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Tilt-induced phase transitions in even-denominator fractional quantum Hall states at the ZnO interface

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Strongly interacting electrons in a ZnO quantum well reveal intriguing electronic properties that were not observed in traditional semiconductor systems, such as in a GaAs heterojunction. In a tilted magnetic field, the even-denominator fractional quantum Hall states in this system exhibit unique phase transitions that necessitate a proper understanding. Since here the Landau level gap is very small, it is essential to consider a screened Coulomb potential in order to include the effects of all Landau levels. We observe an incompressible state–compressible state phase transition induced by the tilted field with different properties for the $\frac{3}{2}$ and $\frac{7}{2}$ filling factors. Additionally, the disappearance of the $\frac{5}{2}$ state in the experiment is most likely due to the presence of a screened Coulomb potential. We, however, propose that a wider quantum well may help to stabilize the incompressible phase of the $\frac{5}{2}$ filling factor, and thereby make the fractional quantum Hall effect observable in this state.

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The "enigmatic" even-denominator fractional quantum Hall effect (FQHE) at the filling factor $\nu = \frac{5}{2}$ was first discovered in GaAs heterojunctions [1,2]. It is a special member in the FQHE family since its ground state and excitations cannot be described by the Laughlin wave function [3,4]. Although there are some aspects of this state that still remain unclear, it is generally believed that a Pfaffian state with non-Abelian excitations is the most likely candidate to describe this extraordinary FQHE [5,6]. Numerical studies of the even-denominator FQHE were also helpful to understand the nature of this state [7–10]. The ground state at this filling factor is incompressible (just as for the odd-denominator states), so that the attributes of the transport experiments are the same as those of the odd-denominator FQHE. Recently, the FQHE has been discovered in an oxide material, a ZnO interface with high mobility [11–13]. There are heightened expectations that the stronger Coulomb interactions in this case will perhaps display unusual effects related to strong electron correlations [14]. Interestingly, in the MgZnO/ZnO interface, the $\nu = \frac{5}{2}$ state was found to be missing. The ZnO quantum well is quite different from the conventional GaAs system because here the Landau level (LL) gap is comparable to the Zeeman gap and the ratio κ of the Coulomb interaction to the LL gap is very large. In order to explain the missing FQHE in the experiment, we earlier introduced the screened Coulomb interaction that includes the effect of all the LLs as required for this system [15]. The system was indeed found to be compressible, thereby explaining the absence of the $\nu = \frac{5}{2}$ state in ZnO.

A tilted magnetic field has been a very powerful means to study the nature of fractional quantum Hall systems [16–18]. It modifies the transport properties and provides additional information about the systems, especially the spin polarization and the excitations associated with electron spins [19,20]. It seems that the ZnO system holds more interesting phase transitions, especially in the even-denominator FQHE states (as compared to, e.g., the GaAs hole system [21]), than the conventional GaAs system. In the experiment of Falson *et al.* involving ZnO [13], the tilted magnetic field reflects a very unusual behavior in the transport properties. For the $\nu = \frac{3}{2}$ filling factor the FQHE was found to appear only when the tilt angle was large. On the other hand, for $\nu = \frac{7}{2}$, the FQHE disappeared when the tilt angle was increased. We believe that there are phase transitions associated with these experimental observations, since the LL energy gap which is only related to the perpendicular component of the magnetic field can be easily exceeded by the Zeeman coupling, which is proportional to the total magnetic field. The change in the kinetic energies can exceed the Coulomb energy when the tilt angle is large enough, which certainly leads to changes of electron occupation in different LLs. In particular, the spin transition is also involved in these phase transitions. The spin polarization measurements could therefore be used to observe these phase transitions. With the increase of the tilt angle, the screening potential is changed. So the transport properties of the electron gas in a variable tilted field must become very different from those in a perpendicular magnetic field alone. Here, we focus only on the even-denominator filling factors.

In our present work, the electron gas is confined in a parabolic potential in the z direction, thereby making the system quasi-two-dimensional. Motivated by experimental work [13], we consider this system to be in a tilted magnetic field. In the ZnO quantum well, unlike in a GaAs system, the LL crossing is easily achieved in a tilted field. Since the Zeeman coupling is about 0.94 of the LL gap in a perpendicular magnetic field, a spin-polarized state may not satisfactorily describe the system. We therefore use a spin-mixed Hamiltonian to study the ground states and the phase transitions involving spin flip.

