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Quantum phase transition of electron-hole liquid in coupled quantum wells

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Many-component electron-hole plasma is considered in the coupled quantum wells. The electrons are assumed to be localized in one quantum well (QW) while the holes are localized in the other QW. It is found that the homogeneous charge distribution within the QWs is unstable if the carrier density is sufficiently small. The instability results in the breakdown of the homogeneous charge distribution into two coexisting phases—a low-density phase and a high-density phase, which is *electron-hole liquid*. In turn, the homogeneous state of the electron-hole liquid is stable if the distance between the quantum wells ℓ is sufficiently small. However, as the distance ℓ increases and reaches a certain critical value $\ell_{\rm cr}$, the plasmon spectrum of the electron-hole liquid becomes unstable. Hereupon, a quantum phase transition occurs, resulting in the appearance of charge-density waves of *finite* amplitude in both quantum wells. Strong mass renormalization and the strong Z-factor renormalization are found for the electron-hole liquid as the quantum phase transition occurs.

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I. INTRODUCTION

For a long time the investigation of the two-dimensional (2D) strongly correlated electron system has attracted a great interest of both theorists and experimentalists (see, e.g., Refs. [1–11]). The electron-hole plasma (EHP) in coupled quantum wells (CQWs), where the electrons are localized in one quantum well and the holes are localized in the other quantum well, occupies a special place among the low-dimensional many-electron systems [12–35]. The interest in the CQW has greatly grown in recent years due to the increasing ability of manufacturing the high-quality quantum-well structures in which electrons and holes are confined in the different spatial regions between which the tunneling can be made negligible [34]. The EHP in the CQW is a nonequilibrium one. However, a probability of the electron-hole annihilation decreases exponentially as the spatial separation between the quantum wells increases. For this reason, the lifetime of both the electron and the holes τ_a increases exponentially as well [12]. At the same time, the equilibrium time $\tau_{\rm eq}$ within the electron subsystem or within the hole subsystem is determined by the interaction with phonons. Since $\tau_{\rm eq} \ll \tau_a$ one can consider the electrons and the holes as equilibrium subsystems for the time interval $\tau_{\rm eq} < t < \tau_a$.

Strong electron-hole correlations in such systems can result in the creation of excitons which are the bound electron-hole states. A possibility of the exciton Bose-Einstein condensation as well as the superfluidity and the superconductivity in the CQW are considered microscopically in Refs. [12,13]. The gas-liquid transition, the features of the liquid exciton phase, and the transition into the superfluid phase are studied as a function of the distance ℓ between the electron and the hole layers in the CQW in Ref. [14]. The strongly nonideal system of the excitons in the CQW considered as structureless bosons was considered in Refs. [24,25,28], the exciton correlation being taken into account in a semiphenomenological way. Correlations in electron-hole layers that result in a transition from the homogeneous liquid to a charge-density wave are considered in Refs. [26,27].

In the present paper we propose a microscopical description of strongly correlated *multicomponent* electron-hole liquid (EHL) which is a nonideal multicomponent plasma (EHP) in the CQW at zero temperature. The electron-hole system in many-valley semiconductors is a typical representative of the multicomponent EHP [29]. The number of different kinds of the electrons and of the holes ν is assumed to be large. As it was shown for the first time in Ref. [36], the multicomponent EHP in bulk semiconductors possesses unconventional Coulomb screening. Such a remarkable feature is connected with occurrence of characteristic momentum p_0 and characteristic energy ω_0 which far exceed the Fermi momentum p_F and the Fermi energy ε_F, respectively. The parameters p_0 and ω_0 determine the region of the plasmon spectrum which is mainly responsible for the unconventional Coulomb screening in the multicomponent EHP [36]. Such a property of the multicomponent EHP was employed for investigation of various features of the EHL [37,38]. The features inherent in the multicomponent EHP are also relevant for the multicomponent electron gas at the uniform positive background and for the EHP and electronic gas with strong anisotropic electron spectrum in the quasi-one-dimensional and quasi-two-dimensional system [36,39–43].

For the first time, a possibility of a bulk phase transition of the EHP into EHL was considered in Ref. [44], followed by numerous experimental and the theoretical investigations (see, e.g., Refs. [43,45]). This phase transition is a consequence of the instability of the neutral homogeneous EHP if it has a density smaller than a certain critical value n_c. The instability results in the appearance of drops of the EHL with the

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equilibrium density $n_{\mathrm{eq}} > n_c$. It is remarkable that, if the bulk EHP is a multicomponent one, both n_c and n_{eq} are completely determined by the number of the component ν [36].

