

Possible Bose-Einstein condensate associated with an orbital degree of freedom in the Mott insulator CaCrO_3

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Whether CaCrO_3 is a Mott insulator or a correlated metal is still controversial. We have performed measurements of magnetization, specific heat, and thermal conductivity on CaCrO_3 samples selected from many batches of high-pressure synthesis. The single-crystal CaCrO_3 sample exhibits an unprecedentedly sharp transition at a Néel temperature $T_N \approx 90$ K. The critical behavior of specific heat cannot be rationalized by the renormalization group theory for a second-order magnetic transition. More surprisingly, the thermal conductivity κ exhibits an anomalous drop on cooling through T_N , which is opposite to all known influence on κ from either spin or orbital ordering. We have argued, on the basis of anomalies found in all three measurements and structural data, for the coexistence of itinerant π -bonding electrons in a c -axis band and localized xy electrons in xy orbitals responsible for type-C antiferromagnetic order below T_N and the occupation of a pure, localized xy orbital undergoing a Bose-Einstein condensate at T_N .

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The perovskite CaCrO_3 has drawn considerable attention in recent years [1–8]. The available experimental results of this perovskite are still controversial. An early-day study on a crystal grain selected from a sample synthesized under high pressure showed a metallic conduction to the lowest temperature with a small jump of resistivity on cooling through T_N [9]. For polycrystalline samples, a high and activated resistivity was found [5]. However, an optical study on polycrystalline samples showed a finite conductivity at $\omega = 0$, which is evidence for a metal [4,5,7]. On the other hand, neutron diffraction on a CaCrO_3 sample has revealed an evolution of a local structural change as a function of temperature and a type-C antiferromagnetic ordering below $T_N \approx 90$ K [4,5]. The finding of a collinear type-C antiferromagnetic ordering, typical for localized electrons, contradicts the conclusion of itinerant electrons from the reflectivity measurement. The local structural distortion also reflects an unusual electronic state and unprecedented orbital dynamics. The orthorhombic $Pbnm$ perovskite has intrinsic distortions that can be extracted from isostructural CaTiO_3 and CaVO_3 where orbital ordering is not an issue [10,11]. The intrinsic distortions bias the orbital ordering in orbitally degenerate systems such as in the t_2 electronic systems of RTiO_3 ($R = \text{rare earth, and Y}$) [12–14]. Neutron diffraction [5] on RTiO_3 showed that an orbital degeneracy survives through a magnetic transition and to lower temperatures. The analysis of critical phenomena of physical properties and an unusual thermal conductivity at T_N found in CaCrO_3 indeed reveal new physics associated with the interaction between spins and orbital dynamics. We argue that while the orbital degeneracy is not removed, a giant number of collective orbital excitations undergo a Bose-Einstein condensate on cooling through the Néel temperature in CaCrO_3 .

The CaCrO_3 samples were made under high pressure. Large cubes (100–200 μm) of CaCrO_3 crystal can be found in some batches of the high-pressure products. Measurements of physical properties were performed on single-crystal samples, polycrystalline samples with embedded crystal grains, and on uniformly polycrystalline samples. Detailed information about the samples' preparation, characterization, and measurements of physical properties can be found in the Supplemental Material (SM) [15]. Figure 1 shows the result of magnetization measurements on CaCrO_3 samples; the transition at T_N is remarkably sharp in the single crystal sample. In type-C magnetic ordering, the easy magnetization axis is along the $\pm y$ direction of the orthorhombic cell according to neutron diffraction studies [4,5]. For the $Pbnm$ crystal structure, the compatible magnetic structure is $F_x C_y$ of the irreducible representation Γ_2 [16]. However, it is impossible for neutron diffraction to pick up a tiny spin canting in the x direction. The magnetization data provide important information about the spin canting. By considering the interatomic spin-orbit coupling, Moriya has put forward the theory of antisymmetric spin coupling in localized magnetic insulators that gives a quantitative description of spin canting and the temperature-dependent magnetization. The magnetic susceptibility for the field perpendicular to the easy axis is expressed as [17]

