin-resolved orbital magnetization as a derivative of this energy with respect to B, taken at $B \to 0$.

III. GREEN'S FUNCTION

The Green's function describing a two-dimensional electron gas with a Rashba interaction in the spin-dependent magnetic field B_s [see Eq. (5)] satisfies the following equation written in the coordinate space,

$$\int d^2\mathbf{r}'\bigg\{\varepsilon + \frac{\hbar^2}{2m}\bigg[\nabla_x^2 + \nabla_y^2 - \frac{2ie}{\hbar}(A_x\nabla_x + A_y\nabla_y)\bigg] + i\alpha[\sigma_x\nabla_y - \sigma_y\nabla_x]\bigg\}\delta(\mathbf{r}-\mathbf{r}')\mathcal{G}(\varepsilon,\mathbf{r}',\mathbf{r}'') = \delta(\mathbf{r}-\mathbf{r}''), \tag{6}$$

where we neglect the diamagnetic term proportional to $\mathbf{A}_s^2$ and a contribution originating from the third term in Hamiltonian (5), which gives a small correction since the Rashba interaction is assumed to be small. Note that for notational brevity we write in this section $\varepsilon \equiv \varepsilon + \mu + i\delta\,\mathrm{sgn}(\varepsilon)$ for the zero-temperature casual Green's function and $\varepsilon \equiv i\varepsilon_n$ for the Matsubara-Green's function.

Similarly as in the case of a constant magnetic field, we make use of the fact that the Green's function may be expressed as the product of the translationally and rotationally invariant core Green's function $\mathcal{G}_0(\varepsilon,\mathbf{r}-\mathbf{r}')$ and an exponential factor [13,14,24,25],

$$\mathcal{G}(\varepsilon,\mathbf{r},\mathbf{r}') = \mathcal{G}_0(\varepsilon,\mathbf{r}-\mathbf{r}')e^{i\mathcal{A}_{\mathbf{rr}'}\sigma_z}, \tag{7}$$

where $\mathcal{A}_{\mathbf{rr}'} \equiv \frac{e}{\hbar}\int_{\mathbf{r}}^{\mathbf{r}'}\mathbf{A}(\mathbf{R})\cdot d\mathbf{R}$ is the Schwinger or Peierls phase factor. The integral of gauge vector potential in this phase factor is along a straight line from $\mathbf{r}$ to $\mathbf{r}'$. Consequently, Eq. (6) can be rewritten in the form

$$\int d^2\mathbf{r}'\bigg\{\varepsilon + \frac{\hbar^2}{2m}[\nabla_x^2 + \nabla_y^2] + i\alpha[\sigma_x\nabla_y - \sigma_y\nabla_x]\bigg\} \times e^{i\sigma_z\mathcal{A}_{\mathbf{rr}'}}\delta(\mathbf{r}-\mathbf{r}')\mathcal{G}_0(\varepsilon,\mathbf{r}'-\mathbf{r}'')e^{i\sigma_z\mathcal{A}_{\mathbf{r}'\mathbf{r}''}} = \delta(\mathbf{r}-\mathbf{r}'')e^{i\sigma_z\mathcal{A}_{\mathbf{rr}''}}. \tag{8}$$

We look for the Green's function $\mathcal{G}_0$ in the following form,

$$\mathcal{G}_0(\varepsilon,\mathbf{r}'-\mathbf{r}'') = \sum_i \mathcal{G}_{0i}(\varepsilon,\mathbf{r}'-\mathbf{r}'')\sigma_i \tag{9}$$

where $i = \{0,x,y,z\}$. Thus, Eq. (8) leads to a set of four equations for the four components of the Green's function, $\mathcal{G}_{0i}(\varepsilon,\mathbf{r}'-\mathbf{r}'')$. Upon performing the Fourier transformation with respect to the space variables, this set of equations can be written in the following form (for details of calculations, see

Appendix A),

$$\Lambda_{\mathbf{k}}(\varepsilon)\begin{pmatrix}\mathcal{G}_{\mathbf{k}0}(\varepsilon)\\ \mathcal{G}_{\mathbf{k}x}(\varepsilon)\\ \mathcal{G}_{\mathbf{k}y}(\varepsilon)\\ \mathcal{G}_{\mathbf{k}z}(\varepsilon)\end{pmatrix}=\begin{pmatrix}a_{\mathbf{k}0}(\varepsilon)\\ a_{\mathbf{k}x}(\varepsilon)\\ a_{\mathbf{k}y}(\varepsilon)\\ a_{\mathbf{k}z}(\varepsilon)\end{pmatrix}, \tag{10}$$

where $\mathcal{G}_{\mathbf{k}i}(\varepsilon)$ (for $i=0,x,y,z$) is the Fourier transform of $\mathcal{G}_{0i}(\varepsilon,\mathbf{r}'-\mathbf{r}'')$, the matrix $\hat{\Lambda}_{\mathbf{k}}(\varepsilon)$ is defined as

$$\hat{\Lambda}_{\mathbf{k}}(\varepsilon)=\begin{pmatrix}[g_{\mathbf{k}0}(\varepsilon)]^{-1} & -\alpha k_y & \alpha k_x & 0\\ -\alpha k_y & [g_{\mathbf{k}0}(\varepsilon)]^{-1} & 0 & i\alpha k_x\\ \alpha k_x & 0 & [g_{\mathbf{k}0}(\varepsilon)]^{-1} & i\alpha k_y\\ 0 & -i\alpha k_x & -i\alpha k_y & [g_{\mathbf{k}0}(\varepsilon)]^{-1}\end{pmatrix}, \tag{11}$$

with $[g_{\mathbf{k}0}(\varepsilon)]^{-1}=\varepsilon-\varepsilon_k$, and the functions $a_{\mathbf{k}i}(\varepsilon)$ on the right-hand side of Eq. (10) have the following form,

$$a_{\mathbf{k}0}(\varepsilon)=1, \tag{12}$$

$$a_{\mathbf{k}x}(\varepsilon)=-\alpha\frac{e}{2\hbar}B\partial_{k_y}G_{\mathbf{k}0}(\varepsilon), \tag{13}$$

$$a_{\mathbf{k}y}(\varepsilon)=\alpha\frac{e}{2\hbar}B\partial_{k_x}G_{\mathbf{k}0}(\varepsilon), \tag{14}$$

$$a_{\mathbf{k}z}(\varepsilon)=i\frac{e}{2\hbar}B\big[\big(\partial_{k_x}[g_{\mathbf{k}0}(\varepsilon)]^{-1}\big)\big(\partial_{k_y}G_{\mathbf{k}0}(\varepsilon)\big) - \big(\partial_{k_y}[g_{\mathbf{k}0}(\varepsilon)]^{-1}\big)\big(\partial_{k_x}G_{\mathbf{k}0}(\varepsilon)\big)\big]. \tag{15}$$

Thus, the core Green's function (9) in the momentum space takes the form

$$\mathcal{G}_{\mathbf{k}}(\varepsilon)=\mathcal{G}_{\mathbf{k}0}(\varepsilon)\sigma_0+\mathcal{G}_{\mathbf{k}x}(\varepsilon)\sigma_x+\mathcal{G}_{\mathbf{k}y}(\varepsilon)\sigma_y+\mathcal{G}_{\mathbf{k}z}(\varepsilon)\sigma_z, \tag{16}$$

where $\mathcal{G}_{\mathbf{k}\alpha}(\varepsilon)$ are solutions of Eq. (10). The expressions for these functions are rather cumbersome and their explicit form is presented in Appendix A.

