Kondo effect in a quantum wire with spin-orbit coupling

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The influence of spin-orbit interactions on the Kondo effect has been under debate recently. Studies conducted recently on a system composed of an Anderson impurity on a two-dimensional electron gas with a Rashba spin orbit have shown that it can enhance or suppress the Kondo temperature ($T_{\rm K}$), depending on the relative energy level position of the impurity with respect to the particle-hole symmetric point. Here, we investigate a system composed of a single Anderson impurity, side coupled to a quantum wire with spin-orbit coupling (SOC). We derive an effective Hamiltonian in which the Kondo coupling is modified by the SOC. In addition, the Hamiltonian contains two other scattering terms, the so-called Dzyaloshinskii-Moriya interaction, known to appear in these systems, and another one describing processes similar to the Elliott-Yafet scattering mechanisms. By performing a renormalization group analysis on the effective Hamiltonian, we find that the correction on the Kondo coupling due to the SOC favors the enhancement of the Kondo temperature even in the particle-hole symmetric point of the Anderson model, agreeing with the numerical renormalization group results. Moreover, away from the particle-hole symmetric point, $T_{\rm K}$ always increases with the SOC, accordingly with a previous renormalization group analysis.

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I. INTRODUCTION

The well-known Kondo effect is a many-body dynamical screening of a localized magnetic moment by the spins of itinerant electrons that occurs at temperatures below the so-called Kondo temperature ($T_{\rm K}$) [1]. Originally observed in bulk magnetic alloys [2] with conspicuous transport features, this effect has been extensively studied in a few magnetic impurities coupled to one-dimensional (1D) [3–5] and two-dimensional (2D) [6–8] systems. Recently, a number of studies has discussed the effect of spin-orbit coupling (SOC) on the Kondo effect on two-dimensional systems. More specifically, the question of how the SOC modifies the Kondo effect in systems with an isolated magnetic impurity has gained more attention [9–16]. The influence of the effect of SOC on the Kondo physics has gained major interest mainly because the former has become remarkably attractive in condensed matter systems [17,18]. For example, SOC is the basic ingredient for many different phenomena, extending from the spin manipulation in the celebrated Datta-Das transistor [19] to more fundamental physics as in the quantum spin-Hall effect [20] and Majorana fermions [21].

Since the Kondo effect involves collectively the spins of the itinerant electrons, it is not surprising that SOC—that locks the electron spin with their momenta—will modify it. In fact, while in Ref. [9] there was found to be no change in the Kondo temperature with SOC, recent studies [10–13] have found a change in the Kondo temperature due to Rashba SOC. Apart from Ref. [12] that addresses the Kondo effect in graphene, the other ones report arguable results about similar systems. On the one hand, in Ref. [9] it was found that the Rashba SOC has essentially no effect on $T_{\rm K}$. On the other, in Ref. [10], by renormalization group analysis (RGA), and in Refs. [11,13], using the numerical renormalization group (NRG), it is reported that $T_{\rm K}$ is dependent on the SOC, although the actual functional dependency obtained by the NRG seems to differ from the RGA approach. This controversy can be attributed to the different regimes in which the analysis was carried out and to some approximations made in the RGA. We should stress that Malecki's idea of studying the effect of SOC on $T_{\rm K}$ using a standard Kondo model was incomplete. This became apparent in Ref. [10], in which it was shown that the standard Kondo model does not include all the scattering phenomena in the system.

Owing to the various studies discussed above, the effect of SOC on the Kondo temperature in two-dimensional systems has been quite well elucidated. In one-dimensional systems, however, the effect of SOC on the Kondo effect may be even more important and has yet to be investigated. The expected importance of SOC on the Kondo effect on 1D systems can be viewed in a simple way. As mentioned above, the Kondo effect is based on electron scatterings accompanied by spin-flip processes involving the spins of the conduction electrons and those of the local magnetic moments. At very low temperatures, energy conserving scatterings become more relevant as compared to nonconserving ones. Contrasting with the 2D case, in which energy conserving skew scatterings are also allowed, in 1D only forward or backward scattering can occur. In situations in which a backward scattering event suffered by the conduction electrons requires a flip of their spins, it is expectable that the SOC has a much stronger influence on the Kondo effect in 1D systems as compared to the 2D ones. Such spin-momentum locking is known to occur in strongly spin-orbit coupled 1D systems, such as InSb nanowires [22] and in the 1D edge states of topological insulators [23].

Motivated by the aforementioned peculiarities of the SOC in one-dimensional systems, we investigate the Kondo effect of a magnetic impurity, side coupled to a quantum wire with both Rashba [24] and Dresselhaus [25] SOC. For the impurity, we restrict ourselves to a spin-$\frac{1}{2}$ magnetic moment and model it as a single-level interacting quantum dot that couples to the conduction electrons in the quantum wire through tunneling matrix elements. By projecting the total Hamiltonian of the system onto a singly occupied subspace of the impurity, we derive an effective Kondo Hamiltonian, which contains the known Dzyaloshinskii-Moriya interaction term and an additional one, analogous to the Elliott-Yafet spin-flip scattering mechanism induced by the SOC [26–30]. Once we have obtained our effective Kondo-like Hamiltonian, we perform a renormalization group analysis (similarly to what was done in Ref. [10]) from which we extract the Kondo temperature.