$$\chi = \frac{Ng^2\mu_B^2 S(S+1)}{3k(T+T_N)} \frac{T-T_0}{T-T_N},$$

$$T_N = \frac{JZS(S+1)}{3k} \left[1 + \left(\frac{D}{J} \right)^2 \right]^{1/2},$$

$$T_0 = \frac{JZS(S+1)}{3k},$$

where Z is the number of nearest neighbors and D is the Dzialoshinski-Moryia coupling constant. In contrast to a regular antiferromagnet, the susceptibility for the antifer-

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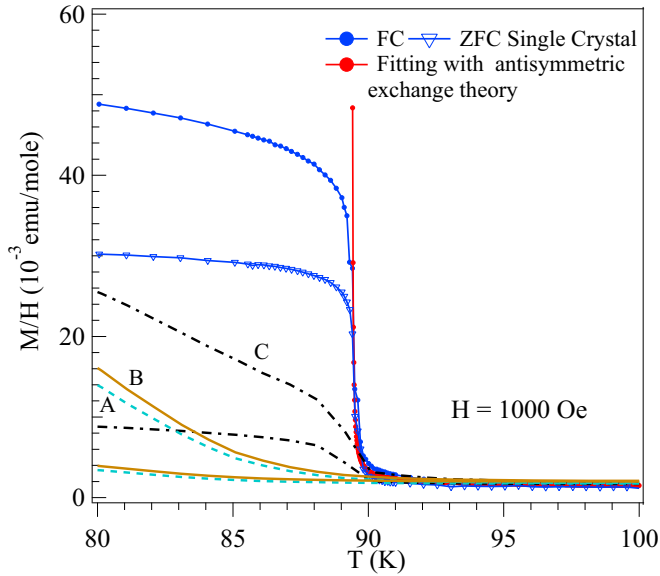


FIG. 1. Temperature dependence of magnetization for single-crystal and polycrystalline samples of CaCrO_3 . Curves labeled with A, B, C are for polycrystalline samples. A is a uniform polycrystalline sample; C is a polycrystalline sample with many embedded single crystal grains; B is between A and C. See the SEM results in SM for the detail.

romagnetic (AF) system with canted spins would exhibit an extremely sharp transition at T_N . The transition area of magnetization data of single crystal CaCrO_3 sample in Fig. 1 can indeed be fit to the expression very well; the fitting gives a $D \approx 4$ K and a spin canting angle $\theta \approx 0.08^\circ$ based on $\tan 2\theta \sim D/2J$ from the theory, J can be obtained from T_N through the mean-field model. In comparison, a metallic antiferromagnet $\text{Nb}_{12}\text{O}_{29}$ shows a broad peak at T_N in the temperature dependence of magnetization [18]. It should be stressed that the antisymmetric exchange interaction is applicable to localized-electron antiferromagnets.

As shown in Fig. 1, the transition at T_N in the polycrystalline samples is relatively broad. A transition at a lower temperature and multiple transitions can also be identified in the results of specific-heat measurements in these samples. A lower T_N is related to oxygen nonstoichiometry in the samples [5]. Figure 2(a) shows the C_p of crystal cubes separated from sample C and the polycrystalline samples with/without embedded crystals. The Néel temperature T_N of these samples can be obtained from a λ -shape anomaly in the plot of C_p versus temperature. The single-crystal sample exhibits the highest T_N among samples studied. As characterized by scanning electron microscopy in the SM [15], sample A is a uniformly polycrystalline sample, which shows a much broader transition at a relatively lower temperature. Sample B, a polycrystalline sample with a few embedded crystals, shows a slightly higher T_N than that of sample A and an anomaly at the temperature corresponding to T_N of the single-crystal sample is also visible. Embedded crystals in sample C occupy more than 1/3 of the sample's volume. A λ -shape anomaly is essentially the same as that at T_N in the single-crystal sample, but the magnitude is lower. All these observations demonstrate that all single-crystal grains found in different batches of