As mentioned above, we also use the screened Coulomb potential in the present case of the FQHE in a tilted magnetic field. With a nonzero tilt angle α_0, it is more realistic to suppose that the electron gas is confined in a parabolic potential with frequency ω_z in the z direction, which is perpendicular to the plane of the electron gas. The advantage of this approximate potential is that the wave function can be analytically obtained for any value of the tilted field. This approximation to a certain extent should be similar to

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other finite-thickness approaches, such as an infinite square well [8] or a triangular well. We choose a Landau gauge with the vector potential $\mathbf{A} = (0, B_z x - B_x z, 0)$, where the wave function can then be found in Refs. [10,22–24], which must be described by two sub-LL indices m,n. One is related to the original LL in a purely two-dimensional (2D) case, while the other one is essentially the energy levels of the parabolic potential. The kinetic energy of the sub-LL (m,n) is $E_{m,n} = (m + \frac{1}{2})\hbar\omega_1 + (n + \frac{1}{2})\hbar\omega_2$, where $\omega_{1,2} = \frac{1}{\sqrt{2}}\sqrt{\omega_b^2 + \omega_\perp^2 \pm \sqrt{(\omega_b^2 - \omega_\perp^2)^2 + 4\omega_\parallel^2\omega_\perp^2}}$, $\omega_{\perp,\parallel} = eB_{z,x}/m^*$, and $\omega_b^2 = \omega_\parallel^2 + \omega_z^2$. The magnetic lengths are $\ell_{\perp,1,2} = \sqrt{\hbar/(m^*\omega_{\perp,1,2})}$ with the effective mass m^*.

The interaction Hamiltonian including spin is given by

$$H = \sum_{\alpha,\beta,m^{(\prime)},n^{(\prime)}} \sum_{j_1\cdots j_4} V_C c^{\alpha\dagger}_{j_1,m,n} c^{\beta\dagger}_{j_2,m',n'} c^{\beta}_{j_3,m',n'} c^{\alpha}_{j_4,m,n},$$

where we only consider one LL or two LLs with different spins α,β. So the Coulomb interaction V_C can be found in Refs. [22,25]. Following Ref. [15], we introduce the static dielectric function in a general screened three-dimensional case, $\epsilon_s(\mathbf{q}) = 1 - \frac{2\pi e^2}{q^2\epsilon}\chi^0_{nn}(\mathbf{q})$, where ϵ is the dielectric constant. Then the screened Coulomb potential becomes $V_C(\mathbf{q})/\epsilon_s(\mathbf{q})$. The noninteracting density-density response function is

$$\chi^0_{nn}(\mathbf{q}) = \frac{N_s}{SL_z} \sum_{\alpha,m^{(\prime)},n^{(\prime)}} \left|G^{mn}_{m'n'}(-\mathbf{q})\right|^2 \frac{\nu_{\alpha,mn} - \nu_{\alpha,m'n'}}{E_{mn} - E_{m'n'}},$$

where $S = L_x L_y$ is the area of the sample, N_s is the degeneracy of a LL, ν is the filling factor, and the form factor is defined as

$$\begin{aligned} G^{mn}_{m'n'}(\mathbf{q}) &= \exp\left[-\frac{1}{2}\left(q_1^2 + q_2^2 + q_-^2 + q_+^2\right)\right] \\ &\quad \times \sqrt{\frac{\min(m,m')!\,\min(n,n')!}{\max(m,m')!\,\max(n,n')!}} \\ &\quad \times \lambda_1(m,m',\mathbf{q})\lambda_2(n,n',\mathbf{q}), \end{aligned}$$

where

$$\begin{aligned} q_1 &= \frac{1}{\sqrt{2}}\frac{\cos\theta}{\ell_1} q_y \ell_\perp^2, \quad q_2 = \frac{1}{\sqrt{2}}\frac{\sin\theta}{\ell_2} q_y \ell_\perp^2, \\ q_\mp &= \mp\frac{1}{\sqrt{2}}(q_{x,z}\cos\theta \mp q_{z,x}\sin\theta)\ell_{1,2}, \\ \lambda_{1,2}(m,m',\mathbf{q}) &= [\mathrm{sgn}(m-m')q_{1,2} \mp i q_\mp]^{|m-m'|} \\ &\quad \times L^{|m-m'|}_{\min(m,m')}\left(q_{1,2}^2 + q_\mp^2\right), \end{aligned}$$

