Criticality revealed through quench dynamics in the Lipkin-Meshkov-Glick model

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We examine the dynamics after a sudden quench in the magnetic field of the Lipkin-Meshkov-Glick model. Starting from the ground state and by employing the time-dependent fidelity, we see manifestly different dynamics are present if the system is quenched through the critical point. Furthermore, we show that the average work shows no sensitivity to the quantum phase transition; however, the free energy and irreversible work show markedly different rates of change in each phase. Finally, we assess the spectral function showing the fundamental excitations that dictate the dynamics of the postquenched system, further highlighting the qualitative differences between the dynamics in the two phases.

I. INTRODUCTION

Phase transitions are an interesting trait of physical systems. In particular, when dealing with quantum many-body setups it is curious that a well defined microscopic description can lead to nontrivial and singular behaviors in the thermodynamic limit. The study of the equilibrium properties of such systems is well established in, for example, exactly solvable one-dimensional spin chains [1]. With the help of several tools from quantum information, notably quantum correlations, the implications of the presence of quantum phase transitions (QPTs) have been explored [2].

Beyond the static properties, there is a growing interest in studying the dynamics of many-body quantum systems. This is further catalyzed by the increasing interest in understanding the thermodynamic properties of genuinely quantum systems [3]. Despite the significantly more involved nature of studying dynamics, remarkable progress has been made in elucidating the behavior of important quantities, e.g., (irreversible) work, entropy production, and residual energy, when a many-body system is evolved through its critical point. The most frequent evolution considered is that of a "sudden quench" of the order parameter [4–9] (however, finite time protocols have also been addressed [10]). Focusing on such a sudden change allows us to capture the salient features of the ensuing nonequilibrium dynamics, while leaving the study of more qualitative differences to a more involved temporal analysis. Typically, the sudden change to the Hamiltonian kicks the system out of equilibrium and can lead to interesting consequences. Notably, for the Ising model the dynamics of a sudden quench have been explored and it has been shown that the irreversible entropy production provides signals of the presence of the equilibrium QPT [5], and also can be used to explain emergent phenomena such as the vanishing gap between ground and first excited energy levels in the thermodynamic limit [6]. Recently, the irreversible work was shown to faithfully capture the critical features even for so-called impurity QPTs [8].

In this work we add to this endeavor by studying the Lipkin-Meshkiv-Glick model [11]. The model has attracted substantial interest as it serves as the paradigmatic example of an infinite range interacting system. It can be solved in the thermodynamic limit and exhibits a complex phase diagram [12,13]. We will be interested in exploring how clear signatures of the equilibrium QPT is manifest in the dynamics when the model is quenched through its critical point. We remark that the evolution of this model through its QPT has been studied previously in Ref. [9] wherein the equal time-order parameter correlation function was examined and Ref. [10] where the adiabatic dynamics were explored. Our study is set apart from these as it seeks to establish a rigorous link between the thermodynamic quantities such as work and free energy, with the presence of the known QPT. By exploiting the time-dependent fidelity we show that the dynamics are manifestly different when the quench is restricted to a particular phase compared to when the system is quenched through the critical point. More interestingly, we explicitly show that while the average work performed on the system due to the quench is blind to the QPT, the free energy, and therefore the irreversible work, appears acutely sensitive, showing markedly different rates of change. Finally, we use the spectral function to further understand the fundamental excitations governing the dynamics of the system.

The remainder of the paper is organized as follows. In Sec. II we present the model and introduce the quantities that will be of interest to our analysis. In Sec. III we explore these quantities for quenching the system across the critical point and show that the time-dependent fidelity exhibits interesting features that reveal the critical nature of the system as well as showing that the irreversible work neatly reveals the QPT. Section IV assesses the spectral function for various quenches. Finally, in Sec. V we present our conclusions and some discussions on our results.

II. THE MODEL AND FIGURES OF MERIT

We consider the ferromagnetic spin-1/2 Lipkin-Meshkov-Glick (LMG) model in a transverse field,

$$\mathcal{H} = -\frac{1}{N}\left(\sum_{i<j}\sigma_x^i \otimes \sigma_x^j + \gamma\sigma_y^i \otimes \sigma_y^j\right) - h\sum_i \sigma_z^i, \quad (1)$$

with $\sigma_{x,y,z}$ the Pauli spin operators, h the magnetic field strength, and γ the anisotropy parameter (which we set to zero for simplicity in our simulations; however, we remark that qualitatively similar results can be obtained for any $0 \leqslant \gamma < 1$; cf. the Appendix). By considering the collective spin operators
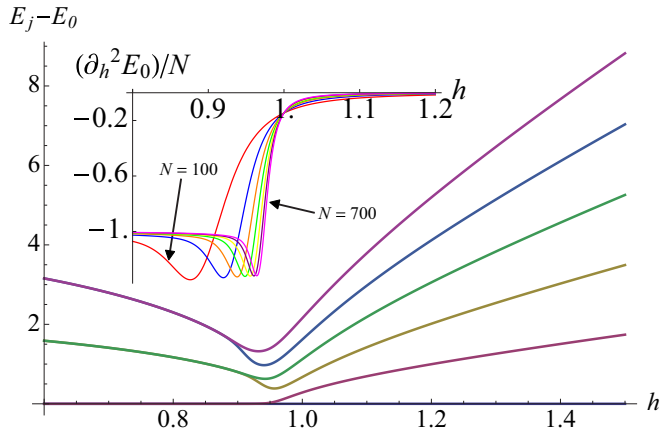
FIG. 1. Energy difference between the ground state and the first five excited states for $N = 400$ plotted against magnetic field strength h. Inset: Second derivative of the ground state energy per site for $N = 100$ (red) to 700 (magenta) in steps of 100.