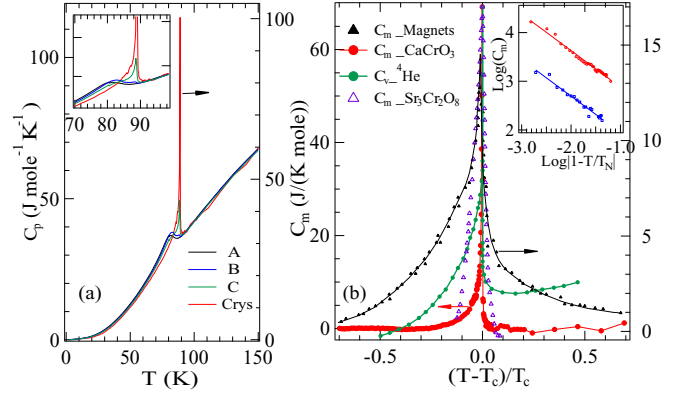


FIG. 2. (a) Temperature dependence of specific heat for a variety of CaCrO_3 samples; inset: a zoom-in plot of the transition region; (b) the magnetic contribution C_m versus reduced temperature $(T-T_c)/T_c$ for single crystal CaCrO_3 , Heisenberg magnets (A collection of many typical magnets), quantum spin system in $\text{Sr}_3\text{Cr}_2\text{O}_8$, and the λ transition in ^4He for comparison. Inset: the plot of $\log_{10} C_m$ versus $\log_{10} |1 - T/T_N|$ for CaCrO_3 .

high-pressure synthesis have the same T_N that is higher than the broad anomaly at T_N in polycrystalline samples. Whereas an extremely sharp transition observed in single crystal CaCrO_3 can be described with a model of antiferromagnetic transition with canted spins, the specific-heat measurement shown below gives a different story about the transition.

The magnetic contribution C_m to the specific heat in Fig. 2(a) of single-crystal CaCrO_3 was obtained by subtracting the lattice contribution obtained by fitting $C_p(T)$ with a combination of the Debye and Einstein formulas at temperatures away from the vicinity of T_N . The $C_m(T)$ versus the reduced temperature $t \equiv (T-T_N)/T_N$ has been compared directly with a $C_m(T)$ of typical Heisenberg magnets [19]. The transition at T_N for a single-crystal CaCrO_3 is significantly narrower than that for a regular second-order magnetic transition. One may be curious whether the super-sharp anomaly at T_N in CaCrO_3 is simply caused by a first-order transition. The structural study [5] indicated that there is no volume change. As shown in the SM [15], a spike with extremely high magnitude and a discontinuity of C_p caused by latent heat at the first-order transition in LuVO_3 clearly distinguish the $C_p(T)$ found in CaCrO_3 from a first-order transition. More importantly, a first-order transition does not come with critical fluctuations. For a second-order magnetic transition, the critical behavior can be described by a power law based on the renormalization group theory. The C_m data are well mapped along two parallel lines in the plot of $\log_{10} C_m$ versus $\log_{10} |1-T/T_N|$ near T_N in Fig. 2(b), which indicates that the critical behavior of CaCrO_3 can be perfectly described by a power law. However, the exponent obtained by linear fitting the plot is $\alpha^+ = -0.75(3)$, $\alpha^- = -0.75(2)$ for the data above and below $T_N (=89.089$ K), which is significantly different from an $\alpha = -0.11$ for a three-dimensional Heisenberg magnet and actually different from any exponents for a variety of magnetic models [20]. On the other hand, $C_m(T)$ of the CaCrO_3 crystal shares stunningly the essential feature of the C_v of ^4He at the λ transition from $^4\text{He I}$ to $^4\text{He II}$ [21]. The quantum liquid of ^4He undergoes a Bose-Einstein condensation (BEC) into the