with a Laguerre polynomial L. In the rectangular space, the discrete momenta are $q_x = \frac{2\pi}{L_x}s$, $q_y = \frac{2\pi}{L_y}t$. For the third dimension, the length in the z direction L_z is difficult to determine in a parabolic potential. In principle, it should be infinity, but for our present purpose we choose a finite value of L_z since the wave functions vanish rapidly in the z direction. The electrons are confined well in the z direction, so it is reasonable to limit L_z within a proper range. Here, we approximate the L_z in terms of the parabolic potential frequency ω_z, $L_z = 2\sqrt{\ln(2)\frac{\omega_\perp}{\omega_z}}\ell_\perp$, which is the width of the wave function of the lowest LL in the z direction. We set $L_z = 1.8$ nm, which corresponds to a relatively narrow quantum well, since the electron gas will be split into a "double" layer system in a wide quantum well [26] and the consequent transport properties would become very different. The density-density response function is calculated in the noninteracting case, so the filling factors ν are also the noninteracting filling factors.

We utilize the exact diagonalization scheme in the standard case of finite-size systems in a periodic rectangular geometry ($L_x = L_y$) [4,27,28] with the screening potential included [15,29]. We have investigated numerically the competition between the kinetic energy and the Coulomb energy. In order to analyze the phase transition near the LL crossing in a tilted field, we take one LL or two LLs with different spin orientations. With an increase of the tilt angle, Landau level crossings occur. The first crossing occurs at about 18°, while the second one occurs at about 62°. We study only the small tilt angles, since the parabolic potential may not be a very good approximation at very large angles. When the quantum well is narrow, the energies of the series of sub-LLs ($m > 0,0$) are much higher than those of sub-LLs ($0,n < 3$). When the tilt angle is $\alpha_0 < 60°$, the LL crossing only happens between the LLs $(0,n;\downarrow)$ and $(0,n+1;\uparrow)$. The magnetic fields of the z component are set the same as in Ref. [13], i.e., $B_z = 6.2$, 3.75, and 2.75 T for $\frac{3}{2}$, $\frac{5}{2}$, $\frac{7}{2}$, respectively.

Phase transition at $\nu = \frac{3}{2}$. We considered the two LLs $(0,0;\downarrow)$ and $(0,1;\uparrow)$ in our exact diagonalization scheme. We did not find any spin coherence, irrespective of the tilt angle. There is a phase transition associated with spin polarization at about $\alpha_1 = 23°$. When $\alpha_0 < \alpha_1$, all electrons are in LL $(0,0;\downarrow)$, even though the kinetic energy of $(0,1;\uparrow)$ is lower when $18° < \alpha_0 < \alpha_1$. The incompressibility of the ground state may not be stable for different sizes of the system. When $\alpha_0 > \alpha_1$, all electrons flip to $(0,1;\uparrow)$. In this LL, the even-denominator FQHE can be found as an incompressible liquid without any LL mixing or screening. For $\nu = \frac{3}{2}$, the screening is weaker than that of the filling factor $\nu = \frac{5}{2}$. The collective modes for $\nu = \frac{3}{2}$ when $\alpha_0 > \alpha_1$ show that the ground state is incompressible. However, in the experiment, the FQHE is only observed when $\alpha_0 > 38°$ [13].

Phase transitions at $\nu = \frac{7}{2}$. When the magnetic field is perpendicular to the electron plane, we have already shown that the FQHE survives the screening potential [15]. In a tilted field, with the parabolic potential we consider the two LLs $(0,1;\downarrow)$ and $(0,2;\uparrow)$ in the exact diagonalization scheme. The phase transition which is found to be at $\alpha_2 = 18°$ is more or less at the same angle where the noninteracting LLs cross. The incompressible ground state is still found when $\alpha_0 < \alpha_2$. All electrons are in LL $(0,1;\downarrow)$, and the collective mode for 11 electrons is shown in Fig. 1. The collective modes do not change much when $\alpha_0 < \alpha_2$. The two minima are at about $q\ell = 2.2$ and 3.8, which is close to the collective mode of the pure 2D case. When $\alpha_0 > \alpha_2$, the screening is changed due to the fact that the noninteracting filling factors are changed, the electrons are flipped to LL $(0,2;\uparrow)$, and the system is no longer incompressible. In the experiment, the phase transition occurs in the range (21°,27°) [13]. The difference between our theoretical work and the measurement is likely due to the LL