Our results show that the dependence of T_K with the SOC strength differs from what was found in Ref. [10]. For instance, we find that the Kondo temperature always increases, even when the system is at the particle-hole symmetric point, which contrasts with the results reported in Ref. [10] but agree with those found in Refs. [11,13]. The disagreement between our results and those of Ref. [10] is attributed to the correction on the effective Kondo coupling due to the SO interaction, neglected in a previous study. It is also noteworthy that the dependence of T_K with the SO coupling is particle-hole asymmetric. We show that an extra scattering term in the effective Hamiltonian is the one responsible for breaking the particle-hole symmetry of the RG equation.

The remainder of this paper is organized as follows: In Sec. II we present the model Hamiltonian and derive an effective Kondo-like Hamiltonian, and in Sec. III we perform a renormalization group analysis with a numerical solution. Finally, in Sec. IV, we summarize our main results. Some of the details of the calculations are shown in the Appendixes.

II. HAMILTONIAN MODEL

For the sake of clarity, we schematically represent our system in Fig. 1, in which the local magnetic moment is modeled by a single-level quantum dot occupied by one electron. The quantum wire is assumed to lie along the x axis and includes both Rashba [24] and linear Dresselhaus SOC [25]. Because of the dimensionality of the wire, both SOCs are treated on the same footing. More formally, our system is described by an Anderson-like model, $H = H_{\rm wire} + H_{\rm dot} + H_{\rm dot-wire}$, where

$$H_{\rm dot} = \sum_s \varepsilon_d d_s^\dagger d_s + U n_\uparrow n_\downarrow \qquad (1)$$

describes the isolated quantum dot, in which $d_s^\dagger$ (d_s) creates (annihilates) an electron with energy ε_d and spin s in the dot and U is the on-site Coulomb repulsion in the quantum dot. We also have defined the number operator $n_s = d_s^\dagger d_s$. The quantum wire is described by

$$H_{\rm wire} = \sum_k \left[\varepsilon_k \delta_{ss'} + k\left(\beta\sigma^x_{ss'} - \alpha\sigma^y_{ss'}\right)\right] c_{ks}^\dagger c_{ks'}, \qquad (2)$$

where k is the momentum along the x axis, and $\varepsilon_k = \hbar^2k^2/2m^*$ the kinetic energy with m^* representing the effective mass of the conduction electrons. The operator $c_{ks}^\dagger$ (c_{ks}) creates (annihilates) an electron with momentum k and spin s in the wire. The Rashba and the linear Dresselhaus spin-orbit interaction coupling is parametrized by the interaction strength α and β, respectively, and σ^ν (with $\nu = x,y,z$) represents the Pauli matrices. Finally,

$$H_{\rm dot-wire} = \sum_{ks} (V_k c_{ks}^\dagger d_s + V_k^* d_s^\dagger c_{ks}) \qquad (3)$$

couples the quantum dot to the wire with an overlap matrix element V_k.

We should keep in mind that we aim to derive an effective Kondo-like Hamiltonian by projecting out the empty and the doubly occupied states of the quantum dot. Before doing so, we want to bring the full Hamiltonian into the single impurity Anderson model (SIAM) form. To accomplished this, we diagonalize $H_{\rm wire}$ by performing the following rotation in the spin space,

$$\begin{pmatrix} c_{k+} \\ c_{k-} \end{pmatrix} = \mathcal{U} \begin{pmatrix} c_{k\uparrow} \\ c_{k\downarrow} \end{pmatrix}, \qquad (4)$$

with

$$\mathcal{U} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & e^{-i\theta} \\ -ie^{i\theta} & i \end{pmatrix}, \qquad (5)$$

where $\theta = \tan^{-1}(\beta/\alpha)$. Under this transformation, the Hamiltonian $H_{\rm wire}$ acquires the diagonal form

$$\tilde{H}_{\rm wire} = \sum_{kh} \varepsilon_{kh} c_{kh}^\dagger c_{kh}, \qquad (6)$$

in which $h = +,-$ is the helical quantum number and $\varepsilon_{kh} = \hbar k^2/2m^* + h|\gamma|k$ with $\gamma = \alpha - i\beta$. By applying the same transformation to the quantum dot operators, we see that the forms of $H_{\rm dot}$ and $H_{\rm dot-wire}$ remain unchanged. Therefore, in the SO basis, the total Hamiltonian acquires the SIAM form

$$\tilde{H} = \sum_h \varepsilon_d d_h^\dagger d_h + U n_+ n_- + \sum_{kh} \varepsilon_{kh} c_{kh}^\dagger c_{kh} + \sum_{kh} (V_k c_{kh}^\dagger d_h + V_k^* d_h^\dagger c_{kh}), \qquad (7)$$

where $\varepsilon_{kh} = \varepsilon_k + h|\gamma|k$. These are the SO bands shown in Fig. 2(a). We are now ready to derive the effective Kondo-like Hamiltonian.

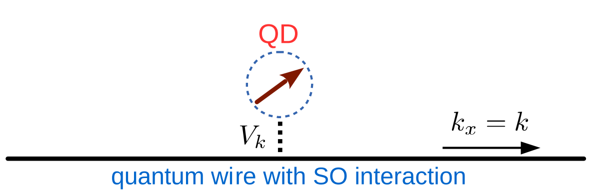

FIG. 1. Schematic representation of a quantum dot, side coupled to a quantum wire with spin-orbit interaction. The wire is assumed to lie along the x direction. V_k represents the hopping of electrons from the quantum into the wire.

