Radiation losses and dark mode for spin-wave propagation through a discrete magnetic micro-waveguide

Yuri Barabanenkov,[1,*] Sergey Osokin,[1,2,†] Dmitry Kalyabin,[1,2,‡] and Sergey Nikitov[1,2,3,§]
[1]*Kotelnikov Institute of Radio-engineering and Electronics of RAS, 11-7 Mokhovaya st., Moscow, 125009, Russia*
[2]*Moscow Institute of Physics and Technology, 9 Instituskij per., Dolgoprudny, 141700, Moscow Region, Russia*
[3]*Laboratory "Metamaterials", Saratov State University, 112 Bol'shaya Kazach'ya, Saratov, 410012, Russia*
(Received 13 August 2016; published 9 November 2016)

This paper presents the quantum mechanical type T-scattering operator approach to studying the forward volume magnetostatic spin-wave multiple scattering by a finite ensemble of cylindrical magnetic inclusions in a ferromagnetic thin film. The approach is applied to the problem of spin-wave excitation transfer along a linear chain of inclusions. The substantial results are deriving the optical theorem for the T-scattering operator and, as a consequence, deriving a formula for collective extinction cross section of inclusion ensemble, where only the first inclusion of the chain is irradiated by an incident narrow spin-wave beam. From this formula it can be shown that only irradiated inclusion makes a direct contribution in the collective extinction cross section of the total number of inclusions. In this case the direct summarized contribution of all the other inclusions from the chain into the spin-wave scattering is invisible; we call such phenomenon the dark mode. Applying a one-multipole and closest neighbor coupling approximation, we reveal a regime of distant resonant transfer for spin-wave excitation along the linear chain of an essentially big but finite number of particles with the dark mode. Because we also found a resonant mechanism of filtering this mode from radiation losses, the revealed regime shows that at resonant conditions the linear chain of magnetic inclusions can play the role of a spin-wave micro-waveguide, which transfers a signal over a big distance in a form of the dark mode, where the controllable level of radiation losses can tend to reach nearly zero values.

DOI: 10.1103/PhysRevB.94.184409

I. INTRODUCTION

Investigations of various magnetic micro- and nanostructures as potential candidates for spintronics and magnetic logic devices have became a hot topic recently [1–7]. In particular, intensive studies of various magnetic structures are performed in order to understand properties of perspective materials for above mentioned applications. This, in turn, requires the study of important physical phenomena related to spin-wave dynamics in magnetic materials and especially in micro- and nanostructured magnetic films. Spin-wave dynamics is very often related to properties of spin-waves propagation in confined magnetic structures or in arrays of magnetic dots, stripes, etc. [8–11]. There are various types of magnetic periodic structures, referred to as magnonic crystals (MC) [12,13] which are suitable for the investigation of propagating in the spin waves with the idea of an information processing and logics. These MC can be arrays of holes (antidots) etched in yttrium-iron-garnet (YIG) films [14,15], dynamic MC [16], and other patterned films [17–19]. The main manifestation of a magnetic film patterning in a spin-wave spectrum is the appearance of wave band structure within the wave spectrum. The properties of the spin-wave band structure can be very complicated and can be governed by variations of external parameters, e.g., an external magnetic field and particularly by a metallization of the structure [20,21]. However, in many of the cited works investigations of interaction of waves with single inhomogeneity in periodic structures are usually left behind consideration. Basically, only the collective influence of magnetic structure at the propagating wave properties is taken into account. The problem of spin-wave scattering by an infinite set of magnetic or nonmagnetic inclusions (cylindrical pillars) embedded in a ferromagnetic thin film (matrix) was considered recently [22]. It was shown that under certain conditions spin-wave edge modes are excited around these inclusions that have nonreciprocal character of propagation with respect to external magnetic field saturating the ferromagnetic matrix and inclusions. Furthermore, investigations of spin-wave edge modes became very popular topic due to the prediction of their existence in various magnetic nanostructures such as ferromagnetic islands and/or circular magnetic thin rings or circular disks, or semi-infinite arrays of dipole coupled magnetic nanopillars [23–27]. Most probably this interest exists due to the analogy with the existence of electrical current edge states in systems with quantum Hall effect [2]. As it has been mentioned, investigations of a spin-wave propagation in MC or other periodic magnetic structures were performed when these structures were considered as an infinite set of periodic perturbations located along the spin-wave propagation path. On the other hand, it is interesting to study and important to understand how spin waves are scattered by a finite array of perturbations located in a ferromagnetic matrix along the propagation path. In particular, such study has been performed for electromagnetic waves within the visible frequency range [28]. Due to the fact that low-dimensional clusters of microsized nonresonating dielectric particles can possess a specific space group resonance [29,30], an ordered array of similar resonant particles can support bound modes with an extremely high quality factor. In our recent paper [31] a general theory was developed of forward volume

*barab624@mail.ru
†OsokinSerg@gmail.com
‡dmitry.kalyabin@phystech.edu
§nikitov@cplire.ru

magnetostatic spin-wave (FVMSWs) multiple scattering by a finite two-dimensional (2D) ensemble of cylindrical magnetic inclusions in a ferromagnetic matrix metallized from both sides. As it turned out, such a finite number of magnetic inclusions arranged periodically along a circle can have spin-wave eigenmodes and can perform as a specific micro spin-wave resonator with a high value of the quality factor. This micro resonator can be considered as an element of a magnonic circuit device. Another element of a magnonic circuit device, which can play the role of a micro spin-wave waveguide, is the linear finite array (chain) of magnetic inclusions embedded into the ferromagnetic matrix. The physical properties of this specific micro spin-wave waveguide is the subject of our current paper.

The first problem in the study of the subject is to show that spin-wave excitation can be transferred along a linear chain over a big distance. The second one is the need to show that the model of discrete waveguide for spin waves can give the possibility to transfer information with as low losses as possible. The first physical problem arising in the study of the spin-wave excitation transfer along a linear chain of magnetic inclusions is solved in the current paper easily. In doing so we apply to the set of self-consistent equations [31] for spin-wave multiple scattering partial amplitudes an iterative method, using as a starting point one-multipole scattering and closest neighbor wave interaction approach. In this way we obtain physically transparent analytical expressions for scattering partial amplitudes of the spin-wave excitation transfer along a linear chain of inclusions, similar to those noted by Rayleigh [32] in his discrete model of string transverse oscillations and known in the theory [33] of electrical filter circuits as combinations of capacitors, inductors, and resistors. With the aid of these analytical expressions in the current paper we show that the spin-wave excitation can have resonant transfer over a big distance along a linear chain with the big number of magnetic inclusions. The second physical problem is closely connected with the productive analytical method evaluating the total amount of spin-wave radiation scattered by inclusions. We solve this problem with an idea borrowed from optics [34] about an extinction cross section characterizing the incident spin-wave energy loss due to scattering and possible absorption by the linear chain inclusions. We use this idea in the study of the forward volume magnetostatic spin-wave multiple scattering by magnetic inclusions embedded inside the ferromagnetic matrix metallized from both sides. We solve the magnetostatic Walker equation [35], that was used in our recent paper [31], by applying the quantum mechanical type T-scattering operator approach, following the quantum mechanical case [36–38]. Our basic result is proving the optical theorem for the T-scattering operator, which describes the spin-wave multiple scattering by magnetic inclusions, and deriving a formula for collective extinction cross section of spin waves by a linear chain of inclusions. The derived formula appears to be especially productive in the case when the incident spin-wave beam irradiates only the first inclusion of the chain. In this particular case the formula shows that only directly irradiated inclusion makes a contribution in collective extinction cross section despite the fact that the total number of inclusions can be big; that makes the summarized contribution of all other inclusions in the spin-wave scattering to be invisible (so called the dark mode). It is valuable to note that we find a resonant mechanism of filtering this mode from radiation losses, transforming thereby the finite linear chain of magnetic inclusions into the waveguide for spin waves, where a controllable level of radiation losses can reach nearly zero values.

The paper consists of two main parts, Secs. II and III. Section II is devoted to general theory of the T-scattering operator method for FVMSWs and includes eight subsections. In subsections A and B Hermiticity property of Walker equation is verified and the T-scattering operator for this equation is introduced. In addition to subsection B the optical theorem for T-scattering operator is proven. In subsection C we transit from a three-dimensional (3D) problem to a two-dimensional (2D) problem for magnetostatic potentials of spin waves by using the expansion along transversal eigenmodes of homogeneous matrix. The extinction and scattering cross sections are also defined in this subsection. In subsection D the defined extinction and scattering cross sections are presented in terms of spin-wave scattering amplitude similar to optics. The spin-wave scattering amplitude is written in terms of dynamic magnetization displacement current excited inside an inclusion by spin-wave scattering. In subsection E an analogy to scattering Watson composition rule for the T-scattering operators of particles is formulated in the case of magnetic inclusion ensemble. Subsection F includes most principal theoretical results concerning extinction cross section for the case of incident narrow spin-wave beam. Namely in this subsection the formula for collective extinction cross section is obtained, which leads to a notion of specific spin-wave dark mode. In subsection G we introduce the self-consistent spin-wave partial scattering amplitudes and find their connection with dynamic magnetization displacement currents excited inside inclusions by spin-wave scattering. Section III aims to apply the general results of the preceding Section II to study spin-wave excitation distant transfer with a dark mode along a linear chain of magnetic inclusions and includes three subsections. In subsection III A the closest neighbor interaction approximation is applied to obtain the analytical solution of the set of equations for partial multiple scattering amplitudes describing spin-wave excitation transfer along a linear chain of magnetic inclusions. Subsection III B analyzes resonant values of inclusions coupling parameter which are most interesting from the point of view of distant spin-wave excitation transfer. Subsection III C translates the resonant values of coupling parameter found in the preceding subsection on spin-wave frequencies using Landay-Lifshitz dispersion equations for tensor magnetic susceptibilities of ferromagnetic matrix and magnetic inclusions. Section IV concludes the paper. Appendix A consists of some details with respect to incident narrow spin-wave beam.