IV. SPIN-RESOLVED ORBITAL MAGNETIZATION

By analogy to ordinary magnetization, we define the spin-resolved orbital magnetization as the derivative of the free energy F with respect to B, $\mathbf{M}=-\partial F/\partial\mathbf{B}$ (see, e.g., Refs. [12,24]). Since the magnetic field is a small perturbation, the induced changes in the free energy F and energy E are approximately equal [26], $\delta F\approx\delta E$. Thus, using the Hellmann-Feynman theorem, we can write (see, for example, Ref. [17]),

$$M^s_{\rm orb}=-\frac{\partial\langle H\rangle}{\partial B}. \tag{17}$$

In the following we will use the above equation to find the spin-resolved orbital magnetization.

The quantum-mechanical average of energy $\langle H\rangle$ for the system in a spin-dependent magnetic field can be found in the Matsubara-Green's function formalism from the following expression,

$$\langle H\rangle=\frac{1}{\beta}\mathrm{Tr}\sum_n\int\frac{d^2\mathbf{k}}{(2\pi)^2}H_R\,\mathcal{G}_{\mathbf{k}}(i\varepsilon_n), \tag{18}$$

where $\beta=1/k_BT$ and the Matsubara energies are defined as $i\varepsilon_n=(2n+1)i\pi k_BT$. The sum over Matsubara energies can be calculated by the method of contour integration [27],

$$\frac{1}{\beta}\sum_n\hat{H}_R\,\mathcal{G}_{\mathbf{k}}(i\varepsilon_n)=-\int_{\mathcal{C}}\frac{dz}{2\pi i}f(z)H_R\mathcal{G}_{\mathbf{k}}(z), \tag{19}$$

where $f(z)$ is a meromorphic function that has simple poles at the odd integers, $z=i\varepsilon_n$, and takes the form $f(z)=(e^{\beta z}+1)^{-1}$, while $\mathcal{C}$ is the appropriate contour of integration [27]. Combining Eqs. (18) and (19), one finds

$$\langle H\rangle=-\mathrm{Tr}\int_{\mathcal{C}}\frac{dz}{2\pi i}\int\frac{d^2\mathbf{k}}{(2\pi)^2}f(z)H_R\,\mathcal{G}_{\mathbf{k}}(z). \tag{20}$$

The integral along the contour $\mathcal{C}$ has a branch cut at the line $z=\varepsilon$, where ε is real. Consequently, one can write

$$\langle H\rangle=i\,\mathrm{Tr}\int\frac{d^2\mathbf{k}}{(2\pi)^2}\int\frac{d\varepsilon}{2\pi}f(\varepsilon)H_R \times[\mathcal{G}_{\mathbf{k}}(\varepsilon+i\delta)-\mathcal{G}_{\mathbf{k}}(\varepsilon-i\delta)], \tag{21}$$

where δ is an infinitesimally small positive number. After analytical continuation, we arrive at the formula

$$\langle H\rangle=i\,\mathrm{Tr}\int\frac{d^2\mathbf{k}}{(2\pi)^2}\int\frac{d\varepsilon}{2\pi}f(\varepsilon)H_R\big[\mathcal{G}^R_{\mathbf{k}}(\varepsilon)-\mathcal{G}^A_{\mathbf{k}}(\varepsilon)\big]. \tag{22}$$

This general expression, in combination with the explicit form of the core Green's function (16), allows one to obtain from Eq. (17) the analytical result for orbital spin-resolved magnetization, which conveniently can be written as a sum of three terms,

$$M^s_{\rm orb}=M^{s(1)}_{\rm orb}+M^{s(2)}_{\rm orb}+M^{s(3)}_{\rm orb}, \tag{23}$$

where

$$M^{s(1)}_{\rm orb}=-2i\int\frac{d^2\mathbf{k}}{(2\pi)^2}\int\frac{d\varepsilon}{2\pi}f(\varepsilon)\varepsilon_k\partial_B\big[\mathcal{G}^R_{\mathbf{k}0}(\varepsilon)-\mathcal{G}^A_{\mathbf{k}0}(\varepsilon)\big], \tag{24}$$

$$M^{s(2)}_{\rm orb}=-2i\int\frac{d^2\mathbf{k}}{(2\pi)^2}\int\frac{d\varepsilon}{2\pi}f(\varepsilon)\alpha k_y\partial_B\big[\mathcal{G}^R_{\mathbf{k}x}(\varepsilon)-\mathcal{G}^A_{\mathbf{k}x}(\varepsilon)\big], \tag{25}$$

$$M^{s(3)}_{\rm orb}=2i\int\frac{d^2\mathbf{k}}{(2\pi)^2}\int\frac{d\varepsilon}{2\pi}f(\varepsilon)\alpha k_x\partial_B\big[\mathcal{G}^R_{\mathbf{k}y}(\varepsilon)-\mathcal{G}^A_{\mathbf{k}y}(\varepsilon)\big]. \tag{26}$$

The explicit forms of the integrals in Eqs. (24)–(26) are given in Appendix B. After integration over ε, one arrives at (for details, see Appendix B)

$$\begin{aligned}M^{s(1)}_{\rm orb}&=\alpha\frac{e}{8\pi\hbar}\int dk\varepsilon_k^2[f''(E_+)-f''(E_-)]\\&\quad-\alpha\frac{e}{8\pi\hbar}\int dk\frac{\varepsilon_k^2}{\alpha k}[f'(E_+)+f'(E_-)]\\&\quad+\alpha\frac{e}{8\pi\hbar}\int dk\frac{\varepsilon_k^2}{\alpha^2k^2}[f(E_+)-f(E_-)]\\&\quad+\alpha\frac{e}{8\pi\hbar}\int dk\frac{\alpha k}{2}\varepsilon_k[f''(E_+)+f''(E_-)]\\&\quad-\alpha\frac{e}{8\pi\hbar}\int dk\frac{\varepsilon_k}{2}[f'(E_+)-f'(E_-)]\end{aligned} \tag{27}$$

and

$$M_{\rm orb}^{s(2)} + M_{\rm orb}^{s(3)} = \alpha\frac{e}{16\pi\hbar}\int dk\varepsilon_k[f'(E_+) - f'(E_-)] + \alpha\frac{e}{16\pi\hbar}\int dk\alpha k\varepsilon_k[f''(E_+) + f''(E_-)] + \alpha\frac{e}{16\pi\hbar}\int dk\alpha k[f'(E_+) + f'(E_-)] + \alpha\frac{e}{16\pi\hbar}\int dk\alpha^2k^2[f''(E_+) - f''(E_-)] - \alpha\frac{e}{16\pi\hbar}\int dk[f(E_+) - f(E_-)], \quad (28)$$

where f' and f'' denote the first and second derivatives of the Fermi distribution function with respect to energy. Upon combining these two equations, one finally gets the general expression for the spin-resolved orbital magnetization of the two-dimensional electron gas with a Rashba interaction,

$$M_{\rm orb}^s = \frac{\alpha e}{16\pi\hbar}\bigg[\int dk\left(\frac{2\varepsilon_k^2}{\alpha^2k^2} - 1\right)[f(E_+) - f(E_-)] - \int dk\alpha k\left(\frac{2\varepsilon_k^2}{\alpha^2k^2} - 1\right)[f'(E_+) + f'(E_-)] + \int dk\alpha^2k^2\left(\frac{2\varepsilon_k^2}{\alpha^2k^2} - 1\right)[f''(E_+) - f''(E_-)] + 2\int dk\alpha k[E_+f''(E_+) + E_-f''(E_-)]\bigg]. \quad (29)$$

