Spatial compression of a particle state in a parabolic potential by spin measurements

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We propose a scheme for engineering compressed spatial states in a two-dimensional parabolic potential with a spin-orbit coupling by selective spin measurements. This sequence of measurements results in a coordinate-dependent density matrix with probability maxima achieved at a set of lines or at a two-dimensional lattice. The resultant probability density depends on the spin-orbit coupling and the potential parameters and allows one to obtain a broad class of localized pure states on demand. The proposed scheme can be realized in spin-orbit-coupled Bose-Einstein condensates.

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I. INTRODUCTION

In recent years, quantum control of individual particles and ensembles has attracted a lot of attention. Experimentally, quantum dots (QD) [1], ion traps [2], and Bose-Einstein condensates (BEC) [3,4] are promising candidates for designing new nano- and microelements for quantum technologies [5]. Of particular interest is the possibility of simultaneous control and manipulation of degrees of freedom of different origins, such as position and spin, by means of spin-orbit coupling (SOC) present in broad variety of solid-state and condensed-matter systems [6]. For instance, spin and position of electron in semiconductor-based QDs may be mutually related due to this coupling. A synthetic SOC realized in BEC [7] demonstrates a variety of phenomena not achievable in a solid-state-based system. One of advantages of SOC is that it is externally tunable, either by a static electric field for semiconductors [8] or by optical fields in the condensates [7].

In many applications, it is important to control a quantum system in a binding potential and prepare on demand a quantum state, for example, initial pure states of qubits for quantum computation. Various methods of quantum system preparations [9] such as logical [10] or temporal labeling [11] were proposed. Coherent state in a parabolic potential has also been shown to be feasible [12]. Due to the high interest in circuit quantum electrodynamics and ion trap experiments, spatially localized states are important for the particle manipulation (see, for example, Refs. [13–15]). A widely used approach to generate and control the quantum state of a target system is by designing a set of tailored *selective* (where the outcome is either discarded or accepted, dependent on the measurement result) measurements of an ancilla coupled to the target. This approach has been theoretically proposed for pure state generation and ground-state cooling [16,17] and entanglement generation [18] and experimentally realized in Ref. [19] for photons. In this paper, we propose a tailored spin measurement to spatially compress the quantum state of a particle with SOC in a two-dimensional (2D) parabolic potential. As examples, two types of compression schemes are studied, where the state is compressed along a certain direction (*linear compressed state*) or into a two-dimensional lattice similar to *lotus-seed* probability distribution. Experimental tolerance against measurement time errors is also analyzed.

The paper is organized as follows. In Sec. II we show how selective spin measurements on a 2D system with SOC can produce a spatially compressed stripelike density distribution. In Sec. III we consider one-dimensional (1D) realization of this compression. In Sec. IV we show that spatial compression can result in a probability lattice which we call a lotus-seed state. In Secs. V and VI we consider efficiency of the proposed technique and discuss the results. Section VII presents the conclusions. The appendix contains mathematical details and extra figures.

II. SPATIAL COMPRESSION BY SPIN MEASUREMENTS

We begin with considering a pseudo-spin-1/2 particle in a 2D parabolic potential described by the Hamiltonian [20,21]

$$H = \frac{p_x^2 + p_y^2}{2M} + \frac{M\omega^2}{2}(x^2 + y^2) + \alpha(\boldsymbol{p}\cdot\boldsymbol{n})(\boldsymbol{\sigma}\cdot\boldsymbol{m}), \tag{1}$$

where $\boldsymbol{p}$ is the momentum operator, $\boldsymbol{n}$ and $\boldsymbol{m}$ are unit vectors ($\boldsymbol{n}$ lies in the XY plane), $\boldsymbol{\sigma} = (\sigma_x,\sigma_y,\sigma_z)$ is the Pauli matrices vector, M is the particle mass, ω is the oscillator frequency, and α is the SOC constant. The Hamiltonian can be diagonalized via a unitary transformation $\mathcal{U}(\alpha) = \cos\chi - i(\boldsymbol{\sigma}\cdot\boldsymbol{m})\sin\chi$, where $\chi \equiv \alpha M\hbar^{-1}(\boldsymbol{r}\cdot\boldsymbol{n})$. The diagonalized Hamiltonian $H' = \mathcal{U}^\dagger(\alpha)H\mathcal{U}(\alpha)$ reads

$$H' = \left[\hbar\omega(a^\dagger a + b^\dagger b + 1) - \frac{\alpha^2 M}{2}\right] \otimes \mathbf{1}, \tag{2}$$

where $a^\dagger(b^\dagger)$ and $a(b)$ are the raising and lowering operators for $x(y)$ directions. It is clear that the presence of SOC causes an energy shift $-\alpha^2 M/2$.