II. BASIC PROPERTIES OF T-SCATTERING OPERATOR FOR FVMSW

A. Hermiticity of Walker equation for FVMSW inside ferromagnetic matrix with inclusions

We consider a problem of FVMSW propagation in the ferromagnetic thin film (matrix) containing the finite 2D array

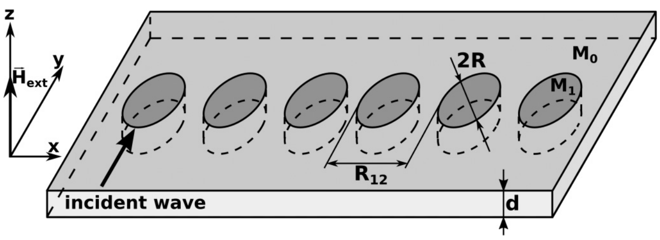

FIG. 1. The ferromagnetic film with linear chain of cylindrical ferromagnetic inclusions.

of magnetic nonintersected cylindrical inclusions (Fig. 1), considering the film's and inclusions' surfaces $z = 0$ and $z = d$ to be metallized. We start as in Ref. [31] with the Maxwell equations in the magnetostatic approximation $\nabla\vec{b} = 0$ and $\nabla \times \vec{h} = 0$ for magnetic induction $\vec{b}$ and magnetic field $\vec{h}$ vectors for the spin wave. The magnetic field can be written in terms of magnetostatic potential Ψ according to $\vec{h} = -\nabla\Psi$.

The magnetic induction vector is defined by the relation $\vec{b} = \overleftrightarrow{\mu}\vec{h}$ where the antisymmetric tensor (dyadic) $\overleftrightarrow{\mu}$ of magnetic susceptibility [35] has a form

$$\overleftrightarrow{\mu} = \begin{bmatrix} \mu_0 & i\mu_a & 0 \\ -i\mu_a & \mu_0 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \qquad (1)$$

The components μ_0 and μ_a in Eq. (1) are evaluated according to the Landay-Lifshitz theory and defined by spin-wave angular frequency ω, the external uniform magnetic field H_{ext} applied along the z axis to the matrix with different values of saturation magnetizations M_s^0 and M_s^1 inside the homogeneous part of the film and inclusions, respectively. Ignoring the thermal dissipations [35], we can consider mentioned diagonal and off diagonal components of magnetic susceptibility dyadic to be real value functions of spatial variables x,y,z inside the volume of the film $0 < z < d$ so that the dyadic of magnetic susceptibility in Eq. (1) is a Hermitian matrix that is invariable to conjugation $\overleftrightarrow{\mu}^+ = \overleftrightarrow{\mu}$. Substitution of the dyadic Eq. (1) in the equation for magnetic induction gives in magnetostatic approximation the Walker equation for our problem

$$L\Psi = \partial_x(\mu_0\partial_x\Psi) + \partial_y(\mu_0\partial_y\Psi) + i[\partial_x(\mu_a\partial_y\Psi) - \partial_y(\mu_a\partial_x\Psi)] + \partial_z^2\Psi = 0. \qquad (2)$$

One can verify that the Walker operator L defined by Eq. (2) is Hermitian operator $L^+ = L$ on functions $\Psi(x,y,z)$ defined inside the matrix with boundary conditions $\partial_z\Psi(x,y,z)|_{z=0,d} = 0$ and a scalar product

$$(\Psi_1,\Psi_2) = \int\int dxdy \int_0^d dz\Psi_1^*(x,y,z)\Psi_2(x,y,z). \qquad (3)$$

Next we suppose that the magnetic susceptibility dyadic $\overleftrightarrow{\mu}(x,y)$ has values $\overleftrightarrow{\mu}^0$ and $\overleftrightarrow{\mu}^1$ inside the matrix and inclusions, respectively, denoting $\delta\overleftrightarrow{\mu}(x,y) = \overleftrightarrow{\mu}(x,y) - \overleftrightarrow{\mu}^0$ the deviation of ferromagnetic matrix magnetic susceptibility with inclusions from magnetic susceptibility of the homogeneous matrix. The Walker operator we write as sum $L = L_0 + L_1$ of unperturbed operator $L_0 = L|_{\overleftrightarrow{\mu}\to\overleftrightarrow{\mu}^0}$ and perturbation $L_1 = L|_{\overleftrightarrow{\mu}\to\delta\overleftrightarrow{\mu}}$. The unperturbed operator describes the magnetostatic spin-wave propagation inside the homogeneous matrix, and perturbation takes into account the spin-wave scattering by inclusions. We write $(1/\mu_0^0)L_1\Psi = -U(x,y)\Psi$ and call $U(x,y)$ the magnetostatic scattering operator of spin waves. This operator is Hermitian $U^+ = U$. Now the Walker Eq. (2) takes a form

$$\left(\partial_x^2 + \partial_y^2\right)\Psi + \frac{1}{\mu_0^0}\partial_z^2\Psi - U\Psi = j(x,y,z), \qquad (4)$$

where a source term $j(x,y,z)$ will be introduced additionally. Note that both the scattering operator and the magnetic susceptibility deviation $\delta\overleftrightarrow{\mu}(x,y)$ do have zero values outside the inclusions' volume.

B. *T*-scattering operator and optical theorem

We denote the Green function $G_0(\vec{r},\vec{r}\,')$, where the vector $\vec{r} = (x,y,z)$ is a point in 3D coordinate space. The Green function for unperturbed Walker Eq. (4) satisfies the equation

$$\left(\partial_x^2 + \partial_y^2 + \frac{1}{\mu_0^0}\partial_z^2\right)G_0(\vec{r},\vec{r}\,') = \delta(\vec{r} - \vec{r}\,'),$$
$$\partial_z G_0(\vec{r},\vec{r}\,')|_{z=0,d} = 0. \qquad (5)$$

The solution to differential Walker Eq. (4) is reduced to integral equation

$$\Psi(\vec{r}) = \Psi_0(\vec{r}) + \int G_0(\vec{r},\vec{r}\,')U(\vec{r}\,')\Psi(\vec{r}\,')d\vec{r}\,', \qquad (6)$$

where the inhomogeneous term on the right hand side (RHS) is the magnetic potential of the incident spin wave

$$\Psi_0(\vec{r}) = \int G_0(\vec{r},\vec{r}\,')j(\vec{r}\,')d\vec{r}\,'. \qquad (7)$$

The T-scattering operator $T(\vec{r},\vec{r}\,')$ for magnetostatic spin waves is introduced by writing the solution to integral Eq. (6) in the form

$$\Psi(\vec{r}) = \Psi_0(\vec{r}) + \int G_0(\vec{r},\vec{r}\,')T(\vec{r}\,',\vec{r}\,'')\Psi_0(\vec{r}\,'')d\vec{r}\,'' \qquad (8)$$

and satisfies the integral Lippmann-Schwinger (LS) equation

$$T(\vec{r},\vec{r}\,') = U(\vec{r})\delta(\vec{r} - \vec{r}\,') + U(\vec{r})\int G_0(\vec{r},\vec{r}\,'')T(\vec{r}\,'',\vec{r}\,')d\vec{r}\,''. \qquad (9)$$

The operator $T(\vec{r},\vec{r}\,')$ is dependent on both its arguments and has nonzero values only inside the inclusions.

On the base of Eqs. (6) and (8) it is useful to introduce a quantity

$$P(\vec{r}) = U(\vec{r})\Psi(\vec{r}) = \int T(\vec{r},\vec{r}\,')\Psi(\vec{r}\,')d\vec{r}\,' \qquad (10)$$

that has a physical meaning of dynamic magnetization displacement current excited inside inclusions by spin-wave scattering. In terms of this current the magnetic field potential scattered by inclusions $\Psi_{sc}(\vec{r})$ is written from Eqs. (6) and (8) as

$$\Psi_{sc}(\vec{r}) = \int G_0(\vec{r},\vec{r}\,')P(\vec{r}\,')d\vec{r}\,'. \qquad (11)$$

Because of the scattering potential U hermiticity one can derive from LS Eq. (9) the optical theorem for T-scattering operator in the form

$$T(\vec{r},\vec{r}') - T^*(\vec{r}',\vec{r}) = \int d\vec{r}'' \int [G_0(\vec{r}'',\vec{r}''') - G_0^*(\vec{r}''',\vec{r}'')] \times T^*(\vec{r}'',\vec{r})T(\vec{r}''',\vec{r}')d\vec{r}'''. \quad (12)$$

This is a fundamental result of the current paper.