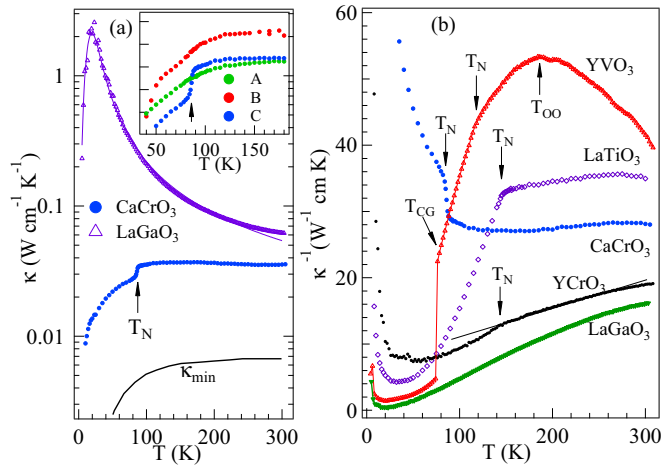


FIG. 3. (a) Temperature dependence of thermal conductivity for a variety of CaCrO_3 samples and LaGaO_3 crystal; inset: a zoom-in plot of the transition region of the CaCrO_3 samples; a solid line inside the data of LaGaO_3 represents the fitting result to the Boltzmann formula; (b) the inverse thermal conductivity versus temperature for the CaCrO_3 sample C and YVO_3 , YCrO_3 , LaTiO_3 , and LaGaO_3 for comparison.

superfluid phase of ^4He II. In the case of the quantum spin-dimer system $\text{Sr}_3\text{Cr}_2\text{O}_8$, an extremely sharp anomaly has also been found where the applied magnetic field H is well above a critical value H_c of the quantum critical point [22]. While the spin ordering state is not different from that of a Néel state, the magnetic transition and its field dependence associated with a degenerate state for triplons can be treated by a BEC [22–24]. In addition to the sharp anomaly, the critical exponents for a BEC are normally different from those predicted by the renormalization group theory [25]. For example, at the λ transition of ^4He , a $C_v = -\alpha \ln[(T_c - T)/T_c] + b$ has been reported instead of the power law [26]. In the case of the BEC in $\text{Sr}_3\text{Cr}_2\text{O}_8$, an $\alpha \approx -0.3$ was obtained by analyzing the data in Ref. [22]. Generally speaking, a small temperature range t_G for critical fluctuations means a longer coherence length ξ_0 based on the Ginzburg criterion $t_G = (T_G - T_c)/T_c \sim \Delta C^{-2} \xi_0^{-1}$, where ΔC is the jump of C_m at T_c or T_N . A smaller t_G is consistent with a condensation-type transition where macroscopic occupation of a quantum state occurs. This analysis together with the anomalous α implies that the CaCrO_3 crystal may actually undergo a BEC near 90 K.

The thermal conductivity κ reveals the dynamics of spins, lattice, and orbitals, which could give an important clue as to what happens at T_N in CaCrO_3 . Figure 3 shows the results of thermal conductivity measurements made on polycrystalline CaCrO_3 samples. It is impossible for us to measure the thermal conductivity on a $200 \mu\text{m}$ grain of CaCrO_3 crystal. However, measurements made on the sample C help to isolate a $\kappa(T)$ that is essentially from the single-crystal CaCrO_3 (see SM [15] for detailed information). A small and weak temperature dependence of κ is found for both polycrystalline and single-crystal CaCrO_3 samples over a broad temperature range above T_N . The $\kappa(T)$ of CaCrO_3 is located between two extreme oxide cases, i.e., an amorphous oxide and a LaGaO_3 crystal. The LaGaO_3 crystal serves as an example of phonon thermal conductivity since it is a diamagnetic insulator. On the other