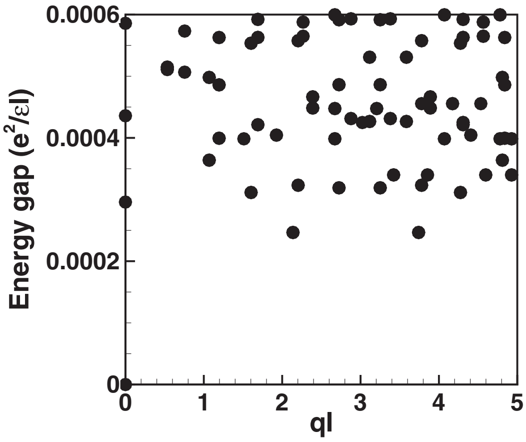

FIG. 1. The collective mode of $\nu = \frac{7}{2}$ for 11 electrons in a tilted magnetic field ($\alpha_0 = 10^\circ$).

broadening induced by the disorder, which is able to shift the LL crossing to a higher tilt angle. Note that the thickness could soften the collective modes shown in Fig. 1. The bare Coulomb potential reduced by the width of the quantum well helps to stabilize the collective modes: The minimum gap of the collective modes increases slightly when the width increases. Somehow the screening plus the thickness is able to compress this gap at this filling factor. The second LL crossing occurs for $\alpha_0 = 62^\circ$, between (0,2; $\uparrow$) and (0,0; $\downarrow$). We determine the collective modes of LL (0,0; $\downarrow$) and the ground state is still compressible. In these phase transitions, the mixed-spin state is also absent. No spin coherence state has a lower energy than the spin-polarized state. All phase transitions are first order.

We note here that all phase transitions involve spin flip. The spin polarization before and after all the phase transitions is changed significantly. If we define the spin polarization as $\langle S_z \rangle = \frac{1}{2}(\nu_\uparrow - \nu_\downarrow)$, then the spin phase transition is as shown in Fig. 2. Therefore, the spin polarization measurement could be a very powerful means to determine the phase transitions. We expect that future experimental work may confirm our present findings.

It seems that either incompressible or compressible states favor a spin-polarized state. Unlike the $\frac{2}{5}$ FQHE [20] or $\frac{1}{2}$ FQHE in a double layer [30] (or a wide well [31]) where the (pseudo)spin-mixed state [32] has a lower energy, the present cases are all spin polarized. It is probably because the spins between two different LLs are difficult to couple even though their kinetic energies are close, which is also supported by a skyrmion study between different LLs with different spins [33].

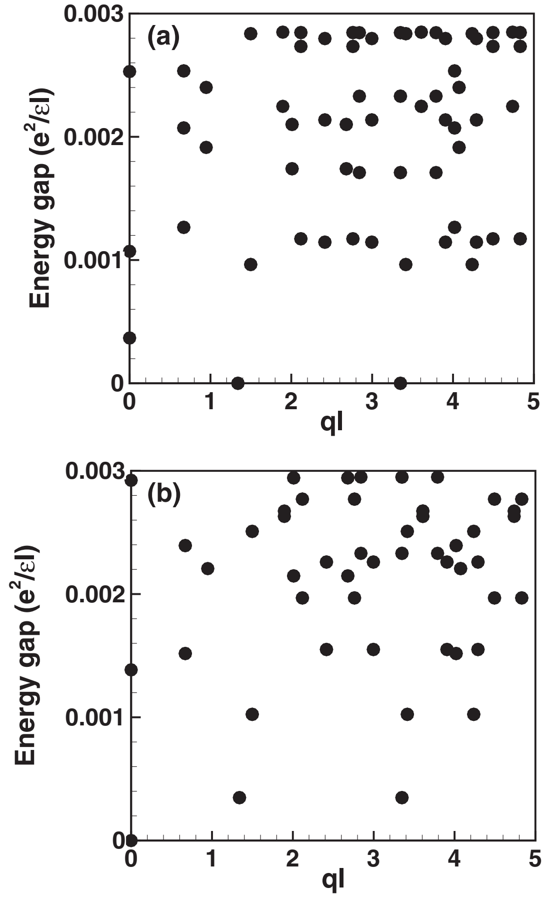

FIG. 3. The collective mode of $\nu = \frac{5}{2}$ for seven electrons in a perpendicular magnetic field. (a) The characterized width of the parabolic potential $L_z = 1.8$ nm and (b) $L_z = 11$ nm.

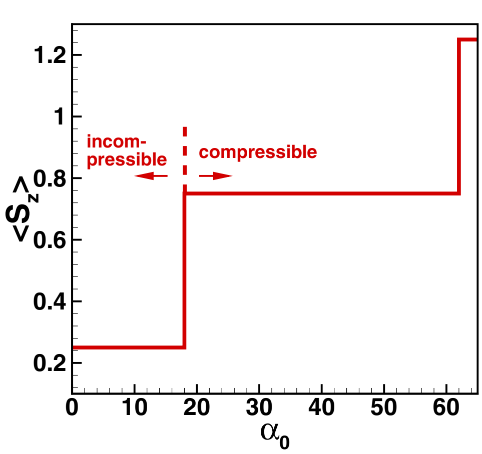

FIG. 2. The phase transitions at the $\nu = \frac{7}{2}$ filling factor.