Now we propose a method to spatially compress a particle state by implementing selective measurements on the spin degree of freedom [22]. In general, the spin measurement can be described by a rotated basis $\{|+\rangle,|-\rangle\}$, where $|+\rangle = \cos\theta/2\,|\uparrow\rangle + e^{i\varphi}\sin\theta/2\,|\downarrow\rangle$, $|-\rangle = \sin\theta/2\,|\uparrow\rangle - e^{i\varphi}\cos\theta/2\,|\downarrow\rangle$, with θ and φ being the polar and azimuthal angles, and $|\uparrow\rangle, |\downarrow\rangle$ denote the spin-up and

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spin-down states. At N evenly spaced times $t = t_1, t_2, \dots, t_N$, we make *selective* measurements described by projection operator $\mathbb{1} \otimes |+\rangle\langle+|$ on the spin state of a particle, which plays the role of an ancilla. After each measurement, we discard the $|-\rangle$ result, thus producing a selective measurement. Between every two consecutive spin measurements, the particle evolves according to the Hamiltonian (1). For an initial particle state described by a factorized density matrix $\rho_0 = \rho_{c0} \otimes |+\rangle\langle+|$, where ρ_{c0} describes the position dependence, after N selective spin measurements, the final density matrix becomes

$$\rho_N = \rho_{cN} \otimes |+\rangle\langle+|, \quad \rho_{cN} = \frac{V^N \rho_{c0} V^{\dagger N}}{P(N)},$$
$$V \equiv \langle+|U(\tau)|+\rangle, \tag{3}$$

where $P(N) \equiv \mathrm{Tr}[V(\tau)^N \rho_{c0} V^{\dagger N}(\tau)]$ is the corresponding survival probability of the system, $U(\tau) = \exp(-iH\tau/\hbar)$ is the evolution operator, and $\tau = t_{i+1} - t_i$ is the interval between two consecutive measurements, assuming instantaneous measurements. We show that by selecting τ in a certain manner, one can obtain the final state ρ_N corresponding to *spatially localized state* of a special kind in the limit $N \to \infty$.

After some algebra (see the appendix) we obtain

$$V(\tau) = [(\cos\chi - if\sin\chi)(\cos\chi' + if\sin\chi') + |g|^2 \sin\chi \sin\chi']e^{-i\tilde{H}\tau/\hbar}, \tag{4}$$

where $\chi' = e^{-i\tilde{H}\tau/\hbar}\chi e^{i\tilde{H}\tau/\hbar}$, $\tilde{H}$ is the spatial part of H', $f \equiv \langle+|(\boldsymbol{\sigma}\cdot\boldsymbol{m})|+\rangle$ and $g \equiv \langle+|(\boldsymbol{\sigma}\cdot\boldsymbol{m})|-\rangle$.

It is noticeable that while V is hard to be calculated even numerically [18], we can have analytical solutions in the following two cases:

(1) $\omega\tau = 2\pi j$, when $\chi' = \chi$,

(2) $\omega\tau = \pi + 2\pi j$, when $\chi' = -\chi$,

where $j's$ $(= 0,1,2,\ldots)$ are integers. It is easy to see that in the first case we have $V(\tau) = e^{-2\pi i j\tilde{H}/\hbar\omega}$, which is not of interest because $V(\tau)$ acts on the oscillator states as unity operator. Henceforth we will consider the second case.