Expression (29) is our final result for the orbital spin-resolved magnetization $M_{\rm orb}^s$, which is valid at arbitrary temperature. Though this formula is rather cumbersome, in the zero-temperature limit it leads to a simple analytical expression for $M_{\rm orb}^s(T=0) = M_{\rm orb}^{s,T=0}$. When both subbands are occupied, i.e., when $\mu > 0$, we find the formula (for details, see Appendix C)

$$M_{\rm orb}^{s,T=0} = -\frac{em\alpha^2}{12\pi\hbar^3}, \quad (30)$$

which means that $M_{\rm orb}^s$ is quadratic in the Rashba parameter α.

In Fig. 1 we show the temperature dependence of the orbital magnetization $M_{\rm orb}^s$ normalized to its zero-temperature value $M_{\rm orb}^{s,T=0}$. Different curves correspond to the indicated values of the Fermi energy μ_0, i.e., the value of the chemical potential at $T=0$. Note that for a fixed particle density ρ, the chemical potential varies with temperature as follows [28]: $\mu = k_BT\ln(e^{\mu_0/k_BT} - 1)$ and $\mu_0 = \pi\hbar^2\rho/m$. It is evident that $M_{\rm orb}^s$ diminishes with increasing temperature, and this decrease depends on the Fermi energy (particle density): It is faster for low values of μ_0. In turn, variation of the normalized magnetization $M_{\rm orb}^s/M_{\rm orb}^{s,T=0}$ with increasing Fermi energy μ_0 is shown explicitly in Fig. 2 for several values of temperature. One can observe a saturation of $M_{\rm orb}^s$ at its low-temperature value when the particle density is sufficiently large.

The physical reason for the appearance of spin-resolved orbital magnetization is related to noncompensated spin currents flowing at the edge of a sample. Note that when the temperature is homogeneous, the spin currents are compensated in the bulk except at the edges. However, when the temperature is nonhomogeneous, the spin currents can also exist in the bulk (see the discussions below). This is a spin analogy to the usual orbital magnetization, which arises due to noncompensated electric currents at the edges.

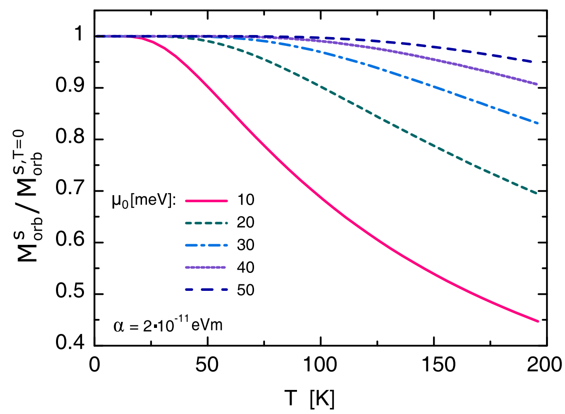

FIG. 1. Spin-resolved orbital magnetization $M_{\rm orb}^s$, normalized to its zero-temperature value $M_{\rm orb}^{s,T=0}$, plotted as a function of temperature for fixed values of the Fermi level μ_0, as indicated. Other parameters are $m = 0.07m_0$ (where m_0 is the electron mass), and $\alpha = 2\times10^{-11}$ eV m.

V. DISCUSSION AND CONCLUSIONS

In our recent paper [22] we used the Matsubara Green's function method to calculate the spin Nernst conductivity $\alpha_{xy}^{s_z}$. This conductivity defines spin current flowing perpendicularly to the temperature gradient. We have shown there that the vertex correction due to scattering on impurities does not cancel the *bare bubble* contribution, contrary to spin Hall conductivity where such a cancellation takes place. As a result, the spin Nernst conductivity in this approximation diverges in the zero-temperature limit.

To remove this divergency, it was necessary to include an additional contribution to the spin current (and also to the spin Nernst conductivity) that follows from spin-resolved orbital

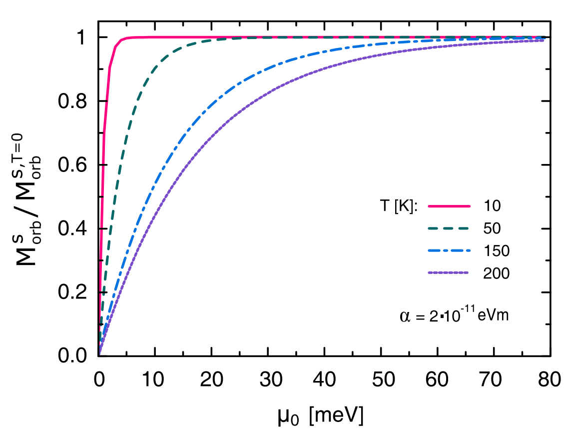

FIG. 2. Spin-resolved orbital magnetization $M_{\rm orb}^s$, normalized to its zero-temperature value $M_{\rm orb}^{s,T=0}$, plotted as a function of the Fermi level μ_0 and for fixed temperatures, as indicated. The other parameters are as in Fig. 1.

magnetization. One can conceive the spin current $\mathbf{J}^z$ as a superposition of spin-up and spin-down currents flowing in opposite directions, $\mathbf{J}^z = \mathbf{J}_\uparrow - \mathbf{J}_\downarrow$. Each of the spin-polarized currents generates a corresponding orbital magnetization. However, the vectors $\mathbf{M}_\uparrow$ and $\mathbf{M}_\downarrow$ are oriented in opposite directions, so the total orbital magnetization vanishes, $\mathbf{M} = \mathbf{M}_\uparrow + \mathbf{M}_\downarrow = \mathbf{0}$, as one can expect from the time-reversal symmetry. In turn, the spin-resolved orbital magnetization $\mathbf{M}^s_{\rm orb} = \mathbf{M}_\uparrow - \mathbf{M}_\downarrow$ is nonzero, $\mathbf{M}^s_{\rm orb} \neq \mathbf{0}$.