$S_\alpha = \sum_i \sigma_\alpha^i / 2$ with $\alpha = \{x, y, z\}$, up to a constant energy shift, the model can be written as

$$\mathcal{H} = -\frac{2}{N}\left(S_x^2 + \gamma S_y^2\right) - 2h S_z. \quad (2)$$

In the following we work in the basis of maximum angular momentum (which is a constant of motion) and using the eigenstates of S_z we can diagonalize Eq. (2) to find the complete spectrum (see, e.g., Refs. [12,13] for further details). In Fig. 1 we show the energy difference between the five lowest excited states and the ground state against h. We see when $h > 1$ each energy level is distinct. As h is decreased the gap between the first excited state and the ground state closes, and similarly the energy gap between subsequent pairs of excited states also closes. However, an important remark, only when $N \to \infty$ does the gap vanish and all eigenstates become doubly degenerate. Hence, for any finite size there is a small difference between the ground and first excited states [10].

The inset in Fig. 1 shows the second derivative with respect to h of the ground state energy (per site) for system sizes ranging from $N = 100$ to $N = 700$. We see the emergence of a discontinuity appearing, thus signaling the known second order QPT at $h = 1$ [12]. We are interested in studying the dynamics when the ground state of one phase is evolved using the propagator of another. In what follows we will assume the system is initialized in the ground state of Eq. (2) corresponding to $h = h_i$. At time $t = 0$, we quench the field strength $h_i \to h_f$ and we evolve the initial state according to the new Hamiltonian $\mathcal{H}_f$, so that

$$|\psi(t)\rangle = e^{-i\mathcal{H}_f t}|\psi(0)\rangle. \quad (3)$$

Using this we can readily evaluate the time-dependent overlap

$$\mathcal{O} = \langle\psi(0)|\psi(t)\rangle. \quad (4)$$

This quantity will be central to our analysis as it allows us to access several important quantities that indicate that signatures of the equilibrium QPT are clearly manifest in the system's evolution.

A particularly important quantity will be the time-dependent fidelity (TDF)

$$\mathcal{L} = |\mathcal{O}|^2, \quad (5)$$

which quantifies how different the evolved state is compared to the initial one (we remark this quantity is sometimes referred to as the Loschmidt echo in the literature). The TDF has already proven to be a useful tool in studying critical dynamics [4]. Additionally, we can determine the average work due to the sudden quench [7,14]

$$\langle W \rangle = \sum_j \left(E_j^f - E_0^i\right)|\langle\psi_0^i|\psi_j^f\rangle|^2, \quad (6)$$

where E_j^f and $|\psi_j^f\rangle$ are the jth eigenenergy and eigenstate of the postquench Hamiltonian, and E_0^i and $|\psi_0^i\rangle$ are ground state energy and ground state for the initial Hamiltonian. The sudden nature of the quench drives the system out of equilibrium, and thus introduces a degree of irreversibility of the process. We can quantitatively define the irreversible work as [8,14,15]

$$\langle W_{\text{irr}} \rangle = \langle W \rangle - \Delta F, \quad (7)$$

where ΔF is the free energy difference. By considering closed dynamics, and since we assume our system begins in the ground state of the initial Hamiltonian, ΔF is simply given by the difference between the post- and prequench ground state energies, i.e., $\Delta F = E_0^f - E_0^i$.

III. QUENCH DYNAMICS ACROSS A QUANTUM CRITICAL POINT

A. From the paramagnetic to the ferromagnetic phase

We begin analyzing the case of quenching from the paramagnetic phase, setting $h_i = 1.5$. In this regime there is a significant energy difference between the ground state and the first excited state (cf. Fig. 1). In Fig. 2 we show the TDF for $N = 400$ and the thick black curve corresponds to
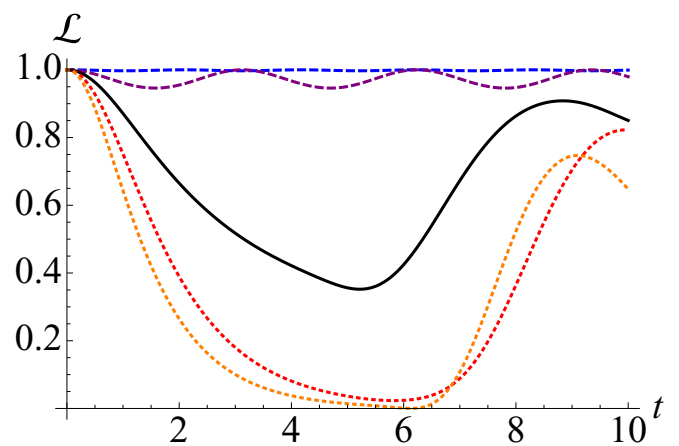