hand, the minimum thermal conductivity κ_{\min} can be calculated from an amorphous oxide in which the mean free path is no longer than the interatomic distance. The κ of CaCrO_3 is close to a phonon thermal conductivity in magnitude near room temperature; but the temperature dependence is characteristic of an amorphous oxide. The thermal conductivity in a solid is described by $\kappa = (\frac{1}{3})Cv\ell$, where C is the specific heat, v is the acoustic sound velocity, and ℓ is the mean free path. In a glass, the temperature dependence of κ is dominated by the C since the ℓ for short-range phonons is small [27]. The κ of CaCrO_3 resembles the temperature dependence of C_p since ℓ is a constant in this amorphouslike oxide. The magnetic transition at T_N causes an abrupt drop of κ , which is clearly observed in sample C. However, the drop is diminished in samples A and B as shown in the inset of Fig. 3(a). An amorphouslike $\kappa(T)$ has been seen in other crystalline oxides with an orbital degeneracy. However, a further reduction from the amorphouslike κ of CaCrO_3 on cooling through T_N is highly unusual.

Inverse κ versus temperature of several Mott insulators are plotted in Fig. 3(b). Each of them offers different stories of how magnetic ordering and orbital ordering influence the thermal conductivity. YCrO_3 has no orbital degeneracy and exhibits type-G AF ordering at T_N . The $\kappa(T)$ of YCrO_3 in the paramagnetic phase resembles that of LaGaO_3 . The κ increases on cooling through T_N because of the magnon contribution and reduced scattering between spin excitations and phonons in the magnetically ordered phase. YVO_3 has the same electronic configuration, i.e., $t_2^2 e^0$, as that of CaCrO_3 ; however, the orbital degeneracy in YVO_3 survives only down to 180 K. Orbital fluctuations contribute a glassy κ for $T > 180$ K. At the orbital ordering transition T_{OO} , the κ bottoms out. A phononlike κ is restored at $T < T_{OO}$ since a long periodicity in the lattice is established due to the orbital ordering. On cooling through $T_N < T_{OO}$, YVO_3 experiences a spin disorder-order transition where the κ is further enhanced. A phonon κ is finally realized through a first-order transition at $T_{CG} < T_N$ [28]. In LaTiO_3 , the orbital degeneracy survives in the entire temperature range of the paramagnetic phase. On cooling through T_N , the spin/orbital ordering restores the phononlike thermal conductivity [29]. These experiments clearly reveal how sensitive the thermal conductivity is as a probe for spin and orbital orderings and show a general trend that orbital ordering and spin ordering restore thermal conductivity. While a glassy κ in the paramagnetic phase of CaCrO_3 is clearly related to the orbital degeneracy, the sharp drop of κ on cooling through T_N distinguishes CaCrO_3 from any oxide systems studied so far. Critical fluctuations would cause a small dent around a magnetic transition temperature, i.e., a small drop and then a coming back of κ ; see the data for CoF_2 for example [30]. But the influence of critical fluctuations on the thermal conductivity is fundamentally different from what happens at T_N in CaCrO_3 . After exhausting all possible consequences of spin/orbital ordering, we focused on whether a BEC can account for the observed drop in κ on cooling through T_N .

As interacting Bose particles condense, it is acknowledged that condensed particles carry no entropy and make no contribution to the thermoelastic properties [31]. Condensed particles normally represent a very small fraction of all particles in a real system. As a result, the impact to the thermal

conductivity from condensed particles is negligible compared to that from normal particles. In ^4He , a five-order jump of κ occurs on cooling through the λ point [32]. In all quantum spin systems, the magnetic state below T_N is not different from a classic Néel state [23]. A jump of κ due to the magnon contribution on cooling through T_N has always been found in these quantum spin systems [33–36]. On the other hand, in heavy metal superconductors like Pb, for example, electrons in states within a range of kT around the Fermi energy are dominant heat carriers. As these electrons form Cooper pairs and condense at $T < T_c$, the κ drops [37], which demonstrates that condensed Bose particles carry no entropy.