To be concrete we consider a realization with $\theta = 0$ and $\boldsymbol{m} = (m_x, m_y, 0)$ and obtain

$$V(\tau) = \cos(2\chi)e^{-2\pi i\tilde{H}(j+1/2)/\hbar\omega}. \tag{5}$$

The eigenstates of operator $e^{-2\pi i\tilde{H}(j+1/2)/\hbar\omega}$ are number states with eigenvalues $A(-1)^k$(see Ref. [23]), where $A = \exp[2\pi i(j+1/2)(\alpha^2 M/2\hbar\omega - 1)]$ and $k = k_x + k_y$ is the total number of excitations related to the x and y spatial degrees of freedom. Therefore, when we substitute Eq. (5) in Eq. (3), we can keep only $\cos(2\chi)$ in $V(\tau)$ when (i) the initial state ρ_{c0} is a pure number state $\rho_{c0} = |k_x,k_y\rangle\langle k_x,k_y|$ or (ii) if the initial mixed state is diagonal in the number states basis: $\rho_{c0} = \sum_{N=0}^{\infty}\sum_{k_x=0}^{N} p(N,k_x)|k_x, N-k_x\rangle\langle k_x, N-k_x|$, exemplified by the thermal state, where $p(N,k_x)$ are classical probabilities and ket (bra) vector contains number of excitations related to the x and y degrees of freedom. In the coordinate representation, our result becomes

$$\rho_{cN}(x,y;x',y') = \cos^N[2\chi(x,y)]\rho_{c0}(x,y;x',y')\cos^N[2\chi(x',y')] \times \left\{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\cos^{2N}[2\chi(x,y)]\rho_{c0}(x,y;x,y)dxdy\right\}^{-1}. \tag{6}$$

As can be seen from Eq. (6), the resultant density matrix $\rho_{cN}(x,y;x',y')$ has sharp maxima in regions where $|\cos[2\chi(x,y)]| = 1$. These regions correspond to lines in the xy plane:

$$(\boldsymbol{r}\cdot\boldsymbol{n}) = \frac{\pi l\hbar}{2\alpha M},\ l = 0, \pm1, \pm2, \ldots, \tag{7}$$

allowing us to achieve the desired *lower-symmetry spatial compression*. Particularly if the SOC is weak (i.e., $\alpha \ll \hbar d^{-1}M^{-1}$, where d is the size of the system), then it is sufficient to take only $l = 0$ in Eq. (7). The distance between the lines in Eq. (7) corresponds to spin-precession length $\hbar/M\alpha$ [8] and does not depend on ω.

Unlike in Refs. [16,17], the resultant state, Eq. (6), of our approach does not increase purity of the initial spatially symmetric (thermal) state of the particle to 1, even if $N \to \infty$. However, it can be used for obtaining pure states out of *nonsymmetric* mixed states. For example, if the spatial part of the initial state is given by density matrix $\rho_{c0} = |0,0\rangle\langle 0,0|/2 + |0,1\rangle\langle 0,1|/2$, with the purity $\mathrm{Tr}\rho_{c0}^2 = 1/2$, then for $\boldsymbol{n} = (0,1,0)$, we obtain $\mathrm{Tr}\rho_{cN}^2 \to 1$ at $N \to \infty$ [24].

To illustrate our proposal, we consider a realization $\alpha = \alpha_0 = \sqrt{\hbar\omega/2M}$ and $\boldsymbol{n} = (1/\sqrt{2}, 1/\sqrt{2}, 0)$. Initial state for the particle is the ground state $\rho_{c0} = |0,0\rangle\langle 0,0|$. In Figs. 1(a)–1(c) we present spatial probability densities $\rho_{cN}(x,x;y,y)$, where a progressively more compressed state is observed for higher number of successful measurements (see Ref. [25] and references therein, e.g., experimental implementation of spin measurements).

This setting can be realized in a QD (with a typical $\hbar\omega \approx 1$ meV) located in an InSb semiconductor layer [26] with a width of about 10^{-6} cm in a perpendicular electric field of 10^5 V/cm. In this case, the general Hamiltonian with both Dresselhaus [27] and Rashba [28] terms can be expressed as (1). However, this QD-based realization of the compression is difficult in terms of the frequency and timing accuracy, which is required to be as higher than 1 ps. Therefore, atomic systems may be much better suitable for obtaining the proposed spatially compressed states. For example, the characteristic frequency ω of trapped BEC is only about 300 s^{-1} and achievable *synthetic* spin-orbit coupling strength is $\alpha \approx 4\alpha_0$ (Ref. [7]). Projective pseudospin measurements on BEC have been used recently in Ref. [29].