FIG. 2. Time-dependent fidelity for a quench from $h_i = 1.5$ to $h_f \in [0.6, 1.4]$ with $N = 400$. The dashed curves are for $h_f > 1$ with $h_f = 1.4$ (topmost, blue) and $h_f = 1.2$ (purple). The lowest two dotted curves are for $h_f < 1$ with $h_f = 0.8$ (red) and $h_f = 0.6$ (bottommost orange). The solid black curve is for $h_f = 1$.
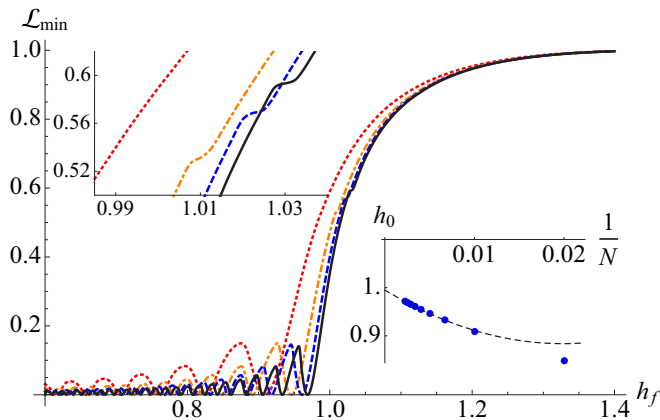
FIG. 3. Main panel: minimum value of TDF achieved for $t \in (0, 10)$ quenching the field from $h_i = 1.5$ to h_f. Each curve from left to right corresponds to an increasing size of $N = 100$ (dotted, red), 200 (dash-dotted, orange), 300 (dashed, blue), and 400 (solid, black). Upper left inset: Zoomed in cross section of the main panel. Lower right inset: Finite size scaling for the first value of field at which the $\mathcal{L}_{\min} = 0$.

$h_f = 1$. The dashed curves above this are for quenches that evolve the initial state taking a value of h_f that is still in the paramagnetic phase. We clearly see the regular oscillatory behavior persists even when quenching close to the critical point. When we evolve the state using $h_f < 1$ (lower dotted curves) we see the dynamics loses the clean periodic behavior, and dynamically the TDF no longer reaches unity. Additionally there is a significant decrease in the values of $\mathcal{L}$, even in some cases reaching exactly 0 indicating that the evolved state is orthogonal to the initial state.

The precise value at which the TDF reaches zero is examined in Fig. 3. We determine the minimum value of TDF observed, $\mathcal{L}_{\min}$, within the same time window for Fig. 2 against h_f. Clearly, when the quench is small, e.g., $h_f \in (1.2, 1.4)$ the minimum value of TDF is still quite large. As the strength of the quench is increased we find this minimum value decreases. Interestingly, $\mathcal{L}_{\min} = 0$ only when $h_f < 1$. We see an oscillatory behavior appearing; however, the amplitude of the oscillations is decreasing as the system size is increased. Furthermore, the first value of h_f where $\mathcal{L}_{\min} = 0$, denoted h_0, shifts closer to 1 as we increase the system size. Through a finite size scaling with a quadratic fit, the lower right inset shows that this accurately determines the critical point. Such a result is remarkable as it clearly indicates that the equilibrium QPT can be witnessed by the occurrence of dynamical orthogonality.

A final peculiarity appears in studying $\mathcal{L}_{\min}$; for $N > 200$ a kink appears close to the critical point as shown in the upper left inset of Fig. 3. However, as the system size is increased, this feature appears to move further from the critical value.