The energy of the ground state and the chemical potential of the multicomponent EHP in the CQW is calculated in Refs. [46–48] as a function of the electron density n (which is the same for the holes), the interplane distance ℓ, and the number of the components $\nu \gg 1$. The critical concentration $n_c = n_c(\ell,\nu)$ is found such that, for the concentration $n < n_c$, a homogeneous in-plane charge distribution is unstable. Such instability results in the breakdown of the homogeneous charge distribution into the rare and the dense phases, the last one being the EHL with the equilibrium density $n_{\mathrm{eq}} = n_{\mathrm{eq}}(\ell,\nu) > n_c$, $n_{\mathrm{eq}} \sim \nu^{3/2}$ if $\ell \ll \nu^{-1} \ll 1$, and $n_{\mathrm{eq}} \sim \ell^{-3/2}$ if $\ell \gg \nu^{-1}$ [46].

It is shown in Ref. [46] that, for the density $n = n_{\mathrm{eq}}$, the in-plane exciton radius is of the order of the average distance between the charge carriers within the quantum well $n_{\mathrm{eq}}^{-1/2}$. This fact does not evidence in favor of an existence of the exciton as a structureless particle in the CQW. Instead, strong electron-hole correlations near the Fermi surface remain. These correlations, in turn, can result in unconventional Coulomb screening (inherent in the multicomponent EHL), and in superconductivity induced due to the Coulomb interaction alone [37,38].

In the present paper we investigate the features of the EHL in the CQW, the existence of which is predicted in Refs. [46–48]. Like these papers, the system of units is used in which the effective electron charge $e^* = e/\sqrt{\kappa_0}$ (κ_0 is the static permittivity), and the bare electron mass m and the Planck constant $\hbar$ are as follows: $e^* = m = \hbar = 1$. For such a system of units, the effective Bohr radius $a_B = \hbar^2/me^{*2} = 1$, which is taken as a length unit. For the sake of simplicity, we assume that the masses of electron and hole are equal. As is shown in Ref. [47], this assumption does not influence the result qualitatively but it simplifies the calculations significantly. According to Ref. [48], the plasmon spectrum of the EHL is stable for $n = n_{\mathrm{eq}}$ if $\ell \ll 1$. In this case, as it is shown in the present paper, both the electron mass and the Z factor for the Green's function experience negligible renormalization induced by the Coulomb interaction. However, as the distance ℓ increases and reaches a certain critical value ℓ_{cr}, the plasmon spectrum of the electron-hole liquid becomes unstable. Hereupon, a quantum phase transition occurs, resulting in the appearance of the charge-density waves of *finite* amplitude in both quantum wells. Strong mass renormalization and strong Z-factor renormalization are found for the electron-hole liquid as the quantum phase transition occurs.

All the results obtained in Refs. [46–48] as well as in the present paper are based on the selection of the diagrams in the small parameters $1/\nu$. However, the results obtained seem to be qualitatively valid if the parameter ν is not too large. A relationship to the experiments available is considered.

II. GREEN'S FUNCTION FOR THE MULTICOMPONENT ELECTRON-HOLE PLASMA

The multicomponent EHP in the CQW is described with the following Hamiltonian of the system $\widehat{H} = \widehat{H}_0 + \widehat{V}_{\mathrm{int}}$:

$$\widehat{H}_0 = \sum_{\alpha\sigma\mathbf{k}} \frac{k^2}{2} a^+_{\alpha\sigma}(\mathbf{k}) a_{\alpha\sigma}(\mathbf{k}), \widehat{V}_{\mathrm{int}} = \frac{1}{2S} \sum_{\substack{\alpha\alpha'\sigma\sigma' \\ \mathbf{k}\mathbf{k}'\mathbf{q}}} V_{\alpha\alpha'}(\mathbf{q}) \times a^+_{\alpha\sigma}(\mathbf{k}) a^+_{\alpha'\sigma'}(\mathbf{k}') a_{\alpha'\sigma'}(\mathbf{k}'-\mathbf{q}) a_{\alpha\sigma}(\mathbf{k}+\mathbf{q}). \quad (1)$$

Here $\alpha = e$ stands for the electrons, while $\alpha = h$ stands for the holes; $\sigma = 1,\ldots,\nu$ labels the kind of the electron or the hole; $a^+_{\alpha\sigma}(\mathbf{k})$ and $a_{\alpha\sigma}(\mathbf{k})$ are the electron or the hole creation and annihilation operators; $\mathbf{k}$, $\mathbf{k}'$, $\mathbf{q}$ are the 2D momenta; S is the area of the QWs. The Coulomb interaction $V_{\alpha\alpha'}$ is assumed to be independent of the kind of the particle, i.e., of the subscripts σ, and

$$V_{\alpha\alpha'}(\mathbf{q}) = \begin{cases} V_{ee}(q) = V_{hh}(q) = V = \frac{2\pi}{q}, & \alpha = \alpha'; \\ V_{eh}(q) = V' = -\frac{2\pi}{q} e^{-q\ell}, & \alpha \neq \alpha' \end{cases}. \quad (2)$$