The effective Hamiltonian

Since we are interested in the Kondo regime of the system in which there is a magnetic moment localized in the quantum dot, we project the Hamiltonian (7) onto the singly occupied subspace of the quantum dot Hilbert space. We follow the same strategy described in Hewson's book [1] (for details, see Appendix A). The resulting effective Hamiltonian can be

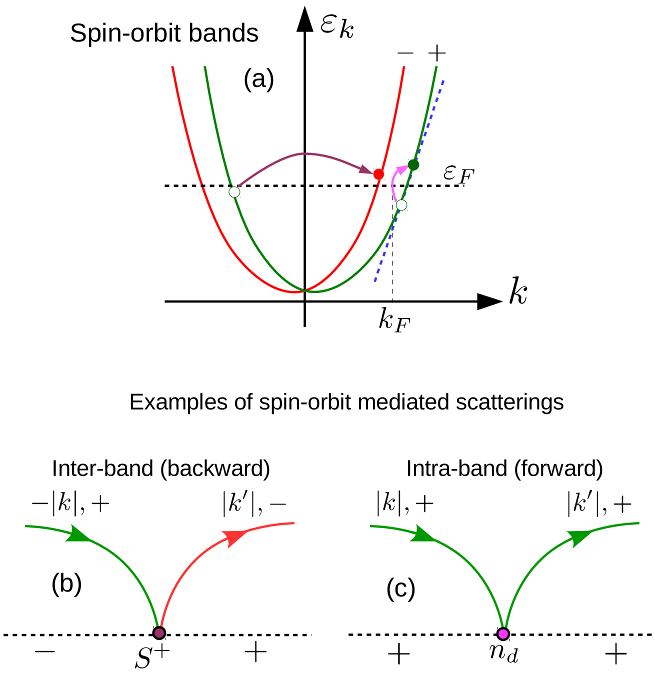

FIG. 2. (a) Spin-orbit bands for the conduction electrons. At low temperature, the allowed processes are those involving energies close to the Fermi level ε_F (horizontal dashed line). The magenta and purple arrows exemplify, respectively, the intraband (forward) and intraband (backward) scatterings. (b) and (c) are representative scattering diagrams describing typical processes contained in the Hamiltonians (28) and (30), respectively.

written in the form

$$H_{\rm eff} = H_0 + H_{\rm K} + H_{\rm DM} + H_{\rm EY}. \tag{8}$$

Here,

$$H_0 = \sum_{k,h} \varepsilon_{kh} c^\dagger_{kh} c_{kh} \tag{9}$$

describes the conduction band on the SO basis,

$$H_{\rm K} = \sum_{kk'} J_{kk'} [(c^\dagger_{k'+} c_{k+} - c^\dagger_{k'-} c_{k-}) \times S_z + c^\dagger_{k'+} c_{k-} S_- + c^\dagger_{k'-} c_{k+} S_+] \tag{10}$$

describes the Kondo coupling, in which

$$J_{kk'} = V_k V^*_{k'} \frac{A_k + A_{k'}}{2}, \tag{11}$$

with

$$A_k = \frac{\varepsilon_k - \varepsilon_d}{(\varepsilon_k - \varepsilon_d)^2 - |\gamma|^2 k^2} + \frac{\varepsilon_d + U - \varepsilon_k}{(\varepsilon_d + U - \varepsilon_k)^2 - |\gamma|^2 k^2}. \tag{12}$$

Observe that $J_{kk'}$ depends on the SO coupling γ. By inspection we see, in the absence of the spin-orbit interaction ($\gamma = 0$), we recover the conventional Kondo coupling, for which $A_k = (\varepsilon_d + U - \varepsilon_k)^{-1} + (\varepsilon_k - \varepsilon_d)^{-1}$. The last two terms of the Hamiltonian (8) are given by

$$H_{\rm DM} = \sum_{kk'} \Gamma_{kk'} (c^\dagger_{k'+} c_{k-} S_- - c^\dagger_{k'-} c_{k+} S_+) \tag{13}$$

and

$$H_{\rm EY} = H^{(1)}_{\rm EY} + H^{(2)}_{\rm EY}. \tag{14}$$

In this last expression,

$$H^{(1)}_{\rm EY} = \sum_{kk'} \Gamma^{(1)}_{kk'} (c^\dagger_{k'+} c_{k+} + c^\dagger_{k'-} c_{k-}) S_z \tag{15}$$

and

$$H^{(2)}_{\rm EY} = \sum_{kk'} \Gamma^{(2)}_{kk'} \frac{n_d}{2} (c^\dagger_{k'+} c_{k+} - c^\dagger_{k'-} c_{k-}). \tag{16}$$

The couplings in the Eqs. (13), (15), and (16) can be written as

$$\Gamma_{kk'} = V_k V^*_{k'} \frac{B^{(+)}_k - B^{(+)}_{k'}}{2}, \tag{17}$$

$$\Gamma^{(1)}_{kk'} = -V_k V^*_{k'} \frac{B^{(+)}_k + B^{(+)}_{k'}}{2}, \tag{18}$$

and

$$\Gamma^{(2)}_{kk'} = V_k V^*_{k'} \frac{B^{(-)}_k + B^{(-)}_{k'}}{2}. \tag{19}$$

Here we have defined

$$B^{(\pm)}_k = \pm|\gamma| k \left[\frac{1}{(\varepsilon_k - \varepsilon_d)^2 - |\gamma|^2 k^2} \mp \frac{1}{(\varepsilon_d + U - \varepsilon_k)^2 - |\gamma|^2 k^2} \right]. \tag{20}$$

The Hamiltonian (13) corresponds to the known Dzyaloshinskii-Moriya interaction while (15) and (16) describe the Elliott-Yafet-like processes [26,27], responsible for spin-flip scatterings of the conduction electrons by the localized magnetic moments [28]. The spin-flip processes involved in the Hamiltonians (15) and (16) are not apparent in the SO basis, but are clearly seen when these Hamiltonians are written in the real spin representation (see Appendix B).