From Fig. 2 it is clear that for small quenches the system dynamically comes close to the initial state; however, for larger quenches this is no longer the case, indicating that a degree of irreversibility has been introduced into the system [16]. In Fig. 4(a) we examine the average work done, $\langle W \rangle$, against h_f. We find $\langle W \rangle$ is linearly dependent on the value of h_f; the larger the quench $h_i \to h_f$, the more work is done. In panels (b) and
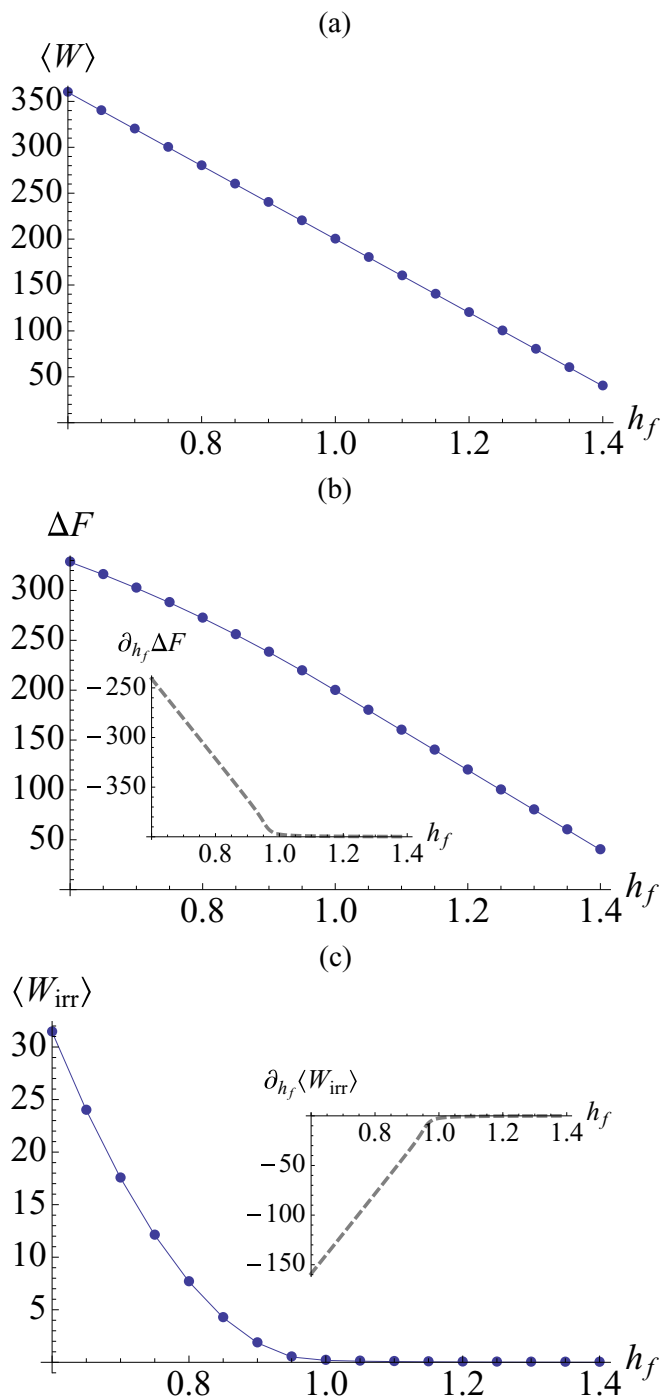

FIG. 4. (a) Average work, (b) Free energy. (c) Irreversible work for a quench from $h_i = 1.5$ to h_f. The insets of panels (b) and (c) are the first derivative of the functions. In all panels $N = 400$.

(c) we show the free energy and average irreversible work, respectively. Again, the free energy increases as we increase the size of the quench. However, in the inset we examine its rate of change, and we see for $h_f > 1$ this rate is linear, and there is a sudden change near $h_f \sim 1$. While it is not surprising that the free energy exhibits a nontrivial behavior as we go through the critical point in light of the fact that it is defined in terms of the ground state energy, the fact that

both $\langle W \rangle$ and ΔF are of the same order of magnitude leads to a trade-off between the two quantities. This has interesting consequences for the irreversibility of the process captured by $\langle W_{\text{irr}} \rangle$. Panel (c) shows that when the quench is small and confined to the same phase as the initial state, $\langle W_{\text{irr}} \rangle = 0$, indicating that the process is fully reversible, as confirmed by the behavior of the TDF which achieves values of unity during the dynamics. For large quenches, when the system is evolved according to a Hamiltonian in the ferromagnetic phase the average irreversible work becomes nonzero, and the degree of irreversibility grows as the magnitude of the quench increases.