A single-particle Green's function $G_{\alpha\sigma}(K)$ depends neither on the subscript α nor on the subscript σ. Then,

$$G_{\alpha\sigma}(K) = G(K) = [i\omega + \mu - k^2/2 - \Sigma(K)]^{-1}, \quad (3)$$

where μ is a chemical potential, $\Sigma(K)$ is a self-energy part (SEP), $K = (i\omega,\mathbf{k})$, ω is the Matzubara frequency, and $\mathbf{k}$ is a 2D momentum. Like Refs. [46–48], the calculation of the Green's function is based on the selection of the diagram in the small parameter $1/\nu \ll 1$. Let us represent the SEP as $\Sigma(K) = \Sigma_H + \Sigma^{(c)}(K)$, where $\Sigma_H = 2\pi n\ell$ is the K-independent Hartree contribution, and $\Sigma^{(c)}(K)$ involves both the exchange and the correlation contributions. Selecting the main sequence of the diagram in the parameter $1/\nu$ one obtains for the SEP $\Sigma^{(c)}(K)$ [46]

$$\Sigma^{(c)}(P) = -\int \frac{d\omega d^2k}{(2\pi)^3} U(K) G^{(0)}(\varepsilon+\omega,\mathbf{p}+\mathbf{k}). \quad (4)$$

Here the Green's function is $G^{(0)}(K) = (i\omega + p_F^2/2 - k^2/2)^{-1}$, $p_F = 2\pi^{1/2}(n/\nu)^{1/2}$ is the Fermi momentum, $\varepsilon_F = 2\pi n/\nu$ is the Fermi energy, and n is the total concentration (the parameters p_F,ε_F, and n are the same for the electrons and the holes). The effective interaction reads

$$U(K) = \frac{V(\mathbf{k})}{1 - \chi(\ell) V(\mathbf{k}) \Pi_0(K)}, \quad (5)$$

where the polarization operator is given by

$$\Pi_0(K) = \nu \int \frac{d\omega_1 d^2k_1}{(2\pi)^3} G^{(0)}(K+K_1) G^{(0)}(K_1).$$

The function $\chi(\ell)$ is a monotonic, continuous, and slowly varying one obeying the condition $\chi(\ell) = 2$ for $\ell \ll 1$ and $\chi(\ell) = 1$ for $\ell \gg 1$ [46].

We are interested in the $\Sigma^{(c)}(P)$ for the momenta and the frequencies which are close to the Fermi ones. On the other hand, as shown in Ref. [46], the main contribution into integral (4) originates from the region around $\omega \sim \omega_0 = n^{2/3}/2 \gg \varepsilon_F$ and $k \sim k_0 = n^{2/3} \gg p_F$. To calculate integral (4), one should take into account that for $\nu \gg 1$ the polarization operator $\Pi_0(K)$ can be substituted with its asymptotics for the momentum $k \gg p_F$ and the frequency $\omega \gg \varepsilon_F$ as

$$\Pi_0(K) = -nk^2/[\omega^2 + \left(k^2/2\right)^2]. \tag{6}$$

Integral (4) is readily calculated by substitution $\mathbf{k} \to (\chi n)^{1/3}\mathbf{k}$, $\omega \to (\chi n)^{2/3}\omega$. Then, the calculation of the $\Sigma^{(c)}(P)$ for $p \ll k_0, \varepsilon \ll k_0^2$ results in the expression for the $\Sigma(P)$ in the form

$$\Sigma(P) = \Sigma(0,p_F) - i\varepsilon I_Z + \xi_p^{(0)} I_m, \tag{7}$$

$$\Sigma(0,p_F) = 2\pi n\ell - C(\chi n)^{1/3}, \xi_p^{(0)} = \left(p^2 - p_F^2\right)/2 \tag{8}$$

where $I_Z = C_Z(\chi n)^{-1/3}$, $I_m = C_m(\chi n)^{-1/3}$, $\xi_p^{(0)} = (p^2 - p_F^2)/2$. The numerical calculation of the constants entering the $\Sigma(P)$ gives $C \approx 1.3$, $C_Z \approx 7.6$, and $C_m = 0.4$.

The chemical potential μ is determined via $\Sigma(0,p_F)$ by the well-known relation

$$\mu = p_F^2/2 + \Sigma(0,p_F) = 2\pi n(1/\nu + \ell) - C(\chi n)^{1/3}, \tag{9}$$

and Green's function (3) reads

$$G(P) = G(i\varepsilon,p) = \frac{Z}{i\varepsilon - \xi_p^*};\ \xi_p^* = \frac{p^2 - p_F^2}{2m^*}; \tag{10}$$

$$Z = \frac{1}{1+I_z};\quad \frac{1}{m^*} = \frac{1+I_m}{1+I_z} < 1. \tag{11}$$

For densities $n < n_c = [\frac{C\chi^{1/3}}{6\pi(\ell+1/\nu)}]^{3/2}$, one has $\partial\mu/\partial n < 0$. This fact means an instability of the homogeneous EHP for sufficiently small densities. Then, chemical potential (9) determines the energy per particle:

$$E = \pi(n/\nu) + \pi n\ell - \tfrac{3}{4}C(\chi n)^{1/3}. \tag{12}$$

This expression has a minimum for the density

$$n_{\rm eq} = \left[\frac{C\chi^{1/3}}{4\pi(\ell + 1/\nu)}\right]^{3/2} > n_c. \tag{13}$$

The minimum corresponds to the vanishing pressure. For this reason, the equilibrium state of the EHP at the density $n = n_{\rm eq}$ is just the EHL.