C. Expansion along matrix transversal eigenmodes

Now we consider vector $\vec{r}$ in cylindrical coordinates $(\vec{\rho},z)$, by translating 2D Cartesian coordinates (x,y) into polar coordinates (ρ,ϕ). With this transformation we introduce a complete and orthogonal set $\Gamma_n(z), n = 0,1,2,\dots$ of homogeneous film (matrix) transversal eigenmodes defined by

$$\Gamma_n(z) = b_n \cos\frac{\pi n z}{d}, \quad b_0 = \frac{1}{\sqrt{d}}, b_n = \sqrt{\frac{2}{d}}, n = 1,2,\dots \quad (13)$$

One can verify that all quantities in Eqs. (5)–(12) can be easily expanded along the set of transversal eigenmodes of Eq. (13). For example the 3D unperturbed Green function $G_0(\vec{r},\vec{r}')$ satisfying the differential Eq. (5) can be expanded as

$$G_0(\vec{r},\vec{r}') = \sum_{n=0}^{\infty} G_n^{(0)}(\vec{\rho}-\vec{\rho}')\Gamma_n(z)\Gamma_n(z'), \quad (14)$$

where 2D unperturbed Green function $G_n^{(0)}(\vec{\rho})$ is evaluated directly in the form

$$G_n^{(0)}(\vec{\rho}) = \frac{1}{4i} H_0^{(1)}\left(k_{rn}^0 \rho\right). \quad (15)$$

On the RHS of this equation $H_m^{(1)}(u)$ denotes the Hankel function of the first kind and order $m = 0$, and $k_{rn}^0 = \sqrt{-1/\mu_0^0}(\pi n/d)$ is a component of the spin-wave wave vector along the (x,y) plane inside the homogeneous matrix for the nth transversal mode. Equation (11) for the magnetic field potential $\Psi_{sc}(\vec{r})$ scattered by inclusions in terms of 2D quantities takes the form

$$\Psi_{scn}(\vec{\rho}) = \int G_n^{(0)}(\vec{\rho}-\vec{\rho}')P_n(\vec{\rho}')d\vec{\rho}', \quad (16)$$

where a 2D current $P_n(\vec{\rho})$ is obtained by transformation of the second equation from Eqs. (10) and is written as

$$P_n(\vec{\rho}) = \int T_n(\vec{\rho},\vec{\rho}')\Psi_{0n}(\vec{\rho}')d\vec{\rho}'. \quad (17)$$

On the RHS of Eq. (17) $T_n(\vec{\rho},\vec{\rho}')$ denotes the 2D scattering operator which is connected with the 3D scattering operator $T(\vec{r},\vec{r}')$ by transformation similar to one in Eq. (14). The quantity $\Psi_{0n}(\vec{\rho})$ appears in expansion of the magnetic potential of the incident spin wave in Eq. (7) along transversal eigenmodes.

Applying this expansion along transversal eigenmodes to the optical theorem Eq. (12) gives

$$\frac{1}{2i}[T_n(\vec{\rho},\vec{\rho}') - T_n^*(\vec{\rho}',\vec{\rho})] = -\frac{1}{8\pi}\int_{2\pi} T_n^*\left(k_{rn}^0\vec{s},\vec{\rho}\right)T_n^*\left(k_{rn}^0\vec{s},\vec{\rho}'\right)d\vec{s}. \quad (18)$$

On the RHS of this equation for 2D scattering operator $T_n(\vec{\rho},\vec{\rho}')$ a 2D Fourier transform is used

$$T_n(\vec{k},\vec{\rho}') = \int e^{-i\vec{k}\vec{\rho}} T_n(\vec{\rho},\vec{\rho}')d\vec{\rho}, \quad (19)$$

where $\vec{s}$ denotes a 2D unit vector. With Fourier transformation $P_n(\vec{k})$ of the current $P_n(\vec{\rho})$ one can rewrite the optical theorem Eq. (18) as follows:

$$Im\left[\int \Psi_{0n}^*(\vec{\rho})P_n(\vec{\rho})d\vec{\rho}\right] = -\frac{1}{8\pi}\int_{2\pi}\left|P_n\left(k_{rn}^0\vec{s}\right)\right|^2 d\vec{s}. \quad (20)$$

Obtained relation is a basic optical theorem for magnetostatic spin waves under consideration. It is useful to rewrite this relation in a form

$$C_{\text{ext}} = C_{sc}, \quad (21)$$

where C_{ext} and C_{sc} are cross sections of extinction and scattering, respectively, for spin-wave scattering by inclusions. These values are defined by relations

$$C_{\text{ext}} = -\frac{1}{k_{rn}^0}\text{Im}\left[\int \Psi_{0n}^*(\vec{\rho})P_n(\vec{\rho})d\vec{\rho}\right], \quad (22)$$

$$C_{sc} = \frac{1}{8\pi k_{rn}^0}\int \left|P_n\left(k_{rn}^0\vec{s}\right)\right|^2 d\vec{s}. \quad (23)$$

In the next sections we show that definitions in Eqs. (22) and (23) coordinate with notions of extinction and scattering cross sections similar to optics [34] at the study of 2D scattering of light when the cross sections have dimensions of a length.

D. Spin-wave scattering amplitude

To clarify the physical meaning of the optical theorem of Eqs. (20)–(23) one can consider the spin-wave magnetic field potential $\Psi_{\text{scn}}(\rho)$ Eq. (16) scattered by single inclusion centered at 2D point $\vec{R}_1 = 0$ (Fig. 2) for observation point $\vec{\rho}$ placed in the far wave zone of an inclusion.

Applying the asymptotics for Hankel function [39] with big argument's value we obtain

$$\Psi_{\text{scn}}(\vec{\rho})|_{\rho\to\infty} \approx e^{-i\pi/4}\sqrt{\frac{2}{\pi k_{rn}^0 \rho}} e^{ik_{rn}^0\rho} T(\vec{s}),$$

$$T(\vec{s}) = \frac{1}{4i}P_n\left(k_{rn}^0\vec{s}\right). \quad (24)$$

Similarly to optics' methods [34] the quantity $T(\vec{s})$ can be called the scattering amplitude in the direction with unit vector $\vec{s} = \vec{\rho}/\rho$. For the special case of plane spin-wave incident magnetic potential $\Psi_{0n}(\rho) = e^{ik_{rn}^0 s_0}$ propagating in the direction of unit vector $\vec{s}_0$ the optical theorem in Eq. (20) is rewritten as

$$\text{Re}T(\vec{s}_0) = -\frac{1}{2\pi}\int_{2\pi}|T(\vec{s})|^2 d\vec{s}. \quad (25)$$

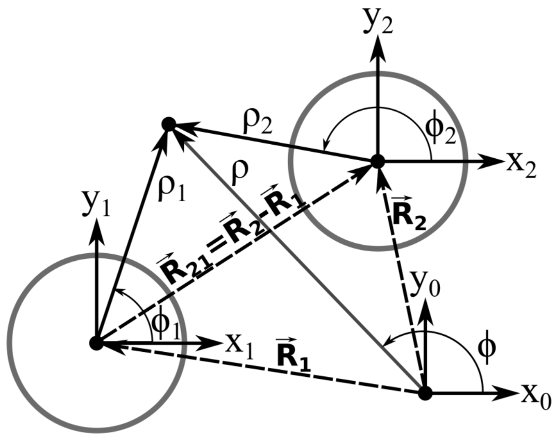

FIG. 2. The sketch showing two ferromagnetic inclusions with their local coordinate systems relative to the laboratory's coordinate system.

That is a traditional form of the optical theorem in optics [34]. To be consistent with this tradition, the extinction and scattering cross sections of spin waves in Eqs. (22) and (23) are written in a usual form

$$C_{\rm ext} = -\frac{4}{k_{rn}^0}{\rm Re}[T(\vec{s}_0)], C_{sc} = \frac{2}{\pi k_{rn}^0}\int_{2\pi}|T(\vec{s})|^2 d\vec{s}. \quad (26)$$

In this subsection we considered the optical theorem in Eqs. (20)–(23) for the case of plane incident spin-wave scattering by a single inclusion. The general optical theorem takes the most interesting form when the plane wave and especially narrow incident spin-wave beam is scattered by an ensemble of inclusions.