B. From the ferromagnetic to the paramagnetic phase

We next consider the complementary case of beginning in the ferromagnetic phase, setting $h_i = 0.5$, and quenching to increasingly larger values of h_f. This scenario is markedly different from that of the previous section as the ground state is now *nearly* degenerate, i.e., there is an exponentially (in N) vanishing gap between the ground state and the first excited state (cf. Fig. 1). Therefore, by quenching the field strength and kicking the system out of equilibrium, it quickly becomes excited and occupies higher order states. In Fig. 5 we see this effect clearly; contrary to the previous section, we see even for moderately small quenches ($h_f \gtrsim 0.6$) the TDF reaches zero, indicating the evolved state is orthogonal. We further remark, the larger the system the smaller the quench required to achieve dynamical orthogonality; again this is a consequence of the fact that the energy gap between the ground and first excited states decreases with increasing N. Thus we cannot use the presence of orthogonality to witness signatures of the QPT in the dynamics. However, there is a clear qualitative difference appearing when the quench is near to or above the critical point. For quenches to $h_f \gtrsim 1.0$ the TDF evolves into fully orthogonal states for a period, before exhibiting short time revivals. The height of these peaks is steadily decreasing in the considered time window, and the width of the revivals broadens. This indicates a sizable increase in the irreversibility
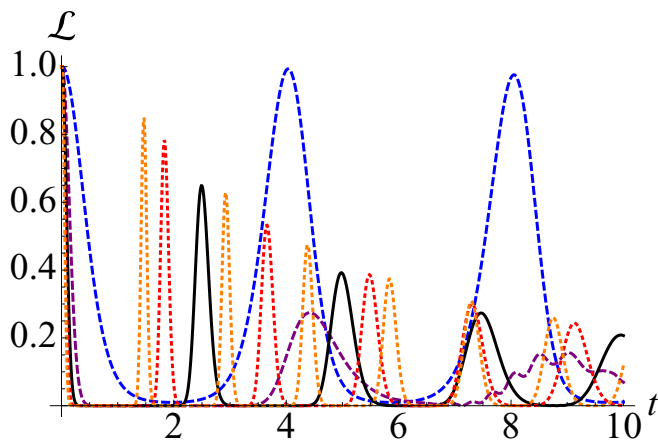
FIG. 5. Time-dependent fidelity for a quench from $h_i = 0.5$ to $h_f \in [0.6, 1.4]$ with $N = 400$. The dashed curves are for $h_f < 1$ with $h_f = 0.6$ (blue) and $h_f = 0.8$ (purple). The dotted curves are for $h_f > 1$ with $h_f = 1.2$ (red) and $h_f = 1.4$ (leftmost, orange). The solid black curve is for $h_f = 1$.
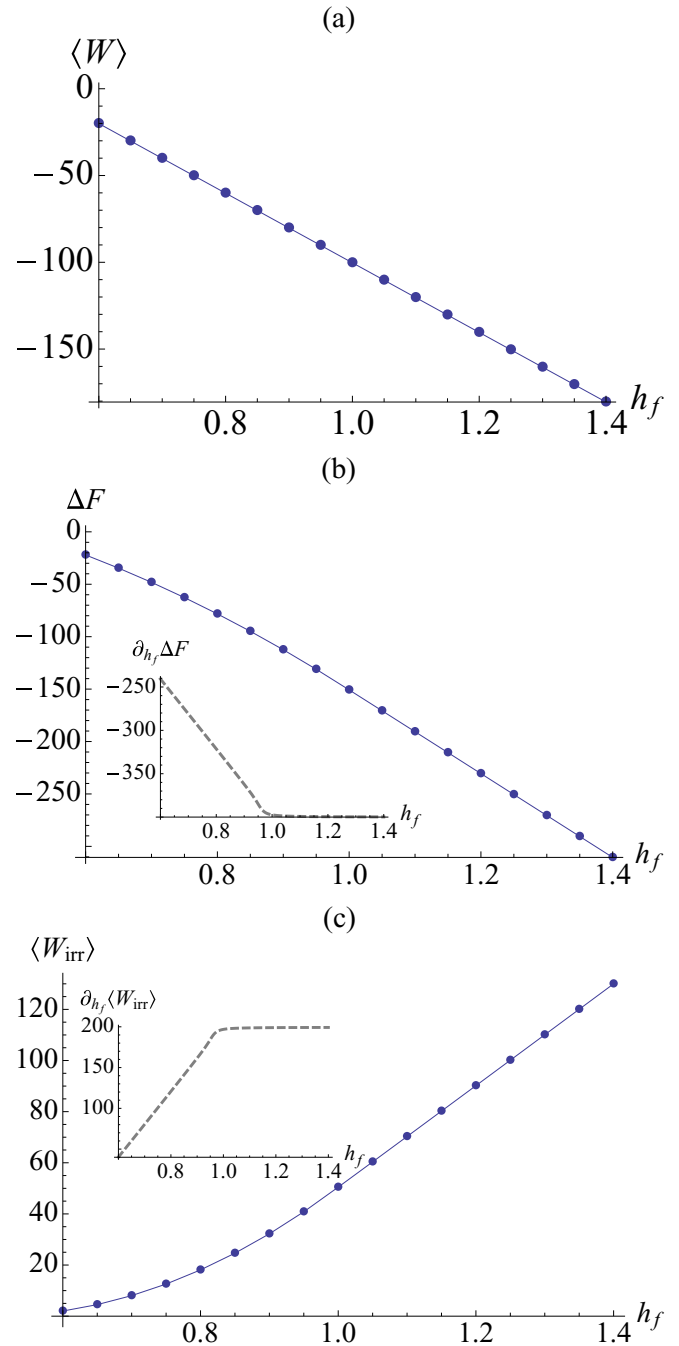
FIG. 6. (a) Average work. (b) Free energy. (c) Irreversible work for a quench from $h_i = 0.5$ to h_f. The insets of panels (b) and (c) are the first derivative of the functions. In all panels $N = 400$.

of the process when quenches into the paramagnetic phase are considered.

We confirm this behavior in Fig. 6. Panel (a) shows that the work is a linear function of the magnitude of the quench. However, the free energy and the irreversibility show the same qualitative behavior as shown in the previous section. Focusing on the average irreversible work, due to the vanishingly small gap between the ground and first excited states, we see even small quenches are accompanied by a degree of irreversibility, the rate of which grows as the magnitude of the quench is

increased. However, beyond the critical point we find the irreversibility grows linearly with the size of the quench.

IV. ANALYSIS BASED ON THE SPECTRAL FUNCTION

Finally we assess the behavior of the spectral function (SF)

$$A(\omega) = 2\text{Re} \int e^{i\omega t} O dt. \tag{8}$$

This gives insight into the fundamental excitations that are governing the evolution, and therefore serves as an informative tool in understanding the dynamics of the system [17]. In Fig. 7 we show the SFs for several quenches, both when the quench remains in the same initial phase, and when it is across the critical point. We restrict ourselves to $N = 50$ for simplicity, although qualitatively similar results hold for larger systems.