Let us consider how the Coulomb interaction affects the effective mass m^* of the quasiparticle and the Z factor of the renormalized Green's function for the EHL, i.e., for the density $n = n_{\rm eq}$. Let $\ell \ll 1$. Then $n_{\rm eq} \gg 1$. It follows from Eq. (11) that $\Delta m = m^* - m \ll m$ and $Z = 1 - \delta, \delta \ll 1$. In the opposite case $\ell \gtrsim 1$, one has $n_{\rm eq} \sim l^{-3/2} \lesssim 1$. Then, according to Eq. (11), $\Delta m = m^* - m \sim m$ and $Z = 1 - \delta, \delta \sim 1$ and the renormalization is significant. Thus, the renormalization induced by the Coulomb interaction is insignificant for $\ell \ll 1$ and is visible for $\ell \gtrsim 1$.

III. VERTEX PART FOR THE MULTICOMPONENT ELECTRON-HOLE PLASMA

To investigate the plasmon spectrum of the EHL and its stability, let us calculate the vertex part $\Gamma_{\alpha\sigma,\alpha_1\sigma_1;\alpha'\sigma',\alpha_1'\sigma_1'}$ with two input fermion ends $\alpha\sigma,\alpha_1\sigma_1$ and two output fermion ends $\alpha'\sigma',\alpha_1'\sigma_1'$. In what follows, for brevity, we will use the notation $\Gamma_{\alpha\beta;\alpha'\beta'}$ instead of $\Gamma_{\alpha\sigma,\alpha_1\sigma_1;\alpha'\sigma',\alpha_1'\sigma_1'}$. Thus, we omit the subscripts $\sigma,\sigma_1\sigma',\sigma_1'$. In particular, the notation α, in fact, implies $\alpha\sigma$. This convention reflects the fact that the value of the vertex part does not depend on the value of the subscripts $\sigma,\sigma_1\sigma',\sigma_1'$ at all. However, the omitted subscripts should be taken into account when the summation over such subscripts is necessary.

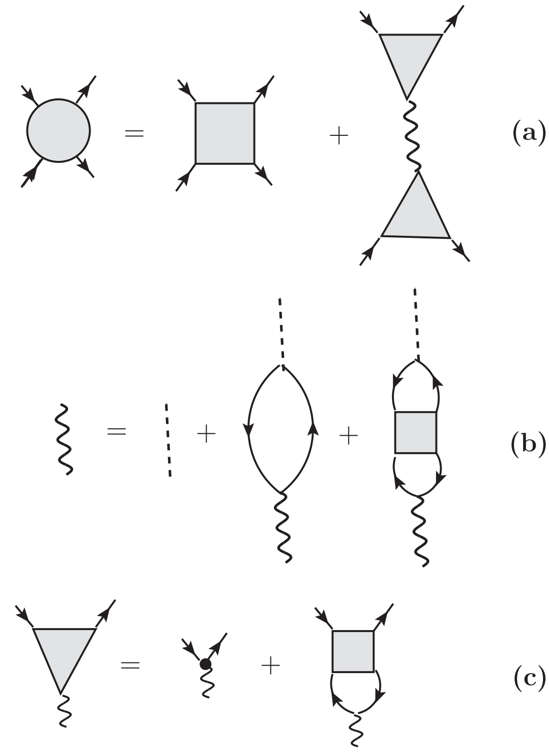

FIG. 1. Exact diagrammatic representation for the vertex part.

The exact diagrammatic representation for the vertex function is given in Fig. 1(a). In this figure the black circle with two input ends and two output ends represents the exact vertex part $\Gamma_{\alpha\beta;\alpha'\beta'}$; the black square with two input ends and two output ends represents the *irreducible* vertex part $\overline{\Gamma}_{\alpha\beta;\alpha'\beta'}$ [any diagram is called an irreducible one if it cannot be cut across one interaction (dotted) line resulting in two uncoupled parts]; the black triangular with one input end, one output end, and one interaction end represents the *irreducible* vertex part $\overline{\Gamma}^3_{\alpha\alpha';\delta}$; the wavy line denotes the effective Coulomb interaction $U_{\delta\delta'}$. In turn, the effective interaction $U_{\delta\delta'}$ is determined by the self-consistent diagrammatic equation in Fig. 1(b), in which the dotted lines denote bare Coulomb interaction (2); the inner lines with arrows denote the exact fermion Green's functions. The diagrammatic equation in Fig. 1(c) is an exact relation between the irreducible vertex parts $\overline{\Gamma}_{\alpha\beta;\alpha'\beta'}$ and $\overline{\Gamma}^3_{\alpha\alpha';\delta}$.