E. Watson composition rule for T-scattering operators

Consider the case of N inclusions centered in points with 2D coordinates $\vec{R}_j (j = 1,2,\ldots)$ (Fig. 2). The 2D T-scattering operator for an ensemble of N inclusions is evaluated with the help of the Watson composition rule [37] as follows

$$T_n(\vec{\rho},\vec{\rho}\,') = \sum_{j=1}^{N} T_n^{(j)}(\vec{\rho},\vec{\rho}\,'), \quad (27)$$

where the self-consistent 2D T-scattering operators $T_n^{(j)}(\vec{\rho},\vec{\rho}\,')$ satisfy the set of equations

$$T_n^{(j)}(\vec{\rho},\vec{\rho}\,') = T_n^{(0)}(\vec{\rho}-\vec{R}_j,\vec{\rho}\,'-\vec{R}_j) + \int d\vec{\rho}\,''\int d\vec{\rho}\,'''T_n^{(0)}(\vec{\rho}-\vec{R}_j,\vec{\rho}\,''-\vec{R}_j) \times G_n^{(0)}(\vec{\rho}\,''-\vec{\rho}\,''')\sum_{j\neq j'=1}^{N} T_n^{(j')}(\vec{\rho}\,''',\vec{\rho}\,'). \quad (28)$$

On the RHS of this set $T_n^{(0)}(\vec{\rho}-\vec{R}_j,\vec{\rho}\,'-\vec{R}_j)$ denotes the 2D T-scattering operator of a single inclusion centered in $\vec{R}_j$. The Eqs. (28) set shows, in particular, that a self-consistent 2D T-scattering operator $T_n^{(j)}(\vec{\rho},\vec{\rho}\,')$ of a jth inclusion is confined in dependence on its first argument ρ inside the jth inclusion. According to Eq. (17) the 2D current $P_n(\vec{\rho})$ in the case of N inclusions takes a form of the sum

$$P_n(\vec{\rho}) = \sum_{j=1}^{N} P_n^{(j)}(\vec{\rho}),$$
$$P_n^{(j)}(\vec{\rho}) = \int T_n^{(j)}(\vec{\rho},\vec{\rho}\,')\Psi_{0n}(\vec{\rho}\,')d\vec{\rho}\,', \quad (29)$$

where $P_n^{(j)}(\vec{\rho})$ is a self-consistent current excited inside a jth inclusion. The scattered spine-wave field in Eq. (16) takes the form of the sum of self-consistent fields $\Psi_{\rm scn}^{(j)}(\vec{\rho})$ scattered by different inclusions

$$\Psi_{\rm scn}(\vec{\rho}) = \sum_{j=1}^{N}\Psi_{\rm scn}^{(j)}(\vec{\rho}),$$
$$\Psi_{\rm scn}^{(j)}(\vec{\rho}) = \int G_n^{(0)}(\vec{\rho}-\vec{\rho}\,')P_n^{(j)}(\vec{\rho}\,')d\vec{\rho}\,'. \quad (30)$$

Transferring the point $\vec{\rho}$ in the far wave zone of all N inclusions and writing $P_n^{(j)}(\vec{\rho}) = \hat{P}_n^{(j)}(\vec{\rho}-\vec{R}_j)$ one obtains similarly with the case in Eq. (24) of single inclusion asymptotics

$$\Psi_{\rm scn}^{(j)}(\vec{\rho})|_{\rho\to\infty} \approx e^{-i\pi/4}\sqrt{\frac{2}{\pi k_{rn}^0\rho}}e^{ik_{rn}^0\rho}T^{(j)}(\vec{s}),$$
$$T^{(j)}(\vec{s}) = \frac{1}{4i}e^{-ik_{rn}^0\vec{s}\vec{\rho}}\hat{P}_n^{(j)}(k_{rn}^0\vec{s}), \quad (31)$$

where 2D Fourier transform $\hat{P}_n^{(j)}(k_{rn}^0\vec{s})$ is evaluated similarly with Eq. (19). Now the general optical theorem in Eqs. (20)–(23) takes for the case of incident plane spin-wave scattering by ensemble of inclusions the form similar to a single inclusion case in Eqs. (25) and (26), with putting

$$T(\vec{s}) = \sum_{j=1}^{N} T^{(j)}(\vec{s}). \quad (32)$$

As we see in the case of incident plane spin-wave scattering by an inclusion ensemble the general optical theorem of Eqs. (20)–(23) takes a traditional in optics form considering 2D light scattering. But in the case of incident narrow beam spin-wave scattering our general optical theorem gives in some sense an unexpected and important result for distant spin-wave excitation transfer along a linear chain of inclusions in the form of an invisible or dark mode.

F. Incident narrow beam spin-wave scattering

In this part we consider propagation of the incident spin-wave narrow beam that irradiates mostly the first inclusion and is scattered by the linear chain of inclusions. The geometry of this case is depicted in Fig. 1, where the linear chain is located along the x axis and the incident wave is propagating along the direction of the unit vector $\vec{s}_0 = \hat{y}$ of the y axis irradiating the first inclusion $j = 1$ located in the $\vec{R}_1 = 0$ point of the film (details concerning such wave beam propagation presented in

Appendix A). In this case we have

$$\int \Psi_{0n}^*(\vec{\rho})P_n(\vec{\rho})d\vec{\rho} \equiv \int \Psi_{0n}^*(\vec{\rho})\sum_{j=1}^{N}\hat{P}_n^{(j)}(\vec{\rho}-\vec{R}_j)d\vec{\rho} \approx \int \Psi_{0n}^*(\vec{\rho})\hat{P}_n^{(1)}(\vec{\rho})d\vec{\rho} \approx \hat{P}_n^{(j)}\big(k_{rn}^0\vec{s}_0\big). \tag{33}$$

The last evaluation in Eqs. (33) was presented because the incident beam $\Psi_{0n}(\rho)$ has the approximate form of a plane wave inside the first inclusion. Substituting this last evaluation in Eq. (22) for a general definition of extinction cross section gives

$$C_{\text{ext}} \approx -\frac{1}{k_{rn}^0}\text{Im}\hat{P}_n^{(1)}\big(k_{rn}^0\vec{s}_0\big) = -\frac{4}{k_{rn}^0}\text{Re}T^{(1)}(\vec{s}_0), \tag{34}$$

where the scattering amplitude $T^{(1)}(\vec{s}_0)$ is defined in Eqs. (31). The major property of Eq. (34) is that the extinction cross section of the linear chain in the case of an incident narrow spin-wave beam irradiating only the first inclusion formally coincides with the extinction cross section in Eq. (26) for a single inclusion. Thus only irradiated inclusion makes a direct contribution in collective extinction cross section despite the total number of inclusions in the linear chain that makes the direct summarized contribution of all other inclusions in spin-wave scattering almost invisible; we call this the dark mode.

G. Self-consistent spin-wave multiple scattering partial amplitudes

In the case of an incident spin-wave multiple scattering by ensemble of N inclusions the total scattered spin-wave field potential $\Psi_{\text{scn}}(\vec{\rho})$ is represented as the sum of N self-consistent field potentials $\Psi_{\text{scn}}^{(j)}(\vec{\rho})$ scattered by different inclusions. One way to write out self-consistent fields scattering by different inclusions is in terms of self-consistent spin-wave multiple scattering partial amplitudes B_{jm} [31].

Spin-wave potential scattered by jth inclusion is written in the form

$$\Psi_{\text{scn}}^{(j)}(\rho_j,\phi_j) = \sum_{m=-\infty}^{\infty} B_{jm}H_m^{(1)}\big(k_{rn}^0\rho_j\big)e^{im\phi_j}. \tag{35}$$

This formula is written in the local coordinate system of the jth inclusion (Fig. 2), with vector $\vec{\rho}_j = \vec{\rho} - \vec{R}_j$. Transferring the point $\vec{\rho}$ in the far wave zone of all N inclusions one can obtain for the Eq. (35) a far wave zone asymptotic similar to the one in Eqs. (31). The comparison enables one to get relations

$$\sum_{m=-\infty}^{\infty} B_{jm}e^{im(\phi_j-\pi/2)} = \frac{1}{4i}\hat{P}_n^{(j)}\big(k_{rn}^0\vec{s}\big). \tag{36}$$

This is the relation between multiple scattering partial amplitudes B_{jm} and 2D Fourier transformation $\hat{P}_n^{(j)}(k_{rn}^0\vec{s})$ of currents excited inside inclusions. Equations (36) enable us to rewrite formulas for the extinction and the scattering cross sections for spin waves in terms of multiple scattering partial amplitudes. In particular, the formula in Eq. (34) takes a form

$$C_{\text{ext}} = -\frac{1}{k_{rn}^0}\text{Re}\sum_{m=-\infty}^{\infty} B_{1m}. \tag{37}$$

