

Systematic construction of spin liquids on the square lattice from tensor networks with SU(2) symmetry

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We elaborate a simple classification scheme of all rank-5 SU(2) spin rotational symmetric tensors according to (i) the onsite physical spin S , (ii) the local Hilbert space $V^{\otimes 4}$ of the four virtual (composite) spins attached to each site, and (iii) the irreducible representations of the C_{4v} point group of the square lattice. We apply our scheme to draw a complete list of all SU(2)-symmetric translationally and rotationally invariant projected entangled pair states (PEPS) with bond dimension $D \leq 6$. All known SU(2)-symmetric PEPS on the square lattice are recovered and simple generalizations are provided in some cases. More generally, to each of our symmetry class can be associated a $(D - 1)$ -dimensional manifold of spin liquids (potentially) preserving lattice symmetries and defined in terms of \mathcal{D} -independent tensors of a given bond dimension D . In addition, generic (low-dimensional) families of PEPS explicitly breaking either (i) particular point-group lattice symmetries (lattice nematics) or (ii) time-reversal symmetry (chiral spin liquids) or (iii) SU(2) spin rotation symmetry down to U(1) (spin nematics or Néel antiferromagnets) can also be constructed. We apply this framework to search for new topological chiral spin liquids characterized by well-defined chiral edge modes, as revealed by their entanglement spectrum. In particular, we show how the symmetrization of a double-layer PEPS leads to a chiral topological state with a gapless edge described by a SU(2)₂ Wess-Zumino-Witten model.

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I. INTRODUCTION

The study of quantum many-body entanglement has provided many key insights into the structure of quantum states of matter. Low-energy states of quantum lattice systems obey typically the so-called “area law” of the entanglement entropy [1–3]. As such, the area law is a huge constraint on the classes of states that capture the relevant properties of matter at low energies. A more refined study has shown that, in fact, those states are captured by the so-called tensor network states, or simply “tensor networks” [4,5]. Such states obey naturally the area law, and are at the basis of many theoretical and numerical developments in the study of quantum many-body systems and beyond [6]. Examples of such states are, e.g., matrix product states (MPS) [7], projected entangled pair states (PEPS) [8,9], and the multiscale entanglement renormalization Ansatz (MERA) [10]. These structures are, respectively, behind the so-called density matrix renormalization group algorithm (DMRG) for one-dimensional (1D) systems [11], PEPS algorithms for two-dimensional (2D) systems [12], and entanglement renormalization [13].

The description of quantum many-body states in terms of tensor networks has several advantages. Apart from naturally obeying the area law (and therefore capturing the correct expected entanglement behavior), tensor network (TN) states can also be manipulated efficiently (either exactly or approximately). Another advantage is the fact that both lattice and internal symmetries can be naturally incorporated. For instance, a description in terms of symmetric tensors [14,15] can lead to important computational advantages [16–20], and helps in the theoretical classification of phases of matter [21]. Moreover, gauge symmetries can also be naturally incorporated [22], hence offering a natural framework to describe lattice gauge theories [23,24].

In a seminal paper [25], Jiang and Ran made the first attempt to organize PEPS into crude classes distinguished

by short-range physics, related to the fractionalization of both onsite symmetries and space-group symmetries. In their work, the authors introduced (quite generally) the notion of projective symmetry group (PSG) for PEPS, enabling to deal *a priori* with gauge equivalence between tensors. Using lattice quantum numbers, the authors predicted a number of district classes for spin- $\frac{1}{2}$ spin liquids on the kagome lattice. More recently, a similar framework was applied to classify (trivial) spin-1 PEPS on the square lattice [26].

Our goal in this paper is to produce a simple classification scheme of all rank-5 SU(2) spin rotational symmetric tensors. We characterize the tensors according to three criteria: (i) the onsite physical spin S , (ii) the local Hilbert space $V^{\otimes 4}$ of the four virtual (composite) spins attached to each site, and (iii) the irreducible representations of the C_{4v} point group of the square lattice. Using this scheme, we produce explicit expressions for all SU(2)-symmetric translationally and rotationally invariant PEPS with bond dimension $D \leq 6$. As we shall see, one can recover all known SU(2)-symmetric PEPS on the square lattice as particular cases in our classification. Generically, to each of our symmetry class can be associated a $(D - 1)$ -dimensional manifold of spin liquids (potentially) preserving lattice symmetries and defined in terms of \mathcal{D} -independent tensors of a given bond dimension D . In addition, generic (low-dimensional) families of PEPS explicitly breaking particular point-group lattice symmetries (lattice nematics) and/or time-reversal symmetry (chiral spin liquids [27,28]) can also be constructed. Finally, we apply this framework to search for new topological chiral spin liquids characterized by well-defined chiral edge modes, as revealed by their entanglement spectrum, and show how the symmetrization of a given double-layer PEPS leads to a chiral topological state with a gapless edge described by a SU(2)₂ Wess-Zumino-Witten (WZW) model [29].

The paper is organized as follows: In Sec. II, we elaborate on the specifics of our classification [30], show how many

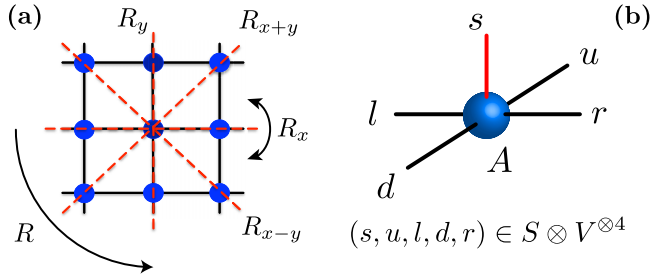


FIG. 1. (a) Square lattice invariant under C_{4v} point-group symmetries (reflection axes are shown). The generators of the point group are, e.g., the 90° rotation R , the reflection R_x , and the inversion $\mathcal{I} = R_x R_y$. (b) A generic rank-5 PEPS tensor with one physical index s and four virtual indices u, l, d, r . $(s, u, l, d, r) \in S \otimes V^{\otimes 4}$

remarkable states of matter fit into it, explain the procedure to construct spin liquids, and point out the connection to previous work. In Sec. III, we explain several attempts to obtain PEPS corresponding to higher-spin chiral topological quantum spin liquids. In particular, we show how a double-layer PEPS leads to a chiral topological state with a gapless edge described by a $SU(2)_2$ WZW model, which we characterize through its entanglement spectrum (ES). We also discuss how the PEPS tensor of such double-layer wave function can be expanded as a sum of “fundamental” $SU(2)$ -invariant PEPS tensors, which we also characterize. In Sec. IV, we wrap up our conclusions and outline several directions for future work. Finally, in Appendix A we review the calculation of entanglement spectrum (ES) from 2D PEPS, and in Appendixes B and C we provide the coefficients for the PEPS tensors of several remarkable and simple states, and of all tensors entering the decomposition of the double-layer CSL, respectively.

II. CLASSIFICATION OF $SU(2)$ -SYMMETRIC PEPS ON THE SQUARE LATTICE

A. Construction

We consider here a square lattice [as shown in Fig. 1(a)] with physical spin- S site degrees of freedom. Hence, $d = 2S + 1$ basis states are assigned on each site. Here, we explicitly consider translationally invariant states described by a PEPS built from a single tensor A , as the one shown in Fig. 1(b). Each physical site has four virtual spins attached labeled on the figure by u, l, d , and r along the up, left, down, and right directions, respectively. The virtual states $|v_\alpha\rangle$ belong to some representation V of $SU(2)$ of total dimension D which, generically, is a direct sum $V = \oplus V_i$ of \mathcal{N} irreducible representations (IRREP) V_i of $SU(2)$, each of partial dimension $2V_i + 1$ and $D = \sum_{i=1}^{\mathcal{N}} (2V_i + 1)$.

The corresponding translationally invariant PEPS [3] is obtained by assigning the same tensor A on every physical site. Physically, the site tensor A simply encodes a projector that maps the virtual space $V^{\otimes 4}$ onto all $2S + 1$ components of the physical spin S . From the bond point of view, every pair of the nearest-neighbor (NN) virtual spins is projected to a block-diagonal *virtual spin-singlet state*. By construction, the obtained wave function is a global spin singlet, i.e., invariant under $SU(2)$ rotations. For a bipartite lattice as the square lattice one can perform a simple spin rotation (by π

around the Y -spin axis) on all sites of a given sublattice that transforms the virtual bond singlets into diagonal maximally entangled NN pair states $|\mathcal{S}\rangle = \sum_{\alpha=1}^D |v_\alpha v_\alpha\rangle$. In this way, the $SU(2)$ -invariant PEPS becomes a simple contraction of the tensor network of the A 's.

Our construction can also be easily generalized to states that are not $SU(2)$ singlets, i.e., have a total (average) spin component. For this, one could eventually build up tensors that transform under $S \neq 0$ IRREPS of $SU(2)$ (sometimes dubbed “covariant” states). In this paper, however, we consider only “invariant” states under $SU(2)$, i.e., singlets, which for simplicity we also call “symmetric.”

The $2S + 1$ components A_s of a tensor A encode the projectors $P_s : V^{\otimes 4} \rightarrow |s\rangle \equiv |S, S_z = s\rangle$ physical state. Hence, the problem of enumerating all $SU(2)$ -invariant PEPS reduces to the finding of all (orthogonal) projectors that map any virtual space $V^{\otimes 4}$ onto any spin- S Hilbert space. Reversely, it amounts to enumerate all D^4 orthogonal spin- S wave functions that can be constructed out of the $(\oplus V_i)^{\otimes 4}$ basis states. In other words, we shall use the one-to-one correspondence between projectors and wave functions and extract the tensor components from the wave functions. One can write the physical state $|s\rangle$ as

$$|s\rangle = \sum_{\alpha_1, \alpha_2, \alpha_3, \alpha_4} A_s(\alpha_1, \alpha_2, \alpha_3, \alpha_4) |\alpha_1, \alpha_2, \alpha_3, \alpha_4\rangle, \quad (1)$$

in terms of the D^4 virtual basis states $|\alpha_1, \alpha_2, \alpha_3, \alpha_4\rangle$ and of the tensor elements $A_s(\alpha_1, \alpha_2, \alpha_3, \alpha_4)$. Since we can always find a set of D^4 orthogonal wave functions, the corresponding basic tensors fulfill the “orthonormalization” property

$$\sum_{\alpha_1, \alpha_2, \alpha_3, \alpha_4} [A_s(\alpha_1, \alpha_2, \alpha_3, \alpha_4)]^* B_{s'}(\alpha_1, \alpha_2, \alpha_3, \alpha_4) = \delta_{ss'} \delta_{AB}, \quad (2)$$

where the left-hand side defines some tensor inner product $A_s \cdot B_{s'}$. We have performed such a program *analytically* using *Mathematica* for all possible virtual spaces with $D \leq 6$. To reduce the cost of the computation, it is advantageous to use spin and (lattice) point-group symmetries. First, it is convenient to decompose the virtual space $V^{\otimes 4}$ into all disconnected subspaces given by the occupations $n_{\text{occ}} = \{n_1, \dots, n_{\mathcal{N}}\}$ of the \mathcal{N} spins V_i ($\sum_{i=1}^{\mathcal{N}} n_i = 4$), each subspace providing a different class of tensors. Second, we use the S_z quantum number of the (physical) wave function. In fact, we start by computing all $S_z = S$ wave functions (or equivalently all projectors onto the maximum $S_z = S$ subspace) and then apply the spin-lowering operator S_- written in the virtual basis states. Simultaneously, we classify the various spin- S wave functions according to their point symmetry, i.e., according to the representations of the C_{4v} point group: A_1 (s wave), B_1 ($d_{x^2-y^2}$ wave), E (doubly degenerate p wave), A_2 (g wave), and B_2 (d_{xy} wave). To accomplish such a purpose, we simply need to diagonalize, in the space spanned by the $(\oplus V_i)^{\otimes 4}$ virtual basis attached to a given site, simultaneously (i) the total spin operator, (ii) the 90° rotation operator R , (iii) the reflection symmetry R_x operator, and (iv) the inversion $\mathcal{I} = R_x R_y$ operator [see Fig. 1(a)]. In practice, to perform this task efficiently, we have constructed

in the $V^{\otimes 4}$ basis, the combined (non-Hermitian) operator

$$\mathcal{O}_{\sigma,\sigma_z,\rho,\delta,\nu} = \sigma \mathbf{S}^2 + \sigma_z S_z + \rho R + \delta R_x + \nu D, \quad (3)$$

where the diagonal operator D has specific diagonal elements characterizing each n_{occ} sector. The (real) coefficients $\sigma, \sigma_z, \rho, \delta, \nu$ are all chosen of very different magnitudes, e.g., $10^8, 10^6, 10^4, 10^2, 1$, in order to sort out the various eigenvalues (of order 1). Note that tensors belonging to the A_1 , B_1 , A_2 , and B_2 symmetry classes are purely real while the E -symmetric tensors are intrinsically complex and come in complex conjugated pairs.

As a simple example, let us consider the case $V = \frac{1}{2}$ which contains $2^4 = 16$ basis states, all in the unique $n_{\text{occ}} = \{4\}$ sector (V here is a simple IRREP). Diagonalizing $\mathcal{O}_{\sigma,\sigma_z,\rho,\delta,\nu}$ (omitting the D part) gives 16 states (or tensor components) which can be grouped into two singlets ($S = 0$), three triplets ($S = 1$), and one quintuplet ($S = 2$), in agreement with the decomposition $(\frac{1}{2})^{\otimes 4} = (0 \oplus 1)^{\otimes 2} = 2(0) \oplus 3(1) \oplus (2)$. The $S = 0$ outcomes correspond to ‘‘classical’’ TN, not considered afterwards. Inspection of the eigenvalues associated to R and R_x reveals that one of the triplet states (or tensors) has B_1 symmetry, while the other two form a complex conjugate pair of E symmetry. On the other hand, the unique $S = 2$ tensor is fully symmetric (A_1 IRREP) and corresponds to the spin-2 AKLT state (see below).

Using the method described above, we have generated all SU(2)-invariant tensors up to $D = 6$. A subset of the list of tensors with $D \leq 5$ and $\frac{1}{2} \leq S \leq 2$ is displayed in Table I, while the complete list for $D \leq 6$ is given as Supplemental Material [30]. The columns correspond to different physical spins (limited to $S \leq 2$). Vertically, the tensors are classified according to the representation $V = \oplus V_i$ of the virtual spins. Each line corresponds to a subclass given by a disconnected set of (virtual) basis states characterized by the occupation numbers n_{occ} of the \mathcal{N} spins V_i .

It should be mentioned that all the tensors produced in our systematic procedure do not lead necessarily to different PEPS due to remaining gauge degrees of freedom. Indeed, imposing SU(2) and point-group symmetries does not completely fix the gauge and some freedom remains. Typically, tensors which have identical nonzero tensor elements up to a sign are gauge equivalent as can be checked case by case. Table I shows explicitly all gauge-equivalent tensors.

B. Remarkable PEPS in this scheme

Some of the low- D PEPS obtained using our systematic construction and listed in Table I have already been introduced in recent literature and/or correspond to well-known states of matter. Let us briefly list a few of them below, together with some simple generalizations (see tensor expressions in Appendix B).

1. Spin- S , S even integer, AKLT states

The simplest 2D Affleck-Kennedy-Lieb-Tasaki (AKLT) [31] state is obtained by decomposing a physical spin 2 on each site into four virtual spin- $\frac{1}{2}$ spins, pairing every NN virtual spin into singlet and, finally, applying the projector $P : \frac{1}{2}^{\otimes 4} \rightarrow 2$

onto the fully symmetric $S = 2$ subspace on every site. This state is known to have short-range correlations, and is given by the A_1 tensor of the $V = \frac{1}{2}$ line/ $S = 2$ column of Table I. This construction can be straightforwardly extended to higher physical spins by projecting four virtual spin- $\frac{k}{2}, k \in \mathbb{N}^*$, onto the fully symmetric spin- $S, S = 2k$, physical subspace. This is realized by the unique $V = \frac{k}{2}, S = 2k, A_1$ tensor of dimension $D = k + 1$ (see Appendix B for $k = 1$ and Supplemental Material [30] for $k = 1, \dots, 5$). The 2D spin- S AKLT states can serve as useful examples of trivial (featureless) states or states with symmetry-protected topological order [32] for k even integer ($S = 4p$) or k odd integer ($S = 4p + 2$), respectively.

2. Spin- S , S odd integer, featureless paramagnets

Starting with four $V = \frac{1}{2}$ virtual spins attached to each site and paired up into NN singlets, as in the AKLT construction, but projecting them onto $S = 1$ onsite physical spins gives rise to a spin-1 featureless paramagnet [33], also with short-range correlations. This is given by the $V = \frac{1}{2}, S = 1, B_1$ tensor of Table I. This construction can be straightforwardly generalized to higher physical spins by simply projecting four virtual spin- $\frac{k}{2}, k \in \mathbb{N}^*$, onto spin- $S, S = 2k - 1$, physical subspace. This is always realized by the unique $V = \frac{k}{2}, S = 2k - 1, B_1$ tensor of dimension $D = k + 1$ (see Appendix B for $k = 1$ and Supplemental Material [30] for $k = 1, \dots, 5$). We believe such states with $S > 1$ are also featureless paramagnets. Note that, for all k , the B_1 tensor comes always in pair with a (complex) E tensor which leads to a *real* wave function and might also have interesting properties.

3. Spin- $\frac{1}{2}$ resonating valence bond state

The resonating valence bond (RVB) state is a spin- $\frac{1}{2}$ spin liquid defined by an equal-weight superposition of all nearest-neighbor (NN) singlet configurations [34]. It is exactly given by the $V = \frac{1}{2} \oplus 0, n_{\text{occ}} = \{1, 3\}, A_1$ (named $\mathcal{A}_1^{(1)}$ from now on) or B_1 tensors (named $\mathcal{B}_1^{(1)}$ from now on) of the $S = \frac{1}{2}$ column of Table I. The NN RVB state was shown to be a \mathbb{Z}_2 topological spin liquid on the kagome lattice [35–37]. On the square lattice it exhibits an extended U(1) gauge symmetry and is critical [36,38] (see below for the discussion of gauge symmetry).

4. Long-range spin- $\frac{1}{2}$ RVB state

The long-range (LR) RVB state is obtained by assuming a distribution of longer-range singlet bonds beyond NN (yet still connecting two different sublattices). It is obtained within the PEPS formalism by linearly combining the $V = \frac{1}{2} \oplus 0, n_{\text{occ}} = \{3, 1\}, A_1$ (named $\mathcal{A}_1^{(2)}$ from now on) tensor with the previous $\mathcal{A}_1^{(1)}$ NN RVB tensor

$$\mathcal{A}_{\text{LR RVB}} = \lambda_1 \mathcal{A}_1^{(1)} + \lambda_2 \mathcal{A}_1^{(2)}, \quad (4)$$

with $\lambda_1, \lambda_2 \in \mathbb{R}$. Alternatively, one can use the gauge-equivalent B_1 tensors, named $\mathcal{B}_1^{(1)}$ and $\mathcal{B}_1^{(2)}$. The singlet bond distribution is controlled by the relative (real) weight between the two tensors. Such a spin liquid *Ansatz* turned

TABLE I. List of all SU(2)-symmetric basic rank-5 tensors for physical spin $\frac{1}{2} \leq S \leq 2$ and bond dimension $D \leq 5$. The virtual (physical) spin degrees of freedom V (S) are displayed vertically (horizontally). Whenever the virtual spin is a direct sum $V = \oplus V_i$, we decompose the virtual space $V^{\otimes 4}$ into all subspaces given by the occupations n_{occ} of the spins V_i . Note that, when two tensor classes are related by “color exchange,” we keep only one of them in the list (e.g., for $V = \frac{1}{2} \oplus 0 \oplus 0$ $n_{\text{occ}} = \{1, 2, 1\}$ is omitted since it is the “color conjugate” of $n_{\text{occ}} = \{1, 1, 2\}$). Tensors are labeled according to their C_{4v} point symmetry, A_1 (s wave), B_1 ($d_{x^2-y^2}$ wave), E (doubly degenerate p wave), A_2 (g wave), B_2 (d_{xy} wave). Subscripts $(a), (b)$, etc., are used to differentiate nonequivalent tensors of the same class. Gauge-equivalent tensors (i.e., giving rise to the same PEPS) are listed between brackets, next to their gauge-related partners. Tensors giving rise to simple known wave functions are highlighted in boldface (see text for a description and Supplemental Material [30] for expressions).

$V \setminus S$	1/2	1	3/2	2
$\frac{1}{2}$		$B_1 E$		A_1
$\frac{1}{2} \oplus 0$				
$n_{\text{occ}} = \{1, 3\}$	$A_1[B_1] E$			
$n_{\text{occ}} = \{2, 2\}$		$A_1^{(a)} A_1^{(b)} [B_1] E B_2$		
$n_{\text{occ}} = \{3, 1\}$	$A_1[B_1] E^{(a,b)} A_2[B_2]$		$A_1[B_1] E$	
1		$B_1 E^{(a,b)} A_2$		$A_1^{(a)} [B_1] A_1^{(b)} E B_2$
$\frac{1}{2} \oplus 0 \oplus 0$				
$n_{\text{occ}} = \{1, 1, 2\}$	$A_1^{(a,b)} [B_1^{(a,b)}]$ $E^{(a-c)} A_2[B_2]$			
$n_{\text{occ}} = \{2, 1, 1\}$		$A_1^{(a,b)} [B_1^{(a,b)}] E^{(a-c)} A_2[B_2]$		
$\frac{1}{2} \oplus \frac{1}{2}$				
$n_{\text{occ}} = \{1, 3\}$		$A_1^{(a,b)} [B_1^{(a,b)}] E^{(a-c)} A_2[B_2]$		$A_1[B_1] E$
$n_{\text{occ}} = \{2, 2\}$		$A_1^{(a,b)} [B_1^{(a,b)}] B_1^{(c)} E^{(a-e)} A_2^{(a)} A_2^{(b)} [B_2]$		$A_1^{(a)} A_1^{(b)} [B_1] E B_2$
1 \oplus 0				
$n_{\text{occ}} = \{1, 3\}$		$A_1[B_1] E$		
$n_{\text{occ}} = \{2, 2\}$		$B_1[A_2] E^{(a,b)}$		$A_1^{(a)} [B_2] A_1^{(b)} [B_1] E$
$n_{\text{occ}} = \{3, 1\}$		$A_1^{(a,b)} [B_1^{(a,b)}] E^{(a-c)} A_2[B_2]$		$A_1[B_1] E$
$\frac{3}{2}$		$B_1^{(a)} B_1^{(b)} E^{(a-c)} A_2$		$A_1^{(a-c)} B_1 E^{(a)} E^{(b)} A_2[B_2^{(a)}] B_2^{(b)}$
$\frac{1}{2} \oplus 0 \oplus 0 \oplus 0$				
$n_{\text{occ}} = \{1, 1, 1, 1\}$	$A_1^{(a-c)} [B_1^{(a-c)}] E^{(a-f)}$ $A_2^{(a,b)} [B_2^{(a,b)}]$			
$\frac{1}{2} \oplus \frac{1}{2} \oplus 0$				
$n_{\text{occ}} = \{1, 2, 1\}$	$A_1^{(a-c)} [B_1^{(a-c)}] E^{(a-f)}$ $A_2^{(a-c)} [B_2^{(a-c)}]$		$A_1^{(a,b)} [B_1^{(a,b)}] E^{(a-c)} A_2[B_2]$	
$n_{\text{occ}} = \{1, 1, 2\}$		$A_1^{(a,b)} [B_1^{(a,b)}] E^{(a-c)} A_2[B_2]$		
1 \oplus 0 \oplus 0				
$n_{\text{occ}} = \{1, 1, 2\}$		$A_1^{(a,b)} [B_1^{(a,b)}] E^{(a-c)} A_2[B_2]$		
$n_{\text{occ}} = \{2, 1, 1\}$		$A_1[B_1] E^{(a-c)} A_2^{(a,b)} [B_2^{(a,b)}]$		$A_1^{(a,b)} [B_1^{(a,b)}] E^{(a-c)} A_2[B_2]$
1 \oplus $\frac{1}{2}$				
$n_{\text{occ}} = \{1, 3\}$	$A_1^{(a,b)} [B_1^{(a,b)}] E^{(a-c)} A_2[B_2]$		$A_1^{(a,b)} [B_1^{(a,b)}] E^{(a-c)} A_2[B_2]$	
$n_{\text{occ}} = \{2, 2\}$		$A_1^{(a-c)} [B_1^{(a-c)}] A_1^{(d,e)} E^{(a-e)}$		$A_1^{(a,b)} [B_1^{(a,b)}] B_1^{(c)} E^{(a-e)}$
$n_{\text{occ}} = \{3, 1\}$	$A_1^{(a,b)} [B_1^{(a,b)}] E^{(a-d)}$ $A_2^{(a,b)} [B_2^{(a,b)}]$	$A_2^{(a,b)} [B_2^{(a,b)}] B_2^{(c,d)}$	$A_1^{(a-c)} [B_1^{(a-c)}] E^{(a-e)}$ $A_2^{(a,b)} [B_2^{(a,b)}]$	$A_2[B_2] B_2$
$\frac{3}{2} \oplus 0$				
$n_{\text{occ}} = \{1, 3\}$			$A_1[B_1] E$	
$n_{\text{occ}} = \{2, 2\}$		$A_1^{(a)} A_1^{(b)} [B_1] E B_2$		$B_1[A_2] E^{(a,b)}$
$n_{\text{occ}} = \{3, 1\}$	$A_1[B_1] E^{(a,b)} A_2[B_2]$		$A_1^{(a,b)} [B_1^{(a,b)}] E^{(a-d)} A_2^{(a,b)} [B_2^{(a,b)}]$	
2		$B_1^{(a,b)} [A_2^{(a,b)}] E^{(a-d)}$		$A_1^{(a,b)} [B_1^{(a,b)}] A_1^{(c,d)}$ $E^{(a-c)} A_2[B_2^{(a)}] B_2^{(b,c)}$

out to be an excellent variational state for the frustrated J_1 - J_2 antiferromagnetic Heisenberg model on the square lattice [39]. It is interesting to notice that this *Ansatz* is in fact the most general $D = 3$ SU(2)-invariant PEPS.

5. NN fermionic spin- $\frac{1}{2}$ RVB

The NN fermionic-RVB (fRVB) is defined as an equal-weight superposition of dimer coverings where each (centrosymmetric) dimer is written in the fermionic representation.

It can be rewritten as a spin- $\frac{1}{2}$ NN RVB state where, e.g., vertical dimers are assigned a complex factor i providing a completely different sign structure [40] than the above NN RVB state. This (real) wave function is given by the unique $S = \frac{1}{2}$, $V = \frac{1}{2} \oplus 0$, $n_{\text{occ}} = \{1,3\}$, complex E tensor [41].

6. Generalized spin- S NN RVB

The spin-1 RVB state can be obtained by attaching a single $S = 1$ virtual spin on every site (accompanied by 3 spin 0). All NN virtual spins $\frac{1}{2}$ are again paired up into singlets which resonate. The $S = 1$, $V = 1 \oplus 0$, $n_{\text{occ}} = \{1,3\}$, A_1 (or B_1) tensor corresponds exactly to such a spin liquid. This scheme can be generalized to any spin- S NN RVB and is always described by a single $V = S \oplus 0$, $n_{\text{occ}} = \{1,3\}$, A_1 (or B_1) tensor. The cases corresponding to $S = 1$ and $\frac{3}{2}$ are highlighted in Table I and the corresponding tensors are given in Appendix B (see Supplemental Material [30] for all physical spin S up to $S = 2$).

7. Spin- S resonating AKLT loop (RAL) state

As shown by Li *et al.* [42], the spin-1 RAL state involves two virtual spin $\frac{1}{2}$ and two virtual spin 0 attached to every site, i.e., the virtual subspace is $V = \frac{1}{2} \oplus 0$, $n_{\text{occ}} = \{2,2\}$. Physically, NN virtual spin $\frac{1}{2}$ are paired up into singlets (as in the RVB state) and all virtual spins are then projected locally onto physical spins 1 to produce AKLT loops. The two $S = 1$, $V = \frac{1}{2} \oplus 0$, $n_{\text{occ}} = \{2,2\}$, $A_1^{(a)}$ and $A_1^{(b)}$ tensors encode the two possible site configurations of the loops with 180° or 90° angles. The RAL state on the square lattice is critical since the dimer-dimer correlations decay as a power law [42]. Since the 1D AKLT chain can be extended to higher physical spin (see above for the 2D case), it is easy to generalize the RAL to a gas of resonating spin- S AKLT chains, for all S integer. It is given by the only two $V = \frac{S}{2} \oplus 0$ ($D = S + 2$), $n_{\text{occ}} = \{2,2\}$, $A_1^{(a)}$ and $A_1^{(b)}$ tensors. The cases corresponding to $S = 1$ and 2 are highlighted in Table I and the corresponding $S = 1$ tensors are given in Appendix B (see Supplemental Material [30] for all integer physical spin S up to $S = 4$).

8. Spin- $\frac{1}{2}$ chiral spin liquid

The chiral spin liquid (CSL) is obtained by linearly combining the two previous $A_1^{(1)}$ and $A_1^{(2)}$ (with real coefficients) and the $V = \frac{1}{2} \oplus 0$, $n_{\text{occ}} = \{3,1\}$, A_2 tensor (named A_2 from now on) with a pure-imaginary coefficient [43,44]. The PEPS obtained from the resulting tensor [see Fig. 2(a)]

$$\mathcal{A}_{\text{chiral}} = \lambda_1 A_1^{(1)} + \lambda_2 A_1^{(2)} + i\lambda_c A_2, \quad (5)$$

with $\lambda_1, \lambda_2, \lambda_c \in \mathbb{R}$, breaks time-reversal symmetry (provided $\lambda_2 \lambda_c \neq 0$) and transforms into its complex conjugate state under any of the reflection symmetries of Fig. 1(a). It exhibits clear $SU(2)_1$ edge modes although there is some evidence for critical (singlet) bulk correlations.

C. Constructing spin liquids and beyond

1. Generic spin liquids

From a few of the previous examples, we see that tensors can be linearly combined to give new interesting states. In

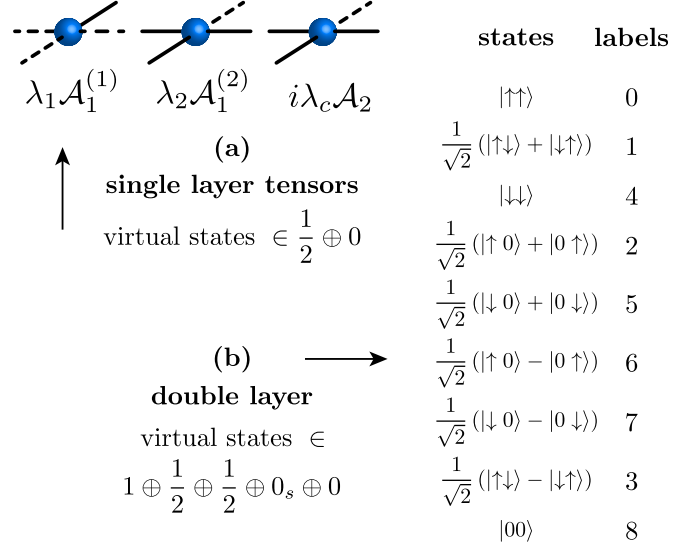


FIG. 2. (a) The three tensors of the spin- $\frac{1}{2}$ CSL involving the three virtual states of the $\frac{1}{2} \oplus 0$ (spin) representation: $|\uparrow\rangle$ and $|\downarrow\rangle$ on the full lines, and $|0\rangle$ on the dotted lines. (b) The natural basis of virtual states of the double-layer tensor $|\uparrow\uparrow\rangle$, $|\uparrow\downarrow\rangle$, $|\downarrow\uparrow\rangle$, and $|\downarrow\downarrow\rangle$ ($\in \frac{1}{2} \otimes \frac{1}{2}$, spin $\frac{1}{2}$ on top and bottom layers), $|\uparrow 0\rangle$, $|\downarrow 0\rangle$, $|0 \uparrow\rangle$, and $|0 \downarrow\rangle$ ($\in \frac{1}{2} \oplus \frac{1}{2}$, spin $\frac{1}{2}$ on top or bottom layer), and $|00\rangle$ ($\in 0$) is transformed (by a simple unitary transformation) into symmetric/antisymmetric states w.r.t to the exchange of layers.

fact, adding \mathcal{D} (real) tensors $T^{(a)}$ as $\sum_{a=1}^{\mathcal{D}} \lambda_a T^{(a)}$ (involving \mathcal{D} real coefficients λ_a) belonging to the same “class,” i.e., characterized by the same physical (S) and virtual (V) degrees of freedom and by the same IRREP of the point group C_{4v} , will lead to a $(\mathcal{D} - 1)$ -dimensional family of completely symmetric spin liquids which (potentially) do not break any symmetry, neither $SU(2)$ nor point-group symmetries. The numbers \mathcal{D} of tensors which can be combined in each $SU(2)$ (V, S) symmetry class to give rise to fully symmetric spin- S spin liquids are given in Table II for $D \leq 6$ and $S \leq 2$. Note that the counting of tensors of a given bond dimension includes all those of smaller bond dimensions which can be combined in each class. Note also that, for a given linear combination of tensors, there is *a priori* no guarantee that all correlations remain short range in such a symmetric state and the absence of spontaneous symmetry breaking in the thermodynamic limit should, in principle, always be verified. We observe that the typical dimensions of the PEPS families do not grow too fast, from $\mathcal{D} \sim 3-10$ for $D = 4$ up to $\mathcal{D} \sim 10-40$ for $D = 6$. Also, it is interesting to notice that a subset of the symmetry classes does not provide a variational representation of half-integer spin S .

2. Lattice nematics

If the \mathcal{D} $T^{(a)}$ tensors belong to (at least) two different IRREP of the point group (while still involving the same virtual and physical degrees of freedom), the resulting PEPS will explicitly break the point-group symmetry. For example, combining A_1 and B_1 tensors, or A_2 and B_2 tensors, will produce a nematic state where vertical and horizontal directions will become nonequivalent (e.g., observables will acquire different mean values). As a concrete example, let us consider the $V = 1$, $S = 2$, gauge-equivalent $A_1^{(a)}$ and B_1 tensors of Table I. It is

TABLE II. Sets of the numbers $\mathcal{D}_1/\mathcal{D}_2/\mathcal{D}_3/\mathcal{D}_4/\mathcal{D}_5$ of basic tensors belonging to the five A_1, B_1, A_2, B_2 and (doubly degenerate) E IRREP of C_{4v} , respectively, which can be combined in each $SU(2)$ (V, S) symmetry class to give rise to fully symmetric spin- S spin liquids, for $D \leq 6$ and $\frac{1}{2} \leq S \leq 2$. Note each class defined by a direct sum $V = \oplus V_i$ of \mathcal{N} $SU(2)$ IRREP includes all basic tensors of all subclasses defined by $n_{\text{occ}} = (n_1, n_2, \dots, n_{\mathcal{N}})$ with $0 \leq n_i \leq 4$ and $\sum_i n_i = 4$. For instance, the $V = \frac{1}{2} \oplus \frac{1}{2} \oplus 0$ ($D = 5$) class includes the $V = \frac{1}{2}$ ($D = 2$), $V = \frac{1}{2} \oplus 0$ ($D = 3$), and $V = \frac{1}{2} \oplus \frac{1}{2}$ ($D = 4$) classes.

$V \setminus S$	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$\frac{1}{2}$	0/0/0/0/0	0/1/0/0/1	0/0/0/0/0	1/0/0/0/0
$\frac{1}{2} \oplus 0$	2/2/1/1/3	2/2/0/1/2	1/1/0/0/1	1/0/0/0/0
1	0/0/0/0/0	0/1/1/0/2	0/0/0/0/0	2/1/0/1/1
$\frac{1}{2} \oplus 0 \oplus 0$	8/8/4/4/12	6/5/1/3/6	2/2/0/0/2	1/0/0/0/0
$\frac{1}{2} \oplus \frac{1}{2}$	0/0/0/0/0	6/9/4/3/13	0/0/0/0/0	6/3/0/1/3
$1 \oplus 0$	0/0/0/0/0	3/5/3/1/8	0/0/0/0/0	5/3/1/3/4
$\frac{3}{2}$	0/0/0/0/0	0/2/1/0/3	0/0/0/0/0	3/1/1/2/2
$\frac{1}{2} \oplus 0 \oplus 0 \oplus 0$	21/21/12/12/33	12/10/3/6/13	3/3/0/0/3	1/0/0/0/0
$\frac{1}{2} \oplus \frac{1}{2} \oplus 0$	10/10/8/8/18	12/13/5/6/18	6/6/2/2/8	6/3/0/1/3
$1 \oplus 0 \oplus 0$	0/0/0/0/0	11/14/9/6/23	0/0/0/0/0	10/7/3/6/10
$1 \oplus \frac{1}{2}$	4/4/3/3/7	5/5/3/4/8	5/5/3/3/8	5/4/2/2/6
$\frac{3}{2} \oplus 0$	1/1/1/1/2	2/3/1/1/4	3/3/2/2/5	3/2/2/2/4
2	0/0/0/0/0	0/2/2/0/4	0/0/0/0/0	4/2/1/3/3
$\frac{1}{2} \oplus 0 \oplus 0 \oplus 0 \oplus 0$	44/44/28/28/72	20/17/6/10/23	4/4/0/0/4	1/0/0/0/0
$\frac{1}{2} \oplus \frac{1}{2} \oplus 0 \oplus 0$	28/28/20/20/48	25/24/11/14/35	12/12/4/4/16	6/3/0/1/3
$\frac{1}{2} \oplus \frac{1}{2} \oplus \frac{1}{2}$	0/0/0/0/0	33/39/24/21/63	0/0/0/0/0	21/15/3/6/18
$1 \oplus 0 \oplus 0 \oplus 0$	0/0/0/0/0	27/31/22/18/53	0/0/0/0/0	17/13/6/10/19
$1 \oplus \frac{1}{2} \oplus 0$	11/11/8/8/19	13/13/8/9/21	11/11/7/7/18	10/8/4/5/12
$1 \oplus 1$	0/0/0/0/0	9/13/13/9/26	0/0/0/0/0	19/15/7/11/22
$\frac{3}{2} \oplus 0 \oplus 0$	2/2/2/2/4	6/6/2/3/8	10/10/6/6/16	4/4/5/4/9
$\frac{3}{2} \oplus \frac{1}{2}$	0/0/0/0/0	7/12/8/5/20	0/0/0/0/0	15/10/7/10/17
$2 \oplus 0$	0/0/0/0/0	1/4/5/2/9	0/0/0/0/0	10/7/3/6/10
$\frac{5}{2}$	0/0/0/0/0	0/3/2/0/5	0/0/0/0/0	5/2/2/4/4

likely that these tensors produce a paramagnet similar to the $S = 2$ AKLT state, although with gapped edge states. The linear combination $A_1^{(a)} + B_1$ ($A_1^{(a)} - B_1$) of the two tensors gives a product of decoupled vertical (horizontal) spin-2 AKLT chains times a collection of independent horizontal (vertical) NN dimers (constructed from pairs of all the remaining virtual spin 1 not involved in the chains). Any partial superposition like $\cos \theta A_1^{(a)} + \sin \theta B_1, \theta \in]0, \pi/2[$ will give a lattice nematic state interpolating between the isotropic paramagnet and the array of AKLT chains.