In panel (a) we assume the system begins in the paramagnetic phase fixing $h_i = 1.5$. For no quench, i.e., $h_f = 1.5$, the SF is a single peak exactly at the ground state energy. For small quenches staying within the paramagnetic phase we see the dynamics continues to be dictated only by the ground state, and this helps us understand why the process is fully reversible. As we approach the critical point a second peak appears, corresponding to the second excited state of the final Hamiltonian. When $h_f \sim 1$ this second peak becomes more prominent. Quenching into the ferromagnetic phase, we see significantly more levels enter into the dynamics of the system. Interestingly, although the ground state still contributes to the dynamics, higher excited states play a significantly more dominant role and this results in dynamical orthogonality. We remark, the model naturally has two distinct subspaces. Since our initial state is in the even excitation subspace, only the even states play a role in the dynamics.

In Fig. 7(b) we show the complementary analysis starting from the ferromagnetic phase $h_i = 0.5$ and quenching to larger values of the field. Again for reference, we see when no quench is performed, the SF is a single peak exactly at the ground state energy. However, now even for small quenches, $h_f = 0.6$, due to the significantly more dense energy spectrum in this phase, more (even excitation) levels play a role in dictating the dynamics of the system, and therefore the system almost immediately witnesses dynamic orthogonality. This in turns allows us to understand the significantly larger irreversibility of the process when quenching from the ferromagnetic into the paramagnetic phase. However, similar to the previous case,

when the quench remains in the same phase, i.e., $h_f \leqslant 1$ in this case, the ground state is still dominant. Quenching (near) to the critical point we see the SF spreads. For h_f deep in the paramagnetic phase the SF is very spread, and again since we have quenched through the QPT, we see higher excited states play the most dominant role in the dynamics.

V. DISCUSSIONS AND CONCLUSIONS

We have examined the dynamics arising by quenching the parameters of the many-body interacting LMG model. Starting from the ground state in a particular phase, and using the time-dependent fidelity, we have shown that manifestly different dynamics occur when the quench is restricted to the same initial phase compared to a quench through the critical point. By employing tools from quantum thermodynamics we have shown that the average work maintains a linear relationship with the magnitude of the quench, regardless if it is through the QPT or not. In contrast, the free energy and irreversible work are acutely sensitive to this difference. We find that quenching through the QPT leads to significant increases in the degree of irreversibility. This result can also help in understanding why controlling such many-body systems is so difficult through their QPTs [18–20], as it is this irreversibility that needs to be controlled. Starting from the paramagnetic phase, where there is a sizable energy gap between the ground and first excited states, we have shown the occurrence of dynamical orthogonality serves as a remarkable witness of criticality in the model. Furthermore, by examining the spectral function we have shown that when the quench is through the critical point, the fundamental excitations that govern the dynamics are no longer dictated primarily by the ground state, but in fact higher excited states play the most prominent role. It is important to remark that our analysis is restricted to zero temperature and a natural question arises regarding the situation for finite temperatures. In the case of a quench from the paramagnetic to the ferromagnetic phase, the presence of the energy gap means that for reasonably small temperatures (i.e., temperatures that fail to provide enough thermal energy to excite the first excited state) the results remain largely unaffected. Conversely, when quenching from the ferromagnetic phase, due to the vanishing energy gap even small temperatures lead to the first excited state becoming populated and thus can significantly change the dynamics. Our results highlight the interesting role the static properties of a many-body system can play in its dynamics.
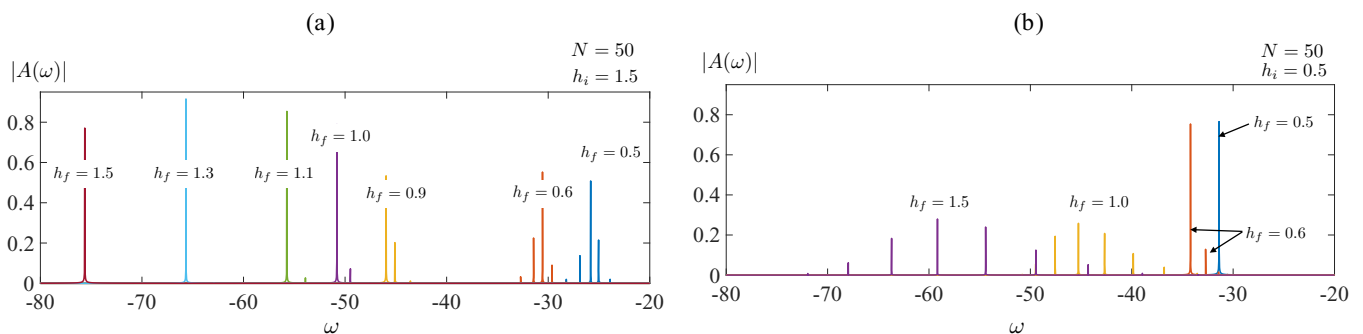

FIG. 7. Spectral function, Eq. (8), for $N = 50$. (a) Starting in the paramagnetic phase with $h_i = 1.5$ we examine the spectral function for several decreasing values of h_f. (b) Spectral function for several quenches to h_f starting from the ferromagnetic phase with $h_i = 0.5$.