The spin current due to spin-resolved orbital magnetization depends on temperature. Therefore, it contributes to spin Nernst conductivity as the corresponding currents flowing at the edges having different temperatures do not cancel each other, though they flow in opposite directions. In turn, these currents do not contribute to the spin Hall conductivity because in a thermally uniform system the currents at the two edges cancel each other. The correction to the spin Nernst conductivity that originates from spin-resolved orbital magnetization is given by the term $(\hbar/e)M^s_{\rm orb}/T$.

A similar situation takes place also in the case of the Nernst effect in systems with no time-reversal symmetry. In that case the absence of time-reversal symmetry admits orbital magnetization. This magnetization in turn contributes to charge current, and the corresponding contribution removes the zero-temperature divergency in the Nernst conductivity (see, e.g., Refs. [6,9,13,14]).

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APPENDIX A: EQUATIONS FOR THE GREEN'S FUNCTION AND ITS FOURIER TRANSFORM

In this Appendix we derive the matrix equation (10). Multiplying Eq. (8) on the right-hand side by $e^{-i\sigma_z \mathcal{A}_{\mathbf{r}\mathbf{r}''}}$ and using commutation relations for the Pauli matrices, we find

$$\begin{aligned} e^{i\sigma_z \mathcal{A}_{\mathbf{r}\mathbf{r}'}} \mathcal{G}_0(\varepsilon,\mathbf{r}'-\mathbf{r}'') e^{i\sigma_z \mathcal{A}_{\mathbf{r}'\mathbf{r}''}} e^{-i\sigma_z \mathcal{A}_{\mathbf{r}\mathbf{r}''}} &= \mathcal{G}_{00}(\varepsilon,\mathbf{r}'-\mathbf{r}'')\sigma_0 e^{i\sigma_z(\mathcal{A}_{\mathbf{r}\mathbf{r}'}+\mathcal{A}_{\mathbf{r}'\mathbf{r}'}-\mathcal{A}_{\mathbf{r}\mathbf{r}''})} + \mathcal{G}_{0z}(\varepsilon,\mathbf{r}'-\mathbf{r}'')\sigma_z e^{i\sigma_z(\mathcal{A}_{\mathbf{r}\mathbf{r}'}+\mathcal{A}_{\mathbf{r}'\mathbf{r}'}-\mathcal{A}_{\mathbf{r}\mathbf{r}''})} \\ &\quad + \mathcal{G}_{0x}(\varepsilon,\mathbf{r}'-\mathbf{r}'')\sigma_x e^{-i\sigma_z(\mathcal{A}_{\mathbf{r}\mathbf{r}'}-\mathcal{A}_{\mathbf{r}'\mathbf{r}''}-\mathcal{A}_{\mathbf{r}\mathbf{r}''})} + \mathcal{G}_{0y}(\varepsilon,\mathbf{r}'-\mathbf{r}'')\sigma_y e^{-i\sigma_z(\mathcal{A}_{\mathbf{r}\mathbf{r}'}-\mathcal{A}_{\mathbf{r}'\mathbf{r}''}-\mathcal{A}_{\mathbf{r}\mathbf{r}''})}. \end{aligned} \tag{A1}$$

The integral along the contour $\mathbf{r}-\mathbf{r}'-\mathbf{r}''-\mathbf{r}$ in the Peierls phase can be transformed into the surface integral,

$$\begin{aligned} \frac{e}{\hbar}\oint \mathbf{A}(\mathbf{R})\cdot d\mathbf{R} &= \frac{e}{\hbar}\left(\int_{\mathbf{r}}^{\mathbf{r}'} \mathbf{A}(\mathbf{R})\cdot d\mathbf{R} + \int_{\mathbf{r}'}^{\mathbf{r}''} \mathbf{A}(\mathbf{R})\cdot d\mathbf{R} + \int_{\mathbf{r}''}^{\mathbf{r}} \mathbf{A}(\mathbf{R})\cdot d\mathbf{R}\right) \\ &= \frac{e}{\hbar}\frac{1}{2}\mathbf{B}\cdot(\mathbf{r}'-\mathbf{r})\times(\mathbf{r}''-\mathbf{r}'), \end{aligned} \tag{A2}$$

so Eq. (A1) takes the form

$$\begin{aligned} e^{i\sigma_z \mathcal{A}_{\mathbf{r}\mathbf{r}'}} \mathcal{G}_0(\varepsilon\mathbf{r}'-\mathbf{r}'') e^{i\sigma_z \mathcal{A}_{\mathbf{r}'\mathbf{r}''}} e^{-i\sigma_z \mathcal{A}_{\mathbf{r}\mathbf{r}''}} &= \mathcal{G}_{00}(\varepsilon,\mathbf{r}'-\mathbf{r}'')\sigma_0 e^{i\sigma_z \frac{e}{\hbar}\mathbf{B}\cdot\frac{1}{2}(\mathbf{r}'-\mathbf{r})\times(\mathbf{r}''-\mathbf{r}')} + \mathcal{G}_{0x}(\varepsilon,\mathbf{r}'-\mathbf{r}'')\sigma_x + \mathcal{G}_{0y}(\varepsilon,\mathbf{r}'-\mathbf{r}'')\sigma_y \\ &\quad + \mathcal{G}_{0z}(\varepsilon,\mathbf{r}'-\mathbf{r}'')\sigma_z e^{i\sigma_z \frac{e}{\hbar}\mathbf{B}\cdot\frac{1}{2}(\mathbf{r}'-\mathbf{r})\times(\mathbf{r}''-\mathbf{r}')}. \end{aligned} \tag{A3}$$

Inserting Eq. (A2) into Eq. (8) we obtain a set of four equations for the four components of the core Green's function $\mathcal{G}_{0i}(\varepsilon,\mathbf{r}'-\mathbf{r}'')$,

$$\begin{aligned} &\int d\mathbf{r}'(\varepsilon-H_0)\mathcal{G}_{00}(\varepsilon,\mathbf{r}'-\mathbf{r}'')\delta(\mathbf{r}-\mathbf{r}') + i\int d\mathbf{r}'(\varepsilon-H_0)\mathcal{G}_{0z}(\varepsilon,\mathbf{r}'-\mathbf{r}'')\frac{e}{2\hbar}\mathbf{B}\cdot(\mathbf{r}'-\mathbf{r})\times(\mathbf{r}''-\mathbf{r}')\delta(\mathbf{r}-\mathbf{r}') \\ &-\alpha\int d\mathbf{r}'\kappa_y\mathcal{G}_{0x}(\varepsilon,\mathbf{r}'-\mathbf{r}'')\delta(\mathbf{r}-\mathbf{r}') + \alpha\int d\mathbf{r}'\kappa_x\mathcal{G}_{0y}(\varepsilon,\mathbf{r}'-\mathbf{r}'')\delta(\mathbf{r}-\mathbf{r}') = \delta(\mathbf{r}-\mathbf{r}''), \end{aligned} \tag{A4a}$$