3. Breaking $SU(2)$ symmetry down to $U(1)$: Spin nematics and Néel antiferromagnets

By breaking the global $SU(2)$ spin rotation invariance down to $U(1)$ one can construct, within our framework, two enlarged families of anisotropic quantum magnets. If TR symmetry and space-group symmetry are independently conserved, one obtains anisotropic spin nematics [45] for which the spin Z axis becomes nonequivalent from the two equivalent X and Y spin axis. If the combination of TR with a unit translation is conserved, one gets Néel-type quantum magnets with a

finite staggered magnetization. In order to achieve this goal, one should remember that our PEPS are defined in a physical basis where all spins on the B sublattice have been rotated by π . Each S_z component (over the $d = 2S + 1$ components) of a given tensor hence contributes to a finite amplitude of the (physical) staggered magnetization $\tilde{S}_z^{\text{stag}} = S_z$ in the original unrotated basis.

The procedure to construct anisotropic magnets is therefore simple: (i) One sorts out all tensors according to the virtual space V (as before) and to some maximum value S_{max} of $|S_z|$, defining the physical Hilbert space $\mathcal{S}_z \in [-S_{\text{max}}, S_{\text{max}}]$, by merging classes of different $SU(2)$ spin S with the same orbital symmetry (i.e., IRREP of C_{4v}). (ii) One groups all tensors into $\mathcal{S}_z = \pm |S_z|$ pairs of tensors $T_{\pm}^{(a)}$ (where a labels the pairs) related by the $S_z \leftrightarrow -S_z$ symmetry (for $S_z \neq 0$). (iii) One then constructs the linear superposition of the (normalized) onsite tensors for each local $\mathcal{S}_z = s |S_z|$ physical index as $\sum_{a,s} \lambda_{a,s} T_s^{(a)}$. Generically, such a family of PEPS exhibits a finite staggered magnetization (in the original unrotated basis) as in a Néel state unless we further impose $\lambda_{a,s} = \lambda_{a,-s}$ to construct nematic states. Note that long-range order (LRO) in the XY (spin) plane may still spontaneously appear in some domains of the parameter space, despite the local $U(1)$ symmetry.

4. Complex E tensors, TR-symmetry breaking, and chiral spin liquids

It is important to notice that, if the \mathcal{D} tensors belong to the A_1 , B_1 , A_2 , or B_2 IRREP, the resulting tensor is real and, hence, invariant under time-reversal (TR) symmetry. For complex coefficients, or when E tensors are combined, the resulting state generically breaks TR, except at fine-tuned parameter subsets. Although chiral spin liquids with protected chiral edge modes have to be searched in these classes of PEPS, we believe they probably span a tiny fraction of the TR-symmetry breaking PEPS manifold. Note also that not all E tensors give complex PEPS. For example, combining the three $S = \frac{1}{2}$, $V = \frac{1}{2} \oplus 0$, complex E tensors (one tensor with $n_{\text{occ}} = \{1,3\}$ and two tensors with $n_{\text{occ}} = \{3,1\}$) surprisingly gives a real [SU(2)-symmetric] wave function. We believe this PEPS family can be viewed physically as an extension of the NN fRVB (see above) in terms of fRVB states with longer-range (fermionic) dimers. Similarly, the two PEPS given by the unique $S = 1$, $V = \frac{1}{2} \oplus 0$, E tensor and by the unique $S = \frac{3}{2}$, $V = \frac{1}{2} \oplus 0$, E tensor are also real. Hence, we believe that the spin- $\frac{1}{2}$, spin-1, and spin- $\frac{3}{2}$ PEPS originated from the $V = \frac{1}{2} \oplus 0$, E tensors can probably all be mapped to real fermionic PEPS (fPEPS).

5. Gauge symmetry and topological order

Whether or not a PEPS of a given family exhibits topological order is a rather subtle issue. The existence of a gauge symmetry, i.e., an invariant gauge group (IGG), plays a crucial role and is often a necessary condition. The AKLT states and the featureless paramagnets above have simple $\mathcal{N} = 1$ virtual spaces with a single spin species in the four directions $n_{\text{occ}} = 4$. This is connected to IGG = \mathbb{I} , characteristic of topologically *trivial* states.

Gauge symmetry [like \mathbb{Z}_2 or U(1)] can also be present, depending on the different n_{occ} sectors involved in the construction of the variational manifold. For instance, the NN RVB state is defined by a unit tensor with $V = \frac{1}{2} \oplus 0$ and $n_{\text{occ}} = \{1,3\}$ which, in practice, implies that one and only one singlet dimer is attached to every site [35]. This local constraint implies that the *number* of dimers cut by a line winding around an infinite cylinder is conserved, hence providing an infinite number of topological sectors associated to a U(1) gauge symmetry [36]. On the other hand, the LR RVB state mixes $n_{\text{occ}} = \{1,3\}$ and $n_{\text{occ}} = \{3,1\}$ tensors so that, then, only the *parity* of the number of dimers cut by a circumference is conserved, hence reducing the number of sectors to two and the gauge symmetry to \mathbb{Z}_2 . More generally, the gauge symmetry can usually be inferred from the set of numbers of occupation $[n_{\text{occ}}^1, n_{\text{occ}}^2, \dots, n_{\text{occ}}^{\mathcal{D}}]$ of the \mathcal{D} superposed tensors. In general, one can always use a *minimal* global virtual basis $V = \oplus V_i$, direct sum of \mathcal{N} IRREP V_i of SU(2), for which all the occupation numbers n_{occ}^i are given by sets of \mathcal{N} numbers, i.e., $n_{\text{occ}}^i = \{n_1^i, \dots, n_{\mathcal{N}}^i\}$ and $\sum_i n_i^j = 4, \forall j$. Subsequently, topological order can be characterized from the symmetry [15]. However, (i) an extended gauge symmetry can emerge in the thermodynamic limit as, e.g., the U(1) symmetry in the case of the \mathbb{Z}_2 LR RVB state; (ii) reversely, a mechanism of “confinement” can suppress the topological order associated to

the underlying gauge symmetry [46]. In any case, topological order can always be inferred from a thorough investigation of the transfer operator [46], and not only from the local symmetry of the tensor.

D. Connection with previous work

Based on the original framework introduced by Jiang and Ran [25] and the notion of projective symmetry group, Lee and Han provided a classification of (trivial) spin-1 PEPS on the square lattice [26], in terms of lattice quantum numbers. In our classification scheme, we have recovered their results which correspond to a subset of our $S = 1$ tensors: the $V = \frac{1}{2}$ ($D = 2$), B_1 tensor, the $V = 1$ ($D = 3$), B_1 and A_2 tensors, the $V = 1 \oplus 0$ ($D = 4$), $B_1, n_{\text{occ}} = \{1,3\}$ (RVB state) and $n_{\text{occ}} = \{2,2\}$ (RAL state) tensors. While each tensor has (emergent) U(1) IGG symmetry, a linear combination of them eliminates this gauge symmetry and produces a trivial state. Note that these authors did not report about either the $S = 1, E$ tensors or the $S = 1, V = 1 \oplus 0$ ($D = 4$), $n_{\text{occ}} = \{3,1\}$ tensors enumerated in Table I. Combining, e.g., the $S = 1, V = 1 \oplus 0$ ($D = 4$), $n_{\text{occ}} = \{1,3\}$, and $n_{\text{occ}} = \{3,1\}$ tensors preserves a \mathbb{Z}_2 IGG and may lead to a topological spin liquid.

III. APPLICATION: SEARCH FOR HIGHER-SPIN

($S > \frac{1}{2}$) CSL

One of the applications of our classification is the systematic construction of CSL beyond the physical $S = \frac{1}{2}$ and virtual $V = \frac{1}{2} \oplus 0$ IRREPs, already realized. Our goal is to construct a family of TR-symmetry breaking PEPS with linear dispersing chiral edge modes described by a CFT beyond SU(2)₁. To characterize the edge modes, we analyze the entanglement spectrum (ES) of the corresponding PEPS when wrapped around an infinite cylinder and splitted in two parts (left and right), as explained in Appendix A. We describe below several natural routes to find such CSL.

A. Complex E tensors

One of the first natural candidates would be the above E tensors which are intrinsically complex. Therefore, their corresponding PEPS can potentially break TR symmetry and, hence, be relevant topological CSL. However, we have found that some of the simplest, small D, E PEPS wave functions are purely real as seen above (hence showing a perfectly momentum-symmetric entanglement spectrum), probably due to the fact that they can be re-casted in the form of real fPEPS wave functions, as it is the case for the fRVB state. In this respect, we found that the ES of the NN fRVB state is completely degenerate, hence saturating the upper bound of the entanglement measures [e.g., the entanglement entropy per unit length $-\text{Tr}(\rho \ln \rho)/N_v$].

In addition, all *complex* E PEPS we have tested turned out to show no well-defined chiral edge modes, although breaking TR symmetry. For example, this is the case of the PEPS associated to the $S = 1$, $V = 1$, $E^{(a)}$ tensor (see Table I and Supplemental Material [30]) whose ES is shown in Fig. 3. In fact, it is not clear, however, whether any E tensor could give rise to a topological CSL whenever $S > \frac{1}{2}$.

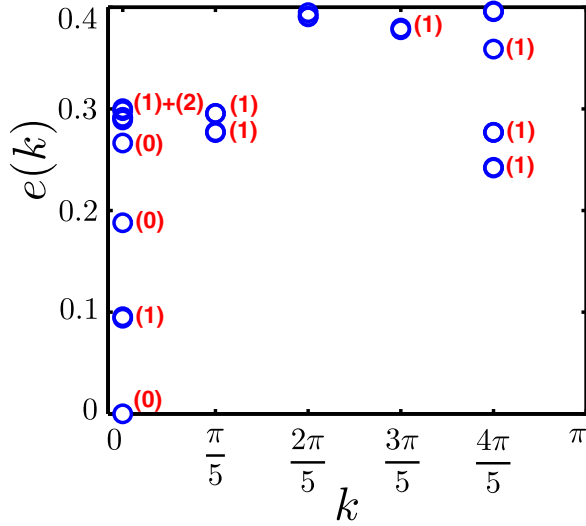


FIG. 3. Entanglement spectrum computed on a $N_v = 10$ (infinite) cylinder (with $\chi = 20$) of the PEPS constructed from the $S = 1$, $V = 1$, $E^{(a)}$ tensor. Data are plotted vs momentum (modulus π) along cylinder circumference. Note the spectrum is not symmetric w.r.t. $k = \pi/2$, reflecting TR-symmetry breaking of the PEPS. States can be grouped into SU(2) multiplets, and (s) labels $2s + 1$ degenerate states. Note the spectrum does not seem to fit a simple (chiral) CFT like SU(2)₂.

B. $A_1 + iA_2$ PEPS

Inspired by previous studies, another promising route to construct CSL is to consider tensors of mixed $A_1 + iA_2$ (point-group) symmetry, where both the real and imaginary components can be a sum of tensors belonging to any given A_1 and A_2 classes involving identical virtual and physical degrees of freedom. This is a direct generalization of Eq. (5). This procedure guarantees that the PEPS, if complex, transforms into its (orthogonal) complex conjugate state under any of the reflection symmetries of Fig. 1(a), a necessary condition for a CSL. However, this construction does not necessarily imply that the PEPS breaks TR symmetry and the wave function can, in some cases, be purely real (up to a global phase). In addition, even if the PEPS breaks TR symmetry, there is no guarantee that it exhibits linear dispersing edge modes described by a CFT characteristic of a topological CSL. Before describing a successful case (the double-layer CSL and PEPS connected to it), we will show below an example of some failure.

C. Naive spin- S generalizations of the spin- $\frac{1}{2}$ CSL

It is known that some families of critical (nonchiral) spin- S chains [47,48] bear a low-energy description in terms of the SU(2) _{k} Wess-Zumino-Witten models with levels $k = 2S$. It is therefore tempting to speculate that (chiral) SU(2) _{k} edge modes can originate from $V_i = \frac{k}{2}$ virtual spins effectively interacting on the edge. Indeed, the $S = \frac{1}{2}$ CSL discussed above, which bears SU(2)₁ edge modes, involves $V = \frac{1}{2} \oplus 0$ virtual states. In fact, inspecting Table I, one sees that the construction for $S = \frac{1}{2}$ can be easily extended for any spin S assuming $V = S \oplus 0$: one can always combine the unique $n_{\text{occ}} = \{1, 3\}$ spin- S RVB tensor (in bold case in Table I) with

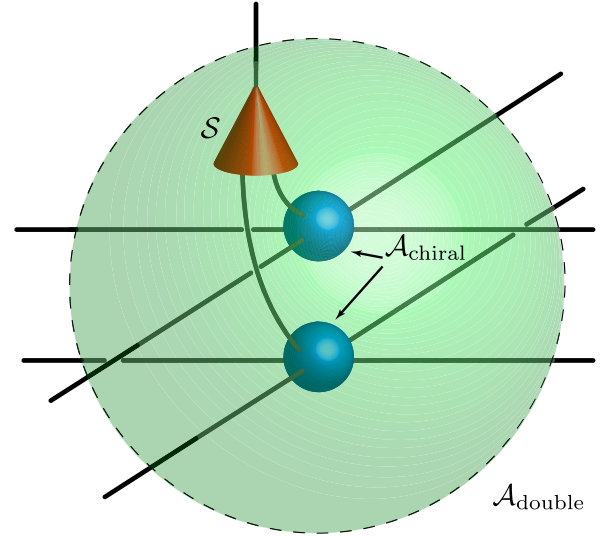


FIG. 4. Diagram corresponding to Eq. (6): the two layers of tensors $\mathcal{A}_{\text{chiral}}$ are symmetrized by the isometry S , which projects the physical dimensions in the three-dimensional spin-1 subspace of $\frac{1}{2} \otimes \frac{1}{2}$, giving rise to the tensor $\mathcal{A}_{\text{double}}$.

both the $n_{\text{occ}} = \{3, 1\}$, A_1 tensors (with a real coefficient) and the $n_{\text{occ}} = \{3, 1\}$, A_2 tensor(s) (with pure-imaginary coefficients). We have computed the ES at a few points of the three-dimensional (four-dimensional) family of PEPS corresponding to the case $S = 1$ ($S = \frac{3}{2}$) which turned out to be always gapped, ruling out chiral topological order.

D. Double-layer CSL and SU(2)₂ edge states

To go beyond the above naive construction, we shall follow here a strategy borrowed from the field of the fractional quantum Hall states (FQHS) [49,50]. Recently, using interpretation of FQHS as conformal blocks in certain rational conformal field theories [50], MPS representations of FQHS were exploited [51,52], providing unprecedented numerical accuracy [53]. While the (Abelian) Laughlin state can be written as a simple MPS, the non-Abelian states can be constructed as multilayer fractional quantum Hall wave functions upon symmetrization over the layer index [54,55]. Very recently, symmetrization of topologically ordered PEPS was shown to be a powerful method for constructing new topological models [56]. Here, we will apply this procedure by considering *two layers* of the CSL defined by Eq. (5). The double-layer tensor $[\mathcal{A}_{\text{chiral}}]^{\otimes 2}$ is symmetrized w.r.t the $\frac{1}{2} \otimes \frac{1}{2}$ physical variables

$$\mathcal{A}_{\text{double}} = S[\mathcal{A}_{\text{chiral}} \otimes \mathcal{A}_{\text{chiral}}], \quad (6)$$

hence projecting onto the $S = 1$ physical state (see Fig. 4). More precisely, its $S_z = +1, 0, -1$ (physical) components are given by

$$\begin{aligned} \mathcal{A}_{\text{double}}|_1 &= \mathcal{A}_{\text{chiral}}|_{\frac{1}{2}} \otimes \mathcal{A}_{\text{chiral}}|_{\frac{1}{2}}, \\ \mathcal{A}_{\text{double}}|_0 &= \frac{1}{\sqrt{2}} \{ \mathcal{A}_{\text{chiral}}|_{\frac{1}{2}} \otimes \mathcal{A}_{\text{chiral}}|_{-\frac{1}{2}} \\ &\quad + \mathcal{A}_{\text{chiral}}|_{-\frac{1}{2}} \otimes \mathcal{A}_{\text{chiral}}|_{\frac{1}{2}} \}, \\ \mathcal{A}_{\text{double}}|_{-1} &= \mathcal{A}_{\text{chiral}}|_{-\frac{1}{2}} \otimes \mathcal{A}_{\text{chiral}}|_{-\frac{1}{2}}, \end{aligned} \quad (7)$$

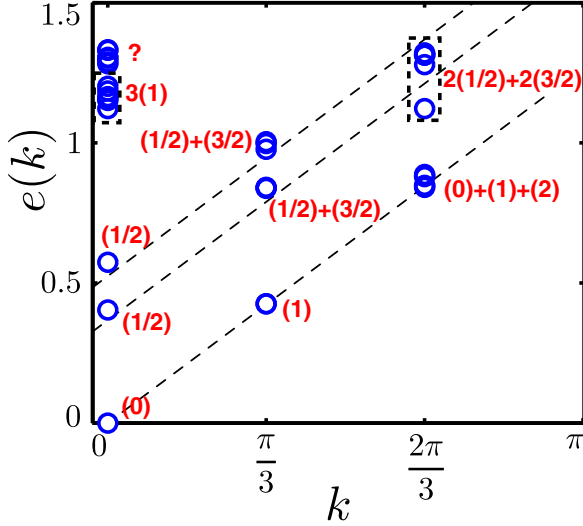


FIG. 5. Same as Fig. 3 for the PEPS obtained from the double-layer tensor of Eq. (6), $N_v = 6$ and $\chi = 14$. Whenever possible, the multiplet content of the levels is shown, following notations of Table III. The dashed lines are guides to the eye emphasizing the linear dispersion of the modes.

where the virtual variables of the double-layer tensor on the left-hand side are given by the tensor product of the virtual variables of the two single-layer CSL tensors on the right-hand side. For convenience, we then realize a (unitary) change of the $D = 9$ virtual basis, from the $[\frac{1}{2} \oplus 0]_{\text{top}} \otimes [\frac{1}{2} \oplus 0]_{\text{bottom}}$ natural basis to the “symmetric” basis $1 \oplus \frac{1}{2} \oplus \frac{1}{2} \oplus 0_s \oplus 0$ described in Fig. 2(b), where the two spin- $\frac{1}{2}$ representations correspond now to spin- $\frac{1}{2}$ states symmetric and antisymmetric w.r.t. layer exchange, respectively, and the spin- 0_s and spin- 0 representations contain the $\frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$ and 00 singlets, respectively. It can be seen easily (see later for details) that $\mathcal{A}_{\text{double}}$ inherits from $\mathcal{A}_{\text{chiral}}$ SU(2) spin rotation symmetry and lattice $A_1 + iA_2$ (orbital) symmetry. It is therefore expected to break TR symmetry while preserving all lattice symmetries, a key property of chiral spin liquids.

We have computed the ES of the double-layer tensor for $\lambda_1 = \lambda_2 = \lambda_c (=1/\sqrt{3})$ on an infinite $N_v = 6$ cylinder and results are shown in Fig. 5. Linearly dispersing branches are clearly seen. A lot of resemblance with the chiral SU(2)₂

TABLE III. Towers of states of the SU(2)₂ WZW model, in each of the three sectors characterized by the primary fields $j = 0, \frac{1}{2}, 1$ (listed in each column) and conformal weights $\frac{1}{4}j(j+1)$. Each line corresponds to a Virasoro level indexed by n . For each sector and each level, the (quasi)degenerate states can be grouped in terms of exact SU(2) multiplets like $n_0(0) + n_1(1) + \dots$ (meaning n_0 singlets, n_1 triplets, etc.). The numbers in square brackets correspond to the total number of states in each group of levels.

$n \setminus j$	0	$\frac{1}{2}$	1
0	(0) [1]	$(\frac{1}{2})$ [2]	(1) [3]
1	(1) [3]	$(\frac{1}{2}) + (\frac{3}{2})$ [6]	(0)+(1) [4]
2	(0)+(1)+(2) [9]	$2(\frac{1}{2}) + 2(\frac{3}{2})$ [12]	(0)+2(1)+(2) [12]
3	(0)+3(1)+(2) [15]	$4(\frac{1}{2}) + 3(\frac{3}{2}) + (\frac{5}{2})$ [26]	2(0)+3(1)+2(2) [21]

bilayer	representation / occupation	tensor(s)
	$\frac{1}{2} \oplus \frac{1}{2}$ $n_{\text{occ.}} = \{2, 2\}$	S_1 G_1
	$\frac{1}{2} \oplus \frac{1}{2} \oplus 0$ $[n_{\text{occ.}}] = \{\{2, 0, 2\}, \{0, 2, 2\}\}$	S_2
	$1 \oplus 0$ $n_{\text{occ.}} = \{1, 3\}$	S_3
	$1 \oplus 0$ $n_{\text{occ.}} = \{3, 1\}$	$S_4^{(a)}, S_4^{(b)}$ G_4
	$1 \oplus 0_s \oplus 0$ $n_{\text{occ.}} = \{1, 2, 1\}$	$S_5^{(a)}, S_5^{(b)}$ G_5
	$\frac{1}{2} \oplus \frac{1}{2} \oplus 0_s$ $[n_{\text{occ.}}] = \{\{2, 0, 2\}, \{0, 2, 2\}\}$	$S_6^{(a)}, S_6^{(b)}$
	$1 \oplus \frac{1}{2} \oplus \frac{1}{2}$ $[n_{\text{occ.}}] = \{\{2, 2, 0\}, \{2, 0, 2\}\}$	$S_7^{(a)}, \dots, S_7^{(e)}$ G_7
	$1 \oplus \frac{1}{2} \oplus \frac{1}{2} \oplus 0_s$ $n_{\text{occ.}} = \{1, 1, 1, 1\}$	$S_8^{(a)}, S_8^{(b)}$ G_8
	$\frac{1}{2} \oplus \frac{1}{2} \oplus 0_s \oplus 0$ $n_{\text{occ.}} = \{1, 1, 1, 1\}$	S_9 G_9
	$1 \oplus \frac{1}{2} \oplus \frac{1}{2} \oplus 0$ $[n_{\text{occ.}}] = \{\{1, 2, 0, 1\}, \{1, 0, 2, 1\}\}$	S_{10} G_{10}

FIG. 6. All combinations (up to global $\pm\pi/2$ or π rotations) obtained by expending the $\mathcal{A}_{\text{chiral}}$ tensors [according to Eq. (5)] simultaneously in the top and bottom layers of the (symmetrized) double-layer CSL (leftmost column), leading to orthonormal tensors (rightmost column) belonging to different representations of the virtual states (middle column).

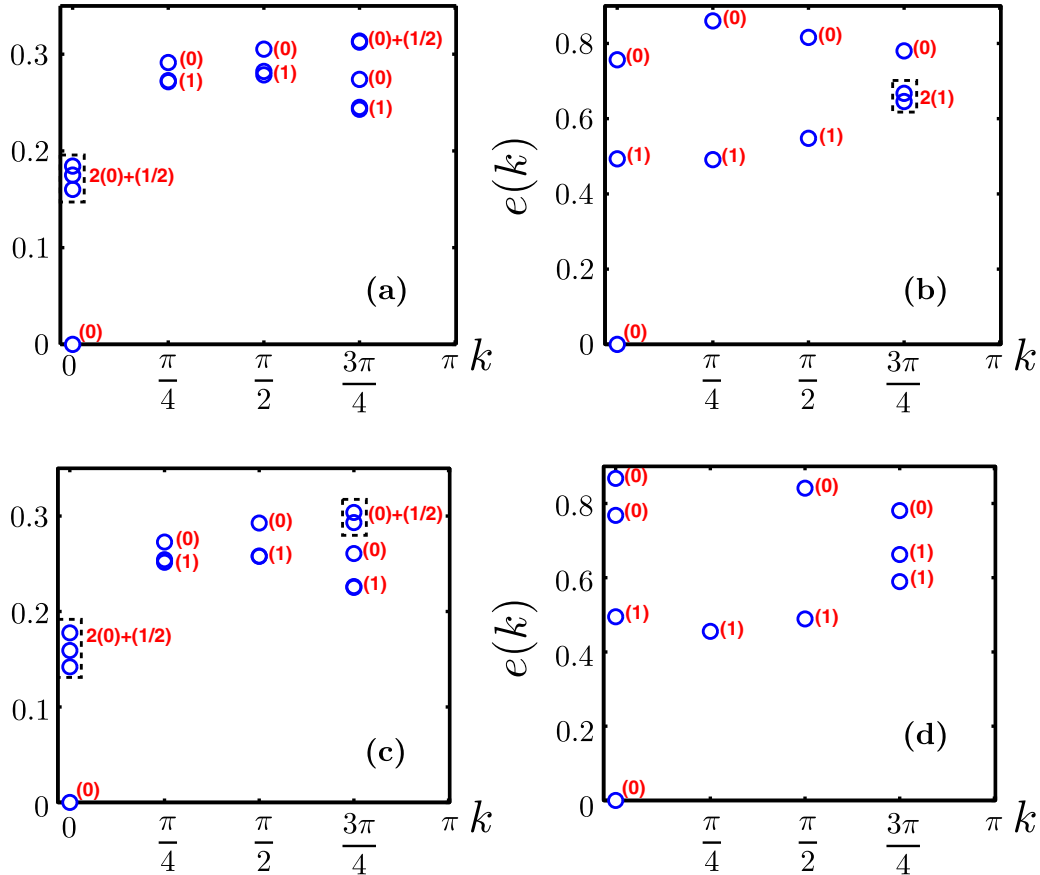


FIG. 7. ES for the PEPS obtained from various subsets of Eq. (8), and $N_v = 8$, $\chi = 20$: (a) S_1 and G_1 tensors ($\frac{1}{2} \oplus \frac{1}{2}, D = 4$); (b) S_3 , S_4 , and G_4 tensors ($1 \oplus 0, D = 4$); (c) S_1 , S_6 , and G_1 tensors ($\frac{1}{2} \oplus \frac{1}{2} \oplus 0_s, D = 5$); (d) S_3 , S_4 , S_5 , G_4 , and G_5 tensors ($1 \oplus 0 \oplus 0_s, D = 5$).

CFT spectrum is seen. The later shown in Table III contains three sectors labeled by the primary fields $j = 0, \frac{1}{2}, 1$. The lowest branch of the ES agrees perfectly with the content of the $j = 0$ sector. We also observe two almost degenerate branches with some energy offset, but with the same slope, each compatible with the theoretical expectation for the $j = \frac{1}{2}$ sector. It should be noted that, although an energy offset of $\frac{3}{16}$ of the (average) level spacing is expected for the $j = \frac{1}{2}$ branch, a larger offset (by a factor ~ 4) is observed which could be plausibly attributed to finite perimeter (N_v) and finite- χ effects in the ES calculation (see Appendix A).

E. Decomposition in terms of elementary tensors

The double-layer tensor involves $D = 9$ virtual states, making hard the computation of the ES on larger cylinders and with larger MPS dimension χ to permit a more definite assignment of the edge theory. In addition, it is not clear whether the observation of exact $SU(2)_2$ edge modes requires some degree of “fine tuning.” For these two reasons, it is a good idea to try to construct simpler (i.e., with lower bond dimension D) PEPS which, potentially, could exhibit chiral edge modes. Our strategy is here to “break up” the double-layer tensor $\mathcal{A}_{\text{double}}$ into independent parts defined by smaller virtual spaces V but still exhibiting $SU(2)$ spin rotation and $A_1 + iA_2$ lattice symmetries. To do so, we expend the $\mathcal{A}_{\text{chiral}}$ tensors according to Eq. (5) simultaneously in the top and bottom

layers. Figure 6 shows all possible combinations depending on the relative orientation of the various contributions in the two layers. In fact, it can be shown that each part leads to the sum of a few orthonormal $SU(2)$ spin symmetric real S and G tensors, whose virtual states belong to a smaller representation, subset of the overall Hilbert space $V = [\frac{1}{2} \oplus 0]^{\otimes 2}$. The S tensors and G tensors belong to the A_1 (s -wave) and A_2 (g -wave) IRREP and appear with real and pure-imaginary coefficients, respectively, so that each part of the decomposition bears an overall $A_1 + iA_2$ symmetry. More precisely, we can write

$$\mathcal{A}_{\text{double}} = \sum_{\alpha} s_{\alpha} S_{\alpha} + \sum_{\beta} g_{\beta} G_{\beta} \quad (8)$$

with

$$\begin{aligned} s_1 &= \frac{\lambda_1 \lambda_2}{\sqrt{2}}, & s_2 &= \frac{1}{2} \sqrt{3} \lambda_1^2, \\ s_3 &= \frac{\lambda_1^2}{2}, & s_4^{(a)} &= \frac{7\lambda_2^2 - 3\lambda_c^2}{12\sqrt{2}}, \\ s_4^{(b)} &= \frac{1}{12} \sqrt{\frac{5}{2}} (\lambda_2^2 + 3\lambda_c^2), & s_5^{(a)} &= \frac{\lambda_2^2}{3\sqrt{2}}, \\ s_5^{(b)} &= \frac{1}{12} (-\lambda_2^2 - 3\lambda_c^2), & s_6^{(a)} &= \frac{5\lambda_2^2 + 3\lambda_c^2}{12\sqrt{2}}, \\ s_6^{(b)} &= \frac{1}{12} (\lambda_2^2 - 3\lambda_c^2), & s_7^{(a)} &= -\frac{\lambda_2^2 + 9\lambda_c^2}{12\sqrt{3}}, \end{aligned}$$

S_1 , S_6 , S_7 , G_1 , and G_7 tensors and $V = 1 \oplus \frac{1}{2} \oplus \frac{1}{2} \oplus 0$ ($D = 8$) involving the S_1 , S_2 , S_3 , S_4 , S_7 , G_1 , G_4 , and G_7 tensors. Results are compared in Figs. 8(a)–8(d). As the number of tensors and the bond dimension increase, one does not observe any systematic trend, rather the ES changes in some erratic fashion. The latter seems nevertheless to remain gapped, although in Fig. 8(b) the (pseudo)energy scale becomes very small. In any case, the ES remains very different from the chiral ES of the $D = 9$ double-layer CSL shown in Fig. 5. This is a clear indication that there is a large degree of fine tuning in the latter wave function. In other words, just imposing $SU(2)$ spin rotation and $A_1 + iA_2$ orbital symmetries, as it was done for the single-layer CSL, is not sufficient to obtain a CSL. One reason might be that for $SU(2)_2$ CFT, (i) spin-1 degrees of freedom would be needed on the boundary but (ii) spin-1 chains are generically in the gapped Haldane phase. A gapless spectrum in a spin-1 chain requires fine tuning [47,48].

IV. CONCLUSIONS AND OUTLOOK

In this paper we have elaborated a classification scheme of all rank-5 $SU(2)$ -symmetric tensors according to the onsite physical spin S , the local Hilbert space of the bond degrees of freedom, and the irreducible representations of the C_{4v} point group of the square lattice. We have shown how many remarkable (Mott-insulating) states of matter fall naturally into this classification. More generally, we have explained how our scheme can be used to systematically construct families of translationally invariant many-body singlet states, preserving or breaking discrete (point-group) lattice symmetries, spin liquids, and (lattice) nematics, respectively. However, we bring here a few words of caution: first, we should mention that LRO [associated, e.g., to spontaneous translation and/or $SU(2)$ symmetry breaking] may still appear in the thermodynamic limit in some parameter regions, the PEPS being in that case a fully symmetric ‘‘Schrödinger cat state.’’ Note that the existence of LRO in our symmetric PEPS can only be diagnosed by a thorough numerical investigation, e.g., inspecting the low-energy spectrum of the transfer operator [46]. Second, it is likely that not all translationally invariant and spin rotationally symmetric spin liquids (on the square lattice) can be expressed in terms of a PEPS based on a single onsite $SU(2)$ -symmetric tensor. However, we believe our classification encompasses a very large manifold of symmetric spin liquids. Spin liquids not generated by our classification may include, e.g., those requiring a two-site (gauge) unit cell such as the (translationally invariant) π -flux PEPS of the PSG classification [25] or those requiring a different type of virtual particles such as fermions, Majoranas, anyons, etc.

We have also used our construction to systematically search for higher-spin ($S > \frac{1}{2}$) topological chiral spin liquids. One of our constructions uses a symmetrization over a double-layer PEPS, showing gapless chiral edge modes corresponding to a non-Abelian $SU(2)_2$ Wess-Zumino-Witten model, which we have determined via the analysis of its entanglement spectrum. This family of CSL can be seen as a two-dimensional manifold (spanned by the parameters λ_2/λ_1 and λ_c/λ_1) imbedded in a much larger PEPS family (characterized by arbitrary superpositions of the S and G tensors). This suggests that, more generally, non-Abelian CSL live on (relatively small)

fine-tuned manifolds of large PEPS families. Also, we believe that our construction can be extended to k layers, then showing $SU(2)_k$ gapless chiral edge modes.

We envisage these results as the first step of a broader research program, concerning the search of quantum spin liquids with tensor networks. Further work will involve, for instance, using the tensors produced in our classification in order to produce *Ansätze* for the numerical simulation of the frustrated Heisenberg model on the square lattice (improving the results of Ref. [39]), extending the classification scheme to the kagome lattice, and using it to propose variational wave functions to search for new spin liquids for the kagome Heisenberg antiferromagnet.

Another straightforward extension of our classification is to enlarge the physical space to more degrees of freedom, e.g., to include extra $U(1)$ -charge degrees of freedom [41] to describe hole doping a Mott (spin liquid or antiferromagnetic) insulator, or more orbital degrees of freedom [$N > 2$ with $SU(N)$ symmetry]. To build wave functions with specified $U(1)$ charge, covariant tensors can be constructed. Lastly, we note that our PEPS are, by construction, translationally invariant. However, it is also straightforward to describe states breaking translation invariance by considering different tensors at every site of a given supercell. All these problems, and more, will be addressed in future work along these lines.

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APPENDIX A: ENTANGLEMENT SPECTRUM FROM 2D PEPS

2D PEPS are a natural arena to study the so-called entanglement spectrum (ES) [57] (see for instance Refs. [58,59]). In this appendix, we review briefly what are the main ingredients of the calculations of ES in the context of 2D PEPS with tensor network methods, which we have used in several sections of this paper to characterize the edge modes.

Consider a 2D PEPS $|\Psi\rangle$ wrapped around a cylinder of circumference N_v , as in Fig. 9(a). For the sake of this calculation, we consider the cylinder to be infinitely long [58]. We now split the cylinder in two parts, say, left (L) and right (R). As explained in Ref. [58], the reduced density matrix of half an infinite cylinder, e.g., for L , is given by

$$\rho = U \sqrt{\sigma_L^T} \sigma_R \sqrt{\sigma_L^T} U^\dagger, \quad (\text{A1})$$

with $\sigma_{L/R}$ the reduced density operators in L/R for the virtual spaces across the bipartition, and U an isometry obtained from the contraction of the PEPS tensors. Technically, $\sigma_{L/R}$ are computed as the dominant left/right eigenvectors of the PEPS transfer matrix T [see Fig. 9(b)]. From this equation, it is

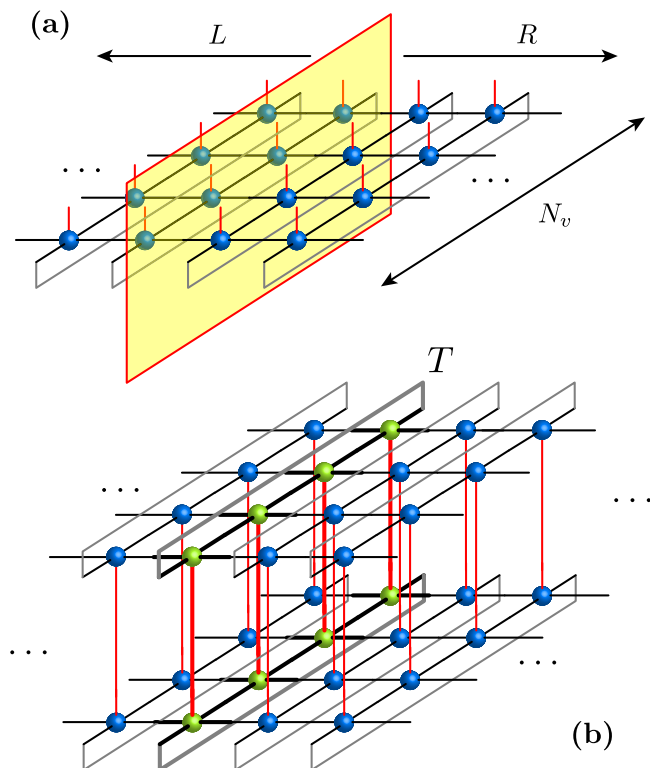


FIG. 9. (a) 2D PEPS wrapped around a cylinder of width N_v , and split into two parts L and R . (b) Transfer matrix T of the PEPS.

clear that ρ has the same eigenvalues than $\sqrt{\sigma_L^T} \sigma_R \sqrt{\sigma_L^T}$ since both operators are related only by an isometry (which leaves the spectrum invariant). Moreover, $\sqrt{\sigma_L^T} \sigma_R \sqrt{\sigma_L^T}$ shares the spectrum with $\sigma_L^T \sigma_R$, and shares the same quantum numbers for the eigenvectors [44]. This is particularly convenient since square roots are difficult to implement in the context of tensor network methods.

The calculation of $\sigma_{L/R}$ on an infinite cylinder for a 2D PEPS is a well-posed tensor network problem that can be solved using many different strategies. Here, we use a similar approach to the one used in Refs. [44,60]. Long story short: we compute this dominant eigenvector using the infinite Time-Evolving Block Decimation (iTEBD) method for nonunitary evolutions [61,62]. More specifically, for a given set of tensors in the transfer matrix T , we compute the tensors for $\sigma_{L/R}$ assuming $N_v \rightarrow \infty$ using iTEBD. The resulting dominant eigenvector can be written as an MPO of bond dimension χ , which is then wrapped around a circle of length N_v , and constitutes our approximated $\sigma_{L/R}$. This approach is remarkably efficient, and in practice provides very accurate results, including the ones in this paper. The computational cost of this calculation is $O(\chi^3 D^6 + \chi^2 D^8)$ [60]. Moreover, in this calculation it is usually possible to fix topological and/or parity sectors if needed, by feeding this as an input in the initial vector of iTEBD.