At the low-temperature regimes we can assume that the scatterings occur only for electrons with momenta close to Fermi momentum k_F. Moreover, for small SO interactions, such that $|\gamma| k_F \ll \hbar k_F^2/2m^*$ (or $|\gamma| \ll \hbar k_F/2m^*$), we can set $\varepsilon_k \approx \varepsilon_{k_F} = 0$ and $V_k = V_{k_F} \equiv V$. With this we can make the approximations

$$J_{kk'} \approx |V|^2 \left[\frac{\varepsilon_d + U}{(\varepsilon_d + U)^2 - |\gamma_F|^2} - \frac{\varepsilon_d}{\varepsilon_d^2 - |\gamma_F|^2} \right] \equiv J, \tag{21}$$

$$\Gamma_{kk'} \approx |V|^2 |\gamma| \frac{k - k'}{2} \left[\frac{1}{\varepsilon_d^2 - |\gamma_F|^2} - \frac{1}{(\varepsilon_d + U - \varepsilon_k)^2 - |\gamma_F|^2} \right], \tag{22}$$

$$\Gamma^{(1)}_{kk'} \approx -|V|^2 |\gamma| \frac{k + k'}{2} \left[\frac{1}{\varepsilon_d^2 - |\gamma_F|^2} - \frac{1}{(\varepsilon_d + U - \varepsilon_k)^2 - |\gamma_F|^2} \right], \tag{23}$$

and

$$\Gamma^{(2)}_{kk'} \approx -|V|^2|\gamma|\frac{(k+k')}{2}\left[\frac{1}{\varepsilon_d^2-|\gamma_F|^2} + \frac{1}{(\varepsilon_d+U-\varepsilon_k)^2-|\gamma_F|^2}\right]. \quad (24)$$

In the above equations we have defined $\gamma_F = \gamma k_F$. To obtain expressions (21)–(24) we have replaced k^2 and k'^2 by k_F^2 but we were careful with the linear terms, keeping k and k' intact. This is because the sums in the Hamiltonian above run for positive and negative momenta. Therefore, considering only scatterings around k_F, we can replace $|k|$ and $|k'|$ by k_F in the couplings (22)–(24). With this, the factor $k-k'$ in Eq. (22) or $k+k'$ in Eqs. (23) and (24) can be approximated by zero or $\pm 2k_F$, depending on the relative sign between k and k'. Bearing this in mind, we see that the coupling (22) contributes only with backward scatterings, whereas Eqs. (23) and (24) contribute only with forward scatterings. Explicitly, at k_F we can write

$$\Gamma = |V|^2\gamma_F\left[\frac{1}{\varepsilon_d^2-|\gamma_F|^2} - \frac{1}{(\varepsilon_d+U)^2-|\gamma_F|^2}\right] = -\Gamma_1 \quad (25)$$

and

$$\Gamma_2 = -|V|^2\gamma_F\left[\frac{1}{\varepsilon_d^2-|\gamma_F|^2} + \frac{1}{(\varepsilon_d+U)^2-|\gamma_F|^2}\right]. \quad (26)$$

Inserting these expressions into Eqs. (10), (13), (15), and (16), we obtain

$$H_{\rm K} = J\sum_{kk'}[(c^\dagger_{k'+}c_{k+} - c^\dagger_{k'-}c_{k-}) \times S_z + c^\dagger_{k'+}c_{k-}S_- + c^\dagger_{k'-}c_{k+}S_+], \quad (27)$$

$$H_{\rm DM} = \Gamma\sum_{kk'>0}(c^\dagger_{-k'+}c_{k-}S_- - c^\dagger_{k'+}c_{-k-}S_- + c^\dagger_{k'-}c_{-k+}S_+ - c^\dagger_{-k'-}c_{k+}S_+), \quad (28)$$

$$H^{(1)}_{\rm EY} = \Gamma_1\sum_{kk'>0}[S_z(c^\dagger_{k'+}c_{k+} - c^\dagger_{-k'+}c_{-k+}) + S_z(c^\dagger_{k'-}c_{k-} - c^\dagger_{-k'-}c_{-k-})], \quad (29)$$

$$H^{(2)}_{\rm EY} = \Gamma_2\sum_{kk'>0}\left[\frac{n_d}{2}(c^\dagger_{k'+}c_{k+} - c^\dagger_{k'-}c_{k-}) + \frac{n_d}{2}(c^\dagger_{-k'-}c_{-k-} - c^\dagger_{-k'+}c_{-k+})\right]. \quad (30)$$