Indeed, such a role has recently been explored in Refs. [21–25] where, for the Ising model and also long-range interacting spin models (including the LMG model), so-called "*dynamical quantum phase transitions*" have been characterized. Our results add further evidence that equilibrium QPTs are clearly manifest in nonequilibrium processes. Finally we remark that we expect similar features to appear for other one-dimensional spin systems such as the Ising model; however, we leave this for a future study.

APPENDIX: FINITE ANISOTROPY

Here we examine a finite value for the anisotropy parameter γ showing that qualitatively the results in the main text are unaffected. We choose $\gamma = 0.5$ and restrict ourselves to the case of quenching from the paramagnetic to the ferromagnetic phase for brevity. In Fig. 8 we show the TDF [panel (a)] and its corresponding dynamical minimum [panel (b)], which are complementary to the results shown in Figs. 2 and 3 of the main text. For finite γ the TDF exhibits the same change
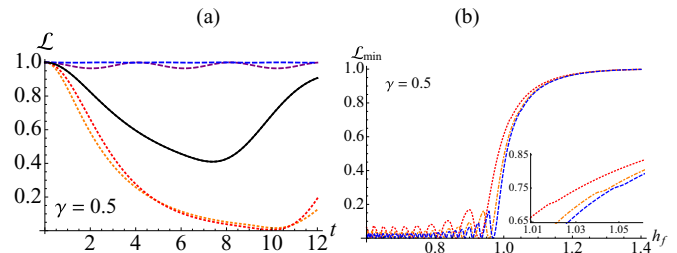

FIG. 8. Finite anisotropy, $\gamma = 0.5$: (a) Time-dependent fidelity for a quench from $h_i = 1.5$ to $h_f \in [0.6, 1.4]$ with $N = 300$. The dashed curves are for $h_f > 1$ with $h_f = 1.4$ (topmost, blue) and $h_f = 1.2$ (purple). The lowest two dotted curves are for $h_f < 1$ with $h_f = 0.8$ (red) and $h_f = 0.6$ (bottommost orange). The solid black curve is for $h_f = 1$. (b) Main panel: minimum value of TDF achieved for $t \in (0,12)$ quenching the field from $h_i = 1.5$ to h_f. Each curve from left to right corresponds to an increasing size of $N = 100$ (dotted, red), 200 (dash-dotted, orange), and 300 (dashed, blue). Inset: Zoomed in cross section of the main panel.

in behavior when the quench is through the critical point. However, by changing γ we are altering the energy of the system, and therefore this will be evidenced by a change in the frequency of the TDF. We clearly see this effect in panel (a) as the time at which the first minimum is achieved is larger than in the $\gamma = 0$ case. When the quench is restricted to a single phase the clean periodic behavior is maintained, while for values of $h < 1$ this feature is lost and we find that the system can become dynamically orthogonal. Interestingly, the "kink" in $\mathcal{L}_{\min}$ is also still present.

[1] S. Sachdev, *Quantum Phase Transitions* (Cambridge University Press, Cambridge, 2011).

[2] A. Osterloh, L. Amico, G. Falci, and R. Fazio, Scaling of entanglement close to a quantum phase transition, Nature (London) **416**, 608 (2002); T. J. Osborne and M. A. Nielsen, Entanglement in a simple quantum phase transition, Phys. Rev. A **66**, 032110 (2002); G. Vidal, J. I. Latorre, E. Rico, and A. Kitaev, Entanglement in Quantum Critical Phenomena, Phys. Rev. Lett. **90**, 227902 (2003).

[3] J. Goold, M. Huber, A. Riera, L. del Rio, and P. Skrzypczyk, The role of quantum information in thermodynamics—A topical review, J. Phys. A: Math. Theor. **49**, 143001 (2016).

[4] H. T. Quan, Z. Song, X. F. Liu, P. Zanardi, and C. P. Sun, Decay of Loschmidt Echo Enhanced by Quantum Criticality, Phys. Rev. Lett. **96**, 140604 (2006); D. Rossini, T. Calarco, V. Giovannetti, S. Montangero, and R. Fazio, Decoherence induced by interacting quantum spin baths, Phys. Rev. A **75**, 032333 (2007); G. De Chiara, T. Calarco, S. Fishman, and G. Morigi, Ramsey interferometry with a spin embedded in a Coulomb chain, *ibid.* **78**, 043414 (2008); F. A. Bayocboc, Jr. and F. N. C. Paraan, Exact work statistics of quantum quenches in the anisotropic *XY* model, Phys. Rev. E **92**, 032142 (2015); A. Bayat, S. Bose, H. Johannesson, and P. Sodano, Universal single-frequency oscillations in a quantum impurity system after a local quench, Phys. Rev. B **92**, 155141 (2015); R. Jafari, Quench dynamics and ground state fidelity of the one-dimensional extended quantum compass model in a transverse field, J. Phys. A: Math. Theor. **49**, 185004 (2016).

[5] A. Silva, Statistics of the Work Done on a Quantum Critical System by Quenching a Control Parameter, Phys. Rev. Lett. **101**, 120603 (2008).