So the analytic representation of the exact diagrammatic equation in Fig. 1(a) is given by

$$\Gamma_{\alpha\beta;\alpha'\beta'} = \overline{\Gamma}_{\alpha\beta;\alpha'\beta'} + \sum_{\delta_1;\delta_2} \overline{\Gamma}^{(3)}_{\alpha;\alpha';\delta_1} \cdot U_{\delta_1;\delta_2} \cdot \overline{\Gamma}^{(3)}_{\beta;\beta';\delta_2}. \tag{14}$$

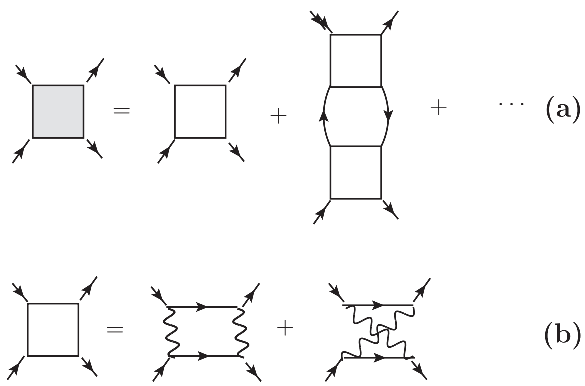

FIG. 2. (a) The main diagrammatic sequence for the $\overline{\Gamma}_{\alpha\beta;\alpha'\beta'}$. (b) The bare irreducible vertex function $\gamma_{\alpha\alpha'}$.

In Fig. 1(b) the self-consistent diagrammatic equation reads the effective interaction $U_{\delta_1;\delta_2}$, which enters Eq. (14). Thus,

$$U_{\delta_1;\delta_2}(K) = V_{\delta_1;\delta_2}(\mathbf{k}) + \sum_{\rho,\eta} V_{\delta_1;\rho}(\mathbf{k})\Pi_{\rho;\eta}(K)U_{\eta;\delta_2}(K). \quad (15)$$

Here $\Pi_{\rho;\eta}(K)$ is the exact polarization operator which, according to Fig. 1(b), reads

$$\Pi_{\rho;\eta}(K) = \Pi_0^*(K)\delta_{\rho\eta} + \Pi_{\rho;\eta}^{(c)}(K). \quad (16)$$

In this equation, the polarization operator $\Pi_0^*(K)$ is determined as follows:

$$\Pi_0^*(K) = \nu \int \frac{d\omega_1 d^2k_1}{(2\pi)^3} G(K+K_1)G(K_1), \quad (17)$$

while the $\Pi_{\rho;\eta}^{(c)}(K)$ is determined via the vertex $\overline{\Gamma}_{\alpha\beta;\alpha'\beta'}$ as is shown in Fig. 1(b). The asymptotic expression for the $\Pi_0^*(K)$ is

$$\Pi_0^*(K) = -\nu \frac{Z^2}{2\pi} \frac{(kv_F)^2/2}{\omega^2 + (kv_F)^2/2}, \quad k \ll p_F, \quad (18)$$

$$\Pi_0^*(K) = -n \frac{2Z^2\xi_k^*}{\omega^2 + (\xi_k^*)^2}, \quad p_F \ll k \ll k_0 \sim n^{1/3}. \quad (19)$$

The main diagrammatic sequence for the $\overline{\Gamma}_{\alpha\beta;\alpha'\beta'}$ in the parameter $1/\nu \ll 1$ is shown in Fig. 2(a). Let $\gamma_{\alpha\beta;\alpha'\beta'}$ (the light square) be the irreducible bare vertex part which generates the main diagrammatic sequence for the vertex part $\overline{\Gamma}_{\alpha\beta;\alpha'\beta'}$. One can show that the $\gamma_{\alpha\beta;\alpha'\beta'}$ is composed of two diagrams shown in Fig. 2(b).

Simple reasoning reveals that $\gamma_{\alpha\beta;\alpha'\beta'}$ may be represented as follows: $\gamma_{\alpha\beta;\alpha'\beta'} = \gamma_{\alpha\beta}\delta_{\alpha\alpha'}\delta_{\beta\beta'}$. Then, e.g.,

$$\begin{aligned}\gamma_{ee} = \gamma_{hh} &= \gamma; \gamma(p_1,p_2,q)\\ &= -\int \frac{d^3p}{(2\pi)^3} U(p)U(k-p)G(p_1-p)\\ &\quad \times [G(p_2+p) + G(p_2+k-p)]. \end{aligned} \quad (20)$$