Note that it is now explicit that the processes in the Hamiltonians $H_{\rm DM}$ and in $H_{\rm EY}$ involve only backward and forward scatterings, respectively. Moreover, we see that the backward scatterings that occur are interband while the forward ones are intraband scatterings. These backward (interband) and forward (intraband) scatterings are exemplified by the diagrams of Figs. 2(b) and 2(c). Because of these very well-defined scattering processes, it is convenient to split the Kondo Hamiltonian (27) likewise. Separating the terms of Eq. (27) involving definite backward and forward processes, we obtain

$$H_{\rm K} = J^{\rm F}_\parallel\sum_{\substack{kk'>0\\kk'<0}}(c^\dagger_{k'+}c_{k+} - c^\dagger_{k'-}c_{k-})S_z + J^{\rm B}_\parallel\sum_{\substack{k>0,k'<0\\k<0,k'>0}}(c^\dagger_{k'+}c_{k+} - c^\dagger_{k'-}c_{k-})S_z + J^{\rm F}_\perp\sum_{\substack{kk'>0\\kk'<0}}[c^\dagger_{k'+}c_{k-}S_- + c^\dagger_{k'-}c_{k+}S_+] + J^{\rm B}_\perp\sum_{\substack{k>0,k'<0\\k<0,k'>0}}[c^\dagger_{k'+}c_{k-}S_- + c^\dagger_{k'-}c_{k+}S_+]. \quad (31)$$

As we will see below, because of the SOC, the several Kondo couplings in Eq. (31) will obey different differential equations in the renormalization group analysis.

III. RENORMALIZATION GROUP ANALYSIS

To study the low-temperature regime of the system we perform a poor-man's scaling analysis of the effective Hamiltonian (8). We follow the original Anderson's approach [31] to obtain the renormalization equations for the effective couplings. After a cumbersome but straightforward calculation (see Appendix C), we find

$$\dot J_{\perp B} = -\rho J_{\perp F}J_{\parallel B} - \rho J_{\perp B}J_{\parallel F} + \rho\Gamma\Gamma_1 - \rho\Gamma\Gamma_2, \quad (32a)$$
$$\dot J_{\perp F} = -\rho J_{\perp F}J_{\parallel F} - \rho J_{\perp B}J_{\parallel B}, \quad (32b)$$
$$\dot J_{\parallel B} = -2\rho J_{\perp F}J_{\perp B}, \quad (32c)$$
$$\dot J_{\parallel F} = -\rho J^2_{\perp F} - \rho J^2_{\perp B} - \rho\Gamma^2, \quad (32d)$$
$$\dot\Gamma = -\rho J_{\parallel F}\Gamma + \rho J_{\perp B}\Gamma_1 - \rho J_{\perp B}\Gamma_2, \quad (32e)$$
$$\dot\Gamma_1 = \rho J_{\perp B}\Gamma + \rho J_{\parallel F}\Gamma_2, \quad (32f)$$
$$\dot\Gamma_2 = \rho J_{\parallel F}\Gamma_1. \quad (32g)$$

Following standard notation, in the equations above we have defined $\dot X \equiv dX/d\ln\Lambda$, where Λ in the reduced bandwidth. We have also denoted $\rho = \rho(0)$ as the density of states of the conduction electrons calculated and the Fermi level, $\varepsilon_F = 0$. For this we had to assume that the Fermi level is far above the bottom of the band. In this limit we can linearize the band about $k = k_F$ as schematically shown in Fig. 2(a).[1] We can verify that in the absence of SO interactions we have the solution for $\Gamma = \Gamma_1 = \Gamma_2 = 0$, provided the condition has $\Gamma(D) = \Gamma_1(D) = \Gamma_2(D) = 0$. With this, by setting $J_{\perp F} = J_{\parallel F} = J_{\perp B} = J_{\parallel B} = J$, the differential equations above reduce to the usual renormalization equation for J in the isotropic Kondo model, $\dot J = -2\rho J^2$, leading to the known expression for the Kondo temperature, $T^0_{\rm K} = D\exp(-1/2\rho J)$.

[1] We should remark here that assuming a constant density of states for the conduction electrons is not valid, in general. For instance, in superconducting wires with a gap, multichannel wires with van Hove singularities near the Fermi level, the conduction band is strongly energy dependent. In these cases we need to improve the poor-man's scaling approach, in a similar manner as in Ref. [33].

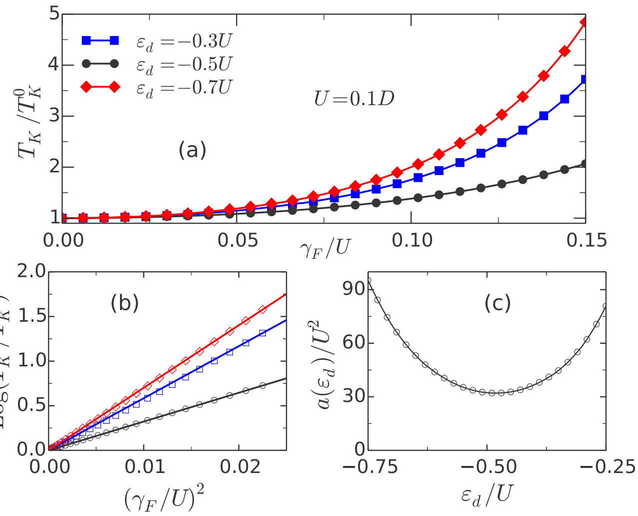

FIG. 3. (a) Scaled Kondo temperature vs γ_F/U for different values of ε_d and $U = 0.1$. $\varepsilon_d = -0.5U$ corresponds exactly to the particle-hole symmetric point of the Anderson model. Note the different behavior of T_K for ε_d above and below 0.05. T_K^0 is the Kondo temperature calculated in the absence of the SO interaction, $\gamma = 0$. (b) $\log(T_K/T_K^0)$ vs γ_F/U (symbols). The solid lines show linear functions connecting the first and the last points of each data set, serving as a guide to the eyes. These lines suggest that T_K depends on γ_F exponentially as $T_K = T_K^0 \exp(a\gamma_F^2)$, in which a is a function of ε_d. (c) a/U^2 vs ε_d/U extracted from the results of (b).