The diagonalization of $\sigma_L^T \sigma_R$ then proceeds by using Krylov-subspace methods (e.g., Lanczos). Such methods rely on matrix-vector multiplications which, in our case, can be done very efficiently since the matrix $\sigma_L^T \sigma_R$ has an explicit tensor network structure in terms of the MPOs for σ_L and σ_R . At

TABLE IV. Conventions for labeling virtual states for $V = S_1 \oplus \dots \oplus S_p$.

Virtual state (S_z)	Tensor index
S_1	0
$S_1 - 1$	1
...	...
$-S_1$	$2S_1$
S_2	$2S_1 + 1$
$S_2 - 1$	$2S_1 + 2$
...	...
$-S_p$	$2(S_1 + \dots + S_p) + (p - 1)$

this point, it is also possible to fix the total z component of the spin, which is a good quantum number in our $SU(2)$ -invariant setting, just by fixing it in the initial vector used in Krylov methods. Finally, momentum in the transverse direction is also a good quantum number, which can be extracted *a posteriori* by checking how the corresponding eigenvectors transform under the action of the translation operator.

APPENDIX B: PEPS TENSORS OF SIMPLE STATES

1. Conventions for labeling virtual states

We list below the tensors of the states described in the text. We first provide the conventions used to label the virtual states $|\alpha\rangle$. Considering a virtual subspace $V = S_1 \oplus \dots \oplus S_p$ the labels run from 0 to $2(S_1 + \dots + S_p) + (p - 1)$ by decreasing order of S_z as explicitly described in Table IV. The resulting tensor coefficients are found in Tables V–XLIII.

2. $S = 2$ AKLT state

TABLE V. The $S = 2$ AKLT state. $V = \frac{1}{2}$, $n_{\text{occ}} = \{4\}$, $S = 2$, $\text{Pg} = A_1$.

$S_z = +2$	$S_z = +1$
$T_2(0,0,0,0) = 1$	$T_1(0,0,0,1) = \frac{1}{2}$
	$T_1(0,0,1,0) = \frac{1}{2}$
	$T_1(0,1,0,0) = \frac{1}{2}$
	$T_1(1,0,0,0) = \frac{1}{2}$
$S_z = 0$	
$T_0(0,0,1,1) = \frac{1}{\sqrt{6}}$	
$T_0(0,1,0,1) = \frac{1}{\sqrt{6}}$	
$T_0(0,1,1,0) = \frac{1}{\sqrt{6}}$	
$T_0(1,0,0,1) = \frac{1}{\sqrt{6}}$	
$T_0(1,0,1,0) = \frac{1}{\sqrt{6}}$	
$T_0(1,1,0,0) = \frac{1}{\sqrt{6}}$	
$S_z = -1$	$S_z = -2$
$T_{-1}(0,1,1,1) = \frac{1}{2}$	$T_{-2}(1,1,1,1) = 1$
$T_{-1}(1,0,1,1) = \frac{1}{2}$	
$T_{-1}(1,1,0,1) = \frac{1}{2}$	
$T_{-1}(1,1,1,0) = \frac{1}{2}$	

3. Spin-1 featureless paramagnet

TABLE VI. The tensor of the spin-1 featureless paramagnet. $V = \frac{1}{2}$, $n_{\text{occ}} = \{4\}$, $S = 1$, $\text{Pg} = B_1$.

$S_z = +1$	$S_z = 0$	$S_z = -1$
$T_1(0,0,0,1) = \frac{1}{2}$	$T_0(0,1,0,1) = \frac{1}{\sqrt{2}}$	$T_{-1}(0,1,1,1) = \frac{1}{2}$
$T_1(0,0,1,0) = -\frac{1}{2}$	$T_0(1,0,1,0) = -\frac{1}{\sqrt{2}}$	$T_{-1}(1,0,1,1) = -\frac{1}{2}$
$T_1(0,1,0,0) = \frac{1}{2}$		$T_{-1}(1,1,0,1) = \frac{1}{2}$
$T_1(1,0,0,0) = -\frac{1}{2}$		$T_{-1}(1,1,1,0) = -\frac{1}{2}$

4. Spin- $\frac{1}{2}$ resonating valence bond state: Gauge-equivalent $\mathcal{A}_1^{(1)}$ and $\mathcal{B}_1^{(1)}$ tensors

TABLE VII. The $\mathcal{A}_1^{(1)}$ tensor of the NN resonating valence bond (NN RVB) state. $V = \frac{1}{2} \oplus 0$, $n_{\text{occ}} = \{1,3\}$, $S = \frac{1}{2}$, $\text{Pg} = A_1$.

$S_z = +1/2$	$S_z = -1/2$
$T_{\frac{1}{2}}(0,2,2,2) = \frac{1}{2}$	$T_{-\frac{1}{2}}(1,2,2,2) = \frac{1}{2}$
$T_{\frac{1}{2}}(2,0,2,2) = \frac{1}{2}$	$T_{-\frac{1}{2}}(2,1,2,2) = \frac{1}{2}$
$T_{\frac{1}{2}}(2,2,0,2) = \frac{1}{2}$	$T_{-\frac{1}{2}}(2,2,1,2) = \frac{1}{2}$
$T_{\frac{1}{2}}(2,2,2,0) = \frac{1}{2}$	$T_{-\frac{1}{2}}(2,2,2,1) = \frac{1}{2}$

TABLE VIII. The $\mathcal{B}_1^{(1)}$ tensor of the NN resonating valence bond (NN RVB) state. $V = \frac{1}{2} \oplus 0$, $n_{\text{occ}} = \{1,3\}$, $S = \frac{1}{2}$, $\text{Pg} = B_1$.

$S_z = +1/2$	$S_z = + - 1/2$
$T_{\frac{1}{2}}(0,2,2,2) = \frac{1}{2}$	$T_{-\frac{1}{2}}(1,2,2,2) = \frac{1}{2}$
$T_{\frac{1}{2}}(2,0,2,2) = -\frac{1}{2}$	$T_{-\frac{1}{2}}(2,1,2,2) = -\frac{1}{2}$
$T_{\frac{1}{2}}(2,2,0,2) = \frac{1}{2}$	$T_{-\frac{1}{2}}(2,2,1,2) = \frac{1}{2}$
$T_{\frac{1}{2}}(2,2,2,0) = -\frac{1}{2}$	$T_{-\frac{1}{2}}(2,2,2,1) = -\frac{1}{2}$

5. Gauge-equivalent $\mathcal{A}_1^{(2)}$ and $\mathcal{B}_1^{(2)}$ RVB tensors

TABLE IX. The $\mathcal{A}_1^{(2)}$ RVB tensor. $V = \frac{1}{2} \oplus 0$, $n_{\text{occ}} = \{3,1\}$, $S = \frac{1}{2}$, $\text{Pg} = A_1$.

$S_z = +1/2$	$S_z = -1/2$
$T_{\frac{1}{2}}(0,0,1,2) = -\frac{1}{2\sqrt{6}}$	$T_{-\frac{1}{2}}(0,1,1,2) = \frac{1}{2\sqrt{6}}$
$T_{\frac{1}{2}}(0,0,2,1) = -\frac{1}{2\sqrt{6}}$	$T_{-\frac{1}{2}}(0,1,2,1) = -\frac{1}{\sqrt{6}}$
$T_{\frac{1}{2}}(0,1,0,2) = \frac{1}{\sqrt{6}}$	$T_{-\frac{1}{2}}(0,2,1,1) = \frac{1}{2\sqrt{6}}$
$T_{\frac{1}{2}}(0,1,2,0) = -\frac{1}{2\sqrt{6}}$	$T_{-\frac{1}{2}}(1,0,1,2) = -\frac{1}{\sqrt{6}}$
$T_{\frac{1}{2}}(0,2,0,1) = \frac{1}{\sqrt{6}}$	$T_{-\frac{1}{2}}(1,0,2,1) = \frac{1}{2\sqrt{6}}$
$T_{\frac{1}{2}}(0,2,1,0) = -\frac{1}{2\sqrt{6}}$	$T_{-\frac{1}{2}}(1,1,0,2) = \frac{1}{2\sqrt{6}}$
$T_{\frac{1}{2}}(1,0,0,2) = -\frac{1}{2\sqrt{6}}$	$T_{-\frac{1}{2}}(1,1,2,0) = \frac{1}{2\sqrt{6}}$
$T_{\frac{1}{2}}(1,0,2,0) = \frac{1}{\sqrt{6}}$	$T_{-\frac{1}{2}}(1,2,0,1) = \frac{1}{2\sqrt{6}}$
$T_{\frac{1}{2}}(1,2,0,0) = -\frac{1}{2\sqrt{6}}$	$T_{-\frac{1}{2}}(1,2,1,0) = -\frac{1}{\sqrt{6}}$
$T_{\frac{1}{2}}(2,0,0,1) = -\frac{1}{2\sqrt{6}}$	$T_{-\frac{1}{2}}(2,0,1,1) = \frac{1}{2\sqrt{6}}$
$T_{\frac{1}{2}}(2,0,1,0) = \frac{1}{\sqrt{6}}$	$T_{-\frac{1}{2}}(2,1,0,1) = -\frac{1}{\sqrt{6}}$
$T_{\frac{1}{2}}(2,1,0,0) = -\frac{1}{2\sqrt{6}}$	$T_{-\frac{1}{2}}(2,1,1,0) = \frac{1}{2\sqrt{6}}$

TABLE X. The $\mathcal{B}_1^{(2)}$ RVB tensor. $V = \frac{1}{2} \oplus 0$, $n_{\text{occ}} = \{3,1\}$, $S = \frac{1}{2}$, $\text{Pg} = B_1$.

$S_z = +1/2$	$S_z = -1/2$
$T_{\frac{1}{2}}(0,0,1,2) = -\frac{1}{2\sqrt{6}}$	$T_{-\frac{1}{2}}(0,1,1,2) = \frac{1}{2\sqrt{6}}$
$T_{\frac{1}{2}}(0,0,2,1) = \frac{1}{2\sqrt{6}}$	$T_{-\frac{1}{2}}(0,1,2,1) = \frac{1}{\sqrt{6}}$
$T_{\frac{1}{2}}(0,1,0,2) = \frac{1}{\sqrt{6}}$	$T_{-\frac{1}{2}}(0,2,1,1) = \frac{1}{2\sqrt{6}}$
$T_{\frac{1}{2}}(0,1,2,0) = \frac{1}{2\sqrt{6}}$	$T_{-\frac{1}{2}}(1,0,1,2) = -\frac{1}{\sqrt{6}}$
$T_{\frac{1}{2}}(0,2,0,1) = \frac{1}{\sqrt{6}}$	$T_{-\frac{1}{2}}(1,0,2,1) = -\frac{1}{2\sqrt{6}}$
$T_{\frac{1}{2}}(0,2,1,0) = -\frac{1}{2\sqrt{6}}$	$T_{-\frac{1}{2}}(1,1,0,2) = \frac{1}{2\sqrt{6}}$
$T_{\frac{1}{2}}(1,0,0,2) = -\frac{1}{2\sqrt{6}}$	$T_{-\frac{1}{2}}(1,1,2,0) = -\frac{1}{2\sqrt{6}}$
$T_{\frac{1}{2}}(1,0,2,0) = -\frac{1}{\sqrt{6}}$	$T_{-\frac{1}{2}}(1,2,0,1) = \frac{1}{2\sqrt{6}}$
$T_{\frac{1}{2}}(1,2,0,0) = -\frac{1}{2\sqrt{6}}$	$T_{-\frac{1}{2}}(1,2,1,0) = -\frac{1}{\sqrt{6}}$
$T_{\frac{1}{2}}(2,0,0,1) = \frac{1}{2\sqrt{6}}$	$T_{-\frac{1}{2}}(2,0,1,1) = -\frac{1}{2\sqrt{6}}$
$T_{\frac{1}{2}}(2,0,1,0) = -\frac{1}{\sqrt{6}}$	$T_{-\frac{1}{2}}(2,1,0,1) = \frac{1}{\sqrt{6}}$
$T_{\frac{1}{2}}(2,1,0,0) = \frac{1}{2\sqrt{6}}$	$T_{-\frac{1}{2}}(2,1,1,0) = -\frac{1}{2\sqrt{6}}$

6. Spin- $\frac{1}{2}$ NN fermionic RVB state (NN fRVB)

TABLE XI. The spin- $\frac{1}{2}$ NN fermionic RVB (NN fRVB) tensor. $V = \frac{1}{2} \oplus 0$, $n_{\text{occ}} = \{1,3\}$, $S = \frac{1}{2}$, $\text{Pg} = E$.

$S_z = +1/2$	$S_z = -1/2$
$T_{\frac{1}{2}}(0,2,2,2) = \frac{1}{2}$	$T_{-\frac{1}{2}}(1,2,2,2) = \frac{1}{2}$
$T_{\frac{1}{2}}(2,0,2,2) = \pm \frac{i}{2}$	$T_{-\frac{1}{2}}(2,1,2,2) = \pm \frac{i}{2}$
$T_{\frac{1}{2}}(2,2,0,2) = -\frac{1}{2}$	$T_{-\frac{1}{2}}(2,2,1,2) = -\frac{1}{2}$
$T_{\frac{1}{2}}(2,2,2,0) = \mp \frac{i}{2}$	$T_{-\frac{1}{2}}(2,2,2,1) = \mp \frac{i}{2}$

7. Gauge-equivalent \mathcal{A}_2 and \mathcal{B}_2 RVB tensors

TABLE XII. The \mathcal{A}_2 RVB tensor. $V = \frac{1}{2} \oplus 0$, $n_{\text{occ}} = \{3,1\}$, $S = \frac{1}{2}$, $\text{Pg} = A_2$.

$S_z = +1/2$	$S_z = -1/2$
$T_{\frac{1}{2}}(0,0,1,2) = \frac{1}{2\sqrt{2}}$	$T_{-\frac{1}{2}}(0,1,1,2) = \frac{1}{2\sqrt{2}}$
$T_{\frac{1}{2}}(0,0,2,1) = -\frac{1}{2\sqrt{2}}$	$T_{-\frac{1}{2}}(0,2,1,1) = -\frac{1}{2\sqrt{2}}$
$T_{\frac{1}{2}}(0,1,2,0) = \frac{1}{2\sqrt{2}}$	$T_{-\frac{1}{2}}(1,0,2,1) = -\frac{1}{2\sqrt{2}}$
$T_{\frac{1}{2}}(0,2,1,0) = -\frac{1}{2\sqrt{2}}$	$T_{-\frac{1}{2}}(1,1,0,2) = -\frac{1}{2\sqrt{2}}$
$T_{\frac{1}{2}}(1,0,0,2) = -\frac{1}{2\sqrt{2}}$	$T_{-\frac{1}{2}}(1,1,2,0) = \frac{1}{2\sqrt{2}}$
$T_{\frac{1}{2}}(1,2,0,0) = \frac{1}{2\sqrt{2}}$	$T_{-\frac{1}{2}}(1,2,0,1) = \frac{1}{2\sqrt{2}}$
$T_{\frac{1}{2}}(2,0,0,1) = \frac{1}{2\sqrt{2}}$	$T_{-\frac{1}{2}}(2,0,1,1) = \frac{1}{2\sqrt{2}}$
$T_{\frac{1}{2}}(2,1,0,0) = -\frac{1}{2\sqrt{2}}$	$T_{-\frac{1}{2}}(2,1,1,0) = -\frac{1}{2\sqrt{2}}$

TABLE XIII. The \mathcal{A}_2 RVB tensor. $V = \frac{1}{2} \oplus 0$, $n_{\text{occ}} = \{3, 1\}$, $S = \frac{1}{2}$, $\text{Pg} = B_2$.

$S_z = +1/2$	$S_z = -1/2$
$T_{\frac{1}{2}}(0, 0, 1, 2) = \frac{1}{2\sqrt{2}}$	$T_{-\frac{1}{2}}(0, 1, 1, 2) = \frac{1}{2\sqrt{2}}$
$T_{\frac{1}{2}}(0, 0, 2, 1) = \frac{1}{2\sqrt{2}}$	$T_{-\frac{1}{2}}(0, 2, 1, 1) = -\frac{1}{2\sqrt{2}}$
$T_{\frac{1}{2}}(0, 1, 2, 0) = -\frac{1}{2\sqrt{2}}$	$T_{-\frac{1}{2}}(1, 0, 2, 1) = \frac{1}{2\sqrt{2}}$
$T_{\frac{1}{2}}(0, 2, 1, 0) = -\frac{1}{2\sqrt{2}}$	$T_{-\frac{1}{2}}(1, 1, 0, 2) = -\frac{1}{2\sqrt{2}}$
$T_{\frac{1}{2}}(1, 0, 0, 2) = -\frac{1}{2\sqrt{2}}$	$T_{-\frac{1}{2}}(1, 1, 2, 0) = -\frac{1}{2\sqrt{2}}$
$T_{\frac{1}{2}}(1, 2, 0, 0) = \frac{1}{2\sqrt{2}}$	$T_{-\frac{1}{2}}(1, 2, 0, 1) = \frac{1}{2\sqrt{2}}$
$T_{\frac{1}{2}}(2, 0, 0, 1) = -\frac{1}{2\sqrt{2}}$	$T_{-\frac{1}{2}}(2, 0, 1, 1) = -\frac{1}{2\sqrt{2}}$
$T_{\frac{1}{2}}(2, 1, 0, 0) = \frac{1}{2\sqrt{2}}$	$T_{-\frac{1}{2}}(2, 1, 1, 0) = \frac{1}{2\sqrt{2}}$

8. Spin-1 resonating loop (RAL) state

 TABLE XIV. The tensors of the spin-1 resonating loop (RAL) state. $V = \frac{1}{2} \oplus 0$, $n_{\text{occ}} = \{2, 2\}$, $S = 1$, $\text{Pg} = A_1$.

$S_z = +1$	$S_z = 0$	$S_z = -1$
$T_1^{(a)}(0, 0, 2, 2) = \frac{1}{2}$	$T_0^{(a)}(0, 1, 2, 2) = \frac{1}{2\sqrt{2}}$	$T_{-1}^{(a)}(1, 1, 2, 2) = \frac{1}{2}$
$T_1^{(a)}(0, 2, 2, 0) = \frac{1}{2}$	$T_0^{(a)}(0, 2, 2, 1) = \frac{1}{2\sqrt{2}}$	$T_{-1}^{(a)}(1, 2, 2, 1) = \frac{1}{2}$
$T_1^{(a)}(2, 0, 0, 2) = \frac{1}{2}$	$T_0^{(a)}(1, 0, 2, 2) = \frac{1}{2\sqrt{2}}$	$T_{-1}^{(a)}(2, 1, 1, 2) = \frac{1}{2}$
$T_1^{(a)}(2, 2, 0, 0) = \frac{1}{2}$	$T_0^{(a)}(1, 2, 2, 0) = \frac{1}{2\sqrt{2}}$	$T_{-1}^{(a)}(2, 2, 1, 1) = \frac{1}{2}$
	$T_0^{(a)}(2, 0, 1, 2) = \frac{1}{2\sqrt{2}}$	
	$T_0^{(a)}(2, 1, 0, 2) = \frac{1}{2\sqrt{2}}$	
	$T_0^{(a)}(2, 2, 0, 1) = \frac{1}{2\sqrt{2}}$	
	$T_0^{(a)}(2, 2, 1, 0) = \frac{1}{2\sqrt{2}}$	
$S_z = +1$	$S_z = 0$	$S_z = -1$
$T_1^{(b)}(0, 2, 0, 2) = \frac{1}{\sqrt{2}}$	$T_0^{(b)}(0, 2, 1, 2) = \frac{1}{2}$	$T_{-1}^{(b)}(1, 2, 1, 2) = \frac{1}{\sqrt{2}}$
$T_1^{(b)}(2, 0, 2, 0) = \frac{1}{\sqrt{2}}$	$T_0^{(b)}(1, 2, 0, 2) = \frac{1}{2}$	$T_{-1}^{(b)}(2, 1, 2, 1) = \frac{1}{\sqrt{2}}$
	$T_0^{(b)}(2, 0, 2, 1) = \frac{1}{2}$	
	$T_0^{(b)}(2, 1, 2, 0) = \frac{1}{2}$	

9. Generalized spin-S NN RVB state

 TABLE XV. The tensor of the generalized spin-1 NN RVB state. $V = 1 \oplus 0$, $n_{\text{occ}} = \{1, 3\}$, $S = 1$, $\text{Pg} = A_1$.

$S_z = +1$	$S_z = 0$	$S_z = -1$
$T_1(0, 3, 3, 3) = \frac{1}{2}$	$T_0(1, 3, 3, 3) = \frac{1}{2}$	$T_{-1}(2, 3, 3, 3) = \frac{1}{2}$
$T_1(3, 0, 3, 3) = \frac{1}{2}$	$T_0(3, 1, 3, 3) = \frac{1}{2}$	$T_{-1}(3, 2, 3, 3) = \frac{1}{2}$
$T_1(3, 3, 0, 3) = \frac{1}{2}$	$T_0(3, 3, 1, 3) = \frac{1}{2}$	$T_{-1}(3, 3, 2, 3) = \frac{1}{2}$
$T_1(3, 3, 3, 0) = \frac{1}{2}$	$T_0(3, 3, 3, 1) = \frac{1}{2}$	$T_{-1}(3, 3, 3, 2) = \frac{1}{2}$

 TABLE XVI. The tensor of the generalized spin-1 NN RVB state. $V = 1 \oplus 0$, $n_{\text{occ}} = \{1, 3\}$, $S = 1$, $\text{Pg} = B_1$.

$S_z = +1$	$S_z = 0$	$S_z = -1$
$T_1(0, 3, 3, 3) = \frac{1}{2}$	$T_0(1, 3, 3, 3) = \frac{1}{2}$	$T_{-1}(2, 3, 3, 3) = \frac{1}{2}$
$T_1(3, 0, 3, 3) = -\frac{1}{2}$	$T_0(3, 1, 3, 3) = -\frac{1}{2}$	$T_{-1}(3, 2, 3, 3) = -\frac{1}{2}$
$T_1(3, 3, 0, 3) = \frac{1}{2}$	$T_0(3, 3, 1, 3) = \frac{1}{2}$	$T_{-1}(3, 3, 2, 3) = \frac{1}{2}$
$T_1(3, 3, 3, 0) = -\frac{1}{2}$	$T_0(3, 3, 3, 1) = -\frac{1}{2}$	$T_{-1}(3, 3, 3, 2) = -\frac{1}{2}$

 TABLE XVII. The tensor of the generalized spin- $\frac{3}{2}$ NN RVB state. $V = \frac{3}{2} \oplus 0$, $n_{\text{occ}} = \{1, 3\}$, $S = \frac{3}{2}$, $\text{Pg} = A_1$.

$S_z = +3/2$	$S_z = +1/2$
$T_{\frac{3}{2}}(0, 4, 4, 4) = \frac{1}{2}$	$T_{\frac{1}{2}}(1, 4, 4, 4) = \frac{1}{2}$
$T_{\frac{3}{2}}(4, 0, 4, 4) = \frac{1}{2}$	$T_{\frac{1}{2}}(4, 1, 4, 4) = \frac{1}{2}$
$T_{\frac{3}{2}}(4, 4, 0, 4) = \frac{1}{2}$	$T_{\frac{1}{2}}(4, 4, 1, 4) = \frac{1}{2}$
$T_{\frac{3}{2}}(4, 4, 4, 0) = \frac{1}{2}$	$T_{\frac{1}{2}}(4, 4, 4, 1) = \frac{1}{2}$
$S_z = -1/2$	$S_z = -3/2$
$T_{-\frac{1}{2}}(2, 4, 4, 4) = \frac{1}{2}$	$T_{-\frac{3}{2}}(3, 4, 4, 4) = \frac{1}{2}$
$T_{-\frac{1}{2}}(4, 2, 4, 4) = \frac{1}{2}$	$T_{-\frac{3}{2}}(4, 3, 4, 4) = \frac{1}{2}$
$T_{-\frac{1}{2}}(4, 4, 2, 4) = \frac{1}{2}$	$T_{-\frac{3}{2}}(4, 4, 3, 4) = \frac{1}{2}$
$T_{-\frac{1}{2}}(4, 4, 4, 2) = \frac{1}{2}$	$T_{-\frac{3}{2}}(4, 4, 4, 3) = \frac{1}{2}$

 TABLE XVIII. The tensor of the generalized spin- $\frac{3}{2}$ NN RVB state. $V = \frac{3}{2} \oplus 0$, $n_{\text{occ}} = \{1, 3\}$, $S = \frac{3}{2}$, $\text{Pg} = B_1$.

$S_z = +3/2$	$S_z = +1/2$
$T_{\frac{3}{2}}(0, 4, 4, 4) = \frac{1}{2}$	$T_{\frac{1}{2}}(1, 4, 4, 4) = \frac{1}{2}$
$T_{\frac{3}{2}}(4, 0, 4, 4) = -\frac{1}{2}$	$T_{\frac{1}{2}}(4, 1, 4, 4) = -\frac{1}{2}$
$T_{\frac{3}{2}}(4, 4, 0, 4) = \frac{1}{2}$	$T_{\frac{1}{2}}(4, 4, 1, 4) = \frac{1}{2}$
$T_{\frac{3}{2}}(4, 4, 4, 0) = -\frac{1}{2}$	$T_{\frac{1}{2}}(4, 4, 4, 1) = -\frac{1}{2}$
$S_z = -1/2$	$S_z = -3/2$
$T_{-\frac{1}{2}}(2, 4, 4, 4) = \frac{1}{2}$	$T_{-\frac{3}{2}}(3, 4, 4, 4) = \frac{1}{2}$
$T_{-\frac{1}{2}}(4, 2, 4, 4) = -\frac{1}{2}$	$T_{-\frac{3}{2}}(4, 3, 4, 4) = -\frac{1}{2}$
$T_{-\frac{1}{2}}(4, 4, 2, 4) = \frac{1}{2}$	$T_{-\frac{3}{2}}(4, 4, 3, 4) = \frac{1}{2}$
$T_{-\frac{1}{2}}(4, 4, 4, 2) = -\frac{1}{2}$	$T_{-\frac{3}{2}}(4, 4, 4, 3) = -\frac{1}{2}$

APPENDIX C: S AND G TENSORS OBTAINED FROM THE DECOMPOSITION OF THE DOUBLE-LAYER CSL

Here, one uses the same set of indices for all tensors, according to Fig. 2(b).

TABLE XIX. S_1 .

$S_z = +1$	$S_z = 0$	$S_z = -1$
$T_1(2,2,6,7) = \frac{1}{4\sqrt{3}}$	$T_0(2,2,7,7) = \frac{1}{2\sqrt{6}}$	$T_{-1}(2,5,7,7) = \frac{1}{4\sqrt{3}}$
$T_1(2,2,7,6) = \frac{1}{4\sqrt{3}}$	$T_0(2,7,2,7) = -\frac{1}{\sqrt{6}}$	$T_{-1}(2,7,5,7) = -\frac{1}{2\sqrt{3}}$
$T_1(2,5,6,6) = -\frac{1}{4\sqrt{3}}$	$T_0(2,7,7,2) = \frac{1}{2\sqrt{6}}$	$T_{-1}(2,7,7,5) = \frac{1}{4\sqrt{3}}$
$T_1(2,6,2,7) = -\frac{1}{2\sqrt{3}}$	$T_0(5,5,6,6) = -\frac{1}{2\sqrt{6}}$	$T_{-1}(5,2,7,7) = \frac{1}{4\sqrt{3}}$
$T_1(2,6,5,6) = \frac{1}{2\sqrt{3}}$	$T_0(5,6,5,6) = \frac{1}{\sqrt{6}}$	$T_{-1}(5,5,6,7) = -\frac{1}{4\sqrt{3}}$
$T_1(2,6,6,5) = -\frac{1}{4\sqrt{3}}$	$T_0(5,6,6,5) = -\frac{1}{2\sqrt{6}}$	$T_{-1}(5,5,7,6) = -\frac{1}{4\sqrt{3}}$
$T_1(2,6,7,2) = \frac{1}{4\sqrt{3}}$	$T_0(6,5,5,6) = -\frac{1}{2\sqrt{6}}$	$T_{-1}(5,6,5,7) = \frac{1}{2\sqrt{3}}$
$T_1(2,7,2,6) = -\frac{1}{2\sqrt{3}}$	$T_0(6,5,6,5) = \frac{1}{\sqrt{6}}$	$T_{-1}(5,6,7,5) = -\frac{1}{4\sqrt{3}}$
$T_1(2,7,6,2) = \frac{1}{4\sqrt{3}}$	$T_0(6,6,5,5) = -\frac{1}{2\sqrt{6}}$	$T_{-1}(5,7,2,7) = -\frac{1}{2\sqrt{3}}$
$T_1(5,2,6,6) = -\frac{1}{4\sqrt{3}}$	$T_0(7,2,2,7) = \frac{1}{2\sqrt{6}}$	$T_{-1}(5,7,5,6) = \frac{1}{2\sqrt{3}}$
$T_1(5,6,2,6) = \frac{1}{2\sqrt{3}}$	$T_0(7,2,7,2) = -\frac{1}{\sqrt{6}}$	$T_{-1}(5,7,6,5) = -\frac{1}{4\sqrt{3}}$
$T_1(5,6,6,2) = -\frac{1}{4\sqrt{3}}$	$T_0(7,7,2,2) = \frac{1}{2\sqrt{6}}$	$T_{-1}(5,7,7,2) = \frac{1}{4\sqrt{3}}$
$T_1(6,2,2,7) = \frac{1}{4\sqrt{3}}$		$T_{-1}(6,5,5,7) = -\frac{1}{4\sqrt{3}}$
$T_1(6,2,5,6) = -\frac{1}{4\sqrt{3}}$		$T_{-1}(6,5,7,5) = \frac{1}{2\sqrt{3}}$
$T_1(6,2,6,5) = \frac{1}{2\sqrt{3}}$		$T_{-1}(6,7,5,5) = -\frac{1}{4\sqrt{3}}$
$T_1(6,2,7,2) = -\frac{1}{2\sqrt{3}}$		$T_{-1}(7,2,5,7) = \frac{1}{4\sqrt{3}}$
$T_1(6,5,2,6) = -\frac{1}{4\sqrt{3}}$		$T_{-1}(7,2,7,5) = -\frac{1}{2\sqrt{3}}$
$T_1(6,5,6,2) = \frac{1}{2\sqrt{3}}$		$T_{-1}(7,5,2,7) = \frac{1}{4\sqrt{3}}$
$T_1(6,6,2,5) = -\frac{1}{4\sqrt{3}}$		$T_{-1}(7,5,5,6) = -\frac{1}{4\sqrt{3}}$
$T_1(6,6,5,2) = -\frac{1}{4\sqrt{3}}$		$T_{-1}(7,5,6,5) = \frac{1}{2\sqrt{3}}$
$T_1(6,7,2,2) = \frac{1}{4\sqrt{3}}$		$T_{-1}(7,5,7,2) = -\frac{1}{2\sqrt{3}}$
$T_1(7,2,2,6) = \frac{1}{4\sqrt{3}}$		$T_{-1}(7,6,5,5) = -\frac{1}{4\sqrt{3}}$
$T_1(7,2,6,2) = -\frac{1}{2\sqrt{3}}$		$T_{-1}(7,7,2,5) = \frac{1}{4\sqrt{3}}$
$T_1(7,6,2,2) = \frac{1}{4\sqrt{3}}$		$T_{-1}(7,7,5,2) = \frac{1}{4\sqrt{3}}$

TABLE XX. G_1 .

$S_z = +1$	$S_z = 0$	$S_z = -1$
$T_1(2,2,6,7) = -\frac{1}{4}$	$T_0(2,5,7,6) = \frac{1}{2\sqrt{2}}$	$T_{-1}(2,5,7,7) = \frac{1}{4}$
$T_1(2,2,7,6) = \frac{1}{4}$	$T_0(2,6,7,5) = -\frac{1}{2\sqrt{2}}$	$T_{-1}(2,7,7,5) = -\frac{1}{4}$
$T_1(2,5,6,6) = \frac{1}{4}$	$T_0(5,2,6,7) = -\frac{1}{2\sqrt{2}}$	$T_{-1}(5,2,7,7) = -\frac{1}{4}$
$T_1(2,6,6,5) = -\frac{1}{4}$	$T_0(5,7,6,2) = \frac{1}{2\sqrt{2}}$	$T_{-1}(5,5,6,7) = -\frac{1}{4}$
$T_1(2,6,7,2) = -\frac{1}{4}$	$T_0(6,2,5,7) = \frac{1}{2\sqrt{2}}$	$T_{-1}(5,5,7,6) = \frac{1}{4}$
$T_1(2,7,6,2) = \frac{1}{4}$	$T_0(6,7,5,2) = -\frac{1}{2\sqrt{2}}$	$T_{-1}(5,6,7,5) = -\frac{1}{4}$
$T_1(5,2,6,6) = -\frac{1}{4}$	$T_0(7,5,2,6) = -\frac{1}{2\sqrt{2}}$	$T_{-1}(5,7,6,5) = \frac{1}{4}$
$T_1(5,6,6,2) = \frac{1}{4}$	$T_0(7,6,2,5) = \frac{1}{2\sqrt{2}}$	$T_{-1}(5,7,7,2) = \frac{1}{4}$
$T_1(6,2,2,7) = \frac{1}{4}$		$T_{-1}(6,5,5,7) = \frac{1}{4}$
$T_1(6,2,5,6) = \frac{1}{4}$		$T_{-1}(6,7,5,5) = -\frac{1}{4}$
$T_1(6,5,2,6) = -\frac{1}{4}$		$T_{-1}(7,2,5,7) = \frac{1}{4}$
$T_1(6,6,2,5) = \frac{1}{4}$		$T_{-1}(7,5,2,7) = -\frac{1}{4}$
$T_1(6,6,5,2) = -\frac{1}{4}$		$T_{-1}(7,5,5,6) = -\frac{1}{4}$
$T_1(6,7,2,2) = -\frac{1}{4}$		$T_{-1}(7,6,5,5) = \frac{1}{4}$
$T_1(7,2,2,6) = -\frac{1}{4}$		$T_{-1}(7,7,2,5) = \frac{1}{4}$
$T_1(7,6,2,2) = \frac{1}{4}$		$T_{-1}(7,7,5,2) = -\frac{1}{4}$

TABLE XXI. S_2 .

$S_z = +1$	$S_z = 0$	$S_z = -1$
$T_1(2,2,8,8) = \frac{1}{2\sqrt{3}}$	$T_0(2,5,8,8) = \frac{1}{2\sqrt{6}}$	$T_{-1}(5,5,8,8) = \frac{1}{2\sqrt{3}}$
$T_1(2,8,2,8) = \frac{1}{2\sqrt{3}}$	$T_0(2,8,5,8) = \frac{1}{2\sqrt{6}}$	$T_{-1}(5,8,5,8) = \frac{1}{2\sqrt{3}}$
$T_1(2,8,8,2) = \frac{1}{2\sqrt{3}}$	$T_0(2,8,8,5) = \frac{1}{2\sqrt{6}}$	$T_{-1}(5,8,8,5) = \frac{1}{2\sqrt{3}}$
$T_1(6,6,8,8) = -\frac{1}{2\sqrt{3}}$	$T_0(5,2,8,8) = \frac{1}{2\sqrt{6}}$	$T_{-1}(7,7,8,8) = -\frac{1}{2\sqrt{3}}$
$T_1(6,8,6,8) = -\frac{1}{2\sqrt{3}}$	$T_0(5,8,2,8) = \frac{1}{2\sqrt{6}}$	$T_{-1}(7,8,7,8) = -\frac{1}{2\sqrt{3}}$
$T_1(6,8,8,6) = -\frac{1}{2\sqrt{3}}$	$T_0(5,8,8,2) = \frac{1}{2\sqrt{6}}$	$T_{-1}(7,8,8,7) = -\frac{1}{2\sqrt{3}}$
$T_1(8,2,2,8) = \frac{1}{2\sqrt{3}}$	$T_0(6,7,8,8) = -\frac{1}{2\sqrt{6}}$	$T_{-1}(8,5,5,8) = \frac{1}{2\sqrt{3}}$
$T_1(8,2,8,2) = \frac{1}{2\sqrt{3}}$	$T_0(6,8,7,8) = -\frac{1}{2\sqrt{6}}$	$T_{-1}(8,5,8,5) = \frac{1}{2\sqrt{3}}$
$T_1(8,6,6,8) = -\frac{1}{2\sqrt{3}}$	$T_0(6,8,8,7) = -\frac{1}{2\sqrt{6}}$	$T_{-1}(8,7,7,8) = -\frac{1}{2\sqrt{3}}$
$T_1(8,6,8,6) = -\frac{1}{2\sqrt{3}}$	$T_0(7,6,8,8) = -\frac{1}{2\sqrt{6}}$	$T_{-1}(8,7,8,7) = -\frac{1}{2\sqrt{3}}$
$T_1(8,8,2,2) = \frac{1}{2\sqrt{3}}$	$T_0(7,8,6,8) = -\frac{1}{2\sqrt{6}}$	$T_{-1}(8,8,5,5) = \frac{1}{2\sqrt{3}}$
$T_1(8,8,6,6) = -\frac{1}{2\sqrt{3}}$	$T_0(7,8,8,6) = -\frac{1}{2\sqrt{6}}$	$T_{-1}(8,8,7,7) = -\frac{1}{2\sqrt{3}}$
	$T_0(8,2,5,8) = \frac{1}{2\sqrt{6}}$	
	$T_0(8,2,8,5) = \frac{1}{2\sqrt{6}}$	
	$T_0(8,5,2,8) = \frac{1}{2\sqrt{6}}$	
	$T_0(8,5,8,2) = \frac{1}{2\sqrt{6}}$	
	$T_0(8,6,7,8) = -\frac{1}{2\sqrt{6}}$	
	$T_0(8,6,8,7) = -\frac{1}{2\sqrt{6}}$	
	$T_0(8,7,6,8) = -\frac{1}{2\sqrt{6}}$	
	$T_0(8,7,8,6) = -\frac{1}{2\sqrt{6}}$	
	$T_0(8,8,2,5) = \frac{1}{2\sqrt{6}}$	
	$T_0(8,8,5,2) = \frac{1}{2\sqrt{6}}$	
	$T_0(8,8,6,7) = -\frac{1}{2\sqrt{6}}$	
	$T_0(8,8,7,6) = -\frac{1}{2\sqrt{6}}$	

TABLE XXII. S_3 .

$S_z = +1$	$S_z = 0$	$S_z = -1$
$T_1(0,8,8,8) = \frac{1}{2}$	$T_0(1,8,8,8) = \frac{1}{2}$	$T_{-1}(4,8,8,8) = \frac{1}{2}$
$T_1(8,0,8,8) = \frac{1}{2}$	$T_0(8,1,8,8) = \frac{1}{2}$	$T_{-1}(8,4,8,8) = \frac{1}{2}$
$T_1(8,8,0,8) = \frac{1}{2}$	$T_0(8,8,1,8) = \frac{1}{2}$	$T_{-1}(8,8,4,8) = \frac{1}{2}$
$T_1(8,8,8,0) = \frac{1}{2}$	$T_0(8,8,8,1) = \frac{1}{2}$	$T_{-1}(8,8,8,4) = \frac{1}{2}$

TABLE XXIII. $S_4^{(a)}$.