The effective interaction $U(K)$ in Eq. (20) is given by Eq. (5). An analysis of the integrand in Eq. (20) [which is similar to the analysis of the integrand in Eq. (4)] shows that the main contribution into integrals (20) originates from the region $k \sim k_0 \sim n^{1/3} \gg p_F$ and $\omega \sim \omega_0 \sim k_0^2 \gg \varepsilon_F$. For this reason, if the components of the external momenta p_1,p_2,q are much smaller than k_0,ω_0, then one can neglect p_1,p_2,q in the integrand. Therefore, the vertex part $\gamma(p_1,p_2,q)$ does not depend on the p_1,p_2,q. After the simple transformation one obtains

$$\gamma = -\frac{1}{2n^2}\int \frac{d^2k d\omega}{(2\pi)^3} U(K)U(-K)[\Pi_0(K)]^2. \quad (21)$$

To calculate integral (21), let us take into account that, for $\nu \gg 1$, Eq. (6) can be used for the polarization operator $\Pi_0(K)$ for large transfer momentum $k \gg p_F$. Then, the integral is readily calculated by substitution $\mathbf{k} \to (\chi n)^{1/3}\mathbf{k}$, $\omega \to (\chi n)^{2/3}\omega$. As a result, one obtains

$$\gamma = -C_\gamma \frac{1}{(\chi n)^{2/3}}, C_\gamma \approx 0.4. \quad (22)$$

Similarly, for small external momenta one has

$$\gamma_{eh} = \gamma'; \gamma' = -\frac{1}{2n^2}\int \frac{d^2k d\omega}{(2\pi)^3} U'(K)U'(-K)[\Pi_0(K)]^2. \quad (23)$$

Here $U'(K)$ is the effective electron-hole interaction. As is mentioned above, integrals like Eq. (23) are determined by the region $k \sim k_0 \sim n^{1/3} \gg p_F$ and $\omega \sim \omega_0 \sim k_0^2 \gg \varepsilon_F$. For this region, the integrand is proportional to $U'(K) \sim V(k_0) \sim \exp(-k_0\ell)$. In what follows, we are interested in the densities $n \sim n_{\text{eq}}$ [see Eq. (13)]. In this case for $\ell \ll 1$ the parameter $k_0\ell \ll 1$ and one has $U'(K) = -U(K)$. In the opposite case $\ell \gg 1$ one has $k_0\ell \sim \ell^{1/2} \gg 1$ and the integrand in Eq. (23) vanishes. Thus, we have

$$\gamma' = \gamma \text{ for } \ell \ll 1; \quad \gamma' = 0 \text{ for } \ell \gg 1. \quad (24)$$

Since the bare vertex parts γ and γ' are constant, the irreducible vertex $\overline{\Gamma}_{\alpha\beta;\alpha'\beta'}$ depends only on the momentum transfer and, thus, $\overline{\Gamma}_{\alpha\beta;\alpha'\beta'} = \overline{\Gamma}_{\alpha\beta;\alpha'\beta'}(k,\omega)$. The main sequence of the diagram in the parameter $1/\nu$ for $\overline{\Gamma}_{\alpha\beta;\alpha'\beta'}$ [see Fig. 2(a)] is easily summed for $k \ll k_0$ and $\omega \ll \omega_0$. Taking into account that the vertex part can be represented in the form $\overline{\Gamma}_{\alpha\beta;\alpha'\beta'} = \overline{\Gamma}_{\alpha\beta}\delta_{\alpha\alpha'}\delta_{\beta\beta'}$, where $\overline{\Gamma}_{ee}(K) = \overline{\Gamma}_{hh}(K) = \overline{\Gamma}(K)$, one has

$$\overline{\Gamma}_{\alpha\beta;\alpha'\beta'} = \overline{\Gamma}_{\alpha\beta}(K)\delta_{\alpha\alpha'}\delta_{\beta\beta'}, K = (i\omega,\mathbf{k}), \quad (25)$$

$$\begin{aligned}\overline{\Gamma}_{ee}(K) &= \overline{\Gamma}_{hh}(K) = \overline{\Gamma}(K)\\ &= \frac{\gamma - (\gamma^2 - \gamma'^2)[\Pi_0^*(K)]^2}{1 - 2\Pi_0^*(K)\gamma + (\gamma^2 - \gamma'^2)[\Pi_0^*(K)]^2}, \end{aligned} \quad (26)$$

$$\overline{\Gamma}_{eh}(K) = \overline{\Gamma}'(K) = \frac{\gamma'}{1 - 2\Pi_0^*(K)\gamma + (\gamma^2 - \gamma'^2)[\Pi_0^*(K)]^2}. \quad (27)$$

These expressions are used to calculate the correlation part of the polarization operator $\Pi_{\rho;\eta}^{(c)}(K)$ [see Eq. (16)] and the vertex $\overline{\Gamma}^{(3)}_{\alpha,\alpha';\delta}$ [see Fig. 1(c)]. As a result, one obtains

$$\Pi_{\rho\eta}^{(c)}(K) = \Pi_0^*(K)\overline{\Gamma}_{\rho\eta}\Pi_0^*(K), \quad (28)$$

$$\overline{\Gamma}^{(3)}_{\alpha,\alpha';\delta} = \overline{\Gamma}^{(3)}_{\alpha;\delta}\delta_{\alpha\alpha'}, \overline{\Gamma}^{(3)}_{\alpha;\delta}(K) = \delta_{\alpha\delta} + \overline{\Gamma}_{\alpha;\delta}(K)\Pi_0^*(K). \quad (29)$$