[6] R. Dorner, J. Goold, C. Cormick, M. Paternostro, and V. Vedral, Emergent Thermodynamics in a Quenched Quantum Many-Body System, Phys. Rev. Lett. **109**, 160601 (2012).

[7] L. Fusco, S. Pigeon, T. J. G. Apollaro, A. Xuereb, L. Mazzola, M. Campisi, A. Ferraro, M. Paternostro, and G. De Chiara, Assessing the Nonequilibrium Thermodynamics in a Quenched Quantum Many-Body System Via Single Projective Measurements, Phys. Rev. X **4**, 031029 (2014).

[8] A. Bayat, T. J. G. Apollaro, S. Paganelli, G. De Chiara, H. Johannesson, S. Bose, and P. Sodano, Nonequilibrium critical scaling in quantum thermodynamics, Phys. Rev. B **93**, 201106(R) (2016).

[9] A. Das, K. Sengupta, D. Sen, and B. K. Chakrabarti, Infinite-range Ising ferromagnet in a time-dependent transverse magnetic field: Quench and ac dynamics near the quantum critical point, Phys. Rev. B **74**, 144423 (2006).

[10] T. Caneva, R. Fazio, and G. E. Santoro, Adiabatic quantum dynamics of the Lipkin-Meshkov-Glick model, Phys. Rev. B **78**, 104426 (2008).

[11] H. J. Lipkin, N. Meshkov, and A. J. Glick, Validity of many-body approximation methods for a solvable model: (I). Exact solutions and perturbation theory, Nucl. Phys. **62**, 188 (1965).

[12] P. Ribeiro, J. Vidal, and R. Mosseri, Thermodynamical Limit of the Lipkin-Meshkov-Glick Model, Phys. Rev. Lett. **99**, 050402 (2007); Exact spectrum of the Lipkin-Meshkov-Glick model

in the thermodynamic limit and finite-size corrections, Phys. Rev. E **78**, 021106 (2008); S. Dusuel and J. Vidal, Continuous unitary transformations and finite-size scaling exponents in the Lipkin-Meshkov-Glick model, Phys. Rev. B **71**, 224420 (2005).

[13] O. Castaños, R. López-Peña, J. G. Hirsch, and E. López-Moreno, Classical and quantum phase transitions in the Lipkin-Meshkov-Glick model, Phys. Rev. B **74**, 104118 (2006).

[14] S. Campbell, M. A. García-March, T. Fogarty, and Th. Busch, Quenching small quantum gases: Genesis of the orthogonality catastrophe, Phys. Rev. A **90**, 013617 (2014); M. A. García-March, T. Fogarty, S. Campbell, Th. Busch, and M. Paternostro, Non-equilibrium thermodynamics of harmonically trapped bosons, New J. Phys. **18**, 103035 (2016).

[15] S. Deffner and E. Lutz, Nonequilibrium Entropy Production for Open Quantum Systems, Phys. Rev. Lett. **107**, 140404 (2011).

[16] T. B. Batalhão, A. M. Souza, R. S. Sarthour, I. S. Oliveira, M. Paternostro, E. Lutz, and R. M. Serra, Irreversibility and the Arrow of Time in a Quenched Quantum System, Phys. Rev. Lett. **115**, 190601 (2015).

[17] L. Campos Venuti and P. Zanardi, Unitary equilibrations: Probability distribution of the Loschmidt echo, Phys. Rev. A **81**, 022113 (2010); T. Keller and T. Fogarty, Probing the out-of-equilibrium dynamics of two interacting atoms, arXiv:1608.03038.

[18] A. del Campo, M. M. Rams, and W. H. Zurek, Assisted Finite-Rate Adiabatic Passage Across a Quantum Critical Point: Exact Solution for the Quantum Ising Model, Phys. Rev. Lett. **109**, 115703 (2012).

[19] S. Campbell, G. De Chiara, M. Paternostro, G. M. Palma, and R. Fazio, Shortcut to Adiabaticity in the Lipkin-Meshkov-Glick Model, Phys. Rev. Lett. **114**, 177206 (2015).

[20] H. Saberi, T. Opatrný, K. Mølmer, and A. del Campo, Adiabatic tracking of quantum many-body dynamics, Phys. Rev. A **90**, 060301(R) (2014).

[21] M. Heyl, A. Polkovnikov, and S. Kehrein, Dynamical Quantum Phase Transitions in the Transverse-Field Ising Model, Phys. Rev. Lett. **110**, 135704 (2013).

[22] M. Heyl, Dynamical Quantum Phase Transitions in Systems with Broken-Symmetry Phases, Phys. Rev. Lett. **113**, 205701 (2014).

[23] M. Heyl, Scaling and Universality at Dynamical Quantum Phase Transitions, Phys. Rev. Lett. **115**, 140602 (2015).

[24] M. Heyl, Quenching a Quantum Critical State by the Order Parameter: Dynamical Quantum Phase Transitions and Quantum Speed Limits, arXiv:1608.06659.

[25] B. Zunkovic, M. Heyl, M. Knap, and A. Silva, Dynamical quantum phase transitions in Sspin chains with long-range interactions: Merging different concepts of non-equilibrium criticality, arXiv:1609.08482.