Corresponding expression for scattering cross section is given by

$$C_{sc} = \frac{2}{\pi k_{rn}^0}\int_{2\pi}\left|\sum_{j=1}^{N} e^{ik_{rn}^0\vec{s}\vec{R}_j}\sum_{m=-\infty}^{\infty} B_{jm}e^{im(\phi_j-\pi/2)}\right|^2 d\vec{s} \tag{38}$$

and should coincide by value with Eq. (37) due to the extinction theorem $C_{\text{ext}} = C_{sc}$. Multiple scattering partial amplitudes satisfy the set of equations that was derived in previous work [31] under the condition of a plane spin-wave incidence. For the case of narrow beam spin-wave incidence similar sets of equations can be derived from the Watson composition rule of Eq. (28)

$$B_{jm} = B_m^{(j)} + \frac{B_m^{(j)}}{\hat{A}_{jm}}\sum_{j\neq j'=1}^{N}\sum_{l=-\infty}^{\infty} G_{m-l}^{jj'}B_{j'l}, \tag{39}$$

with matrix kernel

$$G_{m-l}^{jj'} = H_{l-m}\big(k_r^0 R_{jj'}\big)e^{i(l-m)\arg\vec{R}_{jj'}}, \tag{40}$$

where $\arg R_{jj'}$ denotes the angle between the vector $R_{jj'}$ and the x axis. The quantity $B_m^{(j)}$ denotes the scattering partial amplitude of the jth single isolated inclusion. The ratio $B_m^{(j)}/\hat{A}_{jm} = T_m^{(j)}$ in Eq. (39) defines the scattering matrix $T_m^{(j)}$ of the jth single inclusion and depends only on the inclusion's parameters. The quantities $\hat{A}_{jm}$ are coefficients for the expansion along Bessel functions of the incident spin-wave beam in local coordinate system of the jth inclusion. In this case we are considering only the excitation of the first inclusion, therefore $\hat{A}_{jm} = 0$ for $j \neq 1$ (Appendix A). The scattering matrix of a single inclusion is evaluated in papers [22,31] and can be obtained by solution of the integral Lippmann-Schwinger Eq. (9).

This subsection concludes the consideration of general properties of T-scattering operator for multiple magnetostatic spin-wave scattering by magnetic inclusions inside the ferromagnetic matrix. In Sec. III the general properties of T-scattering operator are applied to the problem of the spin-wave excitation transfer along a linear chain of magnetic inclusions. In this case the principal role will play the formula in Eq. (37) for the extinction cross section of a linear chain when an incident narrow spin-wave beam irradiates only the first inclusion.

III. SPIN-WAVE EXCITATION TRANSFER BY THE DARK MODE ALONG A LINEAR CHAIN OF MAGNETIC INCLUSIONS

A. Closest neighbor interaction approximation

The set of Eqs. (39) and (40) for the partial multiple scattering amplitudes in one-multipole approximation, where

m is fixed, after normalization $B_{jm} = T_m^{(j)} \hat{B}_{jm}$ takes the form

$$\hat{B}_{jm} - \sum_{j \neq j'=1}^{N} a_{jj'}^{(m)} \hat{B}_{j'm} = \hat{A}_{jm}, \quad a_{jj'}^{(m)} = H_0(k_r R_{jj'}) T_m^{(j')}. \tag{41}$$

In Eqs. (41) the quantity $a_{jj'}^{(m)}$ denotes the coupling parameter of inclusions numbered j and j', with $R_{jj'}$ being the distance between their centers. The scattering matrix $T_m^{(j)}$ of the single inclusion is independent from inclusion's number j if all inclusions have identical geometrical and material properties. Further we use another normalization of partial multiple scattering amplitudes $\hat{\hat{B}}_{jm} = \hat{B}_{jm}/\hat{A}_{1m}$. The extinction cross section from Eq. (37) in one-multipole approximation takes a form

$$C_{\text{ext}} = -\frac{1}{k_{rn}^0} \text{Re}(T_m \hat{A}_{1m} \hat{\hat{B}}_{1m}). \tag{42}$$

Defining the extinction cross section of single inclusion as $C_{(1)\text{ext}} = -(4/k_{rn}^0)\text{Re}T_m$ and putting $\hat{A}_{1m} \approx 1$ we present the ratio $C_{\text{ext}}/C_{(1)\text{ext}}$ in a form

$$\frac{C_{\text{ext}}}{C_{(1)\text{ext}}} = \text{Re}F_N - \frac{\text{Im}T_m}{\text{Re}T_m}\text{Im}F_N, \tag{43}$$

where the collective extinction factor F_N is defined by

$$F_N = \hat{\hat{B}}_{1m}. \tag{44}$$

In order to solve Eqs. (41) analytically we apply the closest neighbor interaction approximation putting $a_{jj'} \approx 0$ if $|j - j'| > 1$. With such approximation the matrix of Eq. (41) set becomes the Jacobi matrix [40] and one can use the Rayleigh's like solution [32] (the index m is omitted)

$$\hat{\hat{B}}_j = 2(-1)^{j-1} \cos\theta \frac{\sin[(N+1-j)\theta]}{\sin[(N+1)\theta]}, \quad \cos\theta = -\frac{1}{2a_{12}}. \tag{45}$$

In Eq. (45) a complex value $\theta = \theta' + i\theta''$ satisfies the second equation in Eq. (45), which has the meaning of a dispersion equation. It is interesting to remark that a solution similar to Eqs. (45) one can find in the theory of electrical filter circuits [33]. Substituting the first Eq. (45) in Eq. (44) gives the expression for collective extinction factor F_N

$$F_N = 2\cos\theta \frac{\sin N\theta}{\sin(N+1)\theta}. \tag{46}$$

B. Resonant values of coupling parameter

Now we address the investigation of the distant transfer of spin-wave excitation along the linear chain of coupled magnetic inclusions. The most important point appears to be the resonant case when the imaginary part of the coupling parameter $a_{12} = a'_{12} + ia''_{12}$ becomes equal to zero

$$a''_{12} = 0. \tag{47}$$

As study shows, under this condition it is possible to determine separate cases of small $2|a'_{12}| < 1$ and big $2|a'_{12}| \geqslant 1$ values of the real part of the coupling parameter a'_{12} in the dispersion Eq. (45).

In these two cases solutions of the dispersion Eq. (45) are

$$\theta' = 0, \cosh\theta'' = \frac{1}{2|a'_{12}|} (a''_{12} = 0, -1 < 2a'_{12} < 0), \tag{48}$$

and

$$\theta'' = 0, \cos\theta' = -\frac{1}{2a'_{12}} (a''_{12} = 0, 2a'_{12} < -1). \tag{49}$$

According to Eq. (48) the imaginary part θ'' of complex variable θ changes in a semi-infinite interval $0 \leqslant \theta'' < \infty$, and according to Eq. (49) the value θ' changes in an interval $0 \leqslant \theta' < \pi/2$. If resonant conditions $a''_{12} = 0$ and $a'_{12} \to -0.5 \pm 0$ are met, then both values satisfy conditions $\theta'' \to 0$ and $\theta' \to 0$ in both Eqs. (48) and (49). Therefore, Eqs. (45) and (46) lead to the same limiting formulas

$$\hat{\hat{B}}_j \to (-1)^{j-1} 2\left(1 - \frac{j}{N+1}\right), \quad F_N \to 2\left(1 - \frac{1}{N+1}\right), \theta \to 0. \tag{50}$$

On the other hand, if condition $\theta'' \neq 0$ or even $\theta'' \to \infty$ is satisfied, then Eqs. (45) and (46) give

$$\hat{\hat{B}}_j \to (-1)^{j-1} e^{-(j-1)\theta''}, F_N \to 1. \tag{51}$$

The limiting formulas of Eq. (50) describe the case of the distant resonant transfer of spin-wave excitation along the linear chain of coupled inclusions at resonant value of the coupling parameter $a'_{12} \to -0.5$, which has a linear dependence of excitation decrease on the number of particles. The collective extinction factor according to Eqs. (50) is equal to approximately $F_N \approx 2$ for $N \gg 1$; the fact that $F_N \neq 1$ shows some indirect effect of particles influence with numbers $j > 1$ on the collective extinction factor via influence on self-consistent scattering amplitude $\hat{\hat{B}}_1$ in the first inclusion. The formulas of Eq. (51) describe a short transfer of spin-wave excitation along the chain, with exponential

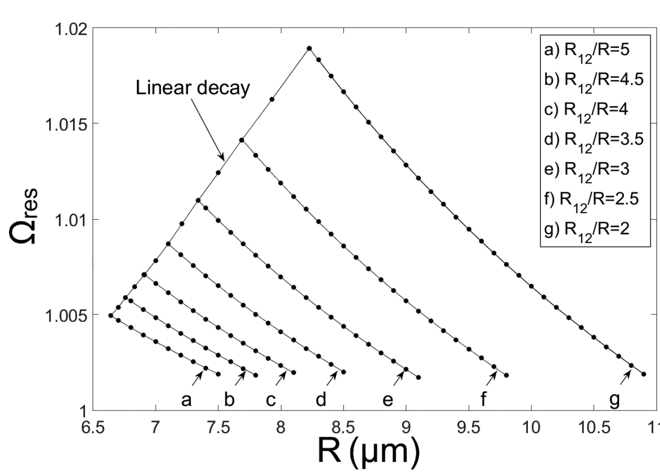

FIG. 3. Curves presenting the dependence of the resonant frequency on the inclusion radius R at fixed ratio R_{12}/R, when $a''_{12} = 0, a'_{12} \leqslant -0.5$.