III. COMPRESSION IN ONE-DIMENSIONAL SYSTEMS

Our proposal can also be applied to a one-dimensional system (such as a quantum wire or a tightly compressed BEC) in a parabolic potential with spin-orbit interaction [30], schematically shown in Fig. 2. In this case, the Hamiltonian (1)

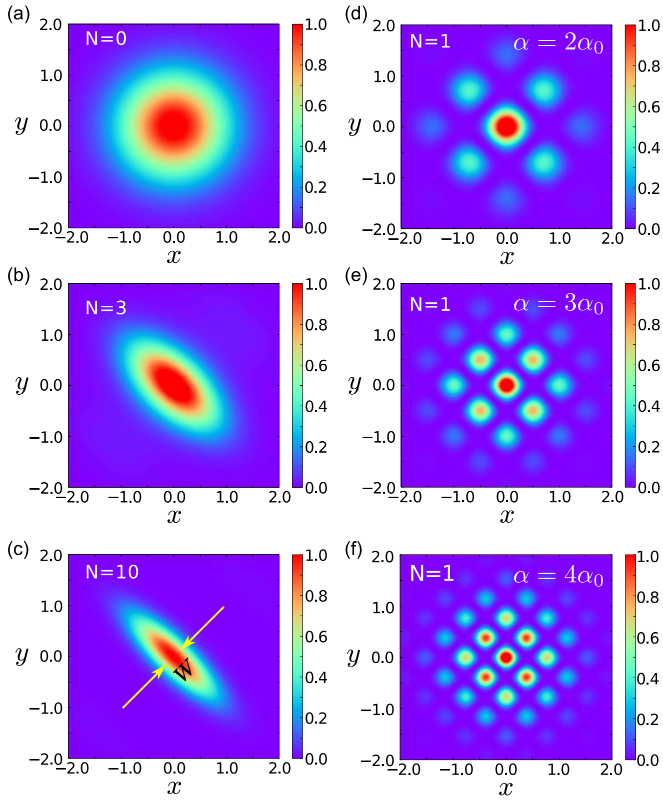

FIG. 1. Density of the spatial probability of finding particle after several successful measurements. Left column: linear compressed state. The probability density of initial ground state (a), after $N = 3$ (b) and $N = 10$ (c) successful measurements. In panel (c) we additionally illustrate definition of half-width W [see Eq. (8)]. Right column: lotus-seed state. We display the probability distributions after two successful spin measurements, one before changing electric field and another one after, with $\alpha = 2\alpha_0$ (d), $\alpha = 3\alpha_0$ (e), and $\alpha = 4\alpha_0$ (f).

can be rewritten as

$$H = \frac{p^2}{2M} + M\omega^2\frac{x^2}{2} + \alpha p(\boldsymbol{\sigma}\cdot\boldsymbol{m}).$$

Note that in the 1D case, this Hamiltonian is valid for *any ratio* of Rashba and Dresselhaus coupling strengths which determine the direction of vector $\boldsymbol{m}$. All results for the wire are similar to those for 2D systems (we again choose

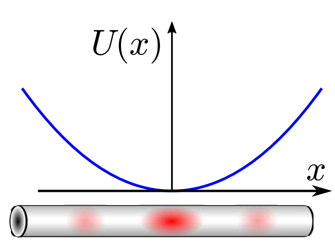

FIG. 2. Schematic illustration of 1D quantum wire with parabolic potential $U(x)$. The density produced by the measurements is illustrated by red color inside the wire.

$\boldsymbol{m} = (m_x, m_y, 0), \theta = 0)$, and Eq. (6) can be rewritten as

$$\rho_{cN}(x;x') = \cos^N[2\chi(x)]\rho_{c0}(x;x')\cos^N[2\chi(x')] \times \left\{\int_{-\infty}^{+\infty}\cos^{2N}[2\chi(x)]\rho_{c0}(x;x)dx\right\}^{-1},$$

where $\chi \equiv \alpha M x/\hbar$. Instead of Eq. (7) for the 2D potential, in 1D we obtain the density distributed near special *points* along the wire: $x_l = \pi l\hbar/2\alpha M$, $l = 0, \pm1, \pm2, \ldots$.