In the presence of SO interactions, an analytical solution for the coupled equations (32) is not available. Fortunately, it can be solved numerically using standard procedures. The numerical solution provides us with the coupling as a function of the reduced bandwidth Λ. As in the conventional Kondo model, the Kondo couplings diverge as $\Lambda \to 0$. It is precisely this divergence that provides a definition for the Kondo temperature within the renormalization group analysis. Using the same idea here, in the presence of the SO interaction, we take as T_K the value of Λ where the numerical solution diverges.[2]

To obtain our results for T_K, we set $U/\Delta = 20$, with $\Delta = \pi V^2/2D$. Here, D is an energy cutoff, within which the band is linearized around $k = k_F$. In Fig. 3(a) we show the Kondo temperature T_K/T_K^0 vs γ_F/U for three different values of ε_d. Here, T_K^0 is the Kondo temperature in the absence of the SO interaction. Note that, similarly to what was obtained in Ref. [10], T_K always increases with γ_F, but it is more pronounced for $\varepsilon_d \neq -U/2$ [squares (blue curves) and diamonds (red curves)]. The increase of T_K with γ_F for $\varepsilon_d = -U/2$ contrasts with the results of Ref. [10] that predict a constant T_K using the same approach but agrees with those obtained in Refs. [11,13,32]. The main reason for the disagreement with the previous RGA is because they have neglected corrections in the Kondo coupling J due to the SO interaction. Another compelling point is that for $\varepsilon_d = -0.7U$ and $\varepsilon_d = -0.3U$ for which the impurity level is placed symmetrically below and above the particle-hole point, respectively, the increase of T_K with γ_F is not symmetric. This behavior disagrees with those of Ref. [10]. This asymmetry is, however, quite different from asymmetry observed in the results of Refs. [11,13,32] because while they considered the Fermi level close to the bottom of the conduction band, here we assume ε_F far away from it.

In the absence of an analytical solution for the set of differential equations (32) we attempt to obtain qualitatively the dependence of T_K on γ_F. To do so, in Fig. 3(b) we plot $\log(T_K/T_K^0)$ vs $(\gamma_G/U)^2$ for the same three values of ε_d as in Fig. 3(a). The symbols correspond to the numerical results as shown in Fig. 3(a) while the solid lines correspond to straight lines connecting the first and the last points of the data. Notably, these linear functions fit quite well all the data. This suggests a dependence of T_K on γ_F as $T_K = T_K^0 \exp(a\gamma_F^2)$, where a is a positive function of the Anderson model parameters (e.g., Δ, U, ε_d). Here, by keeping all the other parameters fixed, a clearly shows a strong dependence on ε_d. To extract a qualitative dependency of a as it varies with ε_d, in Fig. 3(c) we plot a vs ε_d/U. Note that the shape of the curve is almost parabolic with a minimum close to the particle-hole symmetry. It is, however, asymmetric about $\varepsilon_d = -U/2$ because of the particle-hole asymmetry of the renormalization equation introduced by the term $H_{\mathrm{EY}}^{(2)}$ of the effective Hamiltonian.

For a better comprehension of the origin of the particle-hole asymmetry in the results of Fig. 3, let us take a closer look at the renormalization equations (32). We will show that, indeed, the term in the Hamiltonian that breaks particle-hole symmetry of the renormalization equations is $H_{\mathrm{EY}}^{(2)}$, given by Eq. (30). To this end, let us neglect $H_{\mathrm{EY}}^{(2)}$ in the renormalization equations (32). We then remove Eq. (32g) and make $\Gamma_2 = 0$ in all the other equations of the set (32). Now, remember that Γ and Γ_1 are odd functions of ε_d under the change $\varepsilon_d = -U/2 + \delta$ to $\varepsilon_d = -U/2 - \delta$ for any $\delta < U/2$. Therefore, for given equal initial conditions for J's (which is the case, since J is even), we see that by changing $\varepsilon_d = -U/2 + \delta$ to $\varepsilon_d = -U/2 - \delta$, the derivatives of both Γ and Γ_1 just change their signs. Now, because the derivatives of the J's depend on the product $\Gamma\Gamma_1$ or on Γ^2, which are both even, the resulting value of T_K extracted from the solution of Eqs. (32) is particle-hole symmetric, even though Γ and Γ_1 are odd. This shows that indeed it is the additional term $H_{\mathrm{EY}}^{(2)}$ that breaks the particle-hole symmetry of the renormalization equations.