$$\begin{aligned} &-\alpha\int d\mathbf{r}'\kappa_y\mathcal{G}_{00}(\varepsilon,\mathbf{r}'-\mathbf{r}'')\delta(\mathbf{r}-\mathbf{r}') - \alpha\int d\mathbf{r}'\kappa_x\mathcal{G}_{00}(\varepsilon,\mathbf{r}'-\mathbf{r}'')\frac{e}{2\hbar}\mathbf{B}\cdot(\mathbf{r}'-\mathbf{r})\times(\mathbf{r}''-\mathbf{r}')\delta(\mathbf{r}-\mathbf{r}') \\ &-i\alpha\int d\mathbf{r}'\kappa_y\mathcal{G}_{0z}(\varepsilon,\mathbf{r}'-\mathbf{r}'')\frac{e}{2\hbar}\mathbf{B}\cdot(\mathbf{r}'-\mathbf{r})\times(\mathbf{r}''-\mathbf{r}')\delta(\mathbf{r}-\mathbf{r}') + i\alpha\int d\mathbf{r}'\kappa_x\mathcal{G}_{0z}(\varepsilon,\mathbf{r}'-\mathbf{r}'')\delta(\mathbf{r}-\mathbf{r}') \\ &+\int d\mathbf{r}'(\varepsilon-H_0)\mathcal{G}_{0x}(\varepsilon,\mathbf{r}'-\mathbf{r}'')\delta(\mathbf{r}-\mathbf{r}') = 0, \end{aligned} \tag{A4b}$$

$$\begin{aligned} &-\alpha\int d\mathbf{r}'\kappa_y\mathcal{G}_{00}(\varepsilon,\mathbf{r}'-\mathbf{r}'')\frac{e}{2\hbar}\mathbf{B}\cdot(\mathbf{r}'-\mathbf{r})\times(\mathbf{r}''-\mathbf{r}')\delta(\mathbf{r}-\mathbf{r}') + \alpha\int d\mathbf{r}'\kappa_x\mathcal{G}_{00}(\varepsilon,\mathbf{r}'-\mathbf{r}'')\delta(\mathbf{r}-\mathbf{r}') \\ &+i\alpha\int d\mathbf{r}'\kappa_y\mathcal{G}_{0z}(\varepsilon,\mathbf{r}'-\mathbf{r}'')\delta(\mathbf{r}-\mathbf{r}') + i\alpha\int d\mathbf{r}'\kappa_x\mathcal{G}_{0z}(\varepsilon,\mathbf{r}'-\mathbf{r}'')\frac{e}{2\hbar}\mathbf{B}\cdot(\mathbf{r}'-\mathbf{r})\times(\mathbf{r}''-\mathbf{r}')\delta(\mathbf{r}-\mathbf{r}') \\ &+\int d\mathbf{r}'(\varepsilon-H_0)\mathcal{G}_{0y}(\varepsilon,\mathbf{r}'-\mathbf{r}'')\delta(\mathbf{r}-\mathbf{r}') = 0, \end{aligned} \tag{A4c}$$

$$i\int d\mathbf{r}'(\varepsilon - H_0)\mathcal{G}_{00}(\varepsilon,\mathbf{r}'-\mathbf{r}'')\frac{e}{2\hbar}\mathbf{B}\cdot(\mathbf{r}'-\mathbf{r})\times(\mathbf{r}''-\mathbf{r}')\delta(\mathbf{r}-\mathbf{r}') + \int d\mathbf{r}'(\varepsilon - H_0)\mathcal{G}_{0z}(\varepsilon,\mathbf{r}'-\mathbf{r}'')\delta(\mathbf{r}-\mathbf{r}')$$
$$-i\alpha\int d\mathbf{r}'\kappa_x\mathcal{G}_{0x}(\varepsilon,\mathbf{r}'-\mathbf{r}'')\delta(\mathbf{r}-\mathbf{r}') - i\alpha\int d\mathbf{r}'\kappa_y\mathcal{G}_{0y}(\varepsilon,\mathbf{r}'-\mathbf{r}'')\delta(\mathbf{r}-\mathbf{r}') = 0, \quad \text{(A4d)}$$

where $H_0 = \hbar^2(\kappa_x^2+\kappa_y^2)/2m$, with $\kappa_\alpha = -i\nabla_\alpha$, and we expanded the exponential factors to the first order in B. After Fourier transformation, this set of equations takes the form

$$[g_{\mathbf{k}0}(\varepsilon)]^{-1}\mathcal{G}_{\mathbf{k}0}(\varepsilon) - i\frac{e}{2\hbar}B_k\epsilon_{ijk}\left(\frac{\partial}{\partial k_i}[g_{\mathbf{k}0}(\varepsilon)]^{-1}\right)\left(\frac{\partial}{\partial k_j}\mathcal{G}_{\mathbf{k}z}(\varepsilon)\right) - \alpha k_y\mathcal{G}_{\mathbf{k}x}(\varepsilon) + \alpha k_x\mathcal{G}_{\mathbf{k}y}(\varepsilon) = 1, \quad \text{(A5a)}$$

$$-\alpha k_y\mathcal{G}_{\mathbf{k}0}(\varepsilon) + \alpha\frac{e}{2\hbar}B_k\epsilon_{ijk}\delta_{ix}\left(\frac{\partial}{\partial k_j}\mathcal{G}_{\mathbf{k}0}(\varepsilon)\right) + i\alpha\frac{e}{2\hbar}B_k\epsilon_{ijk}\delta_{iy}\left(\frac{\partial}{\partial k_j}\mathcal{G}_{\mathbf{k}z}(\varepsilon)\right) + i\alpha k_x\mathcal{G}_{\mathbf{k}z}(\varepsilon) + [g_{\mathbf{k}0}(\varepsilon)]^{-1}\mathcal{G}_{\mathbf{k}x}(\varepsilon) = 0, \quad \text{(A5b)}$$

$$\alpha\frac{e}{2\hbar}B_k\epsilon_{ijk}\delta_{iy}\left(\frac{\partial}{\partial k_j}\mathcal{G}_{\mathbf{k}0}(\varepsilon)\right) + \alpha k_x\mathcal{G}_{\mathbf{k}0}(\varepsilon) + i\alpha k_y\mathcal{G}_{\mathbf{k}z}(\varepsilon) - i\alpha\frac{e}{2\hbar}B_k\epsilon_{ijk}\delta_{ix}\left(\frac{\partial}{\partial k_j}\mathcal{G}_{\mathbf{k}z}(\varepsilon)\right) + [g_{\mathbf{k}0}(\varepsilon)]^{-1}\mathcal{G}_{\mathbf{k}y}(\varepsilon) = 0, \quad \text{(A5c)}$$

$$-i\frac{e}{2\hbar}B_k\epsilon_{ijk}\left(\frac{\partial}{\partial k_i}[g_{\mathbf{k}0}(\varepsilon)]^{-1}\right)\left(\frac{\partial}{\partial k_j}\mathcal{G}_{\mathbf{k}0}(\varepsilon)\right) + [g_{\mathbf{k}0}(\varepsilon)]^{-1}\mathcal{G}_{\mathbf{k}z}(\varepsilon) - i\alpha k_x\mathcal{G}_{\mathbf{k}x}(\varepsilon) - i\alpha k_y\mathcal{G}_{\mathbf{k}y}(\varepsilon) = 0, \quad \text{(A5d)}$$

where $[g_{\mathbf{k}0}(\varepsilon)]^{-1} = \varepsilon - \varepsilon_k$.