IV. COMPRESSION INTO A LOTUS-SEED STATE

Another interesting possibility is given by changing the SOC term in Hamiltonian (1) after N successful spin measurements and performing N further selective spin measurements with a different SOC term. Physically, it can be realized, for example, by changing the applied electric field across the QD plane to the opposite direction; i.e., after first N measurements we change the vector $\boldsymbol{n}$ in Hamiltonian (1) to $\boldsymbol{n}'$, where $\boldsymbol{n}' \perp \boldsymbol{n}$. In this manner the final spatial probability distribution will be different. Since spatial compression with a given $\boldsymbol{n}$ generates probability lines in Eq. (7), it is easy to understand that by changing $\boldsymbol{n}$ we obtain a state with *lotus-seed spatial distribution* [Figs. 1(d)–1(f)]. Here, peaks in the probability density correspond to the intersections of probability lines in Eq. (7) for different $\boldsymbol{n}$ and $\boldsymbol{n}'$. In Figs. 1(d)–1(f) we show the density of the spatial probability for the same initial ground state as before, $\boldsymbol{m}$ and $\boldsymbol{n}$ as in the previous example for $\alpha = 2\alpha_0$ [Fig. 1(d)], $\alpha = 3\alpha_0$ [Fig. 1(e)], and $\alpha = 4\alpha_0$ [Fig. 1(f)]. Here, after the first ($N = 1$) selective measurement we change the direction of $\boldsymbol{n}$ and after another successful spin measurement with $\boldsymbol{n}'$ we obtain a lotus-seed spatial distribution of electron spatial density. Note that states shown in Figs. 1(d)–1(f) are pure.

V. EFFICIENCY OF SPATIAL COMPRESSION

In order to evaluate the effectiveness of the spatial compression we can calculate characteristic size of the region with essential probability density as a function of the number of successful measurements N. In the case of linear compression [Eq. (6) and Fig. 1] and for large $N \gg \hbar\omega/M\alpha^2$ we obtain half-width $W(N)$ [see Fig. 1(c)] of the spatial probability function as (see the appendix):

$$W(N) = \frac{\hbar}{\alpha M\sqrt{2N}} \propto \frac{1}{\sqrt{N}}. \tag{8}$$

As expected, $W(N)$ tends to zero as $N \to \infty$. In Fig. 3, we present the dependence of survival probability $P(N)$ defined below Eq. (3) on the number of measurements for three different values of the SOC constant for linear compression (the lotus-seed state plot is presented in the appendix).

Experimentally, it is important to take into account the robustness against the measurement errors. As we have noticed, Eq. (6) is valid only when all N measurements are performed with ideal intervals $\tau = 2\pi\omega^{-1}(1/2 + j)$ between each of two consecutive measurements. To achieve a high fidelity, the time interval τ can be different from π/ω, but the absolute time error δt must be small: $\delta t \ll \pi/\omega$. In Fig. 4 we present the time dependence of fidelity F [31] between ideal lotus-seed state achieved by sequentially applying Eq. (6) with different χ and spatial state which results from two

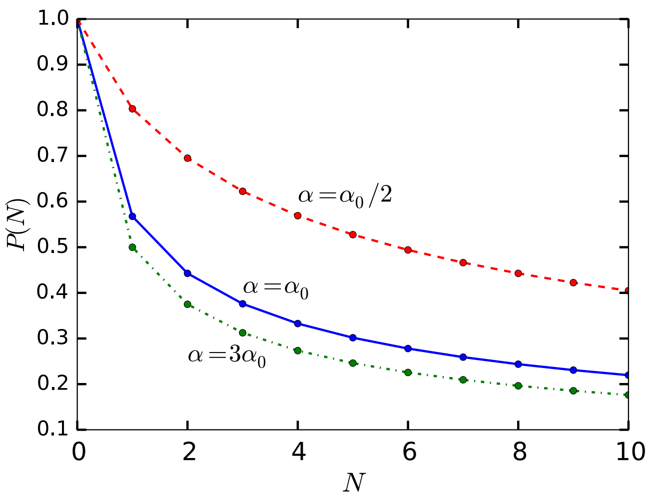

FIG. 3. Survival probability for linear compressed state case: $\alpha = \alpha_0/2$ (dashed red line), $\alpha = \alpha_0$ (solid blue line), and $\alpha = 3\alpha_0$ (dotted green line).

measurements made at nonideal times $\tau_{1,2} = \pi/\omega + \delta t_{1,2}$. The central point of Fig. 4 corresponds to two sequential ideal spin measurements where $F(0,0) = 1$. As can be seen from Fig. 4, F is tolerant against small time errors. Asymmetry with respect to δt_1 and δt_2 in Fig. 4 is due to the dependence of time errors' contribution to the final state on the step number.