Substituting Eqs. (25), (28), and (29) into Eq. (14) for the vertex $\Gamma_{\alpha\beta;\alpha'\beta'}$, one has $\Gamma_{\alpha\beta;\alpha'\beta'} = \Gamma_{\alpha\beta}\delta_{\alpha;\alpha'}\delta_{\beta;\beta'}$, where

$$\Gamma_{ee} = \Gamma_{hh} = \Gamma(K) = \frac{(V+\gamma) - [(V+\gamma)^2 - (V'+\gamma')^2]\Pi_0^*}{[1-(V+V'+\gamma+\gamma')\Pi_0^*][1-(V-V'+\gamma-\gamma')\Pi_0^*]}, \tag{30}$$

$$\Gamma_{eh} = \Gamma'(K) = \frac{V'+\gamma'}{[1-(V+V'+\gamma+\gamma')\Pi_0^*][1-(V-V'+\gamma-\gamma')\Pi_0^*]}. \tag{31}$$

IV. PLASMON SPECTRUM AND INSTABILITY

Let us investigate the plasmon spectrum of the EHP in the CQW which is determined by poles of vertex parts Eqs. (30) and (31). First, let us consider the case $l \ll 1$. Then, it follows from Eqs. (30) and (31) that

$$\Gamma = \frac{(V-\pi\ell)}{[1-2(V-\pi\ell)\Pi_0^*]} + \frac{(\gamma+\pi\ell)}{[1-2(\gamma+\pi\ell)\Pi_0^*]}, \tag{32}$$

$$\Gamma' = \frac{-(V-\pi\ell)}{[1-2(V-\pi\ell)\Pi_0^*]} + \frac{(\gamma+\pi\ell)}{[1-2(\gamma+\pi\ell)\Pi_0^*]}. \tag{33}$$

Let us substitute Eq. (18) into Eqs. (32) and (33) and replace the Matzubara frequency $i\omega$ by the real frequency ω. The pole of the vertex parts Γ and Γ' is given by the second terms in Eq. (32) or Eq. (33). Then, the plasmon spectrum is determined by the equation

$$1 + 2(\gamma+\pi\ell)\frac{\nu}{2\pi}\frac{(kv_F)^2/2}{-\omega^2+(kv_F)^2/2} = 0. \tag{34}$$

The spectrum is stable if ω, which obeys Eq. (34), is real. This takes place if

$$n > \left[\frac{C_\gamma}{\chi^{2/3}\left(\frac{1}{\nu}+l\right)}\right]^{3/2}. \tag{35}$$

Thus, if $n > n_{\rm cr} = [\frac{C\chi^{1/3}}{6\pi(1/\nu+\ell)}]^{3/2}$, the plasmon spectrum is stable. In the opposite case, $n < n_{\rm cr}$, the pole takes place for imaginary ω. This denotes an instability of the plasmon spectrum. This instability just corresponds to the thermodynamic instability of the homogeneous EHP for the densities $n < n_{\rm cr}$ for which one has $\partial\mu/\partial n < 0$ [36]. Let us note that for $\ell \ll 1$ the plasmon spectrum remains stable for the equilibrium EHL which has the density $n_{\rm eq} = [\frac{C\varkappa^{1/3}}{4\pi(1/\nu+\ell)}]^{3/2} > n_c$.

Now let us investigate the plasmon spectrum for the case $\ell \gg 1$. Let us consider momenta and frequencies which obey the limitations $(1/\ell) \ll k \ll k_0$, $\omega \ll \omega_0$. In this case $V'(k)$ vanishes. Also, according to Eq. (24), $\gamma' = 0$. Therefore, it follows from Eq. (31) that $\Gamma' = 0$. So in the case $\ell \gg 1$ Eq. (30) reads

$$\Gamma(K) = \frac{V(k)+\gamma}{1-\Pi_0^*(K)[V(k)+\gamma]}. \tag{36}$$

Let us substitute V, Π_0^*, γ by Eqs. (2), (19), and (22) for the momentums $p_F \ll k \ll k_0$ and change $i\omega$ by ω As a result, one obtains

$$\Gamma(K) = \frac{\left(\frac{2\pi}{k} - \frac{C_\gamma}{n^{2/3}}\right)\left[\omega^2 - \left(\xi_k^*\right)^2\right]}{\omega^2 - \xi_k^*\left[\xi_k^* + nZ^2\left(\frac{2\pi}{k} - \frac{C_\gamma}{n^{2/3}}\right)\right]}. \tag{37}$$