$S_z = +1$	$S_z = 0$	$S_z = -1$
$T_1(0,0,4,8) = \frac{1}{4\sqrt{2}}$	$T_0(0,1,8,4) = \frac{1}{4\sqrt{2}}$	$T_{-1}(0,4,4,8) = \frac{1}{4\sqrt{2}}$
$T_1(0,0,8,4) = \frac{1}{4\sqrt{2}}$	$T_0(0,4,1,8) = \frac{1}{4\sqrt{2}}$	$T_{-1}(0,4,8,4) = \frac{1}{2\sqrt{2}}$
$T_1(0,1,1,8) = -\frac{1}{4\sqrt{2}}$	$T_0(0,4,8,1) = \frac{1}{4\sqrt{2}}$	$T_{-1}(0,8,4,4) = \frac{1}{4\sqrt{2}}$
$T_1(0,4,0,8) = \frac{1}{2\sqrt{2}}$	$T_0(0,8,1,4) = \frac{1}{4\sqrt{2}}$	$T_{-1}(1,1,4,8) = -\frac{1}{4\sqrt{2}}$
$T_1(0,4,8,0) = \frac{1}{4\sqrt{2}}$	$T_0(1,0,4,8) = \frac{1}{4\sqrt{2}}$	$T_{-1}(1,1,8,4) = -\frac{1}{4\sqrt{2}}$
$T_1(0,8,0,4) = \frac{1}{2\sqrt{2}}$	$T_0(1,1,1,8) = -\frac{1}{2\sqrt{2}}$	$T_{-1}(1,4,8,1) = -\frac{1}{4\sqrt{2}}$

TABLE XXIII. (Continued.)

$S_z = +1$	$S_z = 0$	$S_z = -1$
$T_1(0,8,1,1) = -\frac{1}{4\sqrt{2}}$	$T_0(1,1,8,1) = -\frac{1}{2\sqrt{2}}$	$T_{-1}(1,8,4,1) = -\frac{1}{4\sqrt{2}}$
$T_1(0,8,4,0) = \frac{1}{4\sqrt{2}}$	$T_0(1,4,0,8) = \frac{1}{4\sqrt{2}}$	$T_{-1}(4,0,4,8) = \frac{1}{2\sqrt{2}}$
$T_1(1,0,8,1) = -\frac{1}{4\sqrt{2}}$	$T_0(1,8,0,4) = \frac{1}{4\sqrt{2}}$	$T_{-1}(4,0,8,4) = \frac{1}{4\sqrt{2}}$
$T_1(1,1,0,8) = -\frac{1}{4\sqrt{2}}$	$T_0(1,8,1,1) = -\frac{1}{2\sqrt{2}}$	$T_{-1}(4,1,1,8) = -\frac{1}{4\sqrt{2}}$
$T_1(1,1,8,0) = -\frac{1}{4\sqrt{2}}$	$T_0(1,8,4,0) = \frac{1}{4\sqrt{2}}$	$T_{-1}(4,4,0,8) = \frac{1}{4\sqrt{2}}$
$T_1(1,8,0,1) = -\frac{1}{4\sqrt{2}}$	$T_0(4,0,1,8) = \frac{1}{4\sqrt{2}}$	$T_{-1}(4,4,8,0) = \frac{1}{4\sqrt{2}}$
$T_1(4,0,0,8) = \frac{1}{4\sqrt{2}}$	$T_0(4,0,8,1) = \frac{1}{4\sqrt{2}}$	$T_{-1}(4,8,0,4) = \frac{1}{4\sqrt{2}}$
$T_1(4,0,8,0) = \frac{1}{2\sqrt{2}}$	$T_0(4,1,8,0) = \frac{1}{4\sqrt{2}}$	$T_{-1}(4,8,1,1) = -\frac{1}{4\sqrt{2}}$
$T_1(4,8,0,0) = \frac{1}{4\sqrt{2}}$	$T_0(4,8,1,0) = \frac{1}{4\sqrt{2}}$	$T_{-1}(4,8,4,0) = \frac{1}{2\sqrt{2}}$
$T_1(8,0,0,4) = \frac{1}{4\sqrt{2}}$	$T_0(8,0,4,1) = \frac{1}{4\sqrt{2}}$	$T_{-1}(8,0,4,4) = \frac{1}{4\sqrt{2}}$
$T_1(8,0,1,1) = -\frac{1}{4\sqrt{2}}$	$T_0(8,1,0,4) = \frac{1}{4\sqrt{2}}$	$T_{-1}(8,1,1,4) = -\frac{1}{4\sqrt{2}}$
$T_1(8,0,4,0) = \frac{1}{2\sqrt{2}}$	$T_0(8,1,1,1) = -\frac{1}{2\sqrt{2}}$	$T_{-1}(8,4,0,4) = \frac{1}{2\sqrt{2}}$
$T_1(8,1,1,0) = -\frac{1}{4\sqrt{2}}$	$T_0(8,1,4,0) = \frac{1}{4\sqrt{2}}$	$T_{-1}(8,4,1,1) = -\frac{1}{4\sqrt{2}}$
$T_1(8,4,0,0) = \frac{1}{4\sqrt{2}}$	$T_0(8,4,0,1) = \frac{1}{4\sqrt{2}}$	$T_{-1}(8,4,4,0) = \frac{1}{4\sqrt{2}}$

TABLE XXIV. $S_4^{(b)}$.

$S_z = +1$	$S_z = 0$	$S_z = -1$
$T_1(0,0,4,8) = -\frac{3}{4\sqrt{10}}$	$T_0(0,1,4,8) = -\frac{1}{\sqrt{10}}$	$T_{-1}(0,4,4,8) = -\frac{3}{4\sqrt{10}}$
$T_1(0,0,8,4) = -\frac{3}{4\sqrt{10}}$	$T_0(0,1,8,4) = \frac{1}{4\sqrt{10}}$	$T_{-1}(0,4,8,4) = \frac{1}{2\sqrt{10}}$
$T_1(0,1,1,8) = -\frac{1}{4\sqrt{10}}$	$T_0(0,4,1,8) = \frac{1}{4\sqrt{10}}$	$T_{-1}(0,8,4,4) = -\frac{3}{4\sqrt{10}}$
$T_1(0,1,8,1) = \frac{1}{\sqrt{10}}$	$T_0(0,4,8,1) = \frac{1}{4\sqrt{10}}$	$T_{-1}(1,1,4,8) = -\frac{1}{4\sqrt{10}}$
$T_1(0,4,0,8) = \frac{1}{2\sqrt{10}}$	$T_0(0,8,1,4) = \frac{1}{4\sqrt{10}}$	$T_{-1}(1,1,8,4) = -\frac{1}{4\sqrt{10}}$
$T_1(0,4,8,0) = -\frac{3}{4\sqrt{10}}$	$T_0(0,8,4,1) = -\frac{1}{\sqrt{10}}$	$T_{-1}(1,4,1,8) = \frac{1}{\sqrt{10}}$
$T_1(0,8,0,4) = \frac{1}{2\sqrt{10}}$	$T_0(1,0,4,8) = \frac{1}{4\sqrt{10}}$	$T_{-1}(1,4,8,1) = -\frac{1}{4\sqrt{10}}$
$T_1(0,8,1,1) = -\frac{1}{4\sqrt{10}}$	$T_0(1,0,8,4) = -\frac{1}{\sqrt{10}}$	$T_{-1}(1,8,1,4) = \frac{1}{\sqrt{10}}$
$T_1(0,8,4,0) = -\frac{3}{4\sqrt{10}}$	$T_0(1,1,1,8) = \frac{1}{2\sqrt{10}}$	$T_{-1}(1,8,4,1) = -\frac{1}{4\sqrt{10}}$
$T_1(1,0,1,8) = \frac{1}{\sqrt{10}}$	$T_0(1,1,8,1) = \frac{1}{2\sqrt{10}}$	$T_{-1}(4,0,4,8) = \frac{1}{2\sqrt{10}}$
$T_1(1,0,8,1) = -\frac{1}{4\sqrt{10}}$	$T_0(1,4,0,8) = \frac{1}{4\sqrt{10}}$	$T_{-1}(4,0,8,4) = -\frac{3}{4\sqrt{10}}$
$T_1(1,1,0,8) = -\frac{1}{4\sqrt{10}}$	$T_0(1,4,8,0) = -\frac{1}{\sqrt{10}}$	$T_{-1}(4,1,1,8) = -\frac{1}{4\sqrt{10}}$
$T_1(1,1,8,0) = -\frac{1}{4\sqrt{10}}$	$T_0(1,8,0,4) = \frac{1}{4\sqrt{10}}$	$T_{-1}(4,1,8,1) = \frac{1}{\sqrt{10}}$
$T_1(1,8,0,1) = -\frac{1}{4\sqrt{10}}$	$T_0(1,8,1,1) = \frac{1}{2\sqrt{10}}$	$T_{-1}(4,4,0,8) = -\frac{3}{4\sqrt{10}}$
$T_1(1,8,1,0) = \frac{1}{\sqrt{10}}$	$T_0(1,8,4,0) = \frac{1}{4\sqrt{10}}$	$T_{-1}(4,4,8,0) = -\frac{3}{4\sqrt{10}}$
$T_1(4,0,0,8) = -\frac{3}{4\sqrt{10}}$	$T_0(4,0,1,8) = \frac{1}{4\sqrt{10}}$	$T_{-1}(4,8,0,4) = -\frac{3}{4\sqrt{10}}$
$T_1(4,0,8,0) = \frac{1}{2\sqrt{10}}$	$T_0(4,0,8,1) = \frac{1}{4\sqrt{10}}$	$T_{-1}(4,8,1,1) = -\frac{1}{4\sqrt{10}}$
$T_1(4,8,0,0) = -\frac{3}{4\sqrt{10}}$	$T_0(4,1,0,8) = -\frac{1}{\sqrt{10}}$	$T_{-1}(4,8,4,0) = \frac{1}{2\sqrt{10}}$
$T_1(8,0,0,4) = -\frac{3}{4\sqrt{10}}$	$T_0(4,1,8,0) = \frac{1}{4\sqrt{10}}$	$T_{-1}(8,0,4,4) = -\frac{3}{4\sqrt{10}}$
$T_1(8,0,1,1) = -\frac{1}{4\sqrt{10}}$	$T_0(4,8,0,1) = -\frac{1}{\sqrt{10}}$	$T_{-1}(8,1,1,4) = -\frac{1}{4\sqrt{10}}$
$T_1(8,0,4,0) = \frac{1}{2\sqrt{10}}$	$T_0(4,8,1,0) = \frac{1}{4\sqrt{10}}$	$T_{-1}(8,1,4,1) = \frac{1}{\sqrt{10}}$
$T_1(8,1,0,1) = \frac{1}{\sqrt{10}}$	$T_0(8,0,1,4) = -\frac{1}{\sqrt{10}}$	$T_{-1}(8,4,0,4) = \frac{1}{2\sqrt{10}}$
$T_1(8,1,1,0) = -\frac{1}{4\sqrt{10}}$	$T_0(8,0,4,1) = \frac{1}{4\sqrt{10}}$	$T_{-1}(8,4,1,1) = -\frac{1}{4\sqrt{10}}$
$T_1(8,4,0,0) = -\frac{3}{4\sqrt{10}}$	$T_0(8,1,0,4) = \frac{1}{4\sqrt{10}}$	$T_{-1}(8,4,4,0) = -\frac{3}{4\sqrt{10}}$
	$T_0(8,1,1,1) = \frac{1}{2\sqrt{10}}$	
	$T_0(8,1,4,0) = \frac{1}{4\sqrt{10}}$	
	$T_0(8,4,0,1) = \frac{1}{4\sqrt{10}}$	
	$T_0(8,4,1,0) = -\frac{1}{\sqrt{10}}$	

TABLE XXV. G_4 .

$S_z = +1$	$S_z = 0$	$S_z = -1$
$T_1(0,0,4,8) = \frac{1}{4}$	$T_0(0,1,8,4) = -\frac{1}{4}$	$T_{-1}(0,4,4,8) = -\frac{1}{4}$
$T_1(0,0,8,4) = -\frac{1}{4}$	$T_0(0,4,1,8) = -\frac{1}{4}$	$T_{-1}(0,8,4,4) = \frac{1}{4}$
$T_1(0,1,1,8) = -\frac{1}{4}$	$T_0(0,4,8,1) = \frac{1}{4}$	$T_{-1}(1,1,4,8) = \frac{1}{4}$
$T_1(0,4,8,0) = \frac{1}{4}$	$T_0(0,8,1,4) = \frac{1}{4}$	$T_{-1}(1,1,8,4) = -\frac{1}{4}$
$T_1(0,8,1,1) = \frac{1}{4}$	$T_0(1,0,4,8) = \frac{1}{4}$	$T_{-1}(1,4,8,1) = \frac{1}{4}$
$T_1(0,8,4,0) = -\frac{1}{4}$	$T_0(1,4,0,8) = \frac{1}{4}$	$T_{-1}(1,8,4,1) = -\frac{1}{4}$
$T_1(1,0,8,1) = \frac{1}{4}$	$T_0(1,8,0,4) = -\frac{1}{4}$	$T_{-1}(4,0,8,4) = \frac{1}{4}$
$T_1(1,1,0,8) = \frac{1}{4}$	$T_0(1,8,4,0) = -\frac{1}{4}$	$T_{-1}(4,1,1,8) = -\frac{1}{4}$
$T_1(1,1,8,0) = -\frac{1}{4}$	$T_0(4,0,1,8) = -\frac{1}{4}$	$T_{-1}(4,4,0,8) = \frac{1}{4}$
$T_1(1,8,0,1) = -\frac{1}{4}$	$T_0(4,0,8,1) = \frac{1}{4}$	$T_{-1}(4,4,8,0) = -\frac{1}{4}$
$T_1(4,0,0,8) = -\frac{1}{4}$	$T_0(4,1,8,0) = -\frac{1}{4}$	$T_{-1}(4,8,0,4) = -\frac{1}{4}$
$T_1(4,8,0,0) = \frac{1}{4}$	$T_0(4,8,1,0) = \frac{1}{4}$	$T_{-1}(4,8,1,1) = \frac{1}{4}$
$T_1(8,0,0,4) = \frac{1}{4}$	$T_0(8,0,4,1) = -\frac{1}{4}$	$T_{-1}(8,0,4,4) = -\frac{1}{4}$
$T_1(8,0,1,1) = -\frac{1}{4}$	$T_0(8,1,0,4) = \frac{1}{4}$	$T_{-1}(8,1,1,4) = \frac{1}{4}$
$T_1(8,1,1,0) = \frac{1}{4}$	$T_0(8,1,4,0) = \frac{1}{4}$	$T_{-1}(8,4,1,1) = -\frac{1}{4}$
$T_1(8,4,0,0) = -\frac{1}{4}$	$T_0(8,4,0,1) = -\frac{1}{4}$	$T_{-1}(8,4,4,0) = \frac{1}{4}$

TABLE XXVI. $S_5^{(a)}$.

$S_z = +1$	$S_z = 0$	$S_z = -1$
$T_1(0,3,3,8) = \frac{1}{2\sqrt{2}}$	$T_0(1,3,3,8) = \frac{1}{2\sqrt{2}}$	$T_{-1}(3,3,4,8) = \frac{1}{2\sqrt{2}}$
$T_1(0,8,3,3) = \frac{1}{2\sqrt{2}}$	$T_0(1,8,3,3) = \frac{1}{2\sqrt{2}}$	$T_{-1}(3,3,8,4) = \frac{1}{2\sqrt{2}}$
$T_1(3,0,8,3) = \frac{1}{2\sqrt{2}}$	$T_0(3,1,8,3) = \frac{1}{2\sqrt{2}}$	$T_{-1}(3,4,8,3) = \frac{1}{2\sqrt{2}}$
$T_1(3,3,0,8) = \frac{1}{2\sqrt{2}}$	$T_0(3,3,1,8) = \frac{1}{2\sqrt{2}}$	$T_{-1}(3,8,4,3) = \frac{1}{2\sqrt{2}}$
$T_1(3,3,8,0) = \frac{1}{2\sqrt{2}}$	$T_0(3,3,8,1) = \frac{1}{2\sqrt{2}}$	$T_{-1}(4,3,3,8) = \frac{1}{2\sqrt{2}}$
$T_1(3,8,0,3) = \frac{1}{2\sqrt{2}}$	$T_0(3,8,1,3) = \frac{1}{2\sqrt{2}}$	$T_{-1}(4,8,3,3) = \frac{1}{2\sqrt{2}}$
$T_1(8,0,3,3) = \frac{1}{2\sqrt{2}}$	$T_0(8,1,3,3) = \frac{1}{2\sqrt{2}}$	$T_{-1}(8,3,3,4) = \frac{1}{2\sqrt{2}}$
$T_1(8,3,3,0) = \frac{1}{2\sqrt{2}}$	$T_0(8,3,3,1) = \frac{1}{2\sqrt{2}}$	$T_{-1}(8,4,3,3) = \frac{1}{2\sqrt{2}}$

TABLE XXVII. $S_5^{(b)}$.

$S_z = +1$	$S_z = 0$	$S_z = -1$
$T_1(0,3,8,3) = \frac{1}{2}$	$T_0(1,3,8,3) = \frac{1}{2}$	$T_{-1}(3,4,3,8) = \frac{1}{2}$
$T_1(3,0,3,8) = \frac{1}{2}$	$T_0(3,1,3,8) = \frac{1}{2}$	$T_{-1}(3,8,3,4) = \frac{1}{2}$
$T_1(3,8,3,0) = \frac{1}{2}$	$T_0(3,8,3,1) = \frac{1}{2}$	$T_{-1}(4,3,8,3) = \frac{1}{2}$
$T_1(8,3,0,3) = \frac{1}{2}$	$T_0(8,3,1,3) = \frac{1}{2}$	$T_{-1}(8,3,4,3) = \frac{1}{2}$

TABLE XXVIII. G_5 .

$S_z = +1$	$S_z = 0$	$S_z = -1$
$T_1(0,3,3,8) = \frac{1}{2\sqrt{2}}$	$T_0(1,3,3,8) = \frac{1}{2\sqrt{2}}$	$T_{-1}(3,3,4,8) = -\frac{1}{2\sqrt{2}}$
$T_1(0,8,3,3) = -\frac{1}{2\sqrt{2}}$	$T_0(1,8,3,3) = -\frac{1}{2\sqrt{2}}$	$T_{-1}(3,3,8,4) = \frac{1}{2\sqrt{2}}$
$T_1(3,0,8,3) = -\frac{1}{2\sqrt{2}}$	$T_0(3,1,8,3) = -\frac{1}{2\sqrt{2}}$	$T_{-1}(3,4,8,3) = -\frac{1}{2\sqrt{2}}$
$T_1(3,3,0,8) = -\frac{1}{2\sqrt{2}}$	$T_0(3,3,1,8) = -\frac{1}{2\sqrt{2}}$	$T_{-1}(3,8,4,3) = \frac{1}{2\sqrt{2}}$
$T_1(3,3,8,0) = \frac{1}{2\sqrt{2}}$	$T_0(3,3,8,1) = \frac{1}{2\sqrt{2}}$	$T_{-1}(4,3,3,8) = \frac{1}{2\sqrt{2}}$
$T_1(3,8,0,3) = \frac{1}{2\sqrt{2}}$	$T_0(3,8,1,3) = \frac{1}{2\sqrt{2}}$	$T_{-1}(4,8,3,3) = -\frac{1}{2\sqrt{2}}$
$T_1(8,0,3,3) = \frac{1}{2\sqrt{2}}$	$T_0(8,1,3,3) = \frac{1}{2\sqrt{2}}$	$T_{-1}(8,3,3,4) = -\frac{1}{2\sqrt{2}}$
$T_1(8,3,3,0) = -\frac{1}{2\sqrt{2}}$	$T_0(8,3,3,1) = -\frac{1}{2\sqrt{2}}$	$T_{-1}(8,4,3,3) = \frac{1}{2\sqrt{2}}$

TABLE XXIX. $S_6^{(a)}$.

$S_z = +1$	$S_z = 0$	$S_z = -1$
$T_1(2,2,3,3) = -\frac{1}{2\sqrt{2}}$	$T_0(2,3,3,5) = -\frac{1}{4}$	$T_{-1}(3,3,5,5) = -\frac{1}{2\sqrt{2}}$
$T_1(2,3,3,2) = -\frac{1}{2\sqrt{2}}$	$T_0(2,5,3,3) = -\frac{1}{4}$	$T_{-1}(3,3,7,7) = \frac{1}{2\sqrt{2}}$
$T_1(3,2,2,3) = -\frac{1}{2\sqrt{2}}$	$T_0(3,2,5,3) = -\frac{1}{4}$	$T_{-1}(3,5,5,3) = -\frac{1}{2\sqrt{2}}$
$T_1(3,3,2,2) = -\frac{1}{2\sqrt{2}}$	$T_0(3,3,2,5) = -\frac{1}{4}$	$T_{-1}(3,7,7,3) = \frac{1}{2\sqrt{2}}$
$T_1(3,3,6,6) = \frac{1}{2\sqrt{2}}$	$T_0(3,3,5,2) = -\frac{1}{4}$	$T_{-1}(5,3,3,5) = -\frac{1}{2\sqrt{2}}$
$T_1(3,6,6,3) = \frac{1}{2\sqrt{2}}$	$T_0(3,3,6,7) = \frac{1}{4}$	$T_{-1}(5,5,3,3) = -\frac{1}{2\sqrt{2}}$
$T_1(6,3,3,6) = \frac{1}{2\sqrt{2}}$	$T_0(3,3,7,6) = \frac{1}{4}$	$T_{-1}(7,3,3,7) = \frac{1}{2\sqrt{2}}$
$T_1(6,6,3,3) = \frac{1}{2\sqrt{2}}$	$T_0(3,5,2,3) = -\frac{1}{4}$	$T_{-1}(7,7,3,3) = \frac{1}{2\sqrt{2}}$
	$T_0(3,6,7,3) = \frac{1}{4}$	
	$T_0(3,7,6,3) = \frac{1}{4}$	
	$T_0(5,2,3,3) = -\frac{1}{4}$	
	$T_0(5,3,3,2) = -\frac{1}{4}$	
	$T_0(6,3,3,7) = \frac{1}{4}$	
	$T_0(6,7,3,3) = \frac{1}{4}$	
	$T_0(7,3,3,6) = \frac{1}{4}$	
	$T_0(7,6,3,3) = \frac{1}{4}$	

TABLE XXX. $S_6^{(b)}$.

$S_z = +1$	$S_z = 0$	$S_z = -1$
$T_1(2,3,2,3) = -\frac{1}{2}$	$T_0(2,3,5,3) = -\frac{1}{2\sqrt{2}}$	$T_{-1}(3,5,3,5) = -\frac{1}{2}$
$T_1(3,2,3,2) = -\frac{1}{2}$	$T_0(3,2,3,5) = -\frac{1}{2\sqrt{2}}$	$T_{-1}(3,7,3,7) = \frac{1}{2}$
$T_1(3,6,3,6) = \frac{1}{2}$	$T_0(3,5,3,2) = -\frac{1}{2\sqrt{2}}$	$T_{-1}(5,3,5,3) = -\frac{1}{2}$
$T_1(6,3,6,3) = \frac{1}{2}$	$T_0(3,6,3,7) = \frac{1}{2\sqrt{2}}$	$T_{-1}(7,3,7,3) = \frac{1}{2}$
	$T_0(3,7,3,6) = \frac{1}{2\sqrt{2}}$	
	$T_0(5,3,2,3) = -\frac{1}{2\sqrt{2}}$	
	$T_0(6,3,7,3) = \frac{1}{2\sqrt{2}}$	
	$T_0(7,3,6,3) = \frac{1}{2\sqrt{2}}$	

TABLE XXXI. $S_7^{(a)}$.

$S_z = +1$	$S_z = 0$	$S_z = -1$
$T_1(0,2,4,2) = -\frac{1}{2\sqrt{3}}$	$T_0(0,2,4,5) = -\frac{1}{2\sqrt{6}}$	$T_{-1}(0,5,4,5) = -\frac{1}{2\sqrt{3}}$
$T_1(0,6,4,6) = \frac{1}{2\sqrt{3}}$	$T_0(0,5,4,2) = -\frac{1}{2\sqrt{6}}$	$T_{-1}(0,7,4,7) = \frac{1}{2\sqrt{3}}$
$T_1(1,2,1,2) = \frac{1}{2\sqrt{3}}$	$T_0(0,6,4,7) = \frac{1}{2\sqrt{6}}$	$T_{-1}(1,5,1,5) = \frac{1}{2\sqrt{3}}$
$T_1(1,6,1,6) = -\frac{1}{2\sqrt{3}}$	$T_0(0,7,4,6) = \frac{1}{2\sqrt{6}}$	$T_{-1}(1,7,1,7) = -\frac{1}{2\sqrt{3}}$
$T_1(2,0,2,4) = -\frac{1}{2\sqrt{3}}$	$T_0(1,2,1,5) = \frac{1}{2\sqrt{6}}$	$T_{-1}(4,5,0,5) = -\frac{1}{2\sqrt{3}}$
$T_1(2,1,2,1) = \frac{1}{2\sqrt{3}}$	$T_0(1,5,1,2) = \frac{1}{2\sqrt{6}}$	$T_{-1}(4,7,0,7) = \frac{1}{2\sqrt{3}}$
$T_1(2,4,2,0) = -\frac{1}{2\sqrt{3}}$	$T_0(1,6,1,7) = -\frac{1}{2\sqrt{6}}$	$T_{-1}(5,0,5,4) = -\frac{1}{2\sqrt{3}}$
$T_1(4,2,0,2) = -\frac{1}{2\sqrt{3}}$	$T_0(1,7,1,6) = -\frac{1}{2\sqrt{6}}$	$T_{-1}(5,1,5,1) = \frac{1}{2\sqrt{3}}$
$T_1(4,6,0,6) = \frac{1}{2\sqrt{3}}$	$T_0(2,0,5,4) = -\frac{1}{2\sqrt{6}}$	$T_{-1}(5,4,5,0) = -\frac{1}{2\sqrt{3}}$

TABLE XXXI. (Continued.)

$S_z = +1$	$S_z = 0$	$S_z = -1$
$T_1(6,0,6,4) = \frac{1}{2\sqrt{3}}$	$T_0(2,1,5,1) = \frac{1}{2\sqrt{6}}$	$T_{-1}(7,0,7,4) = \frac{1}{2\sqrt{3}}$
$T_1(6,1,6,1) = -\frac{1}{2\sqrt{3}}$	$T_0(2,4,5,0) = -\frac{1}{2\sqrt{6}}$	$T_{-1}(7,1,7,1) = -\frac{1}{2\sqrt{3}}$
$T_1(6,4,6,0) = \frac{1}{2\sqrt{3}}$	$T_0(4,2,0,5) = -\frac{1}{2\sqrt{6}}$	$T_{-1}(7,4,7,0) = \frac{1}{2\sqrt{3}}$
	$T_0(4,5,0,2) = -\frac{1}{2\sqrt{6}}$	
	$T_0(4,6,0,7) = \frac{1}{2\sqrt{6}}$	
	$T_0(4,7,0,6) = \frac{1}{2\sqrt{6}}$	
	$T_0(5,0,2,4) = -\frac{1}{2\sqrt{6}}$	
	$T_0(5,1,2,1) = \frac{1}{2\sqrt{6}}$	
	$T_0(5,4,2,0) = -\frac{1}{2\sqrt{6}}$	
	$T_0(6,0,7,4) = \frac{1}{2\sqrt{6}}$	
	$T_0(6,1,7,1) = -\frac{1}{2\sqrt{6}}$	
	$T_0(6,4,7,0) = \frac{1}{2\sqrt{6}}$	
	$T_0(7,0,6,4) = \frac{1}{2\sqrt{6}}$	
	$T_0(7,1,6,1) = -\frac{1}{2\sqrt{6}}$	
	$T_0(7,4,6,0) = \frac{1}{2\sqrt{6}}$	

TABLE XXXII. $S_7^{(b)}$.

$S_z = +1$	$S_z = 0$	$S_z = -1$
$T_1(0,2,1,5) = -\frac{\sqrt{\frac{3}{10}}}{4}$	$T_0(0,2,4,5) = -\frac{1}{2\sqrt{30}}$	$T_{-1}(0,5,4,5) = \frac{1}{4\sqrt{15}}$
$T_1(0,2,4,2) = \frac{1}{4\sqrt{15}}$	$T_0(0,5,1,5) = \frac{\sqrt{\frac{3}{5}}}{4}$	$T_{-1}(0,7,4,7) = -\frac{1}{4\sqrt{15}}$
$T_1(0,5,0,5) = \frac{\sqrt{\frac{3}{5}}}{2}$	$T_0(0,5,4,2) = -\frac{1}{2\sqrt{30}}$	$T_{-1}(1,2,4,5) = -\frac{\sqrt{\frac{3}{10}}}{4}$
$T_1(0,5,1,2) = -\frac{\sqrt{\frac{3}{10}}}{4}$	$T_0(0,6,4,7) = \frac{1}{2\sqrt{30}}$	$T_{-1}(1,5,1,5) = \frac{1}{2\sqrt{15}}$
$T_1(0,6,1,7) = \frac{\sqrt{\frac{3}{10}}}{4}$	$T_0(0,7,1,7) = -\frac{\sqrt{\frac{3}{5}}}{4}$	$T_{-1}(1,5,4,2) = -\frac{\sqrt{\frac{3}{10}}}{4}$
$T_1(0,6,4,6) = -\frac{1}{4\sqrt{15}}$	$T_0(0,7,4,6) = \frac{1}{2\sqrt{30}}$	$T_{-1}(1,6,4,7) = \frac{\sqrt{\frac{3}{10}}}{4}$
$T_1(0,7,0,7) = -\frac{\sqrt{\frac{3}{5}}}{2}$	$T_0(1,2,1,5) = -\frac{1}{\sqrt{30}}$	$T_{-1}(1,7,1,7) = -\frac{1}{2\sqrt{15}}$
$T_1(0,7,1,6) = \frac{\sqrt{\frac{3}{10}}}{4}$	$T_0(1,2,4,2) = \frac{\sqrt{\frac{3}{5}}}{4}$	$T_{-1}(1,7,4,6) = \frac{\sqrt{\frac{3}{10}}}{4}$
$T_1(1,2,0,5) = -\frac{\sqrt{\frac{3}{10}}}{4}$	$T_0(1,5,0,5) = \frac{\sqrt{\frac{3}{5}}}{4}$	$T_{-1}(2,1,5,4) = -\frac{\sqrt{\frac{3}{10}}}{4}$
$T_1(1,2,1,2) = \frac{1}{2\sqrt{15}}$	$T_0(1,5,1,2) = -\frac{1}{\sqrt{30}}$	$T_{-1}(2,4,2,4) = \frac{\sqrt{\frac{3}{5}}}{2}$
$T_1(1,5,0,2) = -\frac{\sqrt{\frac{3}{10}}}{4}$	$T_0(1,6,1,7) = \frac{1}{\sqrt{30}}$	$T_{-1}(2,4,5,1) = -\frac{\sqrt{\frac{3}{10}}}{4}$
$T_1(1,6,0,7) = \frac{\sqrt{\frac{3}{10}}}{4}$	$T_0(1,6,4,6) = -\frac{\sqrt{\frac{3}{5}}}{4}$	$T_{-1}(4,2,1,5) = -\frac{\sqrt{\frac{3}{10}}}{4}$
$T_1(1,6,1,6) = -\frac{1}{2\sqrt{15}}$	$T_0(1,7,0,7) = -\frac{\sqrt{\frac{3}{5}}}{4}$	$T_{-1}(4,2,4,2) = \frac{\sqrt{\frac{3}{5}}}{2}$
$T_1(1,7,0,6) = \frac{\sqrt{\frac{3}{10}}}{4}$	$T_0(1,7,1,6) = \frac{1}{\sqrt{30}}$	$T_{-1}(4,5,0,5) = \frac{1}{4\sqrt{15}}$
$T_1(2,0,2,4) = \frac{1}{4\sqrt{15}}$	$T_0(2,0,5,4) = -\frac{1}{2\sqrt{30}}$	$T_{-1}(4,5,1,2) = -\frac{\sqrt{\frac{3}{10}}}{4}$
$T_1(2,0,5,1) = -\frac{\sqrt{\frac{3}{10}}}{4}$	$T_0(2,1,2,4) = \frac{\sqrt{\frac{3}{5}}}{4}$	$T_{-1}(4,6,1,7) = \frac{\sqrt{\frac{3}{10}}}{4}$
$T_1(2,1,2,1) = \frac{1}{2\sqrt{15}}$	$T_0(2,1,5,1) = -\frac{1}{\sqrt{30}}$	$T_{-1}(4,6,4,6) = -\frac{\sqrt{\frac{3}{5}}}{2}$
$T_1(2,1,5,0) = -\frac{\sqrt{\frac{3}{10}}}{4}$	$T_0(2,4,2,1) = \frac{\sqrt{\frac{3}{5}}}{4}$	$T_{-1}(4,7,0,7) = -\frac{1}{4\sqrt{15}}$
$T_1(2,4,2,0) = \frac{1}{4\sqrt{15}}$	$T_0(2,4,5,0) = -\frac{1}{2\sqrt{30}}$	$T_{-1}(4,7,1,6) = \frac{\sqrt{\frac{3}{10}}}{4}$
$T_1(4,2,0,2) = \frac{1}{4\sqrt{15}}$	$T_0(4,2,0,5) = -\frac{1}{2\sqrt{30}}$	$T_{-1}(5,0,5,4) = \frac{1}{4\sqrt{15}}$
$T_1(4,6,0,6) = -\frac{1}{4\sqrt{15}}$	$T_0(4,2,1,2) = \frac{\sqrt{\frac{3}{5}}}{4}$	$T_{-1}(5,1,2,4) = -\frac{\sqrt{\frac{3}{10}}}{4}$

TABLE XXXII. (Continued.)

$S_z = +1$	$S_z = 0$	$S_z = -1$
$T_1(5,0,2,1) = -\frac{\sqrt{10}}{4}$	$T_0(4,5,0,2) = -\frac{1}{2\sqrt{30}}$	$T_{-1}(5,1,5,1) = \frac{1}{2\sqrt{15}}$
$T_1(5,0,5,0) = \frac{\sqrt{3}}{2}$	$T_0(4,6,0,7) = \frac{1}{2\sqrt{30}}$	$T_{-1}(5,4,2,1) = -\frac{\sqrt{30}}{4}$
$T_1(5,1,2,0) = -\frac{\sqrt{30}}{4}$	$T_0(4,6,1,6) = -\frac{\sqrt{3}}{4}$	$T_{-1}(5,4,5,0) = \frac{1}{4\sqrt{15}}$
$T_1(6,0,6,4) = -\frac{1}{4\sqrt{15}}$	$T_0(4,7,0,6) = \frac{1}{2\sqrt{30}}$	$T_{-1}(6,1,7,4) = \frac{\sqrt{10}}{4}$
$T_1(6,0,7,1) = \frac{\sqrt{30}}{4}$	$T_0(5,0,2,4) = -\frac{1}{2\sqrt{30}}$	$T_{-1}(6,4,6,4) = -\frac{\sqrt{3}}{2}$
$T_1(6,1,6,1) = -\frac{1}{2\sqrt{15}}$	$T_0(5,0,5,1) = \frac{\sqrt{3}}{4}$	$T_{-1}(6,4,7,1) = \frac{\sqrt{10}}{4}$
$T_1(6,1,7,0) = \frac{\sqrt{30}}{4}$	$T_0(5,1,2,1) = -\frac{1}{4\sqrt{30}}$	$T_{-1}(7,0,7,4) = -\frac{1}{4\sqrt{15}}$
$T_1(6,4,6,0) = -\frac{1}{4\sqrt{15}}$	$T_0(5,1,5,0) = \frac{\sqrt{3}}{4}$	$T_{-1}(7,1,6,4) = \frac{\sqrt{10}}{4}$
$T_1(7,0,6,1) = \frac{\sqrt{30}}{4}$	$T_0(5,4,2,0) = -\frac{1}{2\sqrt{30}}$	$T_{-1}(7,1,7,1) = -\frac{1}{2\sqrt{15}}$
$T_1(7,0,7,0) = -\frac{\sqrt{3}}{2}$	$T_0(6,0,7,4) = \frac{1}{2\sqrt{30}}$	$T_{-1}(7,4,6,1) = \frac{\sqrt{30}}{4}$
$T_1(7,1,6,0) = \frac{\sqrt{30}}{4}$	$T_0(6,1,6,4) = -\frac{\sqrt{3}}{4}$	$T_{-1}(7,4,7,0) = -\frac{1}{4\sqrt{15}}$
	$T_0(6,1,7,1) = \frac{1}{3\sqrt{30}}$	
	$T_0(6,4,6,1) = -\frac{\sqrt{3}}{4}$	
	$T_0(6,4,7,0) = \frac{1}{2\sqrt{30}}$	
	$T_0(7,0,6,4) = \frac{1}{2\sqrt{30}}$	
	$T_0(7,0,7,1) = -\frac{\sqrt{3}}{4}$	
	$T_0(7,1,6,1) = \frac{1}{\sqrt{30}}$	
	$T_0(7,1,7,0) = -\frac{\sqrt{3}}{4}$	
	$T_0(7,4,6,0) = \frac{1}{2\sqrt{30}}$	

TABLE XXXIII. (Continued.)