The thickness effect in the screened electron gas is also important. In particular, at the filling factor $\nu = \frac{5}{2}$, in a pure 2D case, the FQHE is absent because of the screened Coulomb potential due to the other LLs [15]. The FQHE is still absent for a narrow parabolic potential. Interestingly, the $\nu = \frac{5}{2}$ FQHE state could appear, because the ground state becomes incompressible again even with screening, when we tune the width of the quantum well $L_z \geqslant 9$ nm. Figures 3(a) and 3(b) display the collective modes of seven electrons for $L_z = 1.8$ and 11 nm in a zero tilted field, respectively. The softening of the collective modes disappears and the ground state is clearly gapped from the other modes. When the quantum well is narrow, the electron gas behaves similar to that of the pure 2D case, but the collective modes show that incompressibility appears when the width is increased. Moreover, in a higher magnetic field, which means the screening is weaker, the required width of the quantum well to stabilize the $\frac{5}{2}$ FQHE state is narrower, but the magnetic field must reach up to

$B \approx 40$ T so that the incompressible state in a pure 2D case can survive in a weak screening. Therefore, we propose that the finite width of the quantum well can significantly improve the stability of the FQHE state in this case. In GaAs, both theoretical and experimental works have indicated that a wider quantum well helps to stabilize the FQHE for $\nu = \frac{5}{2}$ [8,18]. Our results are therefore compatible with those previous works. We have also checked the collective modes in an infinite square well [8], instead of the parabolic potential, plus the 2D screening as an approximation, and the same thickness effect, i.e., a wider quantum well helps to stabilize the $\frac{5}{2}$ FQHE state when $L_z \geqslant 9$ nm, was found. We should note that the width cannot be too large, since the sub-LL spectrum becomes very different and the approximation of the parabolic potential is not good. If the width of the quantum well in ZnO could be artificially fabricated, then we hope that this width effect predicted here can indeed be observed.

In order to explore the nature of the ground state, we have calculated the wave functions of the even-denominator FQHE states. For simplicity, we consider the pure 2D case. The many-body wave function of the ground state at $\nu = \frac{7}{2}$ in a perpendicular magnetic field is $|\phi^s_{\frac{7}{2}}\rangle$ with our screening potential. The wave function of $\nu = \frac{5}{2}$ or $\nu = \frac{7}{2}$ without screening is $|\phi_{\frac{5}{2}}\rangle = |\phi_{\frac{7}{2}}\rangle$. We find that the overlap of these two wave functions, for $N_e = 7$, is close to unity, $\langle\phi_{\frac{7}{2}}|\phi^s_{\frac{7}{2}}\rangle \approx 0.99$. So the screened Coulomb potential at $\nu = \frac{7}{2}$ does not change the wave function. The pair distribution function is also similar to that of a liquid [7]. The screened Coulomb potential at $\nu = \frac{5}{2}$ completely destroys the incompressibility, the lowest-energy state most likely being a density wave [15].

In summary, we have analyzed the phase transitions between different spins at $\nu = \frac{3}{2}$ and the $\nu = \frac{7}{2}$ of the two LLs with different spin when the magnetic field is tilted from the direction perpendicular to the electron plane. The nature of the ground states is changed significantly in the phase transition. The spin polarization is first-order-like flipped to the other spin orientation when the tilt angle is increased. This spin flip could be observed in a spin-sensitive experiment [34]. The $\nu = \frac{5}{2}$ FQHE was found to be absent in ZnO. However, here we propose that if the width of the quantum well is artificially widened, then this FQHE state should be observable. We have also investigated the changes in the ground-state wave functions with and without the screening. Here we only consider the system sizes $N_e = 5$ and $N_e = 7$ in two different LLs, where the spin-polarized states always have lower energies than those of the unpolarized states. Although a larger system is difficult to analyze, we expect that the first-order spin flipping would be system-size independent, and the spin-polarized state is expected to be more stable, in the inter-LL case. Incidentally, the measured activation gaps in ZnO are usually one order of magnitude smaller than those in GaAs [12]. This can be qualitatively understood as follows: The LL gap of GaAs is about seven times larger than that of ZnO, and therefore the screening of ZnO is seven times stronger than that of the GaAs. Consequently, the excitation energy in ZnO is about one order smaller than that in GaAs.

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