A pole of the $\Gamma(K)$ determines the plasmon spectrum and exists for the frequencies

$$\omega_p^2(k) = \xi_k^*\left[\xi_k^* + nZ^2\left(\frac{2\pi}{k} - \frac{C_\gamma}{n^{2/3}}\right)\right]. \tag{38}$$

Let us investigate a behavior of the plasmon spectrum for the EHL of a density $n \sim n_{\rm eq}$. One can easily see that, for small momenta $k \ll k_0$ which additionally belong to the interval $n^{2/3} \lesssim k \lesssim n^{1/2}$, the plasmon frequency $\omega_p(k)$ becomes imaginary. This means an instability of the homogeneous state of the EHL with respect to an appearance of the spatially inhomogeneous periodic in-plane charge distribution with a period characterized by the wave vector k. Such a charge-density fluctuation describes the charge-density waves (CDWs), which are in phase for the electron and the hole layers. For the equilibrium EHL with $n \sim n_{\rm eq} \sim \ell^{-3/2}$, one has $\frac{1}{\ell} \lesssim k \lesssim \frac{1}{\ell^{3/4}}$ and, thus, the period of the CDW obeys the condition $\ell^{3/4} \lesssim D \lesssim \ell$.

V. CONCLUSION

Thus, for $\ell \ll 1$ the homogeneous state of the EHL with the density $n_{\rm eq} \sim \nu^{3/2}$ is stable. However, as the distance ℓ increases, the plasmon spectrum becomes softer for finite momenta $k \gtrsim \frac{1}{\ell}$. Then, for a certain $\ell_{\rm cr} \sim 1$, there appears a momentum $k = k_{\rm cr} = 1/\ell_{\rm cr}$ for which the plasmon frequency vanishes. As the distance ℓ increases, the plasmon frequencies characterized by the wave-vector interval $\frac{1}{\ell} \lesssim k \lesssim \frac{1}{\ell^{3/4}}$ become imaginary. As a result, the CDW appears. This feature of the plasmon spectrum implies that EHL in the CQW experiences a quantum phase transition in the parameter ℓ.

Note that, according to Refs. [46,48], the homogeneous EHP of the spatially separated charges of the opposite sign in the CQW possesses a negative compressibility if the charge density is smaller than the critical value n_c. Such a feature evidences that the EHP is absolutely thermodynamically unstable. It should be pointed out that such instability is revealed for arbitrary separation distance ℓ (however, the critical density n_c depends on the ℓ). An inevitable presence of impurities serving as condensation centers results in the formation of the EHL drops. It is shown in the present paper that the EHL remains homogeneous if the spatial separation does not exceed the critical value $\ell_{\rm cr}$. Instead, if the separation exceeds the critical value $\ell_{\rm cr}$, the instability of the EHL with respect to creation of the CDW takes place. Thus, the instability of the EHP found in Refs. [46,48] differs from the instability in the EHL discovered in the present paper.

Note that some of the results obtained above are valid for multicomponent electron gas at the positive background [1–4,6,39–42]. In particular, this concerns the effective mass renormalization, the Z-factor renormalization for the single-particle Green's function, the dependence of the ground-state energy, and the chemical potential of the electrons. Also, the conclusion remains valid and leads to an instability of the ground state of the electron gas with respect to appearance of the CDW for sufficiently small density. However, a significant difference takes place: in contrast to EHL, the electron gas at the positive background cannot find the equilibrium density to minimize the ground-state energy since the electron density is settled by the positive background. Instead, Wiegner crystallization of the electron gas can take place.

In this paper, for the sake of simplicity, it is assumed that the bare effective electron and hole masses are the same. First of all, let us note that, if the spatial separation is negligible, i.e., $\ell \ll 1$, the exciton bound energy is determined by the reduced mass of the electron-hole system. For this reason, taking into account the fact that the effective hole mass m_h is much greater than the effective electron mass m_e can change the reduced mass only by a factor of 2. This is insignificant for the results obtained. In the opposite case $\ell \gg 1$, the exciton bound energy is proportional to the factor e^{*2}/ℓ and depends neither on the effective electron mass nor on the effective hole mass. The impact of the difference between the electron and the hole masses on the electron-hole liquid critical parameters is considered in our paper Ref. [47]. It is shown in our paper that correlation energy even increases by the factor $(m_h/m_l)1/3$. Thus, the difference in the masses makes the formation of the EHL more preferable as compared to the exciton gas. So far as the creation of the CDW in the EHL is concerned, the mass difference should not affect the result, as it is due to the strong column correlation which, according to Ref. [47], increases as the mass difference increases.

There are several experiments in which the EHL seems to be observed in CQWs [49–51]. Yet, strictly speaking, our results concern many-component electron-hole systems for which $\nu \gg 1$. Within this circumstance, one can relate the results obtained to the experiments available only qualitatively. However, it is worthwhile mentioning that, if the spatial separation obeys the limitation $\ell \gg 1/\nu$, the dependence of all the critical parameters on the key parameter ν vanishes. For this reason, we expect that our results are connected with Refs. [49–51].

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