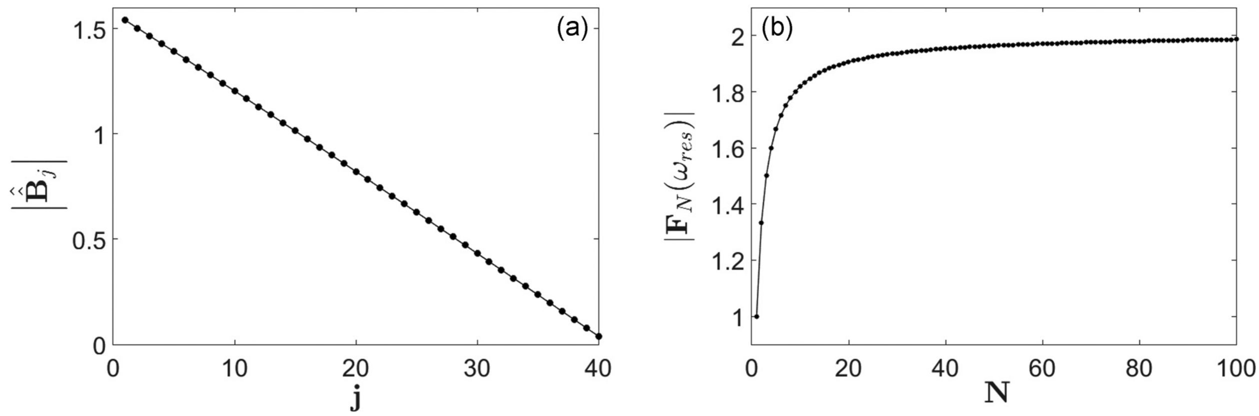

FIG. 4. Linear decay of scattering amplitudes along the linear chain of inclusions. $\Omega_{\rm res} = 1.007, \theta'' = 4.4\,10^{-6}, R = 6.9\,\mu m, R_{12} = 4R, k_r^0 R = 0.38$.

decrease of excitation, and the collective extinction factor has physically more understandable value $F_N \to 1$.

In Fig. 3 we present dependencies of normalized resonance frequency $\Omega_{\rm res} = \omega_{\rm res}/\omega_H$ on geometrical parameters R and R_{12} (Fig. 1) of the linear chain when resonance conditions in Eqs. (49) for coupling parameter are satisfied. As usually, ω_H denotes the ferromagnetic resonance frequency of the ferromagnetic film. All curves in Fig. 3 are obtained by numerical solution of the dispersion Eq. (45) under additional condition $\theta'' \to 0$ in accordance with first Eqs. (49). All calculations where performed for the following material parameters: external magnetic field $H_{\rm ext} = 5k\,Oe$, saturation magnetization of the film and inclusions ferromagnetic materials $M_s^0 = 1620\,Oe$ and $M_s^1 = 1740\,Oe$, and film thickness $d = 10\ \mu m$.

The left boundary linear curve in Fig. 3 is related to the case (50) and presented in Fig. 4. Parameters presented by this curve can be approximately described by the relation $(\Omega_{\rm res} - 1)R_{12}/R = const$. Other data points in Fig. 3 outside the curve, which represent data for linear decay of scattering amplitudes, depict the case of distant transfer with conditions from Eqs. (47) and (49). This case is represented by Fig. 6, where scattering amplitudes and collective extinction factors are described by general equations (45) and (46). Details of oscillation type of transfer will be discussed in the next subsection.

The calculation for the resonant case with small coupling parameters $2|a'_{12}| < 1, a''_{12} = 0$ shows that the condition $\theta'' \to 0$ cannot be met. This means that partial scattering amplitudes will decrease exponentially [Eq. (51)], and the signal will not transfer for a big distance as it is shown in Fig. 5. The deviation in a behavior of the collective extinction factor F_N in Fig. 5 from its approximation in Eqs. (51) is explained by not sufficiently big value of $\theta'' = 0.43$.

C. Dark mode filtering from radiation losses

In the preceding subsection we considered the distant resonant transfer of spin-wave excitation in the form of the dark mode under resonant condition, when the value of the imaginary part of the coupling parameter is equal to zero Eq. (47) and obtained, in particular, formulas for the limiting case in Eqs. (50) under an additional condition on the real part of the coupling parameter in the form $a'_{12} \to -0.5$. As it was stated, Eqs. (50) give for collective extinction factor the approximate value $F_N \approx 2$. Here we will note that mentioned resonant conditions enable one to get substantially less value for the collective extinction factor. It is possible if we try to tend to zero the sine function in the numerator of the ratio in Eq. (46).

We turn to Eqs. (45) and (46) for self-consistent scattering amplitudes $\hat{\hat{B}}_j$ of inclusions and collective extinction factor

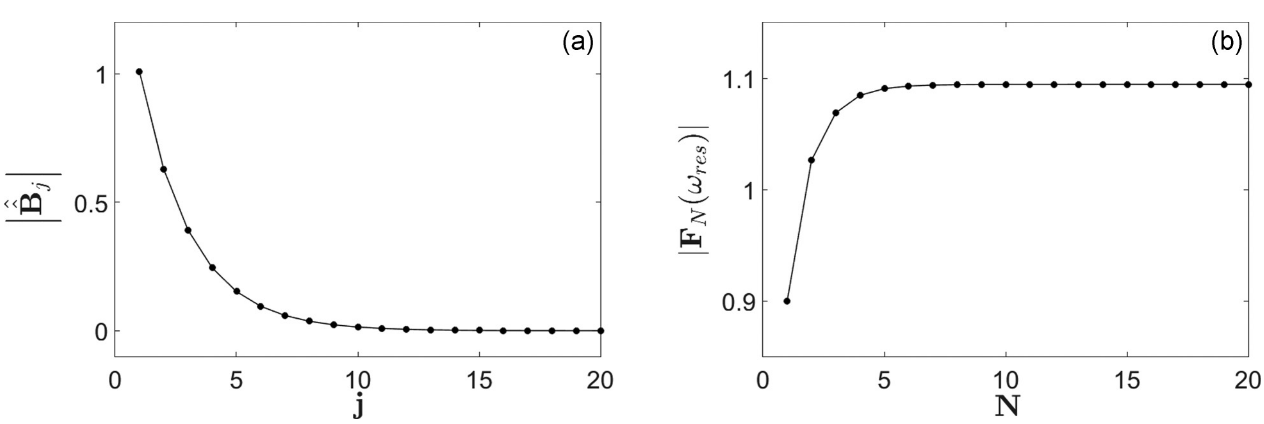

FIG. 5. Exponential decay of scattering amplitudes along the linear chain. $\Omega_{\rm res} = 1.004, \theta'' = 0.43,\ R = 7\,\mu m,\ R_{12} = 3R,\ k_r^0 R = 0.3$.

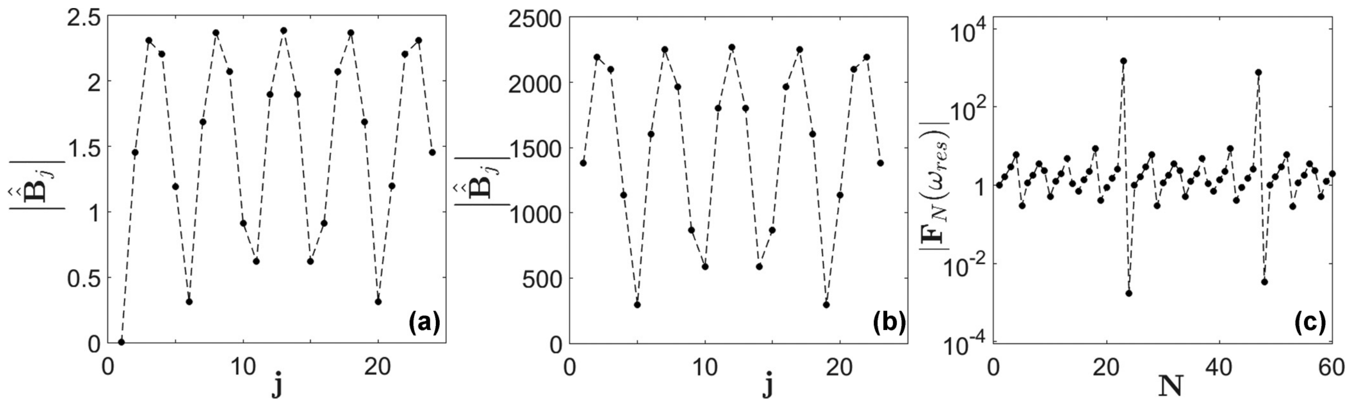

FIG. 6. Illustrations of the behavior of scattering amplitudes $\hat{\hat{B}}_j$ of inclusions depending on the inclusion number j for the case $N = 24$ (a), $N = 23$ (b), and collective extinction factor F_N (c) of a linear chain under condition of dark mode filtering from radiation losses.