$S_z = +1$	$S_z = 0$	$S_z = -1$
$T_1(2,1,0,5) = -\frac{1}{6}$	$T_0(1,4,2,2) = \frac{1}{6\sqrt{2}}$	$T_{-1}(4,4,2,2) = \frac{1}{3\sqrt{2}}$
$T_1(2,1,1,2) = \frac{1}{6\sqrt{2}}$	$T_0(1,4,6,6) = -\frac{1}{6\sqrt{2}}$	$T_{-1}(4,4,6,6) = -\frac{1}{3\sqrt{2}}$
$T_1(2,2,1,1) = \frac{1}{6\sqrt{2}}$	$T_0(1,5,2,1) = -\frac{1}{12}$	$T_{-1}(4,6,6,4) = -\frac{1}{3\sqrt{2}}$
$T_1(2,5,0,1) = -\frac{1}{6}$	$T_0(1,5,5,0) = \frac{1}{6\sqrt{2}}$	$T_{-1}(4,6,7,1) = \frac{1}{6}$
$T_1(5,0,0,5) = \frac{1}{3\sqrt{2}}$	$T_0(1,6,6,4) = -\frac{1}{6\sqrt{2}}$	$T_{-1}(5,1,1,5) = \frac{1}{6\sqrt{2}}$
$T_1(5,0,1,2) = -\frac{1}{6}$	$T_0(1,6,7,1) = \frac{1}{12}$	$T_{-1}(5,1,4,2) = -\frac{1}{6}$
$T_1(5,2,1,0) = -\frac{1}{6}$	$T_0(1,7,6,1) = \frac{1}{12}$	$T_{-1}(5,2,4,1) = -\frac{1}{6}$
$T_1(5,5,0,0) = \frac{1}{3\sqrt{2}}$	$T_0(1,7,7,0) = -\frac{1}{6\sqrt{2}}$	$T_{-1}(5,5,1,1) = \frac{1}{6\sqrt{2}}$
$T_1(6,1,0,7) = \frac{1}{6}$	$T_0(2,1,1,5) = -\frac{1}{12}$	$T_{-1}(6,4,1,7) = \frac{1}{6}$
$T_1(6,1,1,6) = -\frac{1}{6\sqrt{2}}$	$T_0(2,1,4,2) = \frac{1}{6\sqrt{2}}$	$T_{-1}(6,4,4,6) = -\frac{1}{3\sqrt{2}}$
$T_1(6,6,1,1) = -\frac{1}{6\sqrt{2}}$	$T_0(2,2,1,4) = \frac{1}{6\sqrt{2}}$	$T_{-1}(6,6,4,4) = -\frac{1}{3\sqrt{2}}$
$T_1(6,7,0,1) = \frac{1}{6}$	$T_0(2,2,4,1) = \frac{1}{6\sqrt{2}}$	$T_{-1}(6,7,1,4) = \frac{1}{6}$
$T_1(7,0,0,7) = -\frac{1}{3\sqrt{2}}$	$T_0(2,4,0,5) = -\frac{1}{6}$	$T_{-1}(7,1,1,7) = -\frac{1}{6\sqrt{2}}$
$T_1(7,0,1,6) = \frac{1}{6}$	$T_0(2,4,1,2) = \frac{1}{6\sqrt{2}}$	$T_{-1}(7,1,4,6) = \frac{1}{6}$
$T_1(7,6,1,0) = \frac{1}{6}$	$T_0(2,5,0,4) = -\frac{1}{6}$	$T_{-1}(7,6,4,1) = \frac{1}{6}$
$T_1(7,7,0,0) = -\frac{1}{3\sqrt{2}}$	$T_0(2,5,1,1) = -\frac{1}{12}$	$T_{-1}(7,7,1,1) = -\frac{1}{6\sqrt{2}}$
	$T_0(4,0,5,2) = -\frac{1}{6}$	
	$T_0(4,0,7,6) = \frac{1}{6}$	
	$T_0(4,1,2,2) = \frac{1}{6\sqrt{2}}$	
	$T_0(4,1,6,6) = -\frac{1}{6\sqrt{2}}$	
	$T_0(4,2,2,1) = \frac{1}{6\sqrt{2}}$	
	$T_0(4,2,5,0) = -\frac{1}{6}$	
	$T_0(4,6,6,1) = -\frac{1}{6\sqrt{2}}$	
	$T_0(4,6,7,0) = \frac{1}{6}$	
	$T_0(5,0,1,5) = \frac{1}{6\sqrt{2}}$	
	$T_0(5,0,4,2) = -\frac{1}{6}$	
	$T_0(5,1,0,5) = \frac{1}{6\sqrt{2}}$	
	$T_0(5,1,1,2) = -\frac{1}{12}$	
	$T_0(5,2,1,1) = -\frac{1}{12}$	
	$T_0(5,2,4,0) = -\frac{1}{6}$	
	$T_0(5,5,0,1) = \frac{1}{6\sqrt{2}}$	
	$T_0(5,5,1,0) = \frac{1}{6\sqrt{2}}$	
	$T_0(6,1,1,7) = \frac{1}{12}$	
	$T_0(6,1,4,6) = -\frac{1}{6\sqrt{2}}$	
	$T_0(6,4,0,7) = \frac{1}{6}$	
	$T_0(6,4,1,6) = -\frac{1}{6\sqrt{2}}$	
	$T_0(6,6,1,4) = -\frac{1}{6\sqrt{2}}$	
	$T_0(6,6,4,1) = -\frac{1}{6\sqrt{2}}$	
	$T_0(6,7,0,4) = \frac{1}{6}$	
	$T_0(6,7,1,1) = \frac{1}{12}$	
	$T_0(7,0,1,7) = -\frac{1}{6\sqrt{2}}$	
	$T_0(7,0,4,6) = \frac{1}{6}$	
	$T_0(7,1,0,7) = -\frac{1}{6\sqrt{2}}$	
	$T_0(7,1,1,6) = \frac{1}{12}$	
	$T_0(7,6,1,1) = \frac{1}{12}$	
	$T_0(7,6,4,0) = \frac{1}{6}$	
	$T_0(7,7,0,1) = -\frac{1}{6\sqrt{2}}$	
	$T_0(7,7,1,0) = -\frac{1}{6\sqrt{2}}$	

TABLE XXXIII. $S_7^{(c)}$.

$S_z = +1$	$S_z = 0$	$S_z = -1$
$T_1(0,0,5,5) = \frac{1}{3\sqrt{2}}$	$T_0(0,1,5,5) = \frac{1}{6\sqrt{2}}$	$T_{-1}(1,1,5,5) = \frac{1}{6\sqrt{2}}$
$T_1(0,0,7,7) = -\frac{1}{3\sqrt{2}}$	$T_0(0,1,7,7) = -\frac{1}{6\sqrt{2}}$	$T_{-1}(1,1,7,7) = -\frac{1}{6\sqrt{2}}$
$T_1(0,1,2,5) = -\frac{1}{6}$	$T_0(0,4,2,5) = -\frac{1}{6}$	$T_{-1}(1,4,2,5) = -\frac{1}{6}$
$T_1(0,1,6,7) = \frac{1}{6}$	$T_0(0,4,6,7) = \frac{1}{6}$	$T_{-1}(1,4,6,7) = \frac{1}{6}$
$T_1(0,5,2,1) = -\frac{1}{6}$	$T_0(0,5,2,4) = -\frac{1}{6}$	$T_{-1}(1,5,2,4) = -\frac{1}{6}$
$T_1(0,5,5,0) = \frac{1}{3\sqrt{2}}$	$T_0(0,5,5,1) = \frac{1}{6\sqrt{2}}$	$T_{-1}(1,5,5,1) = \frac{1}{6\sqrt{2}}$
$T_1(0,7,6,1) = \frac{1}{6}$	$T_0(0,7,6,4) = \frac{1}{6}$	$T_{-1}(1,7,6,4) = \frac{1}{6}$
$T_1(0,7,7,0) = -\frac{1}{3\sqrt{2}}$	$T_0(0,7,7,1) = -\frac{1}{6\sqrt{2}}$	$T_{-1}(1,7,7,1) = -\frac{1}{6\sqrt{2}}$
$T_1(1,0,5,2) = -\frac{1}{6}$	$T_0(1,0,5,5) = \frac{1}{6\sqrt{2}}$	$T_{-1}(2,2,4,4) = \frac{1}{3\sqrt{2}}$
$T_1(1,0,7,6) = \frac{1}{6}$	$T_0(1,0,7,7) = -\frac{1}{6\sqrt{2}}$	$T_{-1}(2,4,1,5) = -\frac{1}{6}$
$T_1(1,1,2,2) = \frac{1}{6\sqrt{2}}$	$T_0(1,1,2,5) = -\frac{1}{12}$	$T_{-1}(2,4,4,2) = \frac{1}{3\sqrt{2}}$
$T_1(1,1,6,6) = -\frac{1}{6\sqrt{2}}$	$T_0(1,1,5,2) = -\frac{1}{12}$	$T_{-1}(2,5,1,4) = -\frac{1}{6}$
$T_1(1,2,2,1) = \frac{1}{6\sqrt{2}}$	$T_0(1,1,6,7) = \frac{1}{12}$	$T_{-1}(4,1,5,2) = -\frac{1}{6}$
$T_1(1,2,5,0) = -\frac{1}{6}$	$T_0(1,1,7,6) = \frac{1}{12}$	$T_{-1}(4,1,7,6) = \frac{1}{6}$
$T_1(1,6,6,1) = -\frac{1}{6\sqrt{2}}$	$T_0(1,2,2,4) = \frac{1}{6\sqrt{2}}$	$T_{-1}(4,2,2,4) = \frac{1}{3\sqrt{2}}$
$T_1(1,6,7,0) = \frac{1}{6}$	$T_0(1,2,5,1) = -\frac{1}{12}$	$T_{-1}(4,2,5,1) = -\frac{1}{6}$

TABLE XXXIV. $S_7^{(d)}$.

$Sz = +1$	$Sz = 0$	$Sz = -1$
$T_1(0,0,5,5) = \frac{1}{6\sqrt{13}}$	$T_0(0,1,5,5) = \frac{1}{12\sqrt{13}}$	$T_{-1}(0,4,5,5) = \frac{3}{4\sqrt{13}}$
$T_1(0,0,7,7) = -\frac{1}{6\sqrt{13}}$	$T_0(0,1,7,7) = -\frac{1}{12\sqrt{13}}$	$T_{-1}(0,4,7,7) = -\frac{3}{4\sqrt{13}}$
$T_1(0,1,2,5) = -\frac{1}{6\sqrt{26}}$	$T_0(0,2,5,4) = \frac{3}{4\sqrt{26}}$	$T_{-1}(0,5,5,4) = \frac{3}{4\sqrt{13}}$
$T_1(0,1,6,7) = \frac{1}{6\sqrt{26}}$	$T_0(0,4,2,5) = \frac{7}{12\sqrt{26}}$	$T_{-1}(0,7,7,4) = -\frac{3}{4\sqrt{13}}$
$T_1(0,2,2,4) = \frac{3}{4\sqrt{13}}$	$T_0(0,4,5,2) = \frac{3}{4\sqrt{26}}$	$T_{-1}(1,1,5,5) = -\frac{2}{3\sqrt{13}}$
$T_1(0,4,2,2) = \frac{3}{4\sqrt{13}}$	$T_0(0,4,6,7) = -\frac{7}{12\sqrt{26}}$	$T_{-1}(1,1,7,7) = \frac{2}{3\sqrt{13}}$
$T_1(0,4,6,6) = -\frac{3}{4\sqrt{13}}$	$T_0(0,4,7,6) = -\frac{3}{4\sqrt{26}}$	$T_{-1}(1,4,2,5) = -\frac{1}{6\sqrt{26}}$
$T_1(0,5,2,1) = -\frac{1}{6\sqrt{26}}$	$T_0(0,5,2,4) = \frac{7}{12\sqrt{26}}$	$T_{-1}(1,4,6,7) = \frac{1}{6\sqrt{26}}$
$T_1(0,5,5,0) = \frac{1}{6\sqrt{13}}$	$T_0(0,5,5,1) = \frac{1}{12\sqrt{13}}$	$T_{-1}(1,5,2,4) = -\frac{1}{6\sqrt{26}}$
$T_1(0,6,6,4) = -\frac{3}{4\sqrt{13}}$	$T_0(0,6,7,4) = -\frac{3}{4\sqrt{26}}$	$T_{-1}(1,5,5,1) = -\frac{2}{3\sqrt{13}}$
$T_1(0,7,6,1) = \frac{1}{6\sqrt{26}}$	$T_0(0,7,6,4) = -\frac{7}{12\sqrt{26}}$	$T_{-1}(1,7,6,4) = \frac{1}{6\sqrt{26}}$
$T_1(0,7,7,0) = -\frac{1}{6\sqrt{13}}$	$T_0(0,7,7,1) = -\frac{1}{12\sqrt{13}}$	$T_{-1}(1,7,7,1) = \frac{2}{3\sqrt{13}}$
$T_1(1,0,5,2) = -\frac{1}{6\sqrt{26}}$	$T_0(1,0,5,5) = \frac{1}{12\sqrt{13}}$	$T_{-1}(2,2,4,4) = \frac{1}{6\sqrt{13}}$
$T_1(1,0,7,6) = \frac{1}{6\sqrt{26}}$	$T_0(1,0,7,7) = -\frac{1}{12\sqrt{13}}$	$T_{-1}(2,4,1,5) = -\frac{1}{6\sqrt{26}}$
$T_1(1,1,2,2) = -\frac{2}{3\sqrt{13}}$	$T_0(1,1,2,5) = -\frac{5}{6\sqrt{26}}$	$T_{-1}(2,4,4,2) = \frac{1}{6\sqrt{13}}$
$T_1(1,1,6,6) = \frac{2}{3\sqrt{13}}$	$T_0(1,1,5,2) = -\frac{5}{6\sqrt{26}}$	$T_{-1}(2,5,1,4) = -\frac{1}{6\sqrt{26}}$
$T_1(1,2,2,1) = -\frac{2}{3\sqrt{13}}$	$T_0(1,1,6,7) = \frac{5}{6\sqrt{26}}$	$T_{-1}(4,0,5,5) = \frac{3}{4\sqrt{13}}$
$T_1(1,2,5,0) = -\frac{1}{6\sqrt{26}}$	$T_0(1,1,7,6) = \frac{5}{6\sqrt{26}}$	$T_{-1}(4,0,7,7) = -\frac{3}{4\sqrt{13}}$
$T_1(1,6,6,1) = \frac{2}{3\sqrt{13}}$	$T_0(1,2,2,4) = \frac{1}{12\sqrt{13}}$	$T_{-1}(4,1,5,2) = -\frac{1}{6\sqrt{26}}$
$T_1(1,6,7,0) = \frac{1}{6\sqrt{26}}$	$T_0(1,2,5,1) = -\frac{5}{6\sqrt{26}}$	$T_{-1}(4,1,7,6) = \frac{1}{6\sqrt{26}}$
$T_1(2,0,4,2) = \frac{3}{4\sqrt{13}}$	$T_0(1,4,2,2) = \frac{1}{12\sqrt{13}}$	$T_{-1}(4,2,2,4) = \frac{1}{6\sqrt{13}}$
$T_1(2,1,0,5) = -\frac{1}{6\sqrt{26}}$	$T_0(1,4,6,6) = -\frac{1}{12\sqrt{13}}$	$T_{-1}(4,2,5,1) = -\frac{1}{6\sqrt{26}}$
$T_1(2,1,1,2) = -\frac{2}{3\sqrt{13}}$	$T_0(1,5,2,1) = -\frac{5}{6\sqrt{26}}$	$T_{-1}(4,4,2,2) = \frac{1}{6\sqrt{13}}$
$T_1(2,2,0,4) = \frac{3}{4\sqrt{13}}$	$T_0(1,5,5,0) = \frac{1}{12\sqrt{13}}$	$T_{-1}(4,4,6,6) = -\frac{1}{6\sqrt{13}}$
$T_1(2,2,1,1) = -\frac{2}{3\sqrt{13}}$	$T_0(1,6,6,4) = -\frac{1}{12\sqrt{13}}$	$T_{-1}(4,5,5,0) = \frac{3}{4\sqrt{13}}$
$T_1(2,2,4,0) = \frac{3}{4\sqrt{13}}$	$T_0(1,6,7,1) = \frac{5}{6\sqrt{26}}$	$T_{-1}(4,6,6,4) = -\frac{1}{6\sqrt{13}}$
$T_1(2,4,0,2) = \frac{3}{4\sqrt{13}}$	$T_0(1,7,6,1) = \frac{5}{6\sqrt{26}}$	$T_{-1}(4,6,7,1) = \frac{1}{6\sqrt{26}}$
$T_1(2,5,0,1) = -\frac{1}{6\sqrt{26}}$	$T_0(1,7,7,0) = -\frac{1}{12\sqrt{13}}$	$T_{-1}(4,7,7,0) = -\frac{3}{4\sqrt{13}}$
$T_1(4,0,2,2) = \frac{3}{4\sqrt{13}}$	$T_0(2,0,4,5) = \frac{3}{4\sqrt{26}}$	$T_{-1}(5,0,4,5) = \frac{3}{4\sqrt{13}}$
$T_1(4,0,6,6) = -\frac{3}{4\sqrt{13}}$	$T_0(2,1,1,5) = -\frac{5}{6\sqrt{26}}$	$T_{-1}(5,1,1,5) = -\frac{2}{3\sqrt{13}}$
$T_1(4,2,2,0) = \frac{3}{4\sqrt{13}}$	$T_0(2,1,4,2) = \frac{1}{12\sqrt{13}}$	$T_{-1}(5,1,4,2) = -\frac{1}{6\sqrt{26}}$
$T_1(4,6,6,0) = -\frac{3}{4\sqrt{13}}$	$T_0(2,2,1,4) = \frac{1}{12\sqrt{13}}$	$T_{-1}(5,2,4,1) = -\frac{1}{6\sqrt{26}}$
$T_1(5,0,0,5) = \frac{1}{6\sqrt{13}}$	$T_0(2,2,4,1) = \frac{1}{12\sqrt{13}}$	$T_{-1}(5,4,0,5) = \frac{3}{4\sqrt{13}}$
$T_1(5,0,1,2) = -\frac{1}{6\sqrt{26}}$	$T_0(2,4,0,5) = \frac{7}{12\sqrt{26}}$	$T_{-1}(5,5,0,4) = \frac{3}{4\sqrt{13}}$
$T_1(5,2,1,0) = -\frac{1}{6\sqrt{26}}$	$T_0(2,4,1,2) = \frac{1}{12\sqrt{13}}$	$T_{-1}(5,5,1,1) = -\frac{2}{3\sqrt{13}}$
$T_1(5,5,0,0) = \frac{1}{6\sqrt{13}}$	$T_0(2,5,0,4) = \frac{7}{12\sqrt{26}}$	$T_{-1}(5,5,4,0) = \frac{3}{4\sqrt{13}}$
$T_1(6,0,4,6) = -\frac{3}{4\sqrt{13}}$	$T_0(2,5,1,1) = -\frac{5}{6\sqrt{26}}$	$T_{-1}(6,4,1,7) = \frac{1}{6\sqrt{26}}$
$T_1(6,1,0,7) = \frac{1}{6\sqrt{26}}$	$T_0(2,5,4,0) = \frac{3}{4\sqrt{26}}$	$T_{-1}(6,4,4,6) = -\frac{1}{6\sqrt{13}}$
$T_1(6,1,1,6) = \frac{2}{3\sqrt{13}}$	$T_0(4,0,2,5) = \frac{3}{4\sqrt{26}}$	$T_{-1}(6,6,4,4) = -\frac{1}{6\sqrt{13}}$
$T_1(6,4,0,6) = -\frac{3}{4\sqrt{13}}$	$T_0(4,0,5,2) = \frac{7}{12\sqrt{26}}$	$T_{-1}(6,7,1,4) = \frac{1}{6\sqrt{26}}$

TABLE XXXIV. (Continued.)

$Sz = +1$	$Sz = 0$	$Sz = -1$
$T_1(6,6,0,4) = -\frac{3}{4\sqrt{13}}$	$T_0(4,0,6,7) = -\frac{3}{4\sqrt{26}}$	$T_{-1}(7,0,4,7) = -\frac{3}{4\sqrt{13}}$
$T_1(6,6,1,1) = \frac{2}{3\sqrt{13}}$	$T_0(4,0,7,6) = -\frac{7}{12\sqrt{26}}$	$T_{-1}(7,1,1,7) = \frac{2}{3\sqrt{13}}$
$T_1(6,6,4,0) = -\frac{3}{4\sqrt{13}}$	$T_0(4,1,2,2) = \frac{1}{12\sqrt{13}}$	$T_{-1}(7,1,4,6) = \frac{1}{6\sqrt{26}}$
$T_1(6,7,0,1) = \frac{1}{6\sqrt{26}}$	$T_0(4,1,6,6) = -\frac{1}{12\sqrt{13}}$	$T_{-1}(7,4,0,7) = -\frac{3}{4\sqrt{13}}$
$T_1(7,0,0,7) = -\frac{1}{6\sqrt{13}}$	$T_0(4,2,2,1) = \frac{1}{12\sqrt{13}}$	$T_{-1}(7,6,4,1) = \frac{1}{6\sqrt{26}}$
$T_1(7,0,1,6) = \frac{1}{6\sqrt{26}}$	$T_0(4,2,5,0) = \frac{7}{12\sqrt{26}}$	$T_{-1}(7,7,0,4) = -\frac{3}{4\sqrt{13}}$
$T_1(7,6,1,0) = \frac{1}{6\sqrt{26}}$	$T_0(4,5,2,0) = \frac{3}{4\sqrt{26}}$	$T_{-1}(7,7,1,1) = \frac{2}{3\sqrt{13}}$
$T_1(7,7,0,0) = -\frac{1}{6\sqrt{13}}$	$T_0(4,6,6,1) = -\frac{1}{12\sqrt{13}}$	$T_{-1}(7,7,4,0) = -\frac{3}{4\sqrt{13}}$
	$T_0(4,6,7,0) = -\frac{7}{12\sqrt{26}}$	
	$T_0(4,7,6,0) = -\frac{3}{4\sqrt{26}}$	
	$T_0(5,0,1,5) = \frac{1}{12\sqrt{13}}$	
	$T_0(5,0,4,2) = \frac{7}{12\sqrt{26}}$	
	$T_0(5,1,0,5) = \frac{1}{12\sqrt{13}}$	
	$T_0(5,1,1,2) = -\frac{5}{6\sqrt{26}}$	
	$T_0(5,2,0,4) = \frac{3}{4\sqrt{26}}$	
	$T_0(5,2,1,1) = -\frac{5}{6\sqrt{26}}$	
	$T_0(5,2,4,0) = \frac{7}{12\sqrt{26}}$	
	$T_0(5,4,0,2) = \frac{3}{4\sqrt{26}}$	
	$T_0(5,5,0,1) = \frac{1}{12\sqrt{13}}$	
	$T_0(5,5,1,0) = \frac{1}{12\sqrt{13}}$	
	$T_0(6,0,4,7) = -\frac{3}{4\sqrt{26}}$	
	$T_0(6,1,1,7) = \frac{5}{6\sqrt{26}}$	
	$T_0(6,1,4,6) = -\frac{1}{12\sqrt{13}}$	
	$T_0(6,4,0,7) = -\frac{7}{12\sqrt{26}}$	
	$T_0(6,4,1,6) = -\frac{1}{12\sqrt{13}}$	
	$T_0(6,6,1,4) = -\frac{1}{12\sqrt{13}}$	
	$T_0(6,6,4,1) = -\frac{1}{12\sqrt{13}}$	
	$T_0(6,7,0,4) = -\frac{7}{12\sqrt{26}}$	
	$T_0(6,7,1,1) = \frac{5}{6\sqrt{26}}$	
	$T_0(6,7,4,0) = -\frac{3}{4\sqrt{26}}$	
	$T_0(7,0,1,7) = -\frac{1}{12\sqrt{13}}$	
	$T_0(7,0,4,6) = -\frac{7}{12\sqrt{26}}$	
	$T_0(7,1,0,7) = -\frac{1}{12\sqrt{13}}$	
	$T_0(7,1,1,6) = \frac{5}{6\sqrt{26}}$	
	$T_0(7,4,0,6) = -\frac{3}{4\sqrt{26}}$	
	$T_0(7,6,0,4) = -\frac{3}{4\sqrt{26}}$	
	$T_0(7,6,1,1) = \frac{5}{6\sqrt{26}}$	
	$T_0(7,6,4,0) = -\frac{7}{12\sqrt{26}}$	
	$T_0(7,7,0,1) = -\frac{1}{12\sqrt{13}}$	
	$T_0(7,7,1,0) = -\frac{1}{12\sqrt{13}}$	

TABLE XXXV. $S_7^{(e)}$.

$S_z = +1$	$S_z = 0$	$S_z = -1$
$T_1(0,0,5,5) = \frac{3}{2\sqrt{130}}$	$T_0(0,1,5,5) = \frac{3}{4\sqrt{130}}$	$T_{-1}(0,4,5,5) = \frac{1}{4\sqrt{130}}$
$T_1(0,0,7,7) = -\frac{3}{2\sqrt{130}}$	$T_0(0,1,7,7) = -\frac{3}{4\sqrt{130}}$	$T_{-1}(0,4,7,7) = -\frac{1}{4\sqrt{130}}$
$T_1(0,1,2,5) = \frac{7}{8\sqrt{65}}$	$T_0(0,2,5,4) = -\frac{3}{2\sqrt{65}}$	$T_{-1}(0,5,5,4) = \frac{1}{4\sqrt{130}}$
$T_1(0,1,5,2) = -\frac{\sqrt{13}}{8}$	$T_0(0,4,2,5) = \frac{1}{\sqrt{65}}$	$T_{-1}(0,7,7,4) = -\frac{1}{4\sqrt{130}}$
$T_1(0,1,6,7) = -\frac{7}{8\sqrt{65}}$	$T_0(0,4,5,2) = -\frac{3}{2\sqrt{65}}$	$T_{-1}(1,1,5,5) = \frac{1}{2\sqrt{130}}$
$T_1(0,1,7,6) = \frac{\sqrt{13}}{8}$	$T_0(0,4,6,7) = -\frac{1}{\sqrt{65}}$	$T_{-1}(1,1,7,7) = -\frac{1}{2\sqrt{130}}$
$T_1(0,2,2,4) = \frac{1}{4\sqrt{130}}$	$T_0(0,4,7,6) = \frac{3}{2\sqrt{65}}$	$T_{-1}(1,2,5,4) = -\frac{\sqrt{13}}{8}$
$T_1(0,2,5,1) = -\frac{\sqrt{13}}{8}$	$T_0(0,5,2,4) = \frac{1}{\sqrt{65}}$	$T_{-1}(1,4,2,5) = \frac{7}{8\sqrt{65}}$
$T_1(0,4,2,2) = \frac{1}{4\sqrt{130}}$	$T_0(0,5,5,1) = \frac{3}{4\sqrt{130}}$	$T_{-1}(1,4,5,2) = -\frac{\sqrt{13}}{8}$
$T_1(0,4,6,6) = -\frac{1}{4\sqrt{130}}$	$T_0(0,6,7,4) = \frac{3}{2\sqrt{65}}$	$T_{-1}(1,4,6,7) = -\frac{7}{8\sqrt{65}}$
$T_1(0,5,2,1) = \frac{7}{8\sqrt{65}}$	$T_0(0,7,6,4) = -\frac{1}{\sqrt{65}}$	$T_{-1}(1,4,7,6) = \frac{\sqrt{13}}{8}$
$T_1(0,5,5,0) = \frac{3}{2\sqrt{130}}$	$T_0(0,7,7,1) = -\frac{3}{4\sqrt{130}}$	$T_{-1}(1,5,2,4) = \frac{7}{8\sqrt{65}}$
$T_1(0,6,6,4) = -\frac{1}{4\sqrt{130}}$	$T_0(1,0,5,5) = \frac{3}{4\sqrt{130}}$	$T_{-1}(1,5,5,1) = \frac{1}{2\sqrt{130}}$
$T_1(0,6,7,1) = \frac{\sqrt{13}}{8}$	$T_0(1,0,7,7) = -\frac{3}{4\sqrt{130}}$	$T_{-1}(1,6,7,4) = \frac{\sqrt{13}}{8}$
$T_1(0,7,6,1) = -\frac{7}{8\sqrt{65}}$	$T_0(1,1,2,5) = -\frac{1}{2\sqrt{65}}$	$T_{-1}(1,7,6,4) = -\frac{7}{8\sqrt{65}}$
$T_1(0,7,7,0) = -\frac{3}{2\sqrt{130}}$	$T_0(1,1,5,2) = -\frac{1}{2\sqrt{65}}$	$T_{-1}(1,7,7,1) = -\frac{1}{2\sqrt{130}}$
$T_1(1,0,2,5) = -\frac{\sqrt{13}}{8}$	$T_0(1,1,6,7) = \frac{1}{2\sqrt{65}}$	$T_{-1}(2,1,4,5) = -\frac{\sqrt{13}}{8}$
$T_1(1,0,5,2) = \frac{7}{8\sqrt{65}}$	$T_0(1,1,7,6) = \frac{1}{2\sqrt{65}}$	$T_{-1}(2,2,4,4) = \frac{3}{2\sqrt{130}}$
$T_1(1,0,6,7) = \frac{\sqrt{13}}{8}$	$T_0(1,2,2,4) = \frac{3}{4\sqrt{130}}$	$T_{-1}(2,4,1,5) = \frac{7}{8\sqrt{65}}$
$T_1(1,0,7,6) = -\frac{7}{8\sqrt{65}}$	$T_0(1,2,5,1) = -\frac{1}{2\sqrt{65}}$	$T_{-1}(2,4,4,2) = \frac{3}{2\sqrt{130}}$
$T_1(1,1,2,2) = \frac{1}{2\sqrt{130}}$	$T_0(1,4,2,2) = \frac{3}{4\sqrt{130}}$	$T_{-1}(2,5,1,4) = \frac{7}{8\sqrt{65}}$
$T_1(1,1,6,6) = -\frac{1}{2\sqrt{130}}$	$T_0(1,4,6,6) = -\frac{3}{4\sqrt{130}}$	$T_{-1}(2,5,4,1) = -\frac{\sqrt{13}}{8}$
$T_1(1,2,2,1) = \frac{1}{2\sqrt{130}}$	$T_0(1,5,2,1) = -\frac{1}{2\sqrt{65}}$	$T_{-1}(4,0,5,5) = \frac{1}{4\sqrt{130}}$
$T_1(1,2,5,0) = \frac{7}{8\sqrt{65}}$	$T_0(1,5,5,0) = \frac{3}{4\sqrt{130}}$	$T_{-1}(4,0,7,7) = -\frac{1}{4\sqrt{130}}$
$T_1(1,5,2,0) = -\frac{\sqrt{13}}{8}$	$T_0(1,6,6,4) = -\frac{3}{4\sqrt{130}}$	$T_{-1}(4,1,2,5) = -\frac{\sqrt{13}}{8}$
$T_1(1,6,6,1) = -\frac{1}{2\sqrt{130}}$	$T_0(1,6,7,1) = \frac{1}{2\sqrt{65}}$	$T_{-1}(4,1,5,2) = \frac{7}{8\sqrt{65}}$
$T_1(1,6,7,0) = -\frac{7}{8\sqrt{65}}$	$T_0(1,7,6,1) = \frac{1}{2\sqrt{65}}$	$T_{-1}(4,1,6,7) = \frac{\sqrt{13}}{8}$
$T_1(1,7,6,0) = \frac{\sqrt{13}}{8}$	$T_0(1,7,7,0) = -\frac{3}{4\sqrt{130}}$	$T_{-1}(4,1,7,6) = -\frac{7}{8\sqrt{65}}$
$T_1(2,0,1,5) = -\frac{\sqrt{13}}{8}$	$T_0(2,0,4,5) = -\frac{3}{2\sqrt{65}}$	$T_{-1}(4,2,2,4) = \frac{3}{2\sqrt{130}}$
$T_1(2,0,4,2) = \frac{1}{4\sqrt{130}}$	$T_0(2,1,1,5) = -\frac{1}{2\sqrt{65}}$	$T_{-1}(4,2,5,1) = \frac{7}{8\sqrt{65}}$
$T_1(2,1,0,5) = \frac{7}{8\sqrt{65}}$	$T_0(2,1,4,2) = \frac{3}{4\sqrt{130}}$	$T_{-1}(4,4,2,2) = \frac{3}{2\sqrt{130}}$
$T_1(2,1,1,2) = \frac{1}{2\sqrt{130}}$	$T_0(2,2,1,4) = \frac{3}{4\sqrt{130}}$	$T_{-1}(4,4,6,6) = -\frac{3}{2\sqrt{130}}$
$T_1(2,2,0,4) = \frac{1}{4\sqrt{130}}$	$T_0(2,2,4,1) = \frac{3}{4\sqrt{130}}$	$T_{-1}(4,5,2,1) = -\frac{\sqrt{13}}{8}$
$T_1(2,2,1,1) = \frac{1}{2\sqrt{130}}$	$T_0(2,4,0,5) = \frac{1}{\sqrt{65}}$	$T_{-1}(4,5,5,0) = \frac{1}{4\sqrt{130}}$
$T_1(2,2,4,0) = \frac{1}{4\sqrt{130}}$	$T_0(2,4,1,2) = \frac{3}{4\sqrt{130}}$	$T_{-1}(4,6,6,4) = -\frac{3}{2\sqrt{130}}$
$T_1(2,4,0,2) = \frac{1}{4\sqrt{130}}$	$T_0(2,5,0,4) = \frac{1}{\sqrt{65}}$	$T_{-1}(4,6,7,1) = -\frac{7}{8\sqrt{65}}$
$T_1(2,5,0,1) = \frac{7}{8\sqrt{65}}$	$T_0(2,5,1,1) = -\frac{1}{2\sqrt{65}}$	$T_{-1}(4,7,6,1) = \frac{\sqrt{13}}{8}$
$T_1(2,5,1,0) = -\frac{\sqrt{13}}{8}$	$T_0(2,5,4,0) = -\frac{3}{2\sqrt{65}}$	$T_{-1}(4,7,7,0) = -\frac{1}{4\sqrt{130}}$
$T_1(4,0,2,2) = \frac{1}{4\sqrt{130}}$	$T_0(4,0,2,5) = -\frac{3}{2\sqrt{65}}$	$T_{-1}(5,0,4,5) = \frac{1}{4\sqrt{130}}$

TABLE XXXV. (Continued.)

$S_z = +1$	$S_z = 0$	$S_z = -1$
$T_1(4,0,6,6) = -\frac{1}{4\sqrt{130}}$	$T_0(4,0,5,2) = \frac{1}{\sqrt{65}}$	$T_{-1}(5,1,1,5) = \frac{1}{2\sqrt{130}}$
$T_1(4,2,2,0) = \frac{1}{4\sqrt{130}}$	$T_0(4,0,6,7) = \frac{3}{2\sqrt{65}}$	$T_{-1}(5,1,4,2) = \frac{7}{8\sqrt{65}}$
$T_1(4,6,6,0) = -\frac{1}{4\sqrt{130}}$	$T_0(4,0,7,6) = -\frac{1}{\sqrt{65}}$	$T_{-1}(5,2,1,4) = -\frac{\sqrt{13}}{8}$
$T_1(5,0,0,5) = \frac{3}{2\sqrt{130}}$	$T_0(4,1,2,2) = \frac{3}{4\sqrt{130}}$	$T_{-1}(5,2,4,1) = \frac{7}{8\sqrt{65}}$
$T_1(5,0,1,2) = \frac{7}{8\sqrt{65}}$	$T_0(4,1,6,6) = -\frac{3}{4\sqrt{130}}$	$T_{-1}(5,4,0,5) = \frac{1}{4\sqrt{130}}$
$T_1(5,1,0,2) = -\frac{\sqrt{13}}{8}$	$T_0(4,2,2,1) = \frac{3}{4\sqrt{130}}$	$T_{-1}(5,4,1,2) = -\frac{\sqrt{13}}{8}$
$T_1(5,2,0,1) = -\frac{\sqrt{13}}{8}$	$T_0(4,2,5,0) = \frac{1}{\sqrt{65}}$	$T_{-1}(5,5,0,4) = \frac{1}{4\sqrt{130}}$
$T_1(5,2,1,0) = \frac{7}{8\sqrt{65}}$	$T_0(4,5,2,0) = -\frac{3}{2\sqrt{65}}$	$T_{-1}(5,5,1,1) = \frac{1}{2\sqrt{130}}$
$T_1(5,5,0,0) = \frac{3}{2\sqrt{130}}$	$T_0(4,6,6,1) = -\frac{3}{4\sqrt{130}}$	$T_{-1}(5,5,4,0) = \frac{1}{4\sqrt{130}}$
$T_1(6,0,1,7) = \frac{\sqrt{13}}{8}$	$T_0(4,6,7,0) = -\frac{1}{\sqrt{65}}$	$T_{-1}(6,1,4,7) = \frac{\sqrt{13}}{8}$
$T_1(6,0,4,6) = -\frac{1}{4\sqrt{130}}$	$T_0(4,7,6,0) = \frac{3}{2\sqrt{65}}$	$T_{-1}(6,4,1,7) = -\frac{7}{8\sqrt{65}}$
$T_1(6,1,0,7) = -\frac{7}{8\sqrt{65}}$	$T_0(5,0,1,5) = \frac{3}{4\sqrt{130}}$	$T_{-1}(6,4,4,6) = -\frac{3}{2\sqrt{130}}$
$T_1(6,1,1,6) = -\frac{1}{2\sqrt{130}}$	$T_0(5,0,4,2) = \frac{1}{\sqrt{65}}$	$T_{-1}(6,6,4,4) = -\frac{3}{2\sqrt{130}}$
$T_1(6,4,0,6) = -\frac{1}{4\sqrt{130}}$	$T_0(5,1,0,5) = \frac{3}{4\sqrt{130}}$	$T_{-1}(6,7,1,4) = -\frac{7}{8\sqrt{65}}$
$T_1(6,6,0,4) = -\frac{1}{4\sqrt{130}}$	$T_0(5,1,1,2) = -\frac{1}{2\sqrt{65}}$	$T_{-1}(6,7,4,1) = \frac{\sqrt{13}}{8}$
$T_1(6,6,1,1) = -\frac{1}{2\sqrt{130}}$	$T_0(5,2,0,4) = -\frac{3}{2\sqrt{65}}$	$T_{-1}(7,0,4,7) = -\frac{1}{4\sqrt{130}}$
$T_1(6,6,4,0) = -\frac{1}{4\sqrt{130}}$	$T_0(5,2,1,1) = -\frac{1}{2\sqrt{65}}$	$T_{-1}(7,1,1,7) = -\frac{1}{2\sqrt{130}}$
$T_1(6,7,0,1) = -\frac{7}{8\sqrt{65}}$	$T_0(5,2,4,0) = \frac{1}{\sqrt{65}}$	$T_{-1}(7,1,4,6) = -\frac{7}{8\sqrt{65}}$
$T_1(6,7,1,0) = \frac{\sqrt{13}}{8}$	$T_0(5,4,0,2) = -\frac{3}{2\sqrt{65}}$	$T_{-1}(7,4,0,7) = -\frac{1}{4\sqrt{130}}$
$T_1(7,0,0,7) = -\frac{3}{2\sqrt{130}}$	$T_0(5,5,0,1) = \frac{3}{4\sqrt{130}}$	$T_{-1}(7,4,1,6) = \frac{\sqrt{13}}{8}$
$T_1(7,0,1,6) = -\frac{7}{8\sqrt{65}}$	$T_0(5,5,1,0) = \frac{3}{4\sqrt{130}}$	$T_{-1}(7,6,1,4) = \frac{\sqrt{13}}{8}$
$T_1(7,1,0,6) = \frac{\sqrt{13}}{8}$	$T_0(6,0,4,7) = \frac{3}{2\sqrt{65}}$	$T_{-1}(7,6,4,1) = -\frac{7}{8\sqrt{65}}$
$T_1(7,6,0,1) = \frac{\sqrt{13}}{8}$	$T_0(6,1,1,7) = \frac{1}{2\sqrt{65}}$	$T_{-1}(7,7,0,4) = -\frac{1}{4\sqrt{130}}$
$T_1(7,6,1,0) = -\frac{7}{8\sqrt{65}}$	$T_0(6,1,4,6) = -\frac{3}{4\sqrt{130}}$	$T_{-1}(7,7,1,1) = -\frac{1}{2\sqrt{130}}$
$T_1(7,7,0,0) = -\frac{3}{2\sqrt{130}}$	$T_0(6,4,0,7) = -\frac{1}{\sqrt{65}}$	$T_{-1}(7,7,4,0) = -\frac{1}{4\sqrt{130}}$
	$T_0(6,4,1,6) = -\frac{3}{4\sqrt{130}}$	
	$T_0(6,6,1,4) = -\frac{3}{4\sqrt{130}}$	
	$T_0(6,6,4,1) = -\frac{3}{4\sqrt{130}}$	
	$T_0(6,7,0,4) = -\frac{1}{\sqrt{65}}$	
	$T_0(6,7,1,1) = \frac{1}{2\sqrt{65}}$	
	$T_0(6,7,4,0) = \frac{3}{2\sqrt{65}}$	
	$T_0(7,0,1,7) = -\frac{3}{4\sqrt{130}}$	
	$T_0(7,0,4,6) = -\frac{1}{\sqrt{65}}$	
	$T_0(7,1,0,7) = -\frac{3}{4\sqrt{130}}$	
	$T_0(7,1,1,6) = \frac{1}{2\sqrt{65}}$	
	$T_0(7,4,0,6) = \frac{3}{2\sqrt{65}}$	
	$T_0(7,6,0,4) = \frac{3}{2\sqrt{65}}$	
	$T_0(7,6,1,1) = \frac{1}{2\sqrt{65}}$	
	$T_0(7,6,4,0) = -\frac{1}{\sqrt{65}}$	
	$T_0(7,7,0,1) = -\frac{3}{4\sqrt{130}}$	
	$T_0(7,7,1,0) = -\frac{3}{4\sqrt{130}}$	

TABLE XXXVI. G_7 .