Equations (A4a)– (A4d) may be further simplified assuming linear response with respect to B,

$$i\frac{e}{2\hbar}B_k\epsilon_{ijk}\left(\frac{\partial}{\partial k_i}[g_{\mathbf{k}0}(\varepsilon)]^{-1}\right)\left(\frac{\partial}{\partial k_j}\mathcal{G}_{\mathbf{k}z}(\varepsilon)\right) \cong i\frac{e}{2\hbar}B\epsilon_{ijk}\left(\frac{\partial}{\partial k_i}[g_{\mathbf{k}0}(\varepsilon)]^{-1}\right)\left(\frac{\partial}{\partial k_j}G_{\mathbf{k}z}(\varepsilon)\right) = 0, \quad \text{(A6a)}$$

$$\alpha\frac{e}{2\hbar}B_k\epsilon_{ijk}\delta_{ix,y}\left(\frac{\partial}{\partial k_j}\mathcal{G}_{\mathbf{k}0}(\varepsilon)\right) \cong \alpha\frac{e}{2\hbar}B_k\epsilon_{ijk}\delta_{ix,y}\left(\frac{\partial}{\partial k_j}G_{\mathbf{k}0}(\varepsilon)\right), \quad \text{(A6b)}$$

$$i\alpha\frac{e}{2\hbar}B_k\epsilon_{ijk}\delta_{ix,y}\left(\frac{\partial}{\partial k_j}\mathcal{G}_{\mathbf{k}z}\right) \cong i\alpha\frac{e}{2\hbar}B_k\epsilon_{ijk}\delta_{ix,y}\left(\frac{\partial}{\partial k_j}G_{\mathbf{k}z}\right) = 0, \quad \text{(A6c)}$$

$$i\frac{e}{2\hbar}B_k\epsilon_{ijk}\left(\frac{\partial}{\partial k_i}[g_{\mathbf{k}0}(\varepsilon)]^{-1}\right)\left(\frac{\partial}{\partial k_j}\mathcal{G}_{\mathbf{k}0}(\varepsilon)\right) \cong i\frac{e}{2\hbar}B_k\epsilon_{ijk}\left(\frac{\partial}{\partial k_i}[g_{\mathbf{k}0}(\varepsilon)]^{-1}\right)\left(\frac{\partial}{\partial k_j}G_{\mathbf{k}0}(\varepsilon)\right), \quad \text{(A6d)}$$

and finally we obtain the matrix equation (10).

The solution of Eq. (10) takes then the form of Eq. (16), where

$$\mathcal{G}_{\mathbf{k}0} = \frac{1}{\mathcal{D}}\left(\varepsilon - \varepsilon_k - \alpha^2\frac{e}{2\hbar}B\left[\left(\frac{\partial}{\partial k_x}G_{\mathbf{k}0}(\varepsilon)\right)k_x + \left(\frac{\partial}{\partial k_y}G_{\mathbf{k}0}(\varepsilon)\right)k_y\right]\right), \quad \text{(A7)}$$

$$\mathcal{G}_{\mathbf{k}x} = \frac{1}{\mathcal{D}}\left(\alpha k_y - \alpha\frac{e}{2\hbar}B\left[\left(\frac{\partial}{\partial k_y}G_{\mathbf{k}0}(\varepsilon)\right)[g_{\mathbf{k}0}(\varepsilon)]^{-1} - k_x\left[\left(\frac{\partial}{\partial k_x}[g_{\mathbf{k}0}(\varepsilon)]^{-1}\right)\left(\frac{\partial}{\partial k_y}G_{\mathbf{k}0}\right) - \left(\frac{\partial}{\partial k_y}[g_{\mathbf{k}0}(\varepsilon)]^{-1}\right)\left(\frac{\partial}{\partial k_x}G_{\mathbf{k}0}\right)\right]\right]\right), \quad \text{(A8)}$$

$$\mathcal{G}_{\mathbf{k}y} = \frac{1}{\mathcal{D}}\left(-\alpha k_x + \alpha\frac{e}{2\hbar}B\left[\left(\frac{\partial}{\partial k_x}G_{\mathbf{k}0}(\varepsilon)\right)[g_{\mathbf{k}0}(\varepsilon)]^{-1} + k_y\left[\left(\frac{\partial}{\partial k_x}[g_{\mathbf{k}0}(\varepsilon)]^{-1}\right)\left(\frac{\partial}{\partial k_y}G_{\mathbf{k}0}\right) - \left(\frac{\partial}{\partial k_y}[g_{\mathbf{k}0}(\varepsilon)]^{-1}\right)\left(\frac{\partial}{\partial k_x}G_{\mathbf{k}0}\right)\right]\right]\right), \quad \text{(A9)}$$

$$\mathcal{G}_{\mathbf{k}z} = \frac{i}{\mathcal{D}}\frac{e}{2\hbar}B\left(\left[\left(\frac{\partial}{\partial k_x}[g_{\mathbf{k}0}(\varepsilon)]^{-1}\right)\left(\frac{\partial}{\partial k_y}G_{\mathbf{k}0}(\varepsilon)\right) - \left(\frac{\partial}{\partial k_y}[g_{\mathbf{k}0}(\varepsilon)]^{-1}\right)\left(\frac{\partial}{\partial k_x}G_{\mathbf{k}0}(\varepsilon)\right)\right][g_{\mathbf{k}0}(\varepsilon)]^{-1}\right.$$
$$\left.-\alpha^2\left[k_x\left(\frac{\partial}{\partial k_y}G_{\mathbf{k}0}(\varepsilon)\right) - k_y\left(\frac{\partial}{\partial k_x}G_{\mathbf{k}0}(\varepsilon)\right)\right]\right), \quad \text{(A10)}$$

with $\mathcal{D} = (\varepsilon - E_+)(\varepsilon - E_-)$.