VI. DISCUSSION

As mentioned above, the resultant spatial wave function can be expressed as a superposition of high-energy oscillator states (each nondiscarded spin measurement increases the energy by the value of the order of $M\alpha^2$). However, the ideal oscillator spectrum may not be realizable in realistic systems. A QD has only a few low-lying levels with equal energy gaps. Thus, the electron spatial probability will certainly be less sharply defined than in Fig. 1. This also means that there is no sense in performing many measurements if the SOC constant is of the order α_0 or larger.

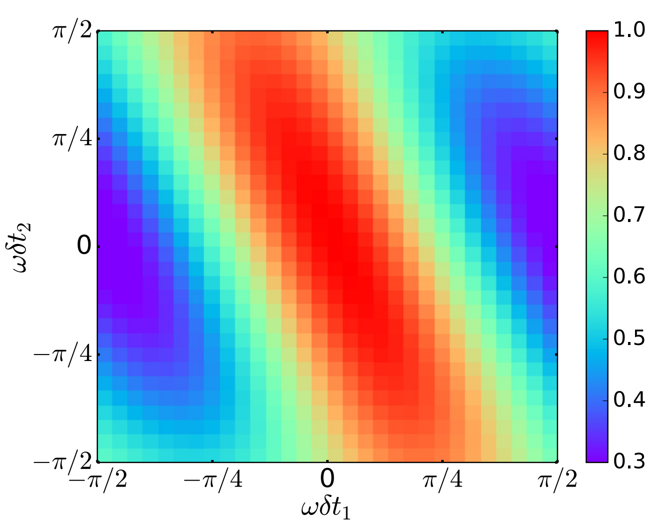

FIG. 4. Fidelity between the ideal lotus seed state ($\alpha = \alpha_0$, two spin measurements after two evolutions with different vectors $\boldsymbol{n}_1, \boldsymbol{n}_2, \boldsymbol{n}_1 \perp \boldsymbol{n}_2$) and another spatial quantum state resulted from two spin measurements with time shifts δt_1 and δt_2 from the ideal time π/ω.

Note that if the last term in Eq. (1) is replaced by $\alpha'(\boldsymbol{r}\cdot\boldsymbol{n})(\boldsymbol{\sigma}\cdot\boldsymbol{m})$ for a spin-coordinate coupling, we can repeat all our calculations and achieve the same results by swapping variables $\boldsymbol{p}$ and $\boldsymbol{r}$. In this case we can generate the compressed states in the momentum space instead of spatial compression described above. This can be realized, for example, by means of an inhomogeneous magnetic field.

In addition to the spatial compression, by using only single-shot selective spin measurement we can achieve a Schrödinger-cat-like spatial superposition state [32] (see the appendix). If the initial state is the ground state with spin $|\uparrow\rangle$ then, as follows from Eq. (1), this state is a superposition of two states [eigenstates of $(\boldsymbol{\sigma}\cdot\boldsymbol{m})$], which have opposite velocities $\pm\alpha/\sqrt{2}$ with directions collinear to $\boldsymbol{n}$ [33]. Therefore, by performing spin measurement at time $\pi/2\omega$, we can achieve separated peaks in the spatial probability distribution (see the appendix). The distance L between these two peaks can be estimated as $L \approx (\alpha/\alpha_0)l_0$, where $l_0 = \sqrt{\hbar/M\omega}$ is the oscillator length. This distance corresponds to the spin separation length [33] arising due to anomalous spin-dependent velocity ($\sim \alpha$) on the timescale of the measurement and can be approximately written as $L \approx \alpha \cdot (\pi/2\omega)$.

For completeness, we mention that our proposal is robust against decoherence effects when the condition $N\tau < T_d$, where T_d is the characteristic decoherence time, holds. This condition, which can be reformulated as $\omega T_d \gg 1$, meaning that the orbital states are well defined, holds for the majority of localized quantum systems weakly coupled to the environment [34,35].