$S_z = +1$	$S_z = 0$	$S_z = -1$
$T_1(0,1,2,5) = \frac{1}{4\sqrt{5}}$	$T_0(0,1,5,5) = \frac{1}{2\sqrt{10}}$	$T_{-1}(0,4,5,5) = \frac{1}{2\sqrt{10}}$
$T_1(0,1,5,2) = \frac{1}{4\sqrt{5}}$	$T_0(0,1,7,7) = -\frac{1}{2\sqrt{10}}$	$T_{-1}(0,4,7,7) = -\frac{1}{2\sqrt{10}}$
$T_1(0,1,6,7) = -\frac{1}{4\sqrt{5}}$	$T_0(0,2,4,5) = -\frac{1}{4\sqrt{5}}$	$T_{-1}(0,5,5,4) = -\frac{1}{2\sqrt{10}}$
$T_1(0,1,7,6) = -\frac{1}{4\sqrt{5}}$	$T_0(0,5,4,2) = \frac{1}{4\sqrt{5}}$	$T_{-1}(0,7,7,4) = \frac{1}{2\sqrt{10}}$
$T_1(0,2,1,5) = -\frac{1}{4\sqrt{5}}$	$T_0(0,5,5,1) = -\frac{1}{2\sqrt{10}}$	$T_{-1}(1,2,4,5) = -\frac{1}{4\sqrt{5}}$
$T_1(0,2,2,4) = \frac{1}{2\sqrt{10}}$	$T_0(0,6,4,7) = \frac{1}{4\sqrt{5}}$	$T_{-1}(1,2,5,4) = \frac{1}{4\sqrt{5}}$
$T_1(0,2,5,1) = -\frac{1}{4\sqrt{5}}$	$T_0(0,7,4,6) = -\frac{1}{4\sqrt{5}}$	$T_{-1}(1,4,2,5) = -\frac{1}{4\sqrt{5}}$
$T_1(0,4,2,2) = -\frac{1}{2\sqrt{10}}$	$T_0(0,7,7,1) = \frac{1}{2\sqrt{10}}$	$T_{-1}(1,4,5,2) = -\frac{1}{4\sqrt{5}}$
$T_1(0,4,6,6) = \frac{1}{2\sqrt{10}}$	$T_0(1,0,5,5) = -\frac{1}{2\sqrt{10}}$	$T_{-1}(1,4,6,7) = \frac{1}{4\sqrt{5}}$
$T_1(0,5,1,2) = \frac{1}{4\sqrt{5}}$	$T_0(1,0,7,7) = \frac{1}{2\sqrt{10}}$	$T_{-1}(1,4,7,6) = \frac{1}{4\sqrt{5}}$
$T_1(0,5,2,1) = -\frac{1}{4\sqrt{5}}$	$T_0(1,2,2,4) = \frac{1}{2\sqrt{10}}$	$T_{-1}(1,5,2,4) = \frac{1}{4\sqrt{5}}$
$T_1(0,6,1,7) = \frac{1}{4\sqrt{5}}$	$T_0(1,4,2,2) = -\frac{1}{2\sqrt{10}}$	$T_{-1}(1,5,4,2) = \frac{1}{4\sqrt{5}}$
$T_1(0,6,6,4) = -\frac{1}{2\sqrt{10}}$	$T_0(1,4,6,6) = \frac{1}{2\sqrt{10}}$	$T_{-1}(1,6,4,7) = \frac{1}{4\sqrt{5}}$
$T_1(0,6,7,1) = \frac{1}{4\sqrt{5}}$	$T_0(1,5,5,0) = \frac{1}{2\sqrt{10}}$	$T_{-1}(1,6,7,4) = -\frac{1}{4\sqrt{5}}$
$T_1(0,7,1,6) = -\frac{1}{4\sqrt{5}}$	$T_0(1,6,6,4) = -\frac{1}{2\sqrt{10}}$	$T_{-1}(1,7,4,6) = -\frac{1}{4\sqrt{5}}$
$T_1(0,7,6,1) = \frac{1}{4\sqrt{5}}$	$T_0(1,7,7,0) = -\frac{1}{2\sqrt{10}}$	$T_{-1}(1,7,6,4) = -\frac{1}{4\sqrt{5}}$
$T_1(1,0,2,5) = -\frac{1}{4\sqrt{5}}$	$T_0(2,0,5,4) = \frac{1}{4\sqrt{5}}$	$T_{-1}(2,1,4,5) = -\frac{1}{4\sqrt{5}}$
$T_1(1,0,5,2) = -\frac{1}{4\sqrt{5}}$	$T_0(2,1,4,2) = -\frac{1}{2\sqrt{10}}$	$T_{-1}(2,1,5,4) = \frac{1}{4\sqrt{5}}$
$T_1(1,0,6,7) = \frac{1}{4\sqrt{5}}$	$T_0(2,2,1,4) = -\frac{1}{2\sqrt{10}}$	$T_{-1}(2,4,1,5) = \frac{1}{4\sqrt{5}}$
$T_1(1,0,7,6) = \frac{1}{4\sqrt{5}}$	$T_0(2,2,4,1) = \frac{1}{2\sqrt{10}}$	$T_{-1}(2,4,5,1) = -\frac{1}{4\sqrt{5}}$
$T_1(1,2,0,5) = \frac{1}{4\sqrt{5}}$	$T_0(2,4,1,2) = \frac{1}{2\sqrt{10}}$	$T_{-1}(2,5,1,4) = -\frac{1}{4\sqrt{5}}$
$T_1(1,2,5,0) = \frac{1}{4\sqrt{5}}$	$T_0(2,4,5,0) = -\frac{1}{4\sqrt{5}}$	$T_{-1}(2,5,4,1) = \frac{1}{4\sqrt{5}}$
$T_1(1,5,0,2) = -\frac{1}{4\sqrt{5}}$	$T_0(4,1,2,2) = \frac{1}{2\sqrt{10}}$	$T_{-1}(4,0,5,5) = -\frac{1}{2\sqrt{10}}$
$T_1(1,5,2,0) = \frac{1}{4\sqrt{5}}$	$T_0(4,1,6,6) = -\frac{1}{2\sqrt{10}}$	$T_{-1}(4,0,7,7) = \frac{1}{2\sqrt{10}}$
$T_1(1,6,0,7) = -\frac{1}{4\sqrt{5}}$	$T_0(4,2,0,5) = \frac{1}{4\sqrt{5}}$	$T_{-1}(4,1,2,5) = \frac{1}{4\sqrt{5}}$
$T_1(1,6,7,0) = -\frac{1}{4\sqrt{5}}$	$T_0(4,2,2,1) = -\frac{1}{2\sqrt{10}}$	$T_{-1}(4,1,5,2) = \frac{1}{4\sqrt{5}}$
$T_1(1,7,0,6) = \frac{1}{4\sqrt{5}}$	$T_0(4,5,0,2) = -\frac{1}{4\sqrt{5}}$	$T_{-1}(4,1,6,7) = -\frac{1}{4\sqrt{5}}$
$T_1(1,7,6,0) = -\frac{1}{4\sqrt{5}}$	$T_0(4,6,0,7) = -\frac{1}{4\sqrt{5}}$	$T_{-1}(4,1,7,6) = -\frac{1}{4\sqrt{5}}$
$T_1(2,0,1,5) = \frac{1}{4\sqrt{5}}$	$T_0(4,6,6,1) = \frac{1}{2\sqrt{10}}$	$T_{-1}(4,2,1,5) = \frac{1}{4\sqrt{5}}$
$T_1(2,0,4,2) = -\frac{1}{2\sqrt{10}}$	$T_0(4,7,0,6) = \frac{1}{4\sqrt{5}}$	$T_{-1}(4,2,5,1) = -\frac{1}{4\sqrt{5}}$
$T_1(2,0,5,1) = \frac{1}{4\sqrt{5}}$	$T_0(5,0,1,5) = \frac{1}{2\sqrt{10}}$	$T_{-1}(4,5,1,2) = -\frac{1}{4\sqrt{5}}$
$T_1(2,1,0,5) = -\frac{1}{4\sqrt{5}}$	$T_0(5,0,2,4) = -\frac{1}{4\sqrt{5}}$	$T_{-1}(4,5,2,1) = -\frac{1}{4\sqrt{5}}$
$T_1(2,1,5,0) = -\frac{1}{4\sqrt{5}}$	$T_0(5,1,0,5) = -\frac{1}{2\sqrt{10}}$	$T_{-1}(4,5,5,0) = \frac{1}{2\sqrt{10}}$
$T_1(2,2,0,4) = -\frac{1}{2\sqrt{10}}$	$T_0(5,4,2,0) = \frac{1}{4\sqrt{5}}$	$T_{-1}(4,6,1,7) = -\frac{1}{4\sqrt{5}}$
$T_1(2,2,4,0) = \frac{1}{2\sqrt{10}}$	$T_0(5,5,0,1) = \frac{1}{2\sqrt{10}}$	$T_{-1}(4,6,7,1) = \frac{1}{4\sqrt{5}}$
$T_1(2,4,0,2) = \frac{1}{2\sqrt{10}}$	$T_0(5,5,1,0) = -\frac{1}{2\sqrt{10}}$	$T_{-1}(4,7,1,6) = \frac{1}{4\sqrt{5}}$
$T_1(2,5,0,1) = \frac{1}{4\sqrt{5}}$	$T_0(6,0,7,4) = -\frac{1}{4\sqrt{5}}$	$T_{-1}(4,7,6,1) = \frac{1}{4\sqrt{5}}$
$T_1(2,5,1,0) = -\frac{1}{4\sqrt{5}}$	$T_0(6,1,4,6) = \frac{1}{2\sqrt{10}}$	$T_{-1}(4,7,7,0) = -\frac{1}{2\sqrt{10}}$
$T_1(4,0,2,2) = \frac{1}{2\sqrt{10}}$	$T_0(6,4,1,6) = -\frac{1}{2\sqrt{10}}$	$T_{-1}(5,0,4,5) = \frac{1}{2\sqrt{10}}$
$T_1(4,0,6,6) = -\frac{1}{2\sqrt{10}}$	$T_0(6,4,7,0) = \frac{1}{4\sqrt{5}}$	$T_{-1}(5,1,2,4) = -\frac{1}{4\sqrt{5}}$
$T_1(4,2,2,0) = -\frac{1}{2\sqrt{10}}$	$T_0(6,6,1,4) = \frac{1}{2\sqrt{10}}$	$T_{-1}(5,1,4,2) = -\frac{1}{4\sqrt{5}}$
$T_1(4,6,6,0) = \frac{1}{2\sqrt{10}}$	$T_0(6,6,4,1) = -\frac{1}{2\sqrt{10}}$	$T_{-1}(5,2,1,4) = -\frac{1}{4\sqrt{5}}$
$T_1(5,0,1,2) = \frac{1}{4\sqrt{5}}$	$T_0(7,0,1,7) = -\frac{1}{2\sqrt{10}}$	$T_{-1}(5,2,4,1) = \frac{1}{4\sqrt{5}}$
$T_1(5,0,2,1) = -\frac{1}{4\sqrt{5}}$	$T_0(7,0,6,4) = \frac{1}{4\sqrt{5}}$	$T_{-1}(5,4,0,5) = -\frac{1}{2\sqrt{10}}$
$T_1(5,1,0,2) = -\frac{1}{4\sqrt{5}}$	$T_0(7,1,0,7) = \frac{1}{2\sqrt{10}}$	$T_{-1}(5,4,1,2) = \frac{1}{4\sqrt{5}}$
$T_1(5,1,2,0) = \frac{1}{4\sqrt{5}}$	$T_0(7,4,6,0) = -\frac{1}{4\sqrt{5}}$	$T_{-1}(5,4,2,1) = \frac{1}{4\sqrt{5}}$
$T_1(5,2,0,1) = \frac{1}{4\sqrt{5}}$	$T_0(7,7,0,1) = -\frac{1}{2\sqrt{10}}$	$T_{-1}(5,5,0,4) = \frac{1}{2\sqrt{10}}$

TABLE XXXVI. (Continued.)

$S_z = +1$	$S_z = 0$	$S_z = -1$
$T_1(5,2,1,0) = -\frac{1}{4\sqrt{5}}$	$T_0(7,7,1,0) = \frac{1}{2\sqrt{10}}$	$T_{-1}(5,5,4,0) = -\frac{1}{2\sqrt{10}}$
$T_1(6,0,1,7) = -\frac{1}{4\sqrt{5}}$		$T_{-1}(6,1,4,7) = \frac{1}{4\sqrt{5}}$
$T_1(6,0,4,6) = \frac{1}{2\sqrt{10}}$		$T_{-1}(6,1,7,4) = -\frac{1}{4\sqrt{5}}$
$T_1(6,0,7,1) = -\frac{1}{4\sqrt{5}}$		$T_{-1}(6,4,1,7) = -\frac{1}{4\sqrt{5}}$
$T_1(6,1,0,7) = \frac{1}{4\sqrt{5}}$		$T_{-1}(6,4,7,1) = \frac{1}{4\sqrt{5}}$
$T_1(6,1,7,0) = \frac{1}{4\sqrt{5}}$		$T_{-1}(6,7,1,4) = \frac{1}{4\sqrt{5}}$
$T_1(6,4,0,6) = -\frac{1}{2\sqrt{10}}$		$T_{-1}(6,7,4,1) = -\frac{1}{4\sqrt{5}}$
$T_1(6,6,0,4) = \frac{1}{2\sqrt{10}}$		$T_{-1}(7,0,4,7) = -\frac{1}{2\sqrt{10}}$
$T_1(6,6,4,0) = -\frac{1}{2\sqrt{10}}$		$T_{-1}(7,1,4,6) = \frac{1}{4\sqrt{5}}$
$T_1(6,7,0,1) = -\frac{1}{4\sqrt{5}}$		$T_{-1}(7,1,6,4) = \frac{1}{4\sqrt{5}}$
$T_1(6,7,1,0) = \frac{1}{4\sqrt{5}}$		$T_{-1}(7,4,0,7) = \frac{1}{2\sqrt{10}}$
$T_1(7,0,1,6) = -\frac{1}{4\sqrt{5}}$		$T_{-1}(7,4,1,6) = -\frac{1}{4\sqrt{5}}$
$T_1(7,0,6,1) = \frac{1}{4\sqrt{5}}$		$T_{-1}(7,4,6,1) = -\frac{1}{4\sqrt{5}}$
$T_1(7,1,0,6) = \frac{1}{4\sqrt{5}}$		$T_{-1}(7,6,1,4) = \frac{1}{4\sqrt{5}}$
$T_1(7,1,6,0) = -\frac{1}{4\sqrt{5}}$		$T_{-1}(7,6,4,1) = -\frac{1}{4\sqrt{5}}$
$T_1(7,6,0,1) = -\frac{1}{4\sqrt{5}}$		$T_{-1}(7,7,0,4) = -\frac{1}{2\sqrt{10}}$
$T_1(7,6,1,0) = \frac{1}{4\sqrt{5}}$		$T_{-1}(7,7,4,0) = \frac{1}{2\sqrt{10}}$

TABLE XXXVII. $S_8^{(a)}$.

$S_z = +1$	$S_z = 0$	$S_z = -1$
$T_1(0,2,3,7) = -\frac{1}{3\sqrt{6}}$	$T_0(0,3,5,7) = -\frac{1}{4\sqrt{3}}$	$T_{-1}(1,3,5,7) = -\frac{1}{4\sqrt{3}}$
$T_1(0,2,7,3) = \frac{1}{6\sqrt{6}}$	$T_0(0,3,7,5) = \frac{1}{4\sqrt{3}}$	$T_{-1}(1,3,7,5) = \frac{1}{4\sqrt{3}}$
$T_1(0,3,2,7) = -\frac{1}{3\sqrt{6}}$	$T_0(0,5,7,3) = \frac{1}{4\sqrt{3}}$	$T_{-1}(1,5,7,3) = \frac{1}{4\sqrt{3}}$
$T_1(0,3,5,6) = -\frac{1}{6\sqrt{6}}$	$T_0(0,7,5,3) = -\frac{1}{4\sqrt{3}}$	$T_{-1}(1,7,5,3) = -\frac{1}{4\sqrt{3}}$
$T_1(0,3,6,5) = \frac{1}{3\sqrt{6}}$	$T_0(1,2,3,7) = -\frac{1}{3\sqrt{6}}$	$T_{-1}(2,3,4,7) = \frac{1}{6\sqrt{6}}$
$T_1(0,3,7,2) = \frac{1}{6\sqrt{6}}$	$T_0(1,2,7,3) = -\frac{1}{12\sqrt{6}}$	$T_{-1}(2,3,7,4) = -\frac{1}{3\sqrt{6}}$
$T_1(0,5,3,6) = \frac{1}{3\sqrt{6}}$	$T_0(1,3,2,7) = -\frac{1}{12\sqrt{6}}$	$T_{-1}(2,4,3,7) = -\frac{1}{3\sqrt{6}}$
$T_1(0,5,6,3) = \frac{1}{3\sqrt{6}}$	$T_0(1,3,5,6) = \frac{1}{12\sqrt{6}}$	$T_{-1}(2,4,7,3) = -\frac{1}{3\sqrt{6}}$
$T_1(0,6,3,5) = \frac{1}{3\sqrt{6}}$	$T_0(1,3,6,5) = \frac{1}{12\sqrt{6}}$	$T_{-1}(2,7,3,4) = -\frac{1}{3\sqrt{6}}$
$T_1(0,6,5,3) = -\frac{1}{6\sqrt{6}}$	$T_0(1,3,7,2) = -\frac{1}{12\sqrt{6}}$	$T_{-1}(2,7,4,3) = \frac{1}{6\sqrt{6}}$
$T_1(0,7,2,3) = -\frac{1}{3\sqrt{6}}$	$T_0(1,5,3,6) = \frac{1}{3\sqrt{6}}$	$T_{-1}(3,1,5,7) = \frac{1}{4\sqrt{3}}$
$T_1(0,7,3,2) = -\frac{1}{3\sqrt{6}}$	$T_0(1,5,6,3) = \frac{1}{12\sqrt{6}}$	$T_{-1}(3,1,7,5) = -\frac{1}{4\sqrt{3}}$
$T_1(1,2,6,3) = -\frac{1}{4\sqrt{3}}$	$T_0(1,6,3,5) = \frac{1}{3\sqrt{6}}$	$T_{-1}(3,2,4,7) = -\frac{1}{3\sqrt{6}}$
$T_1(1,3,2,6) = \frac{1}{4\sqrt{3}}$	$T_0(1,6,5,3) = \frac{1}{12\sqrt{6}}$	$T_{-1}(3,2,7,4) = \frac{1}{6\sqrt{6}}$
$T_1(1,3,6,2) = -\frac{1}{4\sqrt{3}}$	$T_0(1,7,2,3) = -\frac{1}{12\sqrt{6}}$	$T_{-1}(3,4,2,7) = -\frac{1}{3\sqrt{6}}$
$T_1(1,6,2,3) = \frac{1}{4\sqrt{3}}$	$T_0(1,7,3,2) = -\frac{1}{3\sqrt{6}}$	$T_{-1}(3,4,5,6) = -\frac{1}{6\sqrt{6}}$
$T_1(2,0,3,7) = \frac{1}{6\sqrt{6}}$	$T_0(2,1,3,7) = -\frac{1}{12\sqrt{6}}$	$T_{-1}(3,4,6,5) = \frac{1}{3\sqrt{6}}$
$T_1(2,0,7,3) = -\frac{1}{3\sqrt{6}}$	$T_0(2,1,7,3) = -\frac{1}{3\sqrt{6}}$	$T_{-1}(3,4,7,2) = \frac{1}{6\sqrt{6}}$
$T_1(2,1,3,6) = -\frac{1}{4\sqrt{3}}$	$T_0(2,3,1,7) = -\frac{1}{12\sqrt{6}}$	$T_{-1}(3,5,4,6) = \frac{1}{3\sqrt{6}}$
$T_1(2,3,0,7) = -\frac{1}{3\sqrt{6}}$	$T_0(2,3,4,6) = \frac{1}{4\sqrt{3}}$	$T_{-1}(3,5,6,4) = \frac{1}{3\sqrt{6}}$
$T_1(2,3,1,6) = \frac{1}{4\sqrt{3}}$	$T_0(2,3,7,1) = -\frac{1}{3\sqrt{6}}$	$T_{-1}(3,5,7,1) = -\frac{1}{4\sqrt{3}}$
$T_1(2,3,7,0) = -\frac{1}{3\sqrt{6}}$	$T_0(2,4,3,6) = -\frac{1}{4\sqrt{3}}$	$T_{-1}(3,6,4,5) = \frac{1}{3\sqrt{6}}$
$T_1(2,6,1,3) = \frac{1}{4\sqrt{3}}$	$T_0(2,6,3,4) = -\frac{1}{4\sqrt{3}}$	$T_{-1}(3,6,5,4) = -\frac{1}{6\sqrt{6}}$
$T_1(2,6,3,1) = -\frac{1}{4\sqrt{3}}$	$T_0(2,6,4,3) = \frac{1}{4\sqrt{3}}$	$T_{-1}(3,7,2,4) = -\frac{1}{3\sqrt{6}}$

TABLE XXXVII. (Continued.)

$S_z = +1$	$S_z = 0$	$S_z = -1$
$T_1(2,7,0,3) = -\frac{1}{3\sqrt{6}}$	$T_0(2,7,1,3) = -\frac{1}{12\sqrt{6}}$	$T_{-1}(3,7,4,2) = -\frac{1}{3\sqrt{6}}$
$T_1(2,7,3,0) = \frac{1}{6\sqrt{6}}$	$T_0(2,7,3,1) = -\frac{1}{12\sqrt{6}}$	$T_{-1}(3,7,5,1) = \frac{1}{4\sqrt{3}}$
$T_1(3,0,2,7) = \frac{1}{6\sqrt{6}}$	$T_0(3,0,5,7) = \frac{1}{4\sqrt{3}}$	$T_{-1}(4,2,3,7) = -\frac{1}{3\sqrt{6}}$
$T_1(3,0,5,6) = \frac{1}{3\sqrt{6}}$	$T_0(3,0,7,5) = -\frac{1}{4\sqrt{3}}$	$T_{-1}(4,2,7,3) = -\frac{1}{3\sqrt{6}}$
$T_1(3,0,6,5) = -\frac{1}{6\sqrt{6}}$	$T_0(3,1,2,7) = -\frac{1}{12\sqrt{6}}$	$T_{-1}(4,3,2,7) = \frac{1}{6\sqrt{6}}$
$T_1(3,0,7,2) = -\frac{1}{3\sqrt{6}}$	$T_0(3,1,5,6) = \frac{1}{12\sqrt{6}}$	$T_{-1}(4,3,5,6) = \frac{1}{3\sqrt{6}}$
$T_1(3,1,2,6) = -\frac{1}{4\sqrt{3}}$	$T_0(3,1,6,5) = \frac{1}{12\sqrt{6}}$	$T_{-1}(4,3,6,5) = -\frac{1}{6\sqrt{6}}$
$T_1(3,1,6,2) = \frac{1}{4\sqrt{3}}$	$T_0(3,1,7,2) = -\frac{1}{12\sqrt{6}}$	$T_{-1}(4,3,7,2) = -\frac{1}{3\sqrt{6}}$
$T_1(3,2,0,7) = -\frac{1}{3\sqrt{6}}$	$T_0(3,2,1,7) = -\frac{1}{3\sqrt{6}}$	$T_{-1}(4,5,3,6) = \frac{1}{3\sqrt{6}}$
$T_1(3,2,6,1) = \frac{1}{4\sqrt{3}}$	$T_0(3,2,6,4) = \frac{1}{4\sqrt{3}}$	$T_{-1}(4,5,6,3) = -\frac{1}{6\sqrt{6}}$
$T_1(3,2,7,0) = -\frac{1}{3\sqrt{6}}$	$T_0(3,2,7,1) = -\frac{1}{12\sqrt{6}}$	$T_{-1}(4,6,3,5) = \frac{1}{3\sqrt{6}}$
$T_1(3,5,0,6) = \frac{1}{3\sqrt{6}}$	$T_0(3,4,2,6) = -\frac{1}{4\sqrt{3}}$	$T_{-1}(4,6,5,3) = \frac{1}{3\sqrt{6}}$
$T_1(3,5,6,0) = -\frac{1}{6\sqrt{6}}$	$T_0(3,4,6,2) = \frac{1}{4\sqrt{3}}$	$T_{-1}(4,7,2,3) = \frac{1}{6\sqrt{6}}$
$T_1(3,6,0,5) = \frac{1}{3\sqrt{6}}$	$T_0(3,5,1,6) = \frac{1}{3\sqrt{6}}$	$T_{-1}(4,7,3,2) = -\frac{1}{3\sqrt{6}}$
$T_1(3,6,2,1) = -\frac{1}{4\sqrt{3}}$	$T_0(3,5,6,1) = \frac{1}{12\sqrt{6}}$	$T_{-1}(5,1,3,7) = \frac{1}{4\sqrt{3}}$
$T_1(3,6,5,0) = \frac{1}{3\sqrt{6}}$	$T_0(3,5,7,0) = -\frac{1}{4\sqrt{3}}$	$T_{-1}(5,3,1,7) = -\frac{1}{4\sqrt{3}}$
$T_1(3,7,0,2) = -\frac{1}{3\sqrt{6}}$	$T_0(3,6,1,5) = \frac{1}{3\sqrt{6}}$	$T_{-1}(5,3,4,6) = \frac{1}{3\sqrt{6}}$
$T_1(3,7,2,0) = \frac{1}{6\sqrt{6}}$	$T_0(3,6,2,4) = -\frac{1}{4\sqrt{3}}$	$T_{-1}(5,3,6,4) = \frac{1}{3\sqrt{6}}$
$T_1(5,0,3,6) = \frac{1}{3\sqrt{6}}$	$T_0(3,6,5,1) = \frac{1}{12\sqrt{6}}$	$T_{-1}(5,4,3,6) = -\frac{1}{6\sqrt{6}}$
$T_1(5,0,6,3) = \frac{1}{3\sqrt{6}}$	$T_0(3,7,1,2) = -\frac{1}{3\sqrt{6}}$	$T_{-1}(5,4,6,3) = \frac{1}{3\sqrt{6}}$
$T_1(5,3,0,6) = -\frac{1}{6\sqrt{6}}$	$T_0(3,7,2,1) = -\frac{1}{12\sqrt{6}}$	$T_{-1}(5,6,3,4) = -\frac{1}{6\sqrt{6}}$
$T_1(5,3,6,0) = \frac{1}{3\sqrt{6}}$	$T_0(3,7,5,0) = \frac{1}{4\sqrt{3}}$	$T_{-1}(5,6,4,3) = \frac{1}{3\sqrt{6}}$
$T_1(5,6,0,3) = -\frac{1}{6\sqrt{6}}$	$T_0(4,2,6,3) = -\frac{1}{4\sqrt{3}}$	$T_{-1}(5,7,1,3) = -\frac{1}{4\sqrt{3}}$
$T_1(5,6,3,0) = \frac{1}{3\sqrt{6}}$	$T_0(4,3,2,6) = \frac{1}{4\sqrt{3}}$	$T_{-1}(5,7,3,1) = \frac{1}{4\sqrt{3}}$
$T_1(6,0,3,5) = -\frac{1}{6\sqrt{6}}$	$T_0(4,3,6,2) = -\frac{1}{4\sqrt{3}}$	$T_{-1}(6,3,4,5) = -\frac{1}{6\sqrt{6}}$
$T_1(6,0,5,3) = \frac{1}{3\sqrt{6}}$	$T_0(4,6,2,3) = \frac{1}{4\sqrt{3}}$	$T_{-1}(6,3,5,4) = \frac{1}{3\sqrt{6}}$
$T_1(6,1,3,2) = \frac{1}{4\sqrt{3}}$	$T_0(5,0,3,7) = \frac{1}{4\sqrt{3}}$	$T_{-1}(6,4,3,5) = \frac{1}{3\sqrt{6}}$
$T_1(6,2,1,3) = -\frac{1}{4\sqrt{3}}$	$T_0(5,1,3,6) = \frac{1}{12\sqrt{6}}$	$T_{-1}(6,4,5,3) = \frac{1}{3\sqrt{6}}$
$T_1(6,2,3,1) = \frac{1}{4\sqrt{3}}$	$T_0(5,1,6,3) = \frac{1}{3\sqrt{6}}$	$T_{-1}(6,5,3,4) = \frac{1}{3\sqrt{6}}$
$T_1(6,3,0,5) = \frac{1}{3\sqrt{6}}$	$T_0(5,3,0,7) = -\frac{1}{4\sqrt{3}}$	$T_{-1}(6,5,4,3) = -\frac{1}{6\sqrt{6}}$
$T_1(6,3,1,2) = -\frac{1}{4\sqrt{3}}$	$T_0(5,3,1,6) = \frac{1}{12\sqrt{6}}$	$T_{-1}(7,1,3,5) = -\frac{1}{4\sqrt{3}}$
$T_1(6,3,5,0) = \frac{1}{3\sqrt{6}}$	$T_0(5,3,6,1) = \frac{1}{3\sqrt{6}}$	$T_{-1}(7,2,3,4) = \frac{1}{6\sqrt{6}}$
$T_1(6,5,0,3) = \frac{1}{3\sqrt{6}}$	$T_0(5,6,1,3) = \frac{1}{12\sqrt{6}}$	$T_{-1}(7,2,4,3) = -\frac{1}{3\sqrt{6}}$
$T_1(6,5,3,0) = -\frac{1}{6\sqrt{6}}$	$T_0(5,6,3,1) = \frac{1}{12\sqrt{6}}$	$T_{-1}(7,3,1,5) = \frac{1}{4\sqrt{3}}$
$T_1(7,0,2,3) = -\frac{1}{3\sqrt{6}}$	$T_0(5,7,0,3) = -\frac{1}{4\sqrt{3}}$	$T_{-1}(7,3,2,4) = -\frac{1}{3\sqrt{6}}$
$T_1(7,0,3,2) = -\frac{1}{3\sqrt{6}}$	$T_0(5,7,3,0) = \frac{1}{4\sqrt{3}}$	$T_{-1}(7,3,4,2) = -\frac{1}{3\sqrt{6}}$
$T_1(7,2,0,3) = \frac{1}{6\sqrt{6}}$	$T_0(6,1,3,5) = \frac{1}{12\sqrt{6}}$	$T_{-1}(7,4,2,3) = -\frac{1}{3\sqrt{6}}$
$T_1(7,2,3,0) = -\frac{1}{3\sqrt{6}}$	$T_0(6,1,5,3) = \frac{1}{3\sqrt{6}}$	$T_{-1}(7,4,3,2) = \frac{1}{6\sqrt{6}}$
$T_1(7,3,0,2) = \frac{1}{6\sqrt{6}}$	$T_0(6,2,3,4) = \frac{1}{4\sqrt{3}}$	$T_{-1}(7,5,1,3) = \frac{1}{4\sqrt{3}}$
$T_1(7,3,2,0) = -\frac{1}{3\sqrt{6}}$	$T_0(6,2,4,3) = -\frac{1}{4\sqrt{3}}$	$T_{-1}(7,5,3,1) = -\frac{1}{4\sqrt{3}}$
	$T_0(6,3,1,5) = \frac{1}{12\sqrt{6}}$	
	$T_0(6,3,4,2) = -\frac{1}{4\sqrt{3}}$	
	$T_0(6,3,5,1) = \frac{1}{3\sqrt{6}}$	
	$T_0(6,4,3,2) = \frac{1}{4\sqrt{3}}$	
	$T_0(6,5,1,3) = \frac{1}{12\sqrt{6}}$	
	$T_0(6,5,3,1) = \frac{1}{12\sqrt{6}}$	

TABLE XXXVII. (Continued.)

$S_z = +1$	$S_z = 0$	$S_z = -1$
	$T_0(7,0,3,5) = -\frac{1}{4\sqrt{3}}$	
	$T_0(7,1,2,3) = -\frac{1}{3\sqrt{6}}$	
	$T_0(7,1,3,2) = -\frac{1}{12\sqrt{6}}$	
	$T_0(7,2,1,3) = -\frac{1}{12\sqrt{6}}$	
	$T_0(7,2,3,1) = -\frac{1}{12\sqrt{6}}$	
	$T_0(7,3,0,5) = \frac{1}{4\sqrt{3}}$	
	$T_0(7,3,1,2) = -\frac{1}{12\sqrt{6}}$	
	$T_0(7,3,2,1) = -\frac{1}{3\sqrt{6}}$	
	$T_0(7,5,0,3) = \frac{1}{4\sqrt{3}}$	
	$T_0(7,5,3,0) = -\frac{1}{4\sqrt{3}}$	

TABLE XXXVIII. $S_8^{(b)}$.