APPENDIX B: INTEGRATION OVER ε

As follows from Eqs. (27) and (28), we need to calculate 24 integrals over ε:

$$\mathcal{I}_{1,2} = \int d\varepsilon\frac{f(\varepsilon)}{\varepsilon+\mu-E_\pm+i\Gamma} = \mathcal{P}\int d\varepsilon\frac{f(\varepsilon)}{\varepsilon+\mu-E_\pm} - i\pi f(E_\pm), \quad \text{(B1)}$$

$$\mathcal{I}_{3,4} = \int d\varepsilon \frac{f(\varepsilon)}{\varepsilon+\mu-E_\pm-i\Gamma} = \mathcal{P}\int d\varepsilon \frac{f(\varepsilon)}{\varepsilon+\mu-E_\pm} + i\pi f(E_\pm), \tag{B2}$$

$$\mathcal{I}_{5,6} = \int d\varepsilon \frac{f(\varepsilon)}{(\varepsilon+\mu-E_\pm+i\Gamma)^2} = \mathcal{P}\int d\varepsilon \frac{\partial f(\varepsilon)}{\partial\varepsilon}\frac{1}{\varepsilon+\mu-E_\pm} - i\pi f'(E_\pm), \tag{B3}$$

$$\mathcal{I}_{7,8} = \int d\varepsilon \frac{f(\varepsilon)}{(\varepsilon+\mu-E_\pm-i\Gamma)^2} = \mathcal{P}\int d\varepsilon \frac{\partial f(\varepsilon)}{\partial\varepsilon}\frac{1}{\varepsilon+\mu-E_\pm} + i\pi f'(E_\pm), \tag{B4}$$

$$\mathcal{I}_{9,10} = \int d\varepsilon \frac{f(\varepsilon)}{(\varepsilon+\mu-E_\pm+i\Gamma)^3} = \frac{1}{2}\mathcal{P}\int d\varepsilon \frac{\partial^2 f(\varepsilon)}{\partial\varepsilon^2}\frac{1}{\varepsilon+\mu-E_\pm} - i\frac{\pi}{2}f''(E_\pm), \tag{B5}$$

$$\mathcal{I}_{11,12} = \int d\varepsilon \frac{f(\varepsilon)}{(\varepsilon+\mu-E_\pm-i\Gamma)^3} = \frac{1}{2}\mathcal{P}\int d\varepsilon \frac{\partial^2 f(\varepsilon)}{\partial\varepsilon^2}\frac{1}{\varepsilon+\mu-E_\pm} + i\frac{\pi}{2}f''(E_\pm), \tag{B6}$$

$$\mathcal{I}_{13,14} = \int d\varepsilon \frac{(\varepsilon+\mu-\varepsilon_k)f(\varepsilon)}{(\varepsilon+\mu-E_\pm+i\Gamma)^3} = \frac{1}{2}\mathcal{P}\int d\varepsilon[2f'(\varepsilon)+(\varepsilon+\mu-\varepsilon_k)f''(\varepsilon)]\frac{1}{\varepsilon+\mu-E_\pm} \mp i\frac{\pi}{2}[2f'(E_\pm)+\alpha k f''(E_\pm)], \tag{B7}$$

$$\mathcal{I}_{15,16} = \int d\varepsilon \frac{(\varepsilon+\mu-\varepsilon_k)f(\varepsilon)}{(\varepsilon+\mu-E_- -i\Gamma)^3} = \frac{1}{2}\mathcal{P}\int d\varepsilon[2f'(\varepsilon)+(\varepsilon+\mu-\varepsilon_k)f''(\varepsilon)]\frac{1}{\varepsilon+\mu-E_-} \mp i\frac{\pi}{2}[2f'(E_-)-\alpha k f''(E_-)], \tag{B8}$$

$$\mathcal{I}_{17,18} = \int d\varepsilon \frac{(\varepsilon+\mu-\varepsilon_k)f(\varepsilon)}{(\varepsilon+\mu-E_+\pm i\Gamma)^2} = \mathcal{P}\int d\varepsilon[f(\varepsilon)+(\varepsilon+\mu-\varepsilon_k)f'(\varepsilon)]\frac{1}{\varepsilon+\mu-E_+} \mp i\pi[f(E_+)+\alpha k f'(E_+)], \tag{B9}$$

$$\mathcal{I}_{19,20} = \int d\varepsilon \frac{(\varepsilon+\mu-\varepsilon_k)f(\varepsilon)}{(\varepsilon+\mu-E_-\pm i\Gamma)^2} = \mathcal{P}\int d\varepsilon[f(\varepsilon)+(\varepsilon+\mu-\varepsilon_k)f'(\varepsilon)]\frac{1}{\varepsilon+\mu-E_-} \mp i\pi[f(E_-)-\alpha k f'(E_-)], \tag{B10}$$

$$\mathcal{I}_{21,22} = \int d\varepsilon \frac{f(\varepsilon)(\varepsilon+\mu-\varepsilon_k)}{\varepsilon+\mu-E_\pm+i\Gamma} = \mathcal{P}\int d\varepsilon \frac{f(\varepsilon)(\varepsilon+\mu-\varepsilon_k)}{\varepsilon+\mu-E_\pm} \mp i\pi\alpha k f(E_\pm), \tag{B11}$$

$$\mathcal{I}_{23,24} = \int d\varepsilon \frac{f(\varepsilon)(\varepsilon+\mu-\varepsilon_k)}{\varepsilon+\mu-E_\pm-i\Gamma} = \mathcal{P}\int d\varepsilon \frac{f(\varepsilon)(\varepsilon+\mu-\varepsilon_k)}{\varepsilon+\mu-E_\pm} \pm i\pi\alpha k f(E_\pm). \tag{B12}$$

According to the above, we may write $M_{\rm orb}^{s(1)}$ and $M_{\rm orb}^{s(2)} + M_{\rm orb}^{s(3)}$ as follows:

$$\begin{aligned} M_{\rm orb}^{s(1)} &= i\frac{\alpha e}{4\pi\hbar}\int\frac{dk}{2\pi}\varepsilon_k^2\Big[\mathcal{I}_9-\mathcal{I}_{11}+\mathcal{I}_{12}-\mathcal{I}_{10}+\frac{1}{2\alpha k}(-\mathcal{I}_5+\mathcal{I}_7-\mathcal{I}_6+\mathcal{I}_8)+\frac{2}{(2\alpha k)^2}(\mathcal{I}_1-\mathcal{I}_3-\mathcal{I}_2+\mathcal{I}_4)\Big] \\ &\quad + i\frac{\alpha e}{4\pi\hbar}\int\frac{dk}{2\pi}\frac{\alpha k}{2}\varepsilon_k\Big[\mathcal{I}_9-\mathcal{I}_{11}-\mathcal{I}_{12}+\mathcal{I}_{10}+\frac{1}{2\alpha k}(-\mathcal{I}_5+\mathcal{I}_7+\mathcal{I}_6-\mathcal{I}_8)\Big], \end{aligned} \tag{B13}$$

$$\begin{aligned} M_{\rm orb}^{s(2)} + M_{\rm orb}^{s(3)} &= i\frac{\alpha e}{8\pi\hbar}\int\frac{dk}{2\pi}E_+\Big[\mathcal{I}_{13}-\mathcal{I}_{14}+\frac{1}{4\alpha^2k^2}(\mathcal{I}_{21}-\mathcal{I}_{22}-\mathcal{I}_{23}+\mathcal{I}_{24})-\frac{1}{2\alpha k}(\mathcal{I}_{17}-\mathcal{I}_{18})\Big] \\ &\quad + i\frac{\alpha e}{8\pi\hbar}\int\frac{dk}{2\pi}E_-\Big[\mathcal{I}_{16}-\mathcal{I}_{15}-\frac{1}{4\alpha^2k^2}(\mathcal{I}_{22}-\mathcal{I}_{21}-\mathcal{I}_{24}+\mathcal{I}_{23})-\frac{1}{2\alpha k}(\mathcal{I}_{19}-\mathcal{I}_{20})\Big]. \end{aligned} \tag{B14}$$