VII. CONCLUSION

We proposed a technique for achieving two kinds of spatially compressed states in two-dimensional harmonic potential with spin-orbit coupling. It has been shown that tailored *selective* projective measurements of the spin of the particle can dramatically modify the position-dependent probability density. As a result, one can create *on demand* a variety of stripe- and lotus-seed-like density distributions, dependent on the measurement protocols and system parameters. We suggest that spin-orbit-coupled Bose-Einstein condensates are suitable for realization of this proposal.

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APPENDIX

1. Derivation of Eq. (4) using the ladder operators

Using the diagonalized Hamiltonian in Eq. (2), we have

$$V(\tau) = Fe^{-i\tilde{H}\tau/\hbar}F^{\dagger} + Ge^{-i\tilde{H}\tau/\hbar}G^{\dagger}. \quad \text{(A1)}$$

Here $\tilde{H}$ is a spatial part of H' which does not act on the spin degree of freedom, $F \equiv \cos\chi - if\sin\chi$, $G \equiv -ig\sin\chi$, and $f \equiv \langle +|(\boldsymbol{\sigma}\cdot\boldsymbol{m})|+\rangle$, $g \equiv \langle +|(\boldsymbol{\sigma}\cdot\boldsymbol{m})|-\rangle$.

For obtaining expression (4) in the main text, we use the relation

$$V(\tau) = V(\tau)e^{i\tilde{H}\tau/\hbar}e^{-i\tilde{H}\tau/\hbar}, \tag{A2}$$

and the following property of a unitary operator:

$$e^{-i\tilde{H}\tau/\hbar}f(\hat{\chi})e^{i\tilde{H}\tau/\hbar} = f(e^{-i\tilde{H}\tau/\hbar}\hat{\chi}e^{i\tilde{H}\tau/\hbar}), \tag{A3}$$

where the operator $\hat{\chi}$ depends on the ladder operators $a^\dagger$,a,$b^\dagger$,b, and the operator function f in our case is $\sin\hat{\chi}$ or $\cos\hat{\chi}$.

With the above transformations, we can define a new operator

$$\chi' = e^{-i\tilde{H}\tau/\hbar}\chi e^{i\tilde{H}\tau/\hbar}. \tag{A4}$$

The operator χ is a function of coordinate operators $a^\dagger + a$ and $b^\dagger + b$.

To simplify expression for χ', we use the known property of ladder operators:

$$e^{\gamma a^\dagger a}ae^{-\gamma a^\dagger a} = e^{-\gamma}a, \quad e^{\gamma a^\dagger a}a^\dagger e^{-\gamma a^\dagger a} = e^{\gamma}a^\dagger. \tag{A5}$$

If γ in the above relations is $2\pi ij$ ($j = 0,1,2,\ldots$), then it means that

$$e^{2\pi ija^\dagger a}(a^\dagger + a)e^{-2\pi ija^\dagger a} = a^\dagger + a. \tag{A6}$$

The above equation corresponds to the case $\chi = \chi'$ in the main text. If now γ in the above relations is $2\pi j + \pi$, then

$$e^{2\pi i(j+1/2)a^\dagger a}(a^\dagger + a)e^{-2\pi i(j+1/2)a^\dagger a} = -(a^\dagger + a), \tag{A7}$$

corresponding to the $\chi = -\chi'$ case, which is of interest to us. Note that only for these two cases the unitary transformation (A4) transforms one coordinate to another, while for other values of γ, coordinates are transformed into a function of coordinates and momenta.

2. Derivation of the half-width of the probability line

We assume that the initial state is the ground state:

$$\begin{aligned}\rho_0(x,y;x',y') &= \psi_0(x,y)\psi_0^*(x',y')\\ &= \phi_0(x)\phi_0(y)\phi_0^*(x')\phi_0^*(y'),\end{aligned} \tag{A8}$$

where ϕ_0 is the ground-state eigenfunction of a harmonic oscillator and ψ_0 is the initial wave function of a particle in a 2D parabolic potential. Furthermore, we will use wave functions formalism because the purity of the initial state is 1 and it does not change.