$S_z = +1$	$S_z = 0$	$S_z = -1$
$T_1(0,2,3,7) = -\frac{5}{12\sqrt{21}}$	$T_0(0,3,5,7) = -\frac{1}{4\sqrt{3}}$	$T_{-1}(1,3,5,7) = \frac{1}{2\sqrt{42}}$
$T_1(0,2,7,3) = -\frac{11}{12\sqrt{21}}$	$T_0(0,3,7,5) = \frac{1}{4\sqrt{3}}$	$T_{-1}(1,3,7,5) = -\frac{1}{2\sqrt{42}}$
$T_1(0,3,2,7) = -\frac{5}{12\sqrt{21}}$	$T_0(0,5,7,3) = \frac{1}{4\sqrt{3}}$	$T_{-1}(1,5,7,3) = -\frac{1}{2\sqrt{42}}$
$T_1(0,3,5,6) = \frac{11}{12\sqrt{21}}$	$T_0(0,7,5,3) = -\frac{1}{4\sqrt{3}}$	$T_{-1}(1,7,5,3) = \frac{1}{2\sqrt{42}}$
$T_1(0,3,6,5) = \frac{5}{12\sqrt{21}}$	$T_0(1,2,3,7) = -\frac{1}{3\sqrt{6}}$	$T_{-1}(2,3,4,7) = -\frac{11}{12\sqrt{21}}$
$T_1(0,3,7,2) = -\frac{11}{12\sqrt{21}}$	$T_0(1,2,7,3) = -\frac{1}{12\sqrt{6}}$	$T_{-1}(2,3,7,4) = -\frac{5}{12\sqrt{21}}$
$T_1(0,5,3,6) = \frac{5}{12\sqrt{21}}$	$T_0(1,3,2,7) = -\frac{1}{12\sqrt{6}}$	$T_{-1}(2,4,3,7) = -\frac{5}{12\sqrt{21}}$
$T_1(0,5,6,3) = \frac{5}{12\sqrt{21}}$	$T_0(1,3,5,6) = \frac{1}{12\sqrt{6}}$	$T_{-1}(2,4,7,3) = -\frac{5}{12\sqrt{21}}$
$T_1(0,6,3,5) = \frac{5}{12\sqrt{21}}$	$T_0(1,3,6,5) = \frac{1}{12\sqrt{6}}$	$T_{-1}(2,7,3,4) = -\frac{5}{12\sqrt{21}}$
$T_1(0,6,5,3) = \frac{11}{12\sqrt{21}}$	$T_0(1,3,7,2) = -\frac{1}{12\sqrt{6}}$	$T_{-1}(2,7,4,3) = -\frac{11}{12\sqrt{21}}$
$T_1(0,7,2,3) = -\frac{5}{12\sqrt{21}}$	$T_0(1,5,3,6) = \frac{1}{3\sqrt{6}}$	$T_{-1}(3,1,5,7) = -\frac{1}{2\sqrt{42}}$
$T_1(0,7,3,2) = -\frac{5}{12\sqrt{21}}$	$T_0(1,5,6,3) = \frac{1}{12\sqrt{6}}$	$T_{-1}(3,1,7,5) = \frac{1}{2\sqrt{42}}$
$T_1(1,2,6,3) = \frac{1}{2\sqrt{42}}$	$T_0(1,6,3,5) = \frac{1}{3\sqrt{6}}$	$T_{-1}(3,2,4,7) = -\frac{5}{12\sqrt{21}}$
$T_1(1,3,2,6) = -\frac{1}{2\sqrt{42}}$	$T_0(1,6,5,3) = \frac{1}{12\sqrt{6}}$	$T_{-1}(3,2,7,4) = -\frac{11}{12\sqrt{21}}$
$T_1(1,3,6,2) = \frac{1}{2\sqrt{42}}$	$T_0(1,7,2,3) = -\frac{1}{12\sqrt{6}}$	$T_{-1}(3,4,2,7) = -\frac{5}{12\sqrt{21}}$
$T_1(1,6,2,3) = -\frac{1}{2\sqrt{42}}$	$T_0(1,7,3,2) = -\frac{1}{3\sqrt{6}}$	$T_{-1}(3,4,5,6) = \frac{11}{12\sqrt{21}}$
$T_1(2,0,3,7) = -\frac{11}{12\sqrt{21}}$	$T_0(2,1,3,7) = -\frac{1}{12\sqrt{6}}$	$T_{-1}(3,4,6,5) = \frac{5}{12\sqrt{21}}$
$T_1(2,0,7,3) = -\frac{5}{12\sqrt{21}}$	$T_0(2,1,7,3) = -\frac{1}{3\sqrt{6}}$	$T_{-1}(3,4,7,2) = -\frac{11}{12\sqrt{21}}$
$T_1(2,1,3,6) = \frac{1}{2\sqrt{42}}$	$T_0(2,3,1,7) = -\frac{1}{12\sqrt{6}}$	$T_{-1}(3,5,4,6) = \frac{5}{12\sqrt{21}}$
$T_1(2,3,0,7) = -\frac{5}{12\sqrt{21}}$	$T_0(2,3,4,6) = \frac{1}{4\sqrt{3}}$	$T_{-1}(3,5,6,4) = \frac{5}{12\sqrt{21}}$
$T_1(2,3,1,6) = -\frac{1}{2\sqrt{42}}$	$T_0(2,3,7,1) = -\frac{1}{3\sqrt{6}}$	$T_{-1}(3,5,7,1) = \frac{1}{2\sqrt{42}}$
$T_1(2,3,7,0) = -\frac{5}{12\sqrt{21}}$	$T_0(2,4,3,6) = -\frac{1}{4\sqrt{3}}$	$T_{-1}(3,6,4,5) = \frac{5}{12\sqrt{21}}$
$T_1(2,6,1,3) = -\frac{1}{2\sqrt{42}}$	$T_0(2,6,3,4) = -\frac{1}{4\sqrt{3}}$	$T_{-1}(3,6,5,4) = \frac{11}{12\sqrt{21}}$
$T_1(2,6,3,1) = \frac{1}{2\sqrt{42}}$	$T_0(2,6,4,3) = \frac{1}{4\sqrt{3}}$	$T_{-1}(3,7,2,4) = -\frac{5}{12\sqrt{21}}$
$T_1(2,7,0,3) = -\frac{5}{12\sqrt{21}}$	$T_0(2,7,1,3) = -\frac{1}{12\sqrt{6}}$	$T_{-1}(3,7,4,2) = -\frac{5}{12\sqrt{21}}$
$T_1(2,7,3,0) = -\frac{11}{12\sqrt{21}}$	$T_0(2,7,3,1) = -\frac{1}{12\sqrt{6}}$	$T_{-1}(3,7,5,1) = -\frac{1}{2\sqrt{42}}$
$T_1(3,0,2,7) = -\frac{11}{12\sqrt{21}}$	$T_0(3,0,5,7) = \frac{1}{4\sqrt{3}}$	$T_{-1}(4,2,3,7) = -\frac{5}{12\sqrt{21}}$
$T_1(3,0,5,6) = \frac{5}{12\sqrt{21}}$	$T_0(3,0,7,5) = -\frac{1}{4\sqrt{3}}$	$T_{-1}(4,2,7,3) = -\frac{5}{12\sqrt{21}}$
$T_1(3,0,6,5) = \frac{11}{12\sqrt{21}}$	$T_0(3,1,2,7) = -\frac{1}{12\sqrt{6}}$	$T_{-1}(4,3,2,7) = -\frac{11}{12\sqrt{21}}$
$T_1(3,0,7,2) = -\frac{5}{12\sqrt{21}}$	$T_0(3,1,5,6) = \frac{1}{12\sqrt{6}}$	$T_{-1}(4,3,5,6) = \frac{5}{12\sqrt{21}}$
$T_1(3,1,2,6) = -\frac{1}{3\sqrt{6}}$	$T_0(3,1,6,5) = \frac{1}{12\sqrt{6}}$	$T_{-1}(4,3,6,5) = \frac{11}{12\sqrt{21}}$
$T_1(3,1,6,2) = \frac{1}{4\sqrt{3}}$	$T_0(3,1,7,2) = -\frac{1}{12\sqrt{6}}$	
$T_1(3,2,0,7) = -\frac{1}{3\sqrt{6}}$	$T_0(3,2,1,7) = -\frac{1}{3\sqrt{6}}$	
$T_1(3,2,6,1) = \frac{1}{4\sqrt{3}}$	$T_0(3,2,6,4) = \frac{1}{4\sqrt{3}}$	
$T_1(3,2,7,0) = -\frac{1}{3\sqrt{6}}$	$T_0(3,2,7,1) = -\frac{1}{12\sqrt{6}}$	
$T_1(3,5,0,6) = \frac{1}{3\sqrt{6}}$	$T_0(3,4,2,6) = -\frac{1}{4\sqrt{3}}$	
$T_1(3,5,6,0) = -\frac{1}{6\sqrt{6}}$	$T_0(3,4,6,2) = \frac{1}{4\sqrt{3}}$	
$T_1(3,6,0,5) = \frac{1}{3\sqrt{6}}$	$T_0(3,5,1,6) = \frac{1}{3\sqrt{6}}$	
$T_1(3,6,2,1) = -\frac{1}{4\sqrt{3}}$	$T_0(3,5,6,1) = \frac{1}{12\sqrt{6}}$	
$T_1(3,6,5,0) = \frac{1}{3\sqrt{6}}$	$T_0(3,5,7,0) = -\frac{1}{4\sqrt{3}}$	
$T_1(3,7,0,2) = -\frac{1}{3\sqrt{6}}$	$T_0(3,6,1,5) = \frac{1}{3\sqrt{6}}$	
$T_1(3,7,2,0) = \frac{1}{6\sqrt{6}}$	$T_0(3,6,2,4) = -\frac{1}{4\sqrt{3}}$	
$T_1(5,0,3,6) = \frac{1}{3\sqrt{6}}$	$T_0(3,6,5,1) = \frac{1}{12\sqrt{6}}$	
$T_1(5,0,6,3) = \frac{1}{3\sqrt{6}}$	$T_0(3,7,1,2) = -\frac{1}{3\sqrt{6}}$	
$T_1(5,3,0,6) = -\frac{1}{6\sqrt{6}}$	$T_0(3,7,2,1) = -\frac{1}{12\sqrt{6}}$	
$T_1(5,3,6,0) = \frac{1}{3\sqrt{6}}$	$T_0(3,7,5,0) = \frac{1}{4\sqrt{3}}$	
$T_1(5,6,0,3) = -\frac{1}{6\sqrt{6}}$	$T_0(4,2,6,3) = -\frac{1}{4\sqrt{3}}$	
$T_1(5,6,3,0) = \frac{1}{3\sqrt{6}}$	$T_0(4,3,2,6) = \frac{1}{4\sqrt{3}}$	
$T_1(6,0,3,5) = -\frac{1}{6\sqrt{6}}$	$T_0(4,3,6,2) = -\frac{1}{4\sqrt{3}}$	
$T_1(6,0,5,3) = \frac{1}{3\sqrt{6}}$	$T_0(4,6,2,3) = \frac{1}{4\sqrt{3}}$	
$T_1(6,1,3,2) = \frac{1}{4\sqrt{3}}$	$T_0(5,0,3,7) = \frac{1}{4\sqrt{3}}$	
$T_1(6,2,1,3) = -\frac{1}{4\sqrt{3}}$	$T_0(5,1,3,6) = \frac{1}{12\sqrt{6}}$	
$T_1(6,2,3,1) = \frac{1}{4\sqrt{3}}$	$T_0(5,1,6,3) = \frac{1}{3\sqrt{6}}$	
$T_1(6,3,0,5) = \frac{1}{3\sqrt{6}}$	$T_0(5,3,0,7) = -\frac{1}{4\sqrt{3}}$	
$T_1(6,3,1,2) = -\frac{1}{4\sqrt{3}}$	$T_0(5,3,1,6) = \frac{1}{12\sqrt{6}}$	
$T_1(6,3,5,0) = \frac{1}{3\sqrt{6}}$	$T_0(5,3,6,1) = \frac{1}{3\sqrt{6}}$	
$T_1(6,5,0,3) = \frac{1}{3\sqrt{6}}$	$T_0(5,6,1,3) = \frac{1}{12\sqrt{6}}$	
$T_1(6,5,3,0) = -\frac{1}{6\sqrt{6}}$	$T_0(5,6,3,1) = \frac{1}{12\sqrt{6}}$	
$T_1(7,0,2,3) = -\frac{1}{3\sqrt{6}}$	$T_0(5,7,0,3) = -\frac{1}{4\sqrt{3}}$	
$T_1(7,0,3,2) = -\frac{1}{3\sqrt{6}}$	$T_0(5,7,3,0) = \frac{1}{4\sqrt{3}}$	
$T_1(7,2,0,3) = \frac{1}{6\sqrt{6}}$	$T_0(6,1,3,5) = \frac{1}{12\sqrt{6}}$	
$T_1(7,2,3,0) = -\frac{1}{3\sqrt{6}}$	$T_0(6,1,5,3) = \frac{1}{3\sqrt{6}}$	
$T_1(7,3,0,2) = \frac{1}{6\sqrt{6}}$	$T_0(6,2,3,4) = \frac{1}{4\sqrt{3}}$	
$T_1(7,3,2,0) = -\frac{1}{3\sqrt{6}}$	$T_0(6,2,4,3) = -\frac{1}{4\sqrt{3}}$	
	$T_0(6,3,1,5) = \frac{1}{12\sqrt{6}}$	
	$T_0(6,3,4,2) = -\frac{1}{4\sqrt{3}}$	
	$T_0(6,3,5,1) = \frac{1}{3\sqrt{6}}$	
	$T_0(6,4,3,2) = \frac{1}{4\sqrt{3}}$	
	$T_0(6,5,1,3) = \frac{1}{12\sqrt{6}}$	
	$T_0(6,5,3,1) = \frac{1}{12\sqrt{6}}$	

TABLE XXXVIII. (Continued.)

$S_z = +1$	$S_z = 0$	$S_z = -1$
$T_1(3,1,6,2) = -\frac{1}{2\sqrt{42}}$	$T_0(3,1,7,2) = -\frac{1}{12\sqrt{6}}$	$T_{-1}(4,3,7,2) = -\frac{5}{12\sqrt{21}}$
$T_1(3,2,0,7) = -\frac{5}{12\sqrt{21}}$	$T_0(3,2,1,7) = -\frac{1}{3\sqrt{6}}$	$T_{-1}(4,5,3,6) = \frac{5}{12\sqrt{21}}$
$T_1(3,2,6,1) = -\frac{1}{2\sqrt{42}}$	$T_0(3,2,6,4) = \frac{1}{4\sqrt{3}}$	$T_{-1}(4,5,6,3) = \frac{11}{12\sqrt{21}}$
$T_1(3,2,7,0) = -\frac{5}{12\sqrt{21}}$	$T_0(3,2,7,1) = -\frac{1}{12\sqrt{6}}$	$T_{-1}(4,6,3,5) = \frac{5}{12\sqrt{21}}$
$T_1(3,5,0,6) = \frac{5}{12\sqrt{21}}$	$T_0(3,4,2,6) = -\frac{1}{4\sqrt{3}}$	$T_{-1}(4,6,5,3) = \frac{5}{12\sqrt{21}}$
$T_1(3,5,6,0) = \frac{11}{12\sqrt{21}}$	$T_0(3,4,6,2) = \frac{1}{4\sqrt{3}}$	$T_{-1}(4,7,2,3) = -\frac{11}{12\sqrt{21}}$
$T_1(3,6,0,5) = \frac{5}{12\sqrt{21}}$	$T_0(3,5,1,6) = \frac{1}{3\sqrt{6}}$	$T_{-1}(4,7,3,2) = -\frac{5}{12\sqrt{21}}$
$T_1(3,6,2,1) = \frac{1}{2\sqrt{42}}$	$T_0(3,5,6,1) = \frac{1}{12\sqrt{6}}$	$T_{-1}(5,1,3,7) = -\frac{1}{2\sqrt{42}}$
$T_1(3,6,5,0) = \frac{5}{12\sqrt{21}}$	$T_0(3,5,7,0) = -\frac{1}{4\sqrt{3}}$	$T_{-1}(5,3,1,7) = \frac{1}{2\sqrt{42}}$
$T_1(3,7,0,2) = -\frac{5}{12\sqrt{21}}$	$T_0(3,6,1,5) = \frac{1}{3\sqrt{6}}$	$T_{-1}(5,3,4,6) = \frac{5}{12\sqrt{21}}$
$T_1(3,7,2,0) = -\frac{11}{12\sqrt{21}}$	$T_0(3,6,2,4) = -\frac{1}{4\sqrt{3}}$	$T_{-1}(5,3,6,4) = \frac{5}{12\sqrt{21}}$
$T_1(5,0,3,6) = \frac{5}{12\sqrt{21}}$	$T_0(3,6,5,1) = \frac{1}{12\sqrt{6}}$	$T_{-1}(5,4,3,6) = \frac{11}{12\sqrt{21}}$
$T_1(5,0,6,3) = \frac{5}{12\sqrt{21}}$	$T_0(3,7,1,2) = -\frac{1}{3\sqrt{6}}$	$T_{-1}(5,4,6,3) = \frac{5}{12\sqrt{21}}$
$T_1(5,3,0,6) = \frac{11}{12\sqrt{21}}$	$T_0(3,7,2,1) = -\frac{1}{12\sqrt{6}}$	$T_{-1}(5,6,3,4) = \frac{11}{12\sqrt{21}}$
$T_1(5,3,6,0) = \frac{5}{12\sqrt{21}}$	$T_0(3,7,5,0) = \frac{1}{4\sqrt{3}}$	$T_{-1}(5,6,4,3) = \frac{5}{12\sqrt{21}}$
$T_1(5,6,0,3) = \frac{11}{12\sqrt{21}}$	$T_0(4,2,6,3) = -\frac{1}{4\sqrt{3}}$	$T_{-1}(5,7,1,3) = \frac{1}{2\sqrt{42}}$
$T_1(5,6,3,0) = \frac{5}{12\sqrt{21}}$	$T_0(4,3,2,6) = \frac{1}{4\sqrt{3}}$	$T_{-1}(5,7,3,1) = -\frac{1}{2\sqrt{42}}$
$T_1(6,0,3,5) = \frac{11}{12\sqrt{21}}$	$T_0(4,3,6,2) = -\frac{1}{4\sqrt{3}}$	$T_{-1}(6,3,4,5) = \frac{11}{12\sqrt{21}}$
$T_1(6,0,5,3) = \frac{5}{12\sqrt{21}}$	$T_0(4,6,2,3) = \frac{1}{4\sqrt{3}}$	$T_{-1}(6,3,5,4) = \frac{5}{12\sqrt{21}}$
$T_1(6,1,3,2) = -\frac{1}{2\sqrt{42}}$	$T_0(5,0,3,7) = \frac{1}{4\sqrt{3}}$	$T_{-1}(6,4,3,5) = \frac{5}{12\sqrt{21}}$
$T_1(6,2,1,3) = \frac{1}{2\sqrt{42}}$	$T_0(5,1,3,6) = \frac{1}{12\sqrt{6}}$	$T_{-1}(6,4,5,3) = \frac{5}{12\sqrt{21}}$
$T_1(6,2,3,1) = -\frac{1}{2\sqrt{42}}$	$T_0(5,1,6,3) = \frac{1}{3\sqrt{6}}$	$T_{-1}(6,5,3,4) = \frac{5}{12\sqrt{21}}$
$T_1(6,3,0,5) = \frac{5}{12\sqrt{21}}$	$T_0(5,3,0,7) = -\frac{1}{4\sqrt{3}}$	$T_{-1}(6,5,4,3) = \frac{11}{12\sqrt{21}}$
$T_1(6,3,1,2) = \frac{1}{2\sqrt{42}}$	$T_0(5,3,1,6) = \frac{1}{12\sqrt{6}}$	$T_{-1}(7,1,3,5) = \frac{1}{2\sqrt{42}}$
$T_1(6,3,5,0) = \frac{5}{12\sqrt{21}}$	$T_0(5,3,6,1) = \frac{1}{3\sqrt{6}}$	$T_{-1}(7,2,3,4) = -\frac{11}{12\sqrt{21}}$
$T_1(6,5,0,3) = \frac{5}{12\sqrt{21}}$	$T_0(5,6,1,3) = \frac{1}{12\sqrt{6}}$	$T_{-1}(7,2,4,3) = -\frac{5}{12\sqrt{21}}$
$T_1(6,5,3,0) = \frac{11}{12\sqrt{21}}$	$T_0(5,6,3,1) = \frac{1}{12\sqrt{6}}$	$T_{-1}(7,3,1,5) = -\frac{1}{2\sqrt{42}}$
$T_1(7,0,2,3) = -\frac{5}{12\sqrt{21}}$	$T_0(5,7,0,3) = -\frac{1}{4\sqrt{3}}$	$T_{-1}(7,3,2,4) = -\frac{5}{12\sqrt{21}}$
$T_1(7,0,3,2) = -\frac{5}{12\sqrt{21}}$	$T_0(5,7,3,0) = \frac{1}{4\sqrt{3}}$	$T_{-1}(7,3,4,2) = -\frac{5}{12\sqrt{21}}$
$T_1(7,2,0,3) = -\frac{11}{12\sqrt{21}}$	$T_0(6,1,3,5) = \frac{1}{12\sqrt{6}}$	$T_{-1}(7,4,2,3) = -\frac{5}{12\sqrt{21}}$
$T_1(7,2,3,0) = -\frac{5}{12\sqrt{21}}$	$T_0(6,1,5,3) = \frac{1}{3\sqrt{6}}$	$T_{-1}(7,4,3,2) = -\frac{11}{12\sqrt{21}}$
$T_1(7,3,0,2) = -\frac{11}{12\sqrt{21}}$	$T_0(6,2,3,4) = \frac{1}{4\sqrt{3}}$	$T_{-1}(7,5,1,3) = -\frac{1}{2\sqrt{42}}$
$T_1(7,3,2,0) = -\frac{5}{12\sqrt{21}}$	$T_0(6,2,4,3) = -\frac{1}{4\sqrt{3}}$	$T_{-1}(7,5,3,1) = \frac{1}{2\sqrt{42}}$
	$T_0(6,3,1,5) = \frac{1}{12\sqrt{6}}$	
	$T_0(6,3,4,2) = -\frac{1}{4\sqrt{3}}$	
	$T_0(6,3,5,1) = \frac{1}{3\sqrt{6}}$	
	$T_0(6,4,3,2) = \frac{1}{4\sqrt{3}}$	
	$T_0(6,5,1,3) = \frac{1}{12\sqrt{6}}$	
	$T_0(6,5,3,1) = \frac{1}{12\sqrt{6}}$	
	$T_0(7,0,3,5) = -\frac{1}{4\sqrt{3}}$	
	$T_0(7,1,2,3) = -\frac{1}{3\sqrt{6}}$	
	$T_0(7,1,3,2) = -\frac{1}{12\sqrt{6}}$	
	$T_0(7,2,1,3) = -\frac{1}{12\sqrt{6}}$	
	$T_0(7,2,3,1) = -\frac{1}{12\sqrt{6}}$	
	$T_0(7,3,0,5) = \frac{1}{4\sqrt{3}}$	
	$T_0(7,3,1,2) = -\frac{1}{12\sqrt{6}}$	

TABLE XXXVIII. (Continued.)

$S_z = +1$	$S_z = 0$	$S_z = -1$
	$T_0(7,3,2,1) = -\frac{1}{3\sqrt{6}}$	
	$T_0(7,5,0,3) = \frac{1}{4\sqrt{3}}$	
	$T_0(7,5,3,0) = -\frac{1}{4\sqrt{3}}$	

TABLE XXXIX. G_8 .

$S_z = +1$	$S_z = 0$	$S_z = -1$
$T_1(0,2,3,7) = \frac{1}{8}$	$T_0(0,5,3,7) = \frac{1}{4\sqrt{2}}$	$T_{-1}(1,5,3,7) = \frac{1}{4\sqrt{2}}$
$T_1(0,2,7,3) = \frac{1}{8}$	$T_0(0,7,3,5) = -\frac{1}{4\sqrt{2}}$	$T_{-1}(1,7,3,5) = -\frac{1}{4\sqrt{2}}$
$T_1(0,3,2,7) = -\frac{1}{8}$	$T_0(1,2,7,3) = \frac{1}{8}$	$T_{-1}(2,3,4,7) = \frac{1}{8}$
$T_1(0,3,5,6) = \frac{1}{8}$	$T_0(1,3,2,7) = -\frac{1}{8}$	$T_{-1}(2,3,7,4) = -\frac{1}{8}$
$T_1(0,3,6,5) = \frac{1}{8}$	$T_0(1,3,5,6) = \frac{1}{8}$	$T_{-1}(2,4,3,7) = -\frac{1}{8}$
$T_1(0,3,7,2) = -\frac{1}{8}$	$T_0(1,3,6,5) = \frac{1}{8}$	$T_{-1}(2,4,7,3) = \frac{1}{8}$
$T_1(0,5,3,6) = \frac{1}{8}$	$T_0(1,3,7,2) = -\frac{1}{8}$	$T_{-1}(2,7,3,4) = \frac{1}{8}$
$T_1(0,5,6,3) = -\frac{1}{8}$	$T_0(1,5,6,3) = -\frac{1}{8}$	$T_{-1}(2,7,4,3) = -\frac{1}{8}$
$T_1(0,6,3,5) = -\frac{1}{8}$	$T_0(1,6,5,3) = -\frac{1}{8}$	$T_{-1}(3,2,4,7) = \frac{1}{8}$
$T_1(0,6,5,3) = -\frac{1}{8}$	$T_0(1,7,2,3) = \frac{1}{8}$	$T_{-1}(3,2,7,4) = -\frac{1}{8}$
$T_1(0,7,2,3) = \frac{1}{8}$	$T_0(2,1,3,7) = -\frac{1}{8}$	$T_{-1}(3,4,2,7) = \frac{1}{8}$
$T_1(0,7,3,2) = -\frac{1}{8}$	$T_0(2,3,1,7) = \frac{1}{8}$	$T_{-1}(3,4,5,6) = -\frac{1}{8}$
$T_1(1,2,3,6) = -\frac{1}{4\sqrt{2}}$	$T_0(2,3,6,4) = -\frac{1}{4\sqrt{2}}$	$T_{-1}(3,4,6,5) = -\frac{1}{8}$
$T_1(1,6,3,2) = \frac{1}{4\sqrt{2}}$	$T_0(2,4,6,3) = \frac{1}{4\sqrt{2}}$	$T_{-1}(3,4,7,2) = \frac{1}{8}$
$T_1(2,0,3,7) = -\frac{1}{8}$	$T_0(2,7,1,3) = -\frac{1}{8}$	$T_{-1}(3,5,1,7) = -\frac{1}{4\sqrt{2}}$
$T_1(2,0,7,3) = -\frac{1}{8}$	$T_0(2,7,3,1) = \frac{1}{8}$	$T_{-1}(3,5,4,6) = \frac{1}{8}$
$T_1(2,1,6,3) = \frac{1}{4\sqrt{2}}$	$T_0(3,1,2,7) = \frac{1}{8}$	$T_{-1}(3,5,6,4) = \frac{1}{8}$
$T_1(2,3,0,7) = \frac{1}{8}$	$T_0(3,1,5,6) = -\frac{1}{8}$	$T_{-1}(3,6,4,5) = -\frac{1}{8}$
$T_1(2,3,6,1) = -\frac{1}{4\sqrt{2}}$	$T_0(3,1,6,5) = -\frac{1}{8}$	$T_{-1}(3,6,5,4) = \frac{1}{8}$
$T_1(2,3,7,0) = \frac{1}{8}$	$T_0(3,1,7,2) = \frac{1}{8}$	$T_{-1}(3,7,1,5) = \frac{1}{4\sqrt{2}}$
$T_1(2,7,0,3) = -\frac{1}{8}$	$T_0(3,2,4,6) = \frac{1}{4\sqrt{2}}$	$T_{-1}(3,7,2,4) = -\frac{1}{8}$
$T_1(2,7,3,0) = \frac{1}{8}$	$T_0(3,2,7,1) = -\frac{1}{8}$	$T_{-1}(3,7,4,2) = -\frac{1}{8}$
$T_1(3,0,2,7) = \frac{1}{8}$	$T_0(3,5,0,7) = -\frac{1}{4\sqrt{2}}$	$T_{-1}(4,2,3,7) = -\frac{1}{8}$
$T_1(3,0,5,6) = -\frac{1}{8}$	$T_0(3,5,6,1) = \frac{1}{8}$	$T_{-1}(4,2,7,3) = \frac{1}{8}$
$T_1(3,0,6,5) = -\frac{1}{8}$	$T_0(3,6,4,2) = -\frac{1}{4\sqrt{2}}$	$T_{-1}(4,3,2,7) = -\frac{1}{8}$
$T_1(3,0,7,2) = \frac{1}{8}$	$T_0(3,6,5,1) = \frac{1}{8}$	$T_{-1}(4,3,5,6) = \frac{1}{8}$
$T_1(3,2,0,7) = -\frac{1}{8}$	$T_0(3,7,0,5) = \frac{1}{4\sqrt{2}}$	$T_{-1}(4,3,6,5) = \frac{1}{8}$
$T_1(3,2,1,6) = \frac{1}{4\sqrt{2}}$	$T_0(3,7,2,1) = -\frac{1}{8}$	$T_{-1}(4,3,7,2) = -\frac{1}{8}$
$T_1(3,2,7,0) = -\frac{1}{8}$	$T_0(4,2,3,6) = -\frac{1}{4\sqrt{2}}$	$T_{-1}(4,5,3,6) = -\frac{1}{8}$
$T_1(3,5,0,6) = -\frac{1}{8}$	$T_0(4,6,3,2) = \frac{1}{4\sqrt{2}}$	$T_{-1}(4,5,6,3) = -\frac{1}{8}$
$T_1(3,5,6,0) = \frac{1}{8}$	$T_0(5,0,7,3) = -\frac{1}{4\sqrt{2}}$	$T_{-1}(4,6,3,5) = \frac{1}{8}$
$T_1(3,6,1,2) = -\frac{1}{4\sqrt{2}}$	$T_0(5,1,3,6) = \frac{1}{8}$	$T_{-1}(4,6,5,3) = -\frac{1}{8}$
$T_1(3,6,5,0) = \frac{1}{8}$	$T_0(5,3,1,6) = -\frac{1}{8}$	$T_{-1}(4,7,2,3) = \frac{1}{8}$
$T_1(3,7,0,2) = \frac{1}{8}$	$T_0(5,3,7,0) = \frac{1}{4\sqrt{2}}$	$T_{-1}(4,7,3,2) = \frac{1}{8}$
$T_1(3,7,2,0) = -\frac{1}{8}$	$T_0(5,6,1,3) = \frac{1}{8}$	$T_{-1}(5,1,7,3) = -\frac{1}{4\sqrt{2}}$
$T_1(5,0,3,6) = \frac{1}{8}$	$T_0(5,6,3,1) = -\frac{1}{8}$	$T_{-1}(5,3,4,6) = -\frac{1}{8}$
$T_1(5,0,6,3) = -\frac{1}{8}$	$T_0(6,1,3,5) = \frac{1}{8}$	$T_{-1}(5,3,6,4) = -\frac{1}{8}$
$T_1(5,0,6,3) = -\frac{1}{8}$	$T_0(6,3,1,5) = -\frac{1}{8}$	$T_{-1}(5,3,7,1) = \frac{1}{4\sqrt{2}}$
$T_1(5,3,0,6) = -\frac{1}{8}$	$T_0(6,3,2,4) = \frac{1}{4\sqrt{2}}$	$T_{-1}(5,4,3,6) = \frac{1}{8}$
$T_1(5,3,6,0) = \frac{1}{8}$	$T_0(6,4,2,3) = -\frac{1}{4\sqrt{2}}$	$T_{-1}(5,4,6,3) = \frac{1}{8}$
$T_1(5,6,0,3) = \frac{1}{8}$	$T_0(6,5,1,3) = \frac{1}{8}$	$T_{-1}(5,6,3,4) = -\frac{1}{8}$

TABLE XXXIX. (Continued.)

$S_z = +1$	$S_z = 0$	$S_z = -1$
$T_1(5,6,3,0) = -\frac{1}{8}$	$T_0(6,5,3,1) = -\frac{1}{8}$	$T_{-1}(5,6,4,3) = \frac{1}{8}$
$T_1(6,0,3,5) = \frac{1}{8}$	$T_0(7,0,5,3) = \frac{1}{4\sqrt{2}}$	$T_{-1}(6,3,4,5) = -\frac{1}{8}$
$T_1(6,0,5,3) = \frac{1}{8}$	$T_0(7,1,3,2) = -\frac{1}{8}$	$T_{-1}(6,3,5,4) = \frac{1}{8}$
$T_1(6,1,2,3) = -\frac{1}{4\sqrt{2}}$	$T_0(7,2,1,3) = -\frac{1}{8}$	$T_{-1}(6,4,3,5) = \frac{1}{8}$
$T_1(6,3,0,5) = -\frac{1}{8}$	$T_0(7,2,3,1) = \frac{1}{8}$	$T_{-1}(6,4,5,3) = -\frac{1}{8}$
$T_1(6,3,2,1) = \frac{1}{4\sqrt{2}}$	$T_0(7,3,1,2) = \frac{1}{8}$	$T_{-1}(6,5,3,4) = -\frac{1}{8}$
$T_1(6,3,5,0) = -\frac{1}{8}$	$T_0(7,3,5,0) = -\frac{1}{4\sqrt{2}}$	$T_{-1}(6,5,4,3) = \frac{1}{8}$
$T_1(6,5,0,3) = \frac{1}{8}$		$T_{-1}(7,1,5,3) = \frac{1}{4\sqrt{2}}$
$T_1(6,5,3,0) = -\frac{1}{8}$		$T_{-1}(7,2,3,4) = \frac{1}{8}$
$T_1(7,0,2,3) = \frac{1}{8}$		$T_{-1}(7,2,4,3) = -\frac{1}{8}$
$T_1(7,0,3,2) = -\frac{1}{8}$		$T_{-1}(7,3,2,4) = \frac{1}{8}$
$T_1(7,2,0,3) = -\frac{1}{8}$		$T_{-1}(7,3,4,2) = \frac{1}{8}$
$T_1(7,2,3,0) = \frac{1}{8}$		$T_{-1}(7,3,5,1) = -\frac{1}{4\sqrt{2}}$
$T_1(7,3,0,2) = \frac{1}{8}$		$T_{-1}(7,4,2,3) = -\frac{1}{8}$
$T_1(7,3,2,0) = -\frac{1}{8}$		$T_{-1}(7,4,3,2) = -\frac{1}{8}$

TABLE XL. S_9 .

$S_z = +1$	$S_z = 0$	$S_z = -1$
$T_1(2,3,6,8) = -\frac{1}{2\sqrt{3}}$	$T_0(2,3,7,8) = -\frac{1}{2\sqrt{6}}$	$T_{-1}(3,5,7,8) = \frac{1}{4\sqrt{3}}$
$T_1(2,3,8,6) = \frac{1}{4\sqrt{3}}$	$T_0(2,3,8,7) = \frac{1}{4\sqrt{6}}$	$T_{-1}(3,5,8,7) = -\frac{1}{2\sqrt{3}}$
$T_1(2,6,3,8) = \frac{1}{4\sqrt{3}}$	$T_0(2,7,3,8) = \frac{1}{4\sqrt{6}}$	$T_{-1}(3,7,5,8) = \frac{1}{4\sqrt{3}}$
$T_1(2,6,8,3) = \frac{1}{4\sqrt{3}}$	$T_0(2,7,8,3) = \frac{1}{4\sqrt{6}}$	$T_{-1}(3,7,8,5) = -\frac{1}{2\sqrt{3}}$
$T_1(2,8,3,6) = \frac{1}{4\sqrt{3}}$	$T_0(2,8,3,7) = \frac{1}{4\sqrt{6}}$	$T_{-1}(3,8,5,7) = \frac{1}{4\sqrt{3}}$
$T_1(2,8,6,3) = -\frac{1}{2\sqrt{3}}$	$T_0(2,8,7,3) = -\frac{1}{2\sqrt{6}}$	$T_{-1}(3,8,7,5) = \frac{1}{4\sqrt{3}}$
$T_1(3,2,6,8) = \frac{1}{4\sqrt{3}}$	$T_0(3,2,7,8) = \frac{1}{4\sqrt{6}}$	$T_{-1}(5,3,7,8) = -\frac{1}{2\sqrt{3}}$
$T_1(3,2,8,6) = -\frac{1}{2\sqrt{3}}$	$T_0(3,2,8,7) = -\frac{1}{2\sqrt{6}}$	$T_{-1}(5,3,8,7) = \frac{1}{4\sqrt{3}}$
$T_1(3,6,2,8) = \frac{1}{4\sqrt{3}}$	$T_0(3,5,6,8) = \frac{1}{4\sqrt{6}}$	$T_{-1}(5,7,3,8) = \frac{1}{4\sqrt{3}}$
$T_1(3,6,8,2) = -\frac{1}{2\sqrt{3}}$	$T_0(3,5,8,6) = -\frac{1}{2\sqrt{6}}$	$T_{-1}(5,7,8,3) = \frac{1}{4\sqrt{3}}$
$T_1(3,8,2,6) = \frac{1}{4\sqrt{3}}$	$T_0(3,6,5,8) = \frac{1}{4\sqrt{6}}$	$T_{-1}(5,8,3,7) = \frac{1}{4\sqrt{3}}$
$T_1(3,8,6,2) = \frac{1}{4\sqrt{3}}$	$T_0(3,6,8,5) = -\frac{1}{2\sqrt{6}}$	$T_{-1}(5,8,7,3) = -\frac{1}{2\sqrt{3}}$
$T_1(6,2,3,8) = \frac{1}{4\sqrt{3}}$	$T_0(3,7,2,8) = \frac{1}{4\sqrt{6}}$	$T_{-1}(7,3,5,8) = -\frac{1}{2\sqrt{3}}$
$T_1(6,2,8,3) = \frac{1}{4\sqrt{3}}$	$T_0(3,7,8,2) = -\frac{1}{2\sqrt{6}}$	$T_{-1}(7,3,8,5) = \frac{1}{4\sqrt{3}}$
$T_1(6,3,2,8) = -\frac{1}{2\sqrt{3}}$	$T_0(3,8,2,7) = \frac{1}{4\sqrt{6}}$	$T_{-1}(7,5,3,8) = \frac{1}{4\sqrt{3}}$
$T_1(6,3,8,2) = \frac{1}{4\sqrt{3}}$	$T_0(3,8,5,6) = \frac{1}{4\sqrt{6}}$	$T_{-1}(7,5,8,3) = \frac{1}{4\sqrt{3}}$
$T_1(6,8,2,3) = -\frac{1}{2\sqrt{3}}$	$T_0(3,8,6,5) = \frac{1}{4\sqrt{6}}$	$T_{-1}(7,8,3,5) = \frac{1}{4\sqrt{3}}$
$T_1(6,8,3,2) = \frac{1}{4\sqrt{3}}$	$T_0(3,8,7,2) = \frac{1}{4\sqrt{6}}$	$T_{-1}(7,8,5,3) = -\frac{1}{2\sqrt{3}}$
$T_1(8,2,3,6) = -\frac{1}{2\sqrt{3}}$	$T_0(5,3,6,8) = -\frac{1}{2\sqrt{6}}$	$T_{-1}(8,3,5,7) = \frac{1}{4\sqrt{3}}$
$T_1(8,2,6,3) = \frac{1}{4\sqrt{3}}$	$T_0(5,3,8,6) = \frac{1}{4\sqrt{6}}$	$T_{-1}(8,3,7,5) = \frac{1}{4\sqrt{3}}$
$T_1(8,3,2,6) = \frac{1}{4\sqrt{3}}$	$T_0(5,6,3,8) = \frac{1}{4\sqrt{6}}$	$T_{-1}(8,5,3,7) = -\frac{1}{2\sqrt{3}}$
$T_1(8,3,6,2) = \frac{1}{4\sqrt{3}}$	$T_0(5,6,8,3) = \frac{1}{4\sqrt{6}}$	$T_{-1}(8,5,7,3) = \frac{1}{4\sqrt{3}}$
$T_1(8,6,2,3) = \frac{1}{4\sqrt{3}}$	$T_0(5,8,3,6) = \frac{1}{4\sqrt{6}}$	$T_{-1}(8,7,3,5) = -\frac{1}{2\sqrt{3}}$
$T_1(8,6,3,2) = -\frac{1}{2\sqrt{3}}$	$T_0(5,8,6,3) = -\frac{1}{2\sqrt{6}}$	$T_{-1}(8,7,5,3) = \frac{1}{4\sqrt{3}}$
	$T_0(6,3,5,8) = -\frac{1}{2\sqrt{6}}$	
	$T_0(6,3,8,5) = \frac{1}{4\sqrt{6}}$	
	$T_0(6,5,3,8) = \frac{1}{4\sqrt{6}}$	
	$T_0(6,5,8,3) = \frac{1}{4\sqrt{6}}$	

TABLE XL. (Continued.)

$S_z = +1$	$S_z = 0$	$S_z = -1$
	$T_0(6,8,3,5) = \frac{1}{4\sqrt{6}}$	
	$T_0(6,8,5,3) = -\frac{1}{2\sqrt{6}}$	
	$T_0(7,2,3,8) = \frac{1}{4\sqrt{6}}$	
	$T_0(7,2,8,3) = \frac{1}{4\sqrt{6}}$	
	$T_0(7,3,2,8) = -\frac{1}{2\sqrt{6}}$	
	$T_0(7,3,8,2) = \frac{1}{4\sqrt{6}}$	
	$T_0(7,8,2,3) = -\frac{1}{2\sqrt{6}}$	
	$T_0(7,8,3,2) = \frac{1}{4\sqrt{6}}$	
	$T_0(8,2,3,7) = -\frac{1}{2\sqrt{6}}$	
	$T_0(8,2,7,3) = \frac{1}{4\sqrt{6}}$	
	$T_0(8,3,2,7) = \frac{1}{4\sqrt{6}}$	
	$T_0(8,3,5,6) = \frac{1}{4\sqrt{6}}$	
	$T_0(8,3,6,5) = \frac{1}{4\sqrt{6}}$	
	$T_0(8,3,7,2) = \frac{1}{4\sqrt{6}}$	
	$T_0(8,5,3,6) = -\frac{1}{2\sqrt{6}}$	
	$T_0(8,5,6,3) = \frac{1}{4\sqrt{6}}$	
	$T_0(8,6,3,5) = -\frac{1}{2\sqrt{6}}$	
	$T_0(8,6,5,3) = \frac{1}{4\sqrt{6}}$	
	$T_0(8,7,2,3) = \frac{1}{4\sqrt{6}}$	
	$T_0(8,7,3,2) = -\frac{1}{2\sqrt{6}}$	

TABLE XLI. G_9 .