Taking into account explicit forms of the integrals $\mathcal{I}_n$, we find

$$\begin{aligned} M_{\rm orb}^{s(1)} &= i\frac{\alpha e}{4\pi\hbar}\int\frac{dk}{2\pi}\varepsilon_k^2\Big[\pi[f''(E_+)-f''(E_-)]-\frac{\pi}{\alpha k}[f'(E_+)+f'(E_-)]+\frac{\pi}{\alpha^2k^2}[f(E_+)-f(E_-)]\Big] \\ &\quad + \frac{\alpha e}{4\pi\hbar}\int\frac{dk}{2\pi}\frac{\alpha k}{2}\varepsilon_k\Big[\pi[f''(E_+)+f''(E_-)]-\frac{\pi}{\alpha k}[f'(E_+)-f'(E_-)]\Big], \end{aligned} \tag{B15}$$

$$\begin{aligned} M_{\rm orb}^{s(2)} + M_{\rm orb}^{s(3)} &= \frac{\alpha e}{8\pi\hbar}\int\frac{dk}{2\pi}E_+\Big[\pi[2f'(E_+)+\alpha k f''(E_+)]+\frac{\pi}{2\alpha k}[f(E_+)+f(E_-)]-\frac{\pi}{\alpha k}[f(E_+)+\alpha k f'(E_+)]\Big] \\ &\quad + \frac{\alpha e}{8\pi\hbar}\int\frac{dk}{2\pi}E_-\Big[-\pi[2f'(E_-)-\alpha k f''(E_-)]+\frac{\pi}{2\alpha k}[f(E_-)+f(E_+)]-\frac{\pi}{\alpha k}[f(E_-)-\alpha k f'(E_-)]\Big]. \end{aligned} \tag{B16}$$

From Eqs. (B15) and (B16), one finally arrives at Eqs. (27) and (28), respectively.

APPENDIX C: SPIN-RESOLVED ORBITAL MAGNETIZATION IN THE ZERO-TEMPERATURE LIMIT

In the low-temperature limit the orbital magnetization $M^s_{\rm orb}$ can be calculated analytically. To do this, let us write $M^s_{\rm orb}$ in the form [see Eqs. (27) and (28)]

$$M^{s(1)}_{\rm orb} = \sum_{i=1}^{5} \mathcal{M}_i, \tag{C1}$$

$$M^{s(2)}_{\rm orb} + M^{s(3)}_{\rm orb} = \sum_{i=6}^{10} \mathcal{M}_i, \tag{C2}$$

where

$$\begin{aligned}\mathcal{M}_1 &= \frac{e\alpha}{8\pi\hbar}\int dk\varepsilon_k^2[f''(E_+) - f''(E_-)] \\ &= \frac{e\hbar^3\alpha}{32\pi m\sqrt{m^2\alpha^2+2m\mu\hbar^2}}\left[\int dk\frac{\partial}{\partial k}\left(k^4\frac{\partial k}{\partial E_+}\right)\delta(k-k_+) - \int dk\frac{\partial}{\partial k}\left(k^4\frac{\partial k}{\partial E_-}\right)\delta(k-k_-)\right] = -\frac{2m\alpha^2}{4\pi\hbar^3},\end{aligned} \tag{C3}$$

$$\begin{aligned}\mathcal{M}_2 &= -\frac{e\hbar^3}{32\pi m^2}\int dkk^3[f'(E_+) + f'(E_-)] \\ &= \frac{e\hbar^3}{32\pi m\sqrt{m^2\alpha^2+2m\mu\hbar^2}}\int dkk^3[\delta(k-k_+) + \delta(k-k_-)] = \frac{em\alpha^2}{4\pi\hbar^3} + \frac{e\mu}{8\pi\hbar},\end{aligned} \tag{C4}$$

$$\mathcal{M}_3 = \alpha\frac{e}{8\pi\hbar}\int dk\frac{\varepsilon_k^2}{\alpha^2k^2}[f(E_+) - f(E_-)] = \frac{e\hbar^3}{32\pi m^2\alpha}\int_{k_-}^{k_+} dkk^2 = -\frac{em\alpha^2}{12\pi\hbar^3} - \frac{e\mu}{8\pi\hbar}, \tag{C5}$$

$$\begin{aligned}\mathcal{M}_4 &= \frac{e\alpha^2}{16\pi\hbar}\int dkk\varepsilon_k[f''(E_+) + f''(E_-)] \\ &= \frac{e\alpha^2\hbar}{32\pi\sqrt{m^2\alpha^2+2m\mu\hbar^2}}\left[\int dk\frac{\partial}{\partial k}\left(k^3\frac{\partial k}{\partial E_+}\right)\delta(k-k_+) + \int dk\frac{\partial}{\partial k}\left(k^3\frac{\partial k}{\partial E_-}\right)\delta(k-k_-)\right] = \frac{e\alpha^2 m}{8\pi\hbar^3},\end{aligned} \tag{C6}$$

$$\mathcal{M}_5 = -\frac{e\alpha}{16\pi\hbar}\int dk\varepsilon_k[f'(E_+) - f'(E_-)] = \frac{e\alpha\hbar}{32\pi\sqrt{m^2\alpha^2+2m\mu\hbar^2}}\int dkk^2[\delta(k-k_+) - \delta(k-k_-)] = -\frac{em\alpha^2}{8\pi\hbar^3}, \tag{C7}$$

$$\mathcal{M}_6 = -\mathcal{M}_5, \tag{C8}$$

$$\mathcal{M}_7 = \mathcal{M}_4, \tag{C9}$$

$$\begin{aligned}\mathcal{M}_8 &= \frac{e\alpha}{16\pi\hbar}\int dk\alpha k[f'(E_+) + f'(E_-)] \\ &= -\frac{e\alpha^2 m}{16\pi\hbar\sqrt{m^2\alpha^2+2m\mu\hbar^2}}\int dkk[\delta(k-k_+) + \delta(k-k_-)] = -\frac{m\alpha^2 e}{8\pi\hbar^3},\end{aligned} \tag{C10}$$

$$\begin{aligned}\mathcal{M}_9 &= \frac{e\alpha}{16\pi\hbar}\int dk\alpha^2k^2[f''(E_+) - f''(E_-)] \\ &= \frac{\alpha^3 me}{16\pi\hbar\sqrt{m^2\alpha^2+2m\mu\hbar^2}}\left[\int dk\frac{\partial}{\partial k}\left(k^2\frac{\partial k}{\partial E_+}\right)\delta(k-k_+) - \int dk\frac{\partial}{\partial k}\left(k^2\frac{\partial k}{\partial E_-}\right)\delta(k-k_-)\right] = 0,\end{aligned} \tag{C11}$$

$$\mathcal{M}_{10} = -\frac{e\alpha}{16\pi\hbar}\int dk[f(E_+) - f(E_-)] = -\frac{e\alpha}{16\pi\hbar}\int_{k_-}^{k_+} dk = \frac{\alpha^2 me}{8\pi\hbar^3}. \tag{C12}$$

Taking into account the above results, one arrives at formula (30).

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