IV. CONCLUSIONS

Summarizing, we have studied the influence of the Kondo effect of a magnetic impurity side coupled to a quantum wire with a spin-orbit interaction. We start by modeling the system with a single impurity Anderson model (SIAM), in which the conduction electrons move under both Rashba and Dresselhaus spin-orbit couplings. We then derive an effective Kondo model that contains the known Dzyaloshinskii-Moriya (DM) interaction and an additional term describing scattering processes of the same type of Elliott-Yafet (EY) mechanisms responsible for spin relaxation in systems with magnetic impurities. By splitting the total effective 1D Hamiltonian

[2]To check if this is a good estimation of T_K we have compared our numerical results with the analytical solution for $\gamma = 0$ and found a perfect agreement.

into forward and backward scatterings, we are able to perform a poor-man's scaling, providing a set of renormalization equations for the effective couplings. To obtain a Kondo temperature that is dependent on the SO coupling strength, we solve numerically the coupled equations. We find that the spin-orbit interaction modifies substantially the Kondo temperature of the system. Our results show that, even though the DM term vanishes at the particle-hole (p-h) symmetry of the SIAM, and is known to change the Kondo temperature only away from the p-h symmetry, our study shows that the SOC modifies the Kondo temperature even in the p-h symmetry since it modifies the conventional Kondo couplings. Moreover, we find that the contribution from additional EY to the enhancement of the Kondo temperature is asymmetric with respect to the p-h symmetry. Our study shows clearly the scattering mechanisms of the conduction electrons by the magnetic impurity introduced by the SOC in the 1D system. More importantly, we show how these mechanisms affect the Kondo temperature of the system. We believe this work provides a step forward in the comprehension of the influence of SOC in the Kondo effect and is important for future studies, specifically in 1D systems.

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APPENDIX A: DERIVATION OF THE EFFECTIVE HAMILTONIAN

In order to project the total Hamiltonian (7) onto the singly occupied impurity subspace, we define the projector operators

$$P_0 = (1 - d_+^\dagger d_+)(1 - d_-^\dagger d_-), \tag{A1}$$

$$P_1 = d_+^\dagger d_+ + d_-^\dagger d_- - 2d_+^\dagger d_-^\dagger d_- d_+, \tag{A2}$$

$$P_2 = d_+^\dagger d_-^\dagger d_- d_+. \tag{A3}$$

The projected Hamiltonian can be written as

$$\begin{aligned} H_{\text{eff}} = & H_{11} + H_{10}(E - H_{11})^{-1} H_{01} \\ & + H_{12}(E - H_{22})^{-1} H_{21}, \end{aligned} \tag{A4}$$

where $H_{ij} = P_i H P_j$. More explicitly, in terms of the creation and annihilation operators, the Hamiltonian (A4) can be written as

$$\begin{aligned} H = & H_0 + \sum_{\substack{kk' \\ hh'}} \frac{V_{k'}^* V_k}{2} \{[G_h(\varepsilon_d,\varepsilon_k) + G_{h'}(\varepsilon_d,\varepsilon_{k'})] \\ & \times d_{\bar{h}} d_{\bar{h}}^\dagger d_{\bar{h}'} d_{\bar{h}'}^\dagger d_h^\dagger d_{h'} c_{kh} c_{k'h'}^\dagger - [G_h(\varepsilon_d + U,\varepsilon_k) \\ & + G_{h'}(\varepsilon_d + U,\varepsilon_{k'})] n_{d\bar{h}'} c_{k'h'}^\dagger d_{h'} n_{d\bar{h}'} d_h^\dagger c_{kh}\}, \end{aligned} \tag{A5}$$

where

$$G_h(\varepsilon_d,\varepsilon_k) = \frac{1}{\varepsilon_d - \varepsilon_k}\left[1 - \frac{h|\gamma|k}{\varepsilon_d - \varepsilon_k}\right]^{-1}. \tag{A6}$$

In order to perform the summation on h and h' in Eq. (A6) we expand the expression above as a power series of $x = h|\gamma|k(\varepsilon_d - \varepsilon_k)^{-1}$. Summing up the infinite terms of the series we can write

$$\begin{aligned} G_h(\varepsilon_d,\varepsilon_k) & = \frac{\varepsilon_d - \varepsilon_k}{(\varepsilon_d - \varepsilon_k)^2 - |\gamma|^2 k^2} + \frac{h|\gamma|k}{(\varepsilon_d - \varepsilon_k)^2 - |\gamma|^2 k^2} \\ & = G^{(\text{e})}(\varepsilon_d,\varepsilon_k) + hG^{(\text{o})}(\varepsilon_d,\varepsilon_k), \end{aligned} \tag{A7}$$

where the first term corresponds to the even order of the series and the second one corresponds to the odd terms. We also have used the fact that $h^j = 1$ for j even and $h^j = h$ for j odd. It is important to note that the series converges only for $|\gamma|k < (\varepsilon_d - \varepsilon_k)$. This naturally imposes the regime of validity of the expansion, $|\gamma|k_F < (\varepsilon_d - \varepsilon_F)$ and $|\gamma|k_F < (\varepsilon_d + U - \varepsilon_F)$. We can now insert expression (A7) into Eq. (A5) and perform the summation on h and h'. After lengthy and cumbersome operator algebra we see that the even terms will renormalize the Kondo coupling while the odd term will provide additional scattering terms in the effective Hamiltonian. The resulting Hamiltonian can be split into three terms, namely, $H = H_0 + H_{\text{K}} + H_{\text{DM}} + H_{\text{EY}}$. The first describes the free conduction electrons,

$$H_0 = \sum_{k,h} \varepsilon_{kh} c_{kh}^\dagger c_{kh}. \tag{A8}$$

The second term corresponds to the conventional Kondo Hamiltonian,

$$\begin{aligned} H_{\text{K}} = & \sum_{kk'} J_{kk'}[(c_{k'+}^\dagger c_{k+} - c_{k'-}^\dagger c_{k-})S_z \\ & + c_{k'+}^\dagger c_{k-} S_- + c_{k'-}^\dagger c_{k+} S_+], \end{aligned} \tag{A9}$$