$S_z = +1$	$S_z = 0$	$S_z = -1$
$T_1(2,3,8,6) = \frac{1}{4}$	$T_0(2,3,8,7) = \frac{1}{4\sqrt{2}}$	$T_{-1}(3,5,7,8) = -\frac{1}{4}$
$T_1(2,6,3,8) = \frac{1}{4}$	$T_0(2,7,3,8) = \frac{1}{4\sqrt{2}}$	$T_{-1}(3,7,5,8) = -\frac{1}{4}$
$T_1(2,6,8,3) = -\frac{1}{4}$	$T_0(2,7,8,3) = -\frac{1}{4\sqrt{2}}$	$T_{-1}(3,8,5,7) = \frac{1}{4}$
$T_1(2,8,3,6) = -\frac{1}{4}$	$T_0(2,8,3,7) = -\frac{1}{4\sqrt{2}}$	$T_{-1}(3,8,7,5) = \frac{1}{4}$
$T_1(3,2,6,8) = -\frac{1}{4}$	$T_0(3,2,7,8) = -\frac{1}{4\sqrt{2}}$	$T_{-1}(5,3,8,7) = \frac{1}{4}$
$T_1(3,6,2,8) = -\frac{1}{4}$	$T_0(3,5,6,8) = -\frac{1}{4\sqrt{2}}$	$T_{-1}(5,7,8,3) = -\frac{1}{4}$
$T_1(3,8,2,6) = \frac{1}{4}$	$T_0(3,6,5,8) = -\frac{1}{4\sqrt{2}}$	$T_{-1}(5,8,3,7) = -\frac{1}{4}$
$T_1(3,8,6,2) = \frac{1}{4}$	$T_0(3,7,2,8) = -\frac{1}{4\sqrt{2}}$	$T_{-1}(5,8,7,3) = -\frac{1}{4}$
$T_1(6,2,3,8) = \frac{1}{4}$	$T_0(3,8,2,7) = \frac{1}{4\sqrt{2}}$	$T_{-1}(7,3,8,5) = \frac{1}{4}$
$T_1(6,2,8,3) = -\frac{1}{4}$	$T_0(3,8,5,6) = \frac{1}{4\sqrt{2}}$	$T_{-1}(7,5,3,8) = \frac{1}{4}$
$T_1(6,3,8,2) = \frac{1}{4}$	$T_0(3,8,6,5) = \frac{1}{4\sqrt{2}}$	$T_{-1}(7,5,8,3) = -\frac{1}{4}$
$T_1(6,8,3,2) = -\frac{1}{4}$	$T_0(3,8,7,2) = \frac{1}{4\sqrt{2}}$	$T_{-1}(7,8,3,5) = -\frac{1}{4}$
$T_1(8,2,6,3) = \frac{1}{4}$	$T_0(5,3,8,6) = \frac{1}{4\sqrt{2}}$	$T_{-1}(8,3,5,7) = -\frac{1}{4}$
$T_1(8,3,2,6) = -\frac{1}{4}$	$T_0(5,6,3,8) = \frac{1}{4\sqrt{2}}$	$T_{-1}(8,3,7,5) = -\frac{1}{4}$
$T_1(8,3,6,2) = -\frac{1}{4}$	$T_0(5,6,8,3) = -\frac{1}{4\sqrt{2}}$	$T_{-1}(8,5,7,3) = \frac{1}{4}$
$T_1(8,6,2,3) = \frac{1}{4}$	$T_0(5,8,3,6) = -\frac{1}{4\sqrt{2}}$	$T_{-1}(8,7,5,3) = \frac{1}{4}$
	$T_0(6,3,8,5) = \frac{1}{4\sqrt{2}}$	
	$T_0(6,5,3,8) = \frac{1}{4\sqrt{2}}$	
	$T_0(6,5,8,3) = -\frac{1}{4\sqrt{2}}$	
	$T_0(6,8,3,5) = -\frac{1}{4\sqrt{2}}$	
	$T_0(7,2,3,8) = \frac{1}{4\sqrt{2}}$	
	$T_0(7,2,8,3) = -\frac{1}{4\sqrt{2}}$	

TABLE XLI. (Continued.)

$S_z = +1$	$S_z = 0$	$S_z = -1$
	$T_0(7,3,8,2) = \frac{1}{4\sqrt{2}}$	
	$T_0(7,8,3,2) = -\frac{1}{4\sqrt{2}}$	
	$T_0(8,2,7,3) = \frac{1}{4\sqrt{2}}$	
	$T_0(8,3,2,7) = -\frac{1}{4\sqrt{2}}$	
	$T_0(8,3,5,6) = -\frac{1}{4\sqrt{2}}$	
	$T_0(8,3,6,5) = -\frac{1}{4\sqrt{2}}$	
	$T_0(8,3,7,2) = -\frac{1}{4\sqrt{2}}$	
	$T_0(8,5,6,3) = \frac{1}{4\sqrt{2}}$	
	$T_0(8,6,5,3) = \frac{1}{4\sqrt{2}}$	
	$T_0(8,7,2,3) = \frac{1}{4\sqrt{2}}$	

TABLE XLII. S_{10} .

$S_z = +1$	$S_z = 0$	$S_z = -1$
$T_1(0,2,5,8) = -\frac{1}{2\sqrt{30}}$	$T_0(0,5,5,8) = \frac{1}{4\sqrt{15}}$	$T_{-1}(1,5,5,8) = \frac{1}{4\sqrt{15}}$
$T_1(0,2,8,5) = -\frac{1}{2\sqrt{30}}$	$T_0(0,5,8,5) = -\frac{1}{2\sqrt{15}}$	$T_{-1}(1,5,8,5) = -\frac{1}{2\sqrt{15}}$
$T_1(0,5,2,8) = \frac{1}{\sqrt{30}}$	$T_0(0,7,7,8) = \frac{1}{4\sqrt{15}}$	$T_{-1}(1,7,7,8) = \frac{1}{4\sqrt{15}}$
$T_1(0,5,8,2) = -\frac{1}{2\sqrt{30}}$	$T_0(0,7,8,7) = -\frac{1}{2\sqrt{15}}$	$T_{-1}(1,7,8,7) = -\frac{1}{2\sqrt{15}}$
$T_1(0,6,7,8) = -\frac{1}{2\sqrt{30}}$	$T_0(0,8,5,5) = \frac{1}{4\sqrt{15}}$	$T_{-1}(1,8,5,5) = \frac{1}{4\sqrt{15}}$
$T_1(0,6,8,7) = -\frac{1}{2\sqrt{30}}$	$T_0(0,8,7,7) = \frac{1}{4\sqrt{15}}$	$T_{-1}(1,8,7,7) = \frac{1}{4\sqrt{15}}$
$T_1(0,7,6,8) = \frac{1}{\sqrt{30}}$	$T_0(1,2,5,8) = -\frac{\sqrt{\frac{3}{10}}}{4}$	$T_{-1}(2,4,5,8) = \frac{1}{2\sqrt{30}}$
$T_1(0,7,8,6) = -\frac{1}{2\sqrt{30}}$	$T_0(1,5,2,8) = \frac{\sqrt{\frac{3}{10}}}{4}$	$T_{-1}(2,4,8,5) = -\frac{1}{\sqrt{30}}$
$T_1(0,8,2,5) = \frac{1}{\sqrt{30}}$	$T_0(1,6,7,8) = -\frac{\sqrt{\frac{3}{10}}}{4}$	$T_{-1}(2,5,4,8) = \frac{1}{2\sqrt{30}}$
$T_1(0,8,5,2) = -\frac{1}{2\sqrt{30}}$	$T_0(1,7,6,8) = \frac{\sqrt{\frac{3}{10}}}{4}$	$T_{-1}(2,5,8,4) = -\frac{1}{\sqrt{30}}$
$T_1(0,8,6,7) = \frac{1}{\sqrt{30}}$	$T_0(1,8,2,5) = \frac{\sqrt{\frac{3}{10}}}{4}$	$T_{-1}(2,8,4,5) = \frac{1}{2\sqrt{30}}$
$T_1(0,8,7,6) = -\frac{1}{2\sqrt{30}}$	$T_0(1,8,5,2) = -\frac{\sqrt{\frac{3}{10}}}{4}$	$T_{-1}(2,8,5,4) = \frac{1}{2\sqrt{30}}$
$T_1(1,2,2,8) = -\frac{1}{4\sqrt{15}}$	$T_0(1,8,6,7) = \frac{\sqrt{\frac{3}{10}}}{4}$	$T_{-1}(4,2,5,8) = -\frac{1}{\sqrt{30}}$
$T_1(1,2,8,2) = \frac{1}{2\sqrt{15}}$	$T_0(1,8,7,6) = -\frac{\sqrt{\frac{3}{10}}}{4}$	$T_{-1}(4,2,8,5) = \frac{1}{2\sqrt{30}}$
$T_1(1,6,6,8) = -\frac{1}{4\sqrt{15}}$	$T_0(2,1,8,5) = -\frac{\sqrt{\frac{3}{10}}}{4}$	$T_{-1}(4,5,2,8) = \frac{1}{2\sqrt{30}}$
$T_1(1,6,8,6) = \frac{1}{2\sqrt{15}}$	$T_0(2,2,4,8) = -\frac{1}{4\sqrt{15}}$	$T_{-1}(4,5,8,2) = \frac{1}{2\sqrt{30}}$
$T_1(1,8,2,2) = -\frac{1}{4\sqrt{15}}$	$T_0(2,2,8,4) = -\frac{1}{4\sqrt{15}}$	$T_{-1}(4,6,7,8) = -\frac{1}{\sqrt{30}}$
$T_1(1,8,6,6) = -\frac{1}{4\sqrt{15}}$	$T_0(2,4,2,8) = \frac{1}{2\sqrt{15}}$	$T_{-1}(4,6,8,7) = \frac{1}{2\sqrt{30}}$
$T_1(2,0,5,8) = -\frac{1}{2\sqrt{30}}$	$T_0(2,4,8,2) = -\frac{1}{4\sqrt{15}}$	$T_{-1}(4,7,6,8) = \frac{1}{2\sqrt{30}}$
$T_1(2,0,8,5) = -\frac{1}{2\sqrt{30}}$	$T_0(2,5,1,8) = \frac{\sqrt{\frac{3}{10}}}{4}$	$T_{-1}(4,7,8,6) = \frac{1}{2\sqrt{30}}$
$T_1(2,1,2,8) = \frac{1}{2\sqrt{15}}$	$T_0(2,5,8,1) = -\frac{\sqrt{\frac{3}{10}}}{4}$	$T_{-1}(4,8,2,5) = \frac{1}{2\sqrt{30}}$
$T_1(2,1,8,2) = -\frac{1}{4\sqrt{15}}$	$T_0(2,8,1,5) = \frac{\sqrt{\frac{3}{10}}}{4}$	$T_{-1}(4,8,5,2) = -\frac{1}{\sqrt{30}}$
$T_1(2,2,1,8) = -\frac{1}{4\sqrt{15}}$	$T_0(2,8,2,4) = \frac{1}{2\sqrt{15}}$	$T_{-1}(4,8,6,7) = \frac{1}{2\sqrt{30}}$
$T_1(2,2,8,1) = -\frac{1}{4\sqrt{15}}$	$T_0(2,8,4,2) = -\frac{1}{4\sqrt{15}}$	$T_{-1}(4,8,7,6) = -\frac{1}{\sqrt{30}}$
$T_1(2,5,0,8) = \frac{1}{\sqrt{30}}$	$T_0(4,2,2,8) = -\frac{1}{4\sqrt{15}}$	$T_{-1}(5,1,5,8) = -\frac{1}{2\sqrt{15}}$
$T_1(2,5,8,0) = -\frac{1}{2\sqrt{30}}$	$T_0(4,2,8,2) = \frac{1}{2\sqrt{15}}$	$T_{-1}(5,1,8,5) = \frac{1}{4\sqrt{15}}$
$T_1(2,8,0,5) = \frac{1}{\sqrt{30}}$	$T_0(4,6,6,8) = -\frac{1}{4\sqrt{15}}$	$T_{-1}(5,2,4,8) = -\frac{1}{\sqrt{30}}$

TABLE XLII. (Continued.)

$S_z = +1$	$S_z = 0$	$S_z = -1$
$T_1(2,8,1,2) = -\frac{1}{4\sqrt{15}}$	$T_0(4,6,8,6) = \frac{1}{2\sqrt{15}}$	$T_{-1}(5,2,8,4) = \frac{1}{2\sqrt{30}}$
$T_1(2,8,2,1) = \frac{1}{2\sqrt{15}}$	$T_0(4,8,2,2) = -\frac{1}{4\sqrt{15}}$	$T_{-1}(5,4,2,8) = \frac{1}{2\sqrt{30}}$
$T_1(2,8,5,0) = -\frac{1}{2\sqrt{30}}$	$T_0(4,8,6,6) = -\frac{1}{4\sqrt{15}}$	$T_{-1}(5,4,8,2) = \frac{1}{2\sqrt{30}}$
$T_1(5,0,2,8) = -\frac{1}{2\sqrt{30}}$	$T_0(5,0,5,8) = -\frac{1}{2\sqrt{15}}$	$T_{-1}(5,5,1,8) = \frac{1}{4\sqrt{15}}$
$T_1(5,0,8,2) = \frac{1}{\sqrt{30}}$	$T_0(5,0,8,5) = \frac{1}{4\sqrt{15}}$	$T_{-1}(5,5,8,1) = \frac{1}{4\sqrt{15}}$
$T_1(5,2,0,8) = -\frac{1}{2\sqrt{30}}$	$T_0(5,1,8,2) = \frac{\sqrt{\frac{3}{10}}}{4}$	$T_{-1}(5,8,1,5) = \frac{1}{4\sqrt{15}}$
$T_1(5,2,8,0) = \frac{1}{\sqrt{30}}$	$T_0(5,2,1,8) = -\frac{\sqrt{\frac{3}{10}}}{4}$	$T_{-1}(5,8,2,4) = \frac{1}{2\sqrt{30}}$
$T_1(5,8,0,2) = -\frac{1}{2\sqrt{30}}$	$T_0(5,2,8,1) = \frac{\sqrt{\frac{3}{10}}}{4}$	$T_{-1}(5,8,4,2) = -\frac{1}{\sqrt{30}}$
$T_1(5,8,2,0) = -\frac{1}{2\sqrt{30}}$	$T_0(5,5,0,8) = \frac{1}{4\sqrt{15}}$	$T_{-1}(5,8,5,1) = -\frac{1}{2\sqrt{15}}$
$T_1(6,0,7,8) = -\frac{1}{2\sqrt{30}}$	$T_0(5,5,8,0) = \frac{1}{4\sqrt{15}}$	$T_{-1}(6,4,7,8) = \frac{1}{2\sqrt{30}}$
$T_1(6,0,8,7) = -\frac{1}{2\sqrt{30}}$	$T_0(5,8,0,5) = \frac{1}{4\sqrt{15}}$	$T_{-1}(6,4,8,7) = -\frac{1}{\sqrt{30}}$
$T_1(6,1,6,8) = \frac{1}{2\sqrt{15}}$	$T_0(5,8,1,2) = -\frac{\sqrt{\frac{3}{10}}}{4}$	$T_{-1}(6,7,4,8) = \frac{1}{2\sqrt{30}}$
$T_1(6,1,8,6) = -\frac{1}{4\sqrt{15}}$	$T_0(5,8,5,0) = -\frac{1}{2\sqrt{15}}$	$T_{-1}(6,7,8,4) = -\frac{1}{\sqrt{30}}$
$T_1(6,6,1,8) = -\frac{1}{4\sqrt{15}}$	$T_0(6,1,8,7) = -\frac{\sqrt{\frac{3}{10}}}{4}$	$T_{-1}(6,8,4,7) = \frac{1}{2\sqrt{30}}$
$T_1(6,6,8,1) = -\frac{1}{4\sqrt{15}}$	$T_0(6,4,6,8) = \frac{1}{2\sqrt{15}}$	$T_{-1}(6,8,7,4) = \frac{1}{2\sqrt{30}}$
$T_1(6,7,0,8) = \frac{1}{\sqrt{30}}$	$T_0(6,4,8,6) = -\frac{1}{4\sqrt{15}}$	$T_{-1}(7,1,7,8) = -\frac{1}{2\sqrt{15}}$
$T_1(6,7,8,0) = -\frac{1}{2\sqrt{30}}$	$T_0(6,6,4,8) = -\frac{1}{4\sqrt{15}}$	$T_{-1}(7,1,8,7) = \frac{1}{4\sqrt{15}}$
$T_1(6,8,0,7) = \frac{1}{\sqrt{30}}$	$T_0(6,6,8,4) = -\frac{1}{4\sqrt{15}}$	$T_{-1}(7,4,6,8) = \frac{1}{2\sqrt{30}}$
$T_1(6,8,1,6) = -\frac{1}{4\sqrt{15}}$	$T_0(6,7,1,8) = \frac{\sqrt{\frac{3}{10}}}{4}$	$T_{-1}(7,4,8,6) = \frac{1}{2\sqrt{30}}$
$T_1(6,8,6,1) = \frac{1}{2\sqrt{15}}$	$T_0(6,7,8,1) = -\frac{\sqrt{\frac{3}{10}}}{4}$	$T_{-1}(7,6,4,8) = -\frac{1}{\sqrt{30}}$
$T_1(6,8,7,0) = -\frac{1}{2\sqrt{30}}$	$T_0(6,8,1,7) = \frac{\sqrt{\frac{3}{10}}}{4}$	$T_{-1}(7,6,8,4) = \frac{1}{2\sqrt{30}}$
$T_1(7,0,6,8) = -\frac{1}{2\sqrt{30}}$	$T_0(6,8,4,6) = -\frac{1}{4\sqrt{15}}$	$T_{-1}(7,7,1,8) = \frac{1}{4\sqrt{15}}$
$T_1(7,0,8,6) = \frac{1}{\sqrt{30}}$	$T_0(6,8,6,4) = \frac{1}{2\sqrt{15}}$	$T_{-1}(7,7,8,1) = \frac{1}{4\sqrt{15}}$
$T_1(7,6,0,8) = -\frac{1}{2\sqrt{30}}$	$T_0(7,0,7,8) = -\frac{1}{2\sqrt{15}}$	$T_{-1}(7,8,1,7) = \frac{1}{4\sqrt{15}}$
$T_1(7,6,8,0) = \frac{1}{\sqrt{30}}$	$T_0(7,0,8,7) = \frac{1}{4\sqrt{15}}$	$T_{-1}(7,8,4,6) = -\frac{1}{\sqrt{30}}$
$T_1(7,8,0,6) = -\frac{1}{2\sqrt{30}}$	$T_0(7,1,8,6) = \frac{\sqrt{\frac{3}{10}}}{4}$	$T_{-1}(7,8,6,4) = \frac{1}{2\sqrt{30}}$
$T_1(7,8,6,0) = -\frac{1}{2\sqrt{30}}$	$T_0(7,6,1,8) = -\frac{\sqrt{\frac{3}{10}}}{4}$	$T_{-1}(7,8,7,1) = -\frac{1}{2\sqrt{15}}$
$T_1(8,0,2,5) = -\frac{1}{2\sqrt{30}}$	$T_0(7,6,8,1) = \frac{\sqrt{\frac{3}{10}}}{4}$	$T_{-1}(8,1,5,5) = \frac{1}{4\sqrt{15}}$
$T_1(8,0,5,2) = \frac{1}{\sqrt{30}}$	$T_0(7,7,0,8) = \frac{1}{4\sqrt{15}}$	$T_{-1}(8,1,7,7) = \frac{1}{4\sqrt{15}}$
$T_1(8,0,6,7) = -\frac{1}{2\sqrt{30}}$	$T_0(7,7,8,0) = \frac{1}{4\sqrt{15}}$	$T_{-1}(8,2,4,5) = \frac{1}{2\sqrt{30}}$
$T_1(8,0,7,6) = \frac{1}{\sqrt{30}}$	$T_0(7,8,0,7) = \frac{1}{4\sqrt{15}}$	$T_{-1}(8,2,5,4) = \frac{1}{2\sqrt{30}}$
$T_1(8,1,2,2) = -\frac{1}{4\sqrt{15}}$	$T_0(7,8,1,6) = -\frac{\sqrt{\frac{3}{10}}}{4}$	$T_{-1}(8,4,2,5) = -\frac{1}{\sqrt{30}}$
$T_1(8,1,6,6) = -\frac{1}{4\sqrt{15}}$	$T_0(7,8,7,0) = -\frac{1}{2\sqrt{15}}$	$T_{-1}(8,4,5,2) = \frac{1}{2\sqrt{30}}$
$T_1(8,2,0,5) = -\frac{1}{2\sqrt{30}}$	$T_0(8,0,5,5) = \frac{1}{4\sqrt{15}}$	$T_{-1}(8,4,6,7) = -\frac{1}{\sqrt{30}}$
$T_1(8,2,1,2) = \frac{1}{2\sqrt{15}}$	$T_0(8,0,7,7) = \frac{1}{4\sqrt{15}}$	$T_{-1}(8,4,7,6) = \frac{1}{2\sqrt{30}}$
$T_1(8,2,2,1) = -\frac{1}{4\sqrt{15}}$	$T_0(8,1,2,5) = -\frac{\sqrt{\frac{3}{10}}}{4}$	$T_{-1}(8,5,1,5) = -\frac{1}{2\sqrt{15}}$
$T_1(8,2,5,0) = \frac{1}{\sqrt{30}}$	$T_0(8,1,5,2) = \frac{\sqrt{\frac{3}{10}}}{4}$	$T_{-1}(8,5,2,4) = -\frac{1}{\sqrt{30}}$
$T_1(8,5,0,2) = -\frac{1}{2\sqrt{30}}$	$T_0(8,1,6,7) = -\frac{\sqrt{\frac{3}{10}}}{4}$	$T_{-1}(8,5,4,2) = \frac{1}{2\sqrt{30}}$
$T_1(8,5,2,0) = -\frac{1}{2\sqrt{30}}$	$T_0(8,1,7,6) = \frac{\sqrt{\frac{3}{10}}}{4}$	$T_{-1}(8,5,5,1) = \frac{1}{4\sqrt{15}}$
$T_1(8,6,0,7) = -\frac{1}{2\sqrt{30}}$	$T_0(8,2,2,4) = -\frac{1}{4\sqrt{15}}$	$T_{-1}(8,6,4,7) = \frac{1}{2\sqrt{30}}$
$T_1(8,6,1,6) = \frac{1}{2\sqrt{15}}$	$T_0(8,2,4,2) = \frac{1}{2\sqrt{15}}$	$T_{-1}(8,6,7,4) = \frac{1}{2\sqrt{30}}$
$T_1(8,6,6,1) = -\frac{1}{4\sqrt{15}}$	$T_0(8,2,5,1) = \frac{\sqrt{\frac{3}{10}}}{4}$	$T_{-1}(8,7,1,7) = -\frac{1}{2\sqrt{15}}$
$T_1(8,6,7,0) = \frac{1}{\sqrt{30}}$	$T_0(8,4,2,2) = -\frac{1}{4\sqrt{15}}$	$T_{-1}(8,7,4,6) = \frac{1}{2\sqrt{30}}$
$T_1(8,7,0,6) = -\frac{1}{2\sqrt{30}}$	$T_0(8,4,6,6) = -\frac{1}{4\sqrt{15}}$	$T_{-1}(8,7,6,4) = -\frac{1}{\sqrt{30}}$

TABLE XLII. (Continued.)

$S_z = +1$	$S_z = 0$	$S_z = -1$
$T_1(8,7,6,0) = -\frac{1}{2\sqrt{30}}$	$T_0(8,5,0,5) = -\frac{1}{2\sqrt{15}}$	$T_{-1}(8,7,7,1) = \frac{1}{4\sqrt{15}}$
	$T_0(8,5,2,1) = -\frac{\sqrt{\frac{3}{10}}}{4}$	
	$T_0(8,5,5,0) = \frac{1}{4\sqrt{15}}$	
	$T_0(8,6,4,6) = \frac{1}{2\sqrt{15}}$	
	$T_0(8,6,6,4) = -\frac{1}{4\sqrt{15}}$	
	$T_0(8,6,7,1) = \frac{\sqrt{\frac{3}{10}}}{4}$	
	$T_0(8,7,0,7) = -\frac{1}{2\sqrt{15}}$	
	$T_0(8,7,6,1) = -\frac{\sqrt{\frac{3}{10}}}{4}$	
	$T_0(8,7,7,0) = \frac{1}{4\sqrt{15}}$	

TABLE XLIII. G_{10} .

$S_z = +1$	$S_z = 0$	$S_z = -1$
$T_1(0,2,5,8) = \frac{1}{2\sqrt{10}}$	$T_0(0,5,5,8) = \frac{1}{4\sqrt{5}}$	$T_{-1}(1,5,5,8) = \frac{1}{4\sqrt{5}}$
$T_1(0,2,8,5) = -\frac{1}{2\sqrt{10}}$	$T_0(0,7,7,8) = \frac{1}{4\sqrt{5}}$	$T_{-1}(1,7,7,8) = \frac{1}{4\sqrt{5}}$
$T_1(0,5,8,2) = \frac{1}{2\sqrt{10}}$	$T_0(0,8,5,5) = -\frac{1}{4\sqrt{5}}$	$T_{-1}(1,8,5,5) = -\frac{1}{4\sqrt{5}}$
$T_1(0,6,7,8) = \frac{1}{2\sqrt{10}}$	$T_0(0,8,7,7) = -\frac{1}{4\sqrt{5}}$	$T_{-1}(1,8,7,7) = -\frac{1}{4\sqrt{5}}$
$T_1(0,6,8,7) = -\frac{1}{2\sqrt{10}}$	$T_0(1,2,5,8) = \frac{1}{4\sqrt{10}}$	$T_{-1}(2,4,5,8) = \frac{1}{2\sqrt{10}}$
$T_1(0,7,8,6) = \frac{1}{2\sqrt{10}}$	$T_0(1,2,8,5) = -\frac{1}{2\sqrt{10}}$	$T_{-1}(2,5,4,8) = \frac{1}{2\sqrt{10}}$
$T_1(0,8,5,2) = -\frac{1}{2\sqrt{10}}$	$T_0(1,5,2,8) = -\frac{1}{4\sqrt{10}}$	$T_{-1}(2,8,4,5) = -\frac{1}{2\sqrt{10}}$
$T_1(0,8,7,6) = -\frac{1}{2\sqrt{10}}$	$T_0(1,5,8,2) = \frac{1}{2\sqrt{10}}$	$T_{-1}(2,8,5,4) = -\frac{1}{2\sqrt{10}}$
$T_1(1,2,2,8) = -\frac{1}{4\sqrt{5}}$	$T_0(1,6,7,8) = \frac{1}{4\sqrt{10}}$	$T_{-1}(4,2,8,5) = -\frac{1}{2\sqrt{10}}$
$T_1(1,6,6,8) = -\frac{1}{4\sqrt{5}}$	$T_0(1,6,8,7) = -\frac{1}{2\sqrt{10}}$	$T_{-1}(4,5,2,8) = -\frac{1}{2\sqrt{10}}$
$T_1(1,8,2,2) = \frac{1}{4\sqrt{5}}$	$T_0(1,7,6,8) = -\frac{1}{4\sqrt{10}}$	$T_{-1}(4,5,8,2) = \frac{1}{2\sqrt{10}}$
$T_1(1,8,6,6) = \frac{1}{4\sqrt{5}}$	$T_0(1,7,8,6) = \frac{1}{2\sqrt{10}}$	$T_{-1}(4,6,8,7) = -\frac{1}{2\sqrt{10}}$
$T_1(2,0,5,8) = \frac{1}{2\sqrt{10}}$	$T_0(1,8,2,5) = \frac{1}{4\sqrt{10}}$	$T_{-1}(4,7,6,8) = -\frac{1}{2\sqrt{10}}$
$T_1(2,0,8,5) = -\frac{1}{2\sqrt{10}}$	$T_0(1,8,5,2) = -\frac{1}{4\sqrt{10}}$	$T_{-1}(4,7,8,6) = \frac{1}{2\sqrt{10}}$
$T_1(2,1,8,2) = \frac{1}{4\sqrt{5}}$	$T_0(1,8,6,7) = \frac{1}{4\sqrt{10}}$	$T_{-1}(4,8,2,5) = \frac{1}{2\sqrt{10}}$
$T_1(2,2,1,8) = \frac{1}{4\sqrt{5}}$	$T_0(1,8,7,6) = -\frac{1}{4\sqrt{10}}$	$T_{-1}(4,8,6,7) = \frac{1}{2\sqrt{10}}$
$T_1(2,2,8,1) = -\frac{1}{4\sqrt{5}}$	$T_0(2,1,5,8) = \frac{1}{2\sqrt{10}}$	$T_{-1}(5,1,8,5) = -\frac{1}{4\sqrt{5}}$
$T_1(2,5,8,0) = \frac{1}{2\sqrt{10}}$	$T_0(2,1,8,5) = -\frac{1}{4\sqrt{10}}$	$T_{-1}(5,2,8,4) = -\frac{1}{2\sqrt{10}}$
$T_1(2,8,1,2) = -\frac{1}{4\sqrt{5}}$	$T_0(2,2,4,8) = \frac{1}{4\sqrt{5}}$	$T_{-1}(5,4,2,8) = -\frac{1}{2\sqrt{10}}$
$T_1(2,8,5,0) = -\frac{1}{2\sqrt{10}}$	$T_0(2,2,8,4) = -\frac{1}{4\sqrt{5}}$	$T_{-1}(5,4,8,2) = \frac{1}{2\sqrt{10}}$
$T_1(5,0,2,8) = -\frac{1}{2\sqrt{10}}$	$T_0(2,4,8,2) = \frac{1}{4\sqrt{5}}$	$T_{-1}(5,5,1,8) = -\frac{1}{4\sqrt{5}}$
$T_1(5,2,0,8) = -\frac{1}{2\sqrt{10}}$	$T_0(2,5,1,8) = \frac{1}{4\sqrt{10}}$	$T_{-1}(5,5,8,1) = \frac{1}{4\sqrt{5}}$
$T_1(5,8,0,2) = \frac{1}{2\sqrt{10}}$	$T_0(2,5,8,1) = \frac{1}{4\sqrt{10}}$	$T_{-1}(5,8,1,5) = \frac{1}{4\sqrt{5}}$
$T_1(5,8,2,0) = \frac{1}{2\sqrt{10}}$	$T_0(2,8,1,5) = -\frac{1}{4\sqrt{10}}$	$T_{-1}(5,8,2,4) = \frac{1}{2\sqrt{10}}$
$T_1(6,0,7,8) = \frac{1}{2\sqrt{10}}$	$T_0(2,8,4,2) = -\frac{1}{4\sqrt{5}}$	$T_{-1}(6,4,7,8) = \frac{1}{2\sqrt{10}}$
$T_1(6,0,8,7) = -\frac{1}{2\sqrt{10}}$	$T_0(2,8,5,1) = -\frac{1}{2\sqrt{10}}$	$T_{-1}(6,7,4,8) = \frac{1}{2\sqrt{10}}$
$T_1(6,1,8,6) = \frac{1}{4\sqrt{5}}$	$T_0(4,2,2,8) = -\frac{1}{4\sqrt{5}}$	$T_{-1}(6,8,4,7) = -\frac{1}{2\sqrt{10}}$
$T_1(6,6,1,8) = \frac{1}{4\sqrt{5}}$	$T_0(4,6,6,8) = -\frac{1}{4\sqrt{5}}$	$T_{-1}(6,8,7,4) = -\frac{1}{2\sqrt{10}}$
$T_1(6,6,8,1) = -\frac{1}{4\sqrt{5}}$	$T_0(4,8,2,2) = \frac{1}{4\sqrt{5}}$	$T_{-1}(7,1,8,7) = -\frac{1}{4\sqrt{5}}$
$T_1(6,7,8,0) = \frac{1}{2\sqrt{10}}$	$T_0(4,8,6,6) = \frac{1}{4\sqrt{5}}$	$T_{-1}(7,4,6,8) = -\frac{1}{2\sqrt{10}}$
$T_1(6,8,1,6) = -\frac{1}{4\sqrt{5}}$	$T_0(5,0,8,5) = -\frac{1}{4\sqrt{5}}$	$T_{-1}(7,4,8,6) = \frac{1}{2\sqrt{10}}$
$T_1(6,8,7,0) = -\frac{1}{2\sqrt{10}}$	$T_0(5,1,2,8) = -\frac{1}{2\sqrt{10}}$	$T_{-1}(7,6,8,4) = -\frac{1}{2\sqrt{10}}$

TABLE XLIII. (Continued.)

$S_z = +1$	$S_z = 0$	$S_z = -1$
$T_1(7,0,6,8) = -\frac{1}{2\sqrt{10}}$	$T_0(5,1,8,2) = \frac{1}{4\sqrt{10}}$	$T_{-1}(7,7,1,8) = -\frac{1}{4\sqrt{5}}$
$T_1(7,6,0,8) = -\frac{1}{2\sqrt{10}}$	$T_0(5,2,1,8) = -\frac{1}{4\sqrt{10}}$	$T_{-1}(7,7,8,1) = \frac{1}{4\sqrt{5}}$
$T_1(7,8,0,6) = \frac{1}{2\sqrt{10}}$	$T_0(5,2,8,1) = -\frac{1}{4\sqrt{10}}$	$T_{-1}(7,8,1,7) = \frac{1}{4\sqrt{5}}$
$T_1(7,8,6,0) = \frac{1}{2\sqrt{10}}$	$T_0(5,5,0,8) = -\frac{1}{4\sqrt{5}}$	$T_{-1}(7,8,6,4) = \frac{1}{4\sqrt{5}}$
$T_1(8,0,2,5) = \frac{1}{2\sqrt{10}}$	$T_0(5,5,8,0) = \frac{1}{4\sqrt{5}}$	$T_{-1}(8,1,5,5) = \frac{1}{4\sqrt{5}}$
$T_1(8,0,6,7) = \frac{1}{2\sqrt{10}}$	$T_0(5,8,0,5) = \frac{1}{4\sqrt{5}}$	$T_{-1}(8,1,7,7) = \frac{1}{4\sqrt{5}}$
$T_1(8,1,2,2) = -\frac{1}{4\sqrt{5}}$	$T_0(5,8,1,2) = \frac{1}{4\sqrt{10}}$	$T_{-1}(8,2,4,5) = \frac{1}{2\sqrt{10}}$
$T_1(8,1,6,6) = -\frac{1}{4\sqrt{5}}$	$T_0(5,8,2,1) = \frac{1}{2\sqrt{10}}$	$T_{-1}(8,2,5,4) = \frac{1}{2\sqrt{10}}$
$T_1(8,2,0,5) = \frac{1}{2\sqrt{10}}$	$T_0(6,1,7,8) = \frac{1}{2\sqrt{10}}$	$T_{-1}(8,4,5,2) = -\frac{1}{2\sqrt{10}}$
$T_1(8,2,2,1) = \frac{1}{4\sqrt{5}}$	$T_0(6,1,8,7) = -\frac{1}{4\sqrt{10}}$	$T_{-1}(8,4,7,6) = -\frac{1}{2\sqrt{10}}$
$T_1(8,5,0,2) = -\frac{1}{2\sqrt{10}}$	$T_0(6,4,8,6) = \frac{1}{4\sqrt{5}}$	$T_{-1}(8,5,4,2) = -\frac{1}{2\sqrt{10}}$
$T_1(8,5,2,0) = -\frac{1}{2\sqrt{10}}$	$T_0(6,6,4,8) = \frac{1}{4\sqrt{5}}$	$T_{-1}(8,5,5,1) = -\frac{1}{4\sqrt{5}}$
$T_1(8,6,0,7) = \frac{1}{2\sqrt{10}}$	$T_0(6,6,8,4) = -\frac{1}{4\sqrt{5}}$	$T_{-1}(8,6,4,7) = \frac{1}{2\sqrt{10}}$
$T_1(8,6,6,1) = \frac{1}{4\sqrt{5}}$	$T_0(6,7,1,8) = \frac{1}{4\sqrt{10}}$	$T_{-1}(8,6,7,4) = \frac{1}{2\sqrt{10}}$
$T_1(8,7,0,6) = -\frac{1}{2\sqrt{10}}$	$T_0(6,7,8,1) = \frac{1}{4\sqrt{10}}$	$T_{-1}(8,7,4,6) = -\frac{1}{2\sqrt{10}}$
$T_1(8,7,6,0) = -\frac{1}{2\sqrt{10}}$	$T_0(6,8,1,7) = -\frac{1}{4\sqrt{10}}$	$T_{-1}(8,7,7,1) = -\frac{1}{4\sqrt{5}}$
	$T_0(6,8,4,6) = -\frac{1}{4\sqrt{5}}$	
	$T_0(6,8,7,1) = -\frac{1}{2\sqrt{10}}$	
	$T_0(7,0,8,7) = -\frac{1}{4\sqrt{5}}$	
	$T_0(7,1,6,8) = -\frac{1}{2\sqrt{10}}$	
	$T_0(7,1,8,6) = \frac{1}{4\sqrt{10}}$	
	$T_0(7,6,1,8) = -\frac{1}{4\sqrt{10}}$	
	$T_0(7,6,8,1) = -\frac{1}{4\sqrt{10}}$	
	$T_0(7,7,0,8) = -\frac{1}{4\sqrt{5}}$	
	$T_0(7,7,8,0) = \frac{1}{4\sqrt{5}}$	
	$T_0(7,8,0,7) = \frac{1}{4\sqrt{5}}$	
	$T_0(7,8,1,6) = \frac{1}{4\sqrt{10}}$	
	$T_0(7,8,6,1) = \frac{1}{2\sqrt{10}}$	
	$T_0(8,0,5,5) = \frac{1}{4\sqrt{5}}$	
	$T_0(8,0,7,7) = \frac{1}{4\sqrt{5}}$	
	$T_0(8,1,2,5) = \frac{1}{4\sqrt{10}}$	
	$T_0(8,1,5,2) = -\frac{1}{4\sqrt{10}}$	
	$T_0(8,1,6,7) = \frac{1}{4\sqrt{10}}$	
	$T_0(8,1,7,6) = -\frac{1}{4\sqrt{10}}$	
	$T_0(8,2,1,5) = \frac{1}{2\sqrt{10}}$	
	$T_0(8,2,2,4) = \frac{1}{4\sqrt{5}}$	
	$T_0(8,2,5,1) = \frac{1}{4\sqrt{10}}$	
	$T_0(8,4,2,2) = -\frac{1}{4\sqrt{5}}$	
	$T_0(8,4,6,6) = -\frac{1}{4\sqrt{5}}$	
	$T_0(8,5,1,2) = -\frac{1}{2\sqrt{10}}$	
	$T_0(8,5,2,1) = -\frac{1}{4\sqrt{10}}$	
	$T_0(8,5,5,0) = -\frac{1}{4\sqrt{5}}$	
	$T_0(8,6,1,7) = \frac{1}{2\sqrt{10}}$	
	$T_0(8,6,6,4) = \frac{1}{4\sqrt{5}}$	
	$T_0(8,6,7,1) = \frac{1}{4\sqrt{10}}$	
	$T_0(8,7,1,6) = -\frac{1}{2\sqrt{10}}$	
	$T_0(8,7,6,1) = -\frac{1}{4\sqrt{10}}$	
	$T_0(8,7,7,0) = -\frac{1}{4\sqrt{5}}$	

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