F_N, considering these equations under resonance conditions in Eqs. (49) when $a''_{12} = 0$ and $a'_{12} = -0.5 - \Delta a'_{12}$, with $\Delta a'_{12} \geqslant 0$ being a small positive quantity. In this case from solution of the dispersion Eq. (45) we have $\theta'' = 0$ and $\theta' \approx 2(\Delta a'_{12})^{1/2}$. The quantity $\Delta a'_{12}$ is dependent on magnetic and geometrical parameters of the chain. On the assumption that we are able to choose these parameters in such a way that the following filtering dark mode from the radiation losses condition is fulfilled, we get

$$\theta' = \frac{n\pi}{N} + \delta\theta', n = 1,2,\ldots, \tag{52}$$

where $\delta\theta'$ is the detuning factor. The substitution of Eq. (52) in Eq. (46) gives

$$F_N = \frac{2}{1 + \Delta a'_{12}} \frac{\sin N\delta\theta'}{\sin\left[\frac{n\pi}{N} + (N+1)\delta\theta'\right]}. \tag{53}$$

As one can see at detuning values $\delta\theta' \to 0$ the collective extinction factor value tends to zero $F_N \to 0$ and the dark mode is filtered from radiation losses. Under this condition the self-consistent scattering amplitudes of coupled inclusions according to Eq. (45) take the form

$$\hat{\hat{B}}_j = (-1)^j \frac{2}{1 + \Delta a'_{12}} \frac{\sin(j-1)\frac{n\pi}{N}}{\sin\frac{n\pi}{N}}. \tag{54}$$

In particular at $n = 1$ and N taking even values we obtain several values

$$\hat{\hat{B}}_2 = \hat{\hat{B}}_N \approx 2, \quad \hat{\hat{B}}_{1+N/2} \approx (-1)^{1+N/2} \frac{1}{\sin\frac{\pi}{N}}. \tag{55}$$

Thus, under condition Eq. (52) of radiation losses filtering with zero detuning the finite linear chain of magnetic inclusions plays the role of a waveguide for spin wave without radiation losses. The formulas in Eqs. (53) and (54) under conditions of Eq. (52) of radiation losses filtering are illustrated in Fig. 6.

Analyzing Fig. 6 we need to note that the collective extinction factor value at $N = 23{,}47$ becomes as big as $F_N \approx 10^3$, that is caused by a small value of the sine function in the denominator of the ratio in the Eq. (46) formula and consequently leads to big values of scattering amplitudes according to Eq. (45). On the other hand, one can see at $N = 24{,}48$ Fig. 6(a) the extinction factor can have values down to $F_N \approx 10^{-3}$ due to the same properties of Eqs. (45) and (46).

IV. CONCLUSION

We have developed the quantum mechanical type T-scattering operator approach to study multiple scattering of the forward volume magnetostatic spin wave by a finite ensemble of inclusions in a ferromagnetic metallized thin film. The approach is applied to the problem of spin-wave excitation transfer along the linear chain of magnetic cylindrical inclusions. Substantial results of developed approach are the deriving of an optical theorem for the T-scattering operator and, as a consequence, the obtaining of a formula for collective extinction cross section of the inclusion ensemble, when only the first inclusion of the chain is irradiated by the incident narrow spin-wave beam. This formula shows that only directly irradiated inclusion makes a contribution in the collective extinction cross section despite that the total number of inclusions can be big; that makes the direct summarized contribution of all other inclusions in spin-wave scattering invisible (dark mode). Applying the developed T-scattering operator approach to the problem of spin-wave excitation distant transfer along a linear chain of magnetic inclusions we found a resonant regime, when excitation propagation along a linear chain is described by linear decreasing of excitation depending on the number of inclusions. Corresponding to this resonant regime collective extinction cross section is approximately two times bigger than extinction cross section of single irradiated inclusion in the long chain. Also we found a resonant mechanism of filtering the dark mode from radiation losses, that makes the linear chain of magnetic inclusions a micro-waveguide for spin waves. This waveguide can transfer information over a long distance in the form of the dark mode with low radiation losses.

ACKNOWLEDGMENTS

Authors acknowledge Russian Science Foundation Grant No. 14-19-00760 and No. 13-07-12416, Ministry of Education and Science of the Russian Federation Grant No. 14.607.21.0141 and Russian Foundation for Basic Research (RFBR) Grant No. 15-07-08152 for financial support.

APPENDIX: FORMING INCIDENT SPIN-WAVE BEAM

In subsection II F we considered the case when the incident spin-wave narrow beam propagates along the direction of the unit vector of the y axis and irradiates an inclusion in the film and is scattered by the linear chain of inclusions aligned along the x axis (see Fig. 2). This appendix shows that such a way of incidence is most suitable to irradiate only one inclusion from the chain.

We consider the incident 2D narrow spin-wave beam propagation under some angle $\overline{\alpha}$ to the x axis and aims to irradiate the first inclusion $j=1$ with center $\vec{R}_1=0$ from an ensemble of N inclusions (see Figs. 2 and 1). We form the magnetostatic potential $\Psi^{\rm inc}(\vec{\rho})$ of the incident beam as a linear combination of plane waves $e^{i\vec{k}_r\vec{\rho}}$ propagating with wave vectors $\vec{k}_r=k_r\vec{s}(\alpha)$, where $k_r=k^0_{rn}$ and $\vec{s}(\alpha)=(\cos\alpha,\sin\alpha)$ is a 2D unit vector in the x,y plane under angles α to x_j axes of all local coordinate systems. These plane waves have amplitudes $f(\alpha)$ with not zero values in small enough angular interval $\Delta\alpha$ near $\overline{\alpha}$. Further we write

$$\Psi^{\rm inc}(\vec{\rho})=\int_{\overline{\alpha}-\Delta\alpha}^{\overline{\alpha}+\Delta\alpha}d\alpha f(\alpha)e^{i\vec{k}_r\vec{\rho}}. \tag{A1}$$

For the convenience the angular variable is changed to $\alpha=\overline{\alpha}+\alpha'$, where α' is small. The angular amplitude function $f(\alpha')$ is written in Gauss form

$$f(\alpha')=\frac{1}{\alpha_0\sqrt{\pi}}e^{-\alpha'^2/\alpha_0^2}, \tag{A2}$$

where α_0 is an angular half width of the spin-wave beam. Now substitution of Eq. (A2) into Eq. (A1) and integration of the result gives

$$\Psi^{\rm inc}(\vec{\rho}_1)\approx e^{ik_r\vec{s}(\overline{\alpha})\vec{\rho}_1}e^{-(\vec{s}_\perp(\overline{\alpha})\vec{\rho}_1)^2k_r^2\alpha_0^2/4}. \tag{A3}$$

Here on the RHS a unit vector $\vec{s}_\perp=(-\sin\alpha,\cos\alpha)$ is orthogonal to the unit vector $\vec{s}(\overline{\alpha})$. In particular the incident spin-wave beam angular brightness $\alpha_0=2/k_rR$ where R is the radius of an inclusion, with that one can rewrite Eq. (A3) in the local coordinate system of the first inclusion as

$$\Psi^{\rm inc}(\vec{\rho}_1)\approx e^{ik_r\vec{s}(\overline{\alpha})\vec{\rho}_1}e^{-(\vec{s}_\perp(\overline{\alpha})\vec{\rho}_1)^2/R^2}. \tag{A4}$$

The obtained formula describes a collimated spin-wave beam in the form of a plane wave propagating with wave vector $k_r\vec{s}(\overline{\alpha})$ and spatially modulated in a direction perpendicular to direction $\vec{s}(\overline{\alpha})$ of propagation. The spin-wave beam spatial brightness is defined by the inclusion radius according to Eq. (A4). We need to present the magnetic potential of the incident spin-wave beam in the cylindrical coordinates $\Psi^{\rm inc}(\rho_1,\phi)$ of the local coordinate system of the first inclusion. With this aim we write $\vec{k}_r\vec{\rho}_1=k_r\rho_1\cos(\phi_1-\alpha)$ and use the plane wave expansion [41] along Bessel functions $J_m(u)$ as follows

$$e^{ik_r\rho_1\cos(\phi_1-\alpha)}=\sum_{m=-\infty}^{\infty}i^mJ_m(k_r\rho_1)e^{im(\phi_1-\alpha)}. \tag{A5}$$

Substituting Eq. (A5) in the integrand of Eq. (A1) RHS and performing integration with the amplitude function of Eq. (A2) gives

$$\Psi^{\rm inc}(\rho_1,\phi_1)=\sum_{m=-\infty}^{\infty}\hat{A}_{1m}J_m(k_r\rho_1)e^{im\phi_1}, \tag{A6}$$

where

$$\hat{A}_{1m}=i^me^{-im\overline{\alpha}}e^{-m^2/(k_r^2R^2)}. \tag{A7}$$

It is interesting to note that multipole index module $|m|$ should not be greater than the wave parameter of inclusion $|m|\lesssim k_rR$, according to Eq. (A7).