Wave function after N nondiscarded measurements can be written as

$$\psi_N(x,y) = \frac{\cos^N(2\chi)\psi_0(x,y)}{\sqrt{P(N)}}. \tag{A9}$$

For this choice, the maximum of the probability density is at the (0,0) point. The value of the probability density in this point is

$$|\psi_N(0,0)|^2 = \frac{|\psi_0(0,0)|^2}{P(N)}. \tag{A10}$$

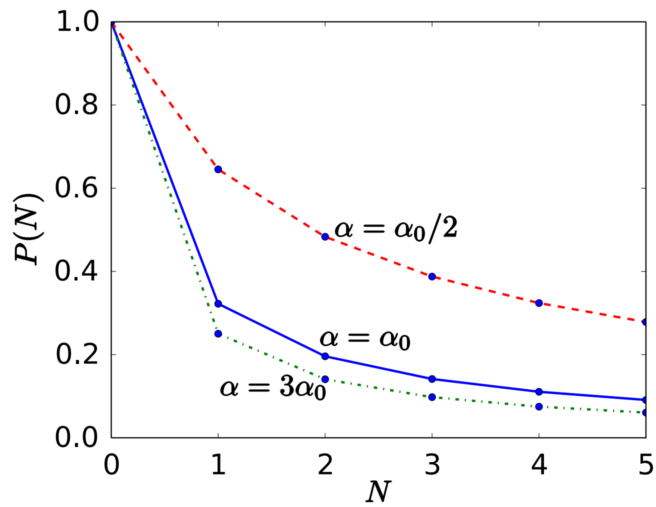

FIG. 5. Survival probability for compressed lotus-seed state. Here the symbol N means N measurements with the initial applied electric field direction and N measurements with the opposite one.

We define a half-width W as follows:

$$|\psi_N(W)|^2 = \frac{|\psi_N(0,0)|^2}{2}. \tag{A11}$$

Here we skip N in $W(N)$ and write $W(N)$ as W. By using the definition of χ in the main text, we write

$$\chi(W) = \alpha M\hbar^{-1}W/2. \tag{A12}$$

We assume that for a large N, we have $W \to 0$ and $\phi_0(0) \approx \phi_0(W)$. Thus, by using the above definitions we obtain the following equation for an unknown W:

$$\cos^{2N}[2\chi(W)] = 1/2, \tag{A13}$$

which can be solved by using the expansion $\cos^{2N}[2\chi(W)] \approx 1 - 4N\chi^2(W)$, valid at $W \to 0$.

In Fig. 5 we present the survival probability for the lotus-seed compressing case. As can be seen from comparison of

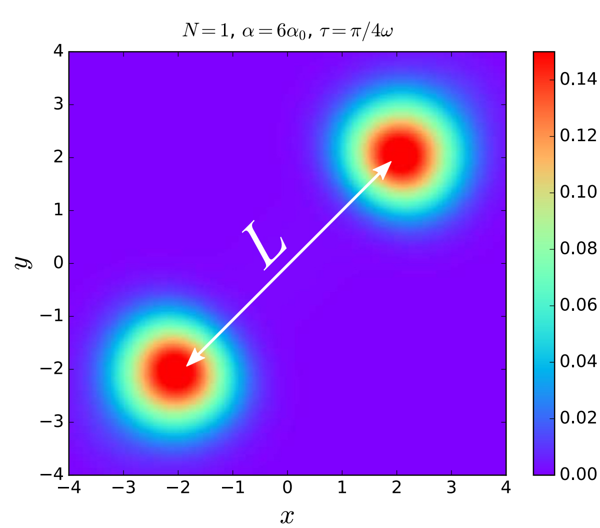

FIG. 6. Schrödinger cat state corresponding to the probability density distribution after one successful measurement at the special time $\tau = \pi\omega^{-1}/4$. Here spin-orbit coupling is strong: $\alpha = 6\alpha_0$.

Figs. 3 and 5, survival probability for the lotus-seed state is lower than that for the linear compression.

3. Schrödinger cat state

If the initial state of a particle is the ground state then we can achieve a Schrödinger cat state by a single-shot measurement at time $\omega\tau = \pi/4$. In Fig. 6, we present the probability density after this measurement. The distance L between two separated peaks of probability is proportional to the spin-orbit coupling, i.e., $L \propto \alpha$. Note that the probability density in Fig. 6 corresponds to a pure state of the particle.

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