In order to find possible particles/quasiparticles in CaCrO_3 that condense at T_N , we start from the local crystal structure since it reflects the electron and the orbital state. CrO_6 octahedra in CaCrO_3 are slightly compressed, i.e., $c/a < 1$, at room temperature [5]. This local site distortion is contrary to the intrinsic distortions in a $Pbnm$ perovskite structure that would bias an orbital ordering leading to long, medium, and short Cr-O bonds [13,14]. A compressed CrO_6 octahedron is compatible with the orbital occupation of $xy(l_z = 0)$ and $yz \pm izx(l_z = \pm 1)$ for a t_2^2 electron manifold. The spin-orbit coupling would make the spin easy axis along the c direction if the orbital angular momentum is not quenched. The observation of an easy magnetic axis along the b axis [5], however, indicates that the orbital angular momentum is fully quenched. Moreover, an abrupt collapse of the Cr-O bond length along the c axis, and an expansion in the ab plane on cooling through T_N in a type- C antiferromagnet, are just opposite to what would be predicted for a localized-electron magnetostrictive distortion. We are, therefore, forced to conclude that, at least below T_N , there are itinerant yz , zx electrons in a narrow c -axis π^* band that is $1/4$ -filled, which is consistent with ferromagnetic c -axis coupling of the localized xy spins that are antiferromagnetically coupled in the basal plane. Although the Cr-O bond lengths as well as the lattice parameters (excluding the volume) change abruptly at T_N , the magnetic ordering and the c/a changes leave the transition second order as indicated by the critical fluctuations seen in the specific-heat data. Since a classic orbital ordering is clearly not an option here, we turn to the possibility that the xy orbital occupation undergoes a BEC to account for the drop in the κ on cooling through T_N .

Whether particles can undergo a BEC depends critically on number conservation [23]. In a typical orbitally ordered

phase, as is found in the paramagnetic phase of LaMnO_3 below 750 K, the orbital excitations are described as orbitons [38]. The number of orbitons is not conserved. Analogous to quantum magnets where a BEC causes a sharp increase of magnetization, the Bose density in CaCrO_3 corresponds to the a/c ratio of an octahedron. The xy orbital occupation and/or localization lead to abrupt expansion of the ab plane and an increase of $a/c > 1$. Thermal vibrations involve all possible atomic displacements allowed by the crystal symmetry, which include modes associated with the orbital occupation. In quantum magnets [33–36], heat flux carried by magnetic excitations is negligible as compared to that by lattice vibrations. A jump of κ at T_c may reflect the influence of a BEC to the lattice. In CaCrO_3 , however, the lattice vibrations associated with the xy orbital occupation no longer contribute to the κ at $T \leq T_N$. A slightly smaller C_p of the crystal sample than that of polycrystalline samples at $T < T_N$ [see the inset of Fig. 2(a)] may indicate fewer degrees of freedom available for heat carriers. The analysis of thermal conductivity is highly consistent with a possible BEC from the critical behavior of C_p . The extremely sharp magnetic transition in single-crystal CaCrO_3 may be interpreted on the basis of a BEC. As a matter of fact, the BEC model for TlCuCl_3 predicts an abrupt jump of magnetization on cooling through T_c [39].

In summary, oxygen-stoichiometric single-crystal CaCrO_3 exhibits three unprecedented features: (1) localized and itinerant electrons coexist in π -bonding t_2 orbitals on the same atom in the structure; (2) the bias effect by the intrinsic distortion in the $Pbnm$ perovskite structure is entirely overwhelmed by the coexistence of itinerant yz , zx electrons and localized xy electrons; (3) a drop in thermal conductivity on cooling through T_N . We believe the experimental data manifest a BEC at T_N in CaCrO_3 . A BEC of the orbital occupation accounting for (3) relies largely on the peculiar electron and orbital status in (1) and (2).

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