We generalize Eqs. (A4), (A6), and (A7) for the magnetic potential of the incident spin-wave beam presentation in the arbitrary inclusion local coordinate system. The generalization is performed using a geometrical relation $\vec{R}_j-\vec{R}_1+\vec{\rho}_j=\vec{\rho}_1$ (Fig. 2). The substitution of the relation in Eq. (A4) gives

$$\Psi^{\rm inc}(\vec{\rho}_j)\approx e^{ik_r\vec{s}(\overline{\alpha})(\vec{R}_j+\vec{\rho}_j)}e^{-(\vec{s}_\perp(\overline{\alpha})(\vec{R}_j+\vec{\rho}_j))^2/R^2}. \tag{A8}$$

In the case of an incident spin-wave beam propagating along the y axis the angle $\overline{\alpha}=\pi/2$ and Eq. (A8) give

$$\Psi^{\rm inc}(\vec{\rho}_j)\approx e^{ik_ry_j}e^{-(X_j+x_j)^2/R^2}. \tag{A9}$$

Because $|x_j|\leqslant R$ inside the jth inclusion on the RHS of Eq. (A9) becomes exponentially small at $|X_j|\gg R$ the incident spin-wave beam practically does not radiate the jth inclusion. In the case of the incident spin-wave beam propagating along the x axis with angle $\overline{\alpha}=0$ and Eq. (A8) takes a form

$$\Psi^{\rm inc}(\vec{\rho}_j)\approx e^{ik_r(X_j+x_j)}e^{-y_j^2/R^2}. \tag{A10}$$

According to this equation the incident beam irradiates all inclusions. The Eqs. (A6) and (A7) are generalized for the arbitrary inclusion local coordinate system as follows

$$\Psi^{\rm inc}(\rho_j,\phi_j)=\sum_{m=-\infty}^{\infty}\hat{A}_{jm}J_m(k_r\rho_j)e^{im\phi_j}, \tag{A11}$$

where

$$\hat{A}_{jm}=i^me^{-im\overline{\alpha}+ik_rX_j\cos\overline{\alpha}}\exp\left[-\left(\frac{m+k_rX_j\sin\overline{\alpha}}{k_rR}\right)^2\right]. \tag{A12}$$

In the case of an incident spin-wave beam propagating along the y axis ($\overline{\alpha}=\pi/2$) the module of an amplitude in Eq. (A12) becomes exponentially small under conditions $|X_j|\gg R$ and $|m|\lesssim k_rR$. In the case of an incident spin-wave beam propagating along the x axis ($\overline{\alpha}=0$) the module of an amplitude in Eq. (A12) is independent of the number of inclusion.

[1] R. L. Stamps, S. Breitkreutz, J. Akerman, A. V. Chumak, Y. Otani, G. E. W. Bauer, J.-U. Thiele, M. Bowen, S. A. Majetich, M. Kläui *et al.*, J. Phys. D **47**, 333001 (2014).

[2] S. Blügel, D. Bürgler, M. Morgenstern, C. M. Schneider, and R. Waser (eds.), *Lecture Notes of the 40th Spring School, March 9–20, 2009* (Jülich, Germany, 2009).

[3] I. Zutic and M. Fuhrer, Nat. Phys. **1**, 85 (2005).
[4] D. H. Morris, D. M. Bromberg, J.-G. Zhu, and L. Pileggi, International Journal of High Speed Electronics and Systems **21**, 1250005 (2012).
[5] F. Pulizzi, Nat. Mater. **11**, 367 (2012).
[6] V. V. Kruglyak, P. S. Keatley, A. Neudert, R. J. Hicken, J. R. Childress, and J. A. Katine, Phys. Rev. Lett. **104**, 027201 (2010).
[7] J. Ding and A. Adeyeye, Appl. Phys. Lett. **101**, 103117 (2012).
[8] J. Jorzick, S. O. Demokritov, B. Hillebrands, M. Bailleul, C. Fermon, K. Y. Guslienko, A. N. Slavin, D. V. Berkov, and N. L. Gorn, Phys. Rev. Lett. **88**, 047204 (2002).
[9] G. Gubbiotti, M. Conti, G. Carlotti, P. Candeloro, E. D. Fabrizio, K. Y. Guslienko, A. Andre, C. Bayer, and A. N. Slavin, J. Phys.: Condens. Matter **16**, 7709 (2004).
[10] V. V. Kruglyak, S. O. Demokritov, and D. Grundler, J. Phys. D **43**, 264001 (2010.
[11] F. Ciubotaru, A. V. Chumak, B. Obry, A. A. Serga, and B. Hillebrands, Phys. Rev. B **88**, 134406 (2013).
[12] H. P. M. Krawczyk, Acta Phys. Pol. A **93**, 805 (1998).
[13] S. A. Nikitov, P. Tailhades, and C. S. Tsai, J. Magn. Magn. Mater. **236**, 320 (2001).
[14] Y. V. Gulyaev, S. A. Nikitov, L. V. Zhivotovskii, A. A. Klimov, Ph. Tailhades, L. Presmanes, C. Bonningue, C. S. Tsai, S. L. Vysotskii, and Yu. A. Filimonov, JETP Lett. **77**, 567 (2003).
[15] S. L. Vysotskii, S. A. Nikitov, and Y. A. Filimonov, J. Exp. Theor. Phys. **101**, 547 (2005).
[16] A. V. Chumak, T. Neumann, A. A. Serga, B. Hillebrands, and M. P. Kostylev, J. Phys. D **42**, 205005 (2009).
[17] M. P. Kostylev, A. A. Serga, T. Schneider, T. Neumann, B. Leven, B. Hillebrands, and R. L. Stamps, Phys. Rev. B **76**, 184419 (2007).
[18] V. E. Demidov, U.-H. Hansen, and S. O. Demokritov, Phys. Rev. Lett. **98**, 157203 (2007).
[19] A. V. Chumak, A. A. Serga, B. Hillebrands, and M. P. Kostylev, Appl. Phys. Lett. **93**, 022508 (2008).
[20] M. Mruczkiewicz, M. Krawczyk, G. Gubbiotti, S. Tacchi, Y. A. Filimonov, D. V. Kalyabin, I. V. Lisenkov, and S. A. Nikitov, New J. Phys. **15**, 113023 (2013).
[21] M. Mruczkiewicz, E. S. Pavlov, S. L. Vysotsky, M. Krawczyk, Y. A. Filimonov, and S. A. Nikitov, Phys. Rev. B **90**, 174416 (2014).
[22] I. Lisenkov, D. Kalyabin, and S. Nikitov, Appl. Phys. Lett. **103**, 202402 (2013).
[23] R. Shindou, J.-i. Ohe, R. Matsumoto, S. Murakami, and E. Saitoh, Phys. Rev. B **87**, 174402 (2013).
[24] R. Shindou and J.-i. Ohe, Phys. Rev. B **89**, 054412 (2014).
[25] I. Lisenkov, V. Tyberkevych, A. Slavin, P. Bondarenko, B. A. Ivanov, E. Bankowski, T. Meitzler, and S. Nikitov, Phys. Rev. B **90**, 104417 (2014).
[26] R. Shindou, R. Matsumoto, S. Murakami, and J.-i. Ohe, Phys. Rev. B **87**, 174427 (2013).
[27] T. Sebastian, T. Bracher, P. Pirro, A. A. Serga, B. Hillebrands, T. Kubota, H. Naganuma, M. Oogane, and Y. Ando, Phys. Rev. Lett. **110**, 067201 (2013).
[28] A. Burin, Phys. Rev. E **73**, 066614 (2006).
[29] Y. N. Barabanenkov and V. V. Shlyapin, Phys. Lett. A **170**, 239 (1992).
[30] Y. N. Barabanenkov and V. V. Shlyapin, Phys. Rev. B **58**, 3034 (1998).
[31] Y. Barabanenkov, S. Osokin, D. Kalyabin, and S. Nikitov, Phys. Rev. B **91**, 214419 (2015).
[32] J. W. S. Rayleigh, *Theory of Sound* (MacMillan, London, 1984).
[33] M. A. Lavrentev and B. V. Sabat, *Methods of the Theory of Functions of a Complex Variable 2nd ed.* (Izdat. Fiz.-Mat. Lit., Moscow, 1958).
[34] H. van de Hulst, *Light Scattering by Small Particles* (John Wiley and Sons, Inc., New York, 1957).
[35] A. G. Gurevich and G. A. Melkov, *Magnetization, Oscillations and Waves* (CRC Press, New York, 1996).
[36] B. A. Lippmann and J. Schwinger, Phys. Rev. **79**, 469 (1950).
[37] M. Goldberger and K. Watson, *Collision Theory* (Wiley, New York, 1965).
[38] J. Taylor, *The Quantum Theory of Nonrelativistic Collisions* (Wiley, New York, 1985).
[39] G. Korn and T. Korn, *Mathematical Handbook for Scientists and Engineers* (McGraw-Hill, New York, 1961).
[40] P. Lankaster, *Theory of Matrices* (Academic Press, New York, 1969).
[41] J. Stratton, *Electromagnetic Theory* (McGraw-Hill, New York, 1941).