with a renormalized Kondo coupling,

$$J_{kk'} = V_k V_{k'}^* \frac{A_k + A_{k'}}{2}, \tag{A10}$$

where

$$\begin{aligned} A_k & = -G^{(\text{e})}(\varepsilon_d,\varepsilon_k) + G^{(\text{e})}(\varepsilon_d + U,\varepsilon_k) \\ & = \frac{\varepsilon_k - \varepsilon_d}{(\varepsilon_k - \varepsilon_d)^2 - |\gamma|^2 k^2} + \frac{\varepsilon_d + U - \varepsilon_k}{(\varepsilon_d + U - \varepsilon_k)^2 - |\gamma|^2 k^2}. \end{aligned} \tag{A11}$$

The third term describes the Dzyaloshinskii-Moriya scattering processes and can be written as

$$H_{\text{DM}} = \sum_{kk'} \Gamma_{kk'}(c_{k'+}^\dagger c_{k-} S_- - c_{k'-}^\dagger c_{k+} S_+), \tag{A12}$$

where the coupling $\Gamma_{kk'}$ is given by

$$\Gamma_{kk'} = V_k V_{k'}^* \frac{B_k^{(+)} - B_{k'}^{(+)}}{2}, \tag{A13}$$

in which we have defined

$$\begin{aligned} B_k^{\pm} & = \pm G^{(\text{o})}(\varepsilon_d,\varepsilon_k) - G^{(\text{o})}(\varepsilon_d + U,\varepsilon_k) \\ & = \pm|\gamma|k\left[\frac{1}{(\varepsilon_k - \varepsilon_d)^2 - |\gamma|^2 k^2} \right. \\ & \left. \mp \frac{1}{(\varepsilon_d + U - \varepsilon_k)^2 - |\gamma|^2 k^2}\right]. \end{aligned} \tag{A14}$$

Finally, the fourth term has the form

$$H_{\rm EY}^{(1)} = \sum_{kk'} \Gamma_{kk'}^{(1)} (c_{k'+}^\dagger c_{k+} + c_{k'-}^\dagger c_{k-}) S_z, \tag{A15}$$

$$H_{\rm EY}^{(2)} = \sum_{kk'} \Gamma_{kk'}^{(2)} (c_{k'+}^\dagger c_{k+} - c_{k'-}^\dagger c_{k-}) \frac{n_d}{2}, \tag{A16}$$

with

$$\Gamma_{kk'}^{(1)} = -V_k V_{k'}^* \frac{B_k^{(+)} + B_{k'}^{(+)}}{2} \tag{A17}$$

and

$$\Gamma_{kk'}^{(2)} = V_k V_{k'}^* \frac{B_k^{(-)} + B_{k'}^{(-)}}{2}, \tag{A18}$$

This term can be thought as describing the Elliott-Yafet-like scattering processes in which a electron real spin of the conduction band is flipped upon being scattered by the magnetic impurity. This can be better seen if we write the Hamiltonian (A15) in the real spin basis, as shown in Appendix B.

APPENDIX B: REAL SPIN REPRESENTATION OF THE SPIN-ORBIT SCATTERING TERMS

It is instructive to see what the effective Hamiltonian looks like in the real spin basis. To represent the Hamiltonian back to the real spin basis, we use the inverse of the transformation (4). Although in this transformation the Kondo Hamiltonian (10) is invariant, the spin-orbit scattering terms in the effective Hamiltonian acquire a different form. After some algebra the spin-orbit scattering terms (A12) and (A15) acquire, respectively, the form

$$H_{\rm DM} = \frac{i}{2} \sum_{kk'} \Gamma_{kk'} [(c_{k'\uparrow}^\dagger c_{k\uparrow} - c_{k'\downarrow}^\dagger c_{k\downarrow})(e^{-i\theta} d_\uparrow^\dagger d_\downarrow + e^{i\theta} d_\downarrow^\dagger d_\uparrow) - (d_\uparrow^\dagger d_\uparrow - d_\downarrow^\dagger d_\downarrow)(e^{-i\theta} c_{k'\uparrow}^\dagger c_{k\downarrow} + e^{i\theta} c_{k'\downarrow}^\dagger c_{k\uparrow})] \tag{B1}$$

and

$$H_{\rm EY} = \frac{i}{2} \sum_{kk'} \big[\Gamma_{kk'}^{(1)} (c_{k'\uparrow}^\dagger c_{k\uparrow} + c_{k'\downarrow}^\dagger c_{k\downarrow})(e^{-i\theta} d_\uparrow^\dagger d_\downarrow - e^{i\theta} d_\downarrow^\dagger d_\uparrow) + \Gamma_{kk'}^{(2)} (e^{-i\theta} c_{k'\uparrow}^\dagger c_{k\downarrow} - e^{i\theta} c_{k'\downarrow}^\dagger c_{k\uparrow})(d_\uparrow^\dagger d_\uparrow + d_\downarrow^\dagger d_\downarrow)\big]. \tag{B2}$$

The phase factor $e^{\pm i\theta}$ appearing in these two last expressions can be fully gauged away by the gauge transformation $c_{k\uparrow} \to e^{-i\theta/2} c_{k\uparrow}$ and $c_{k\downarrow} \to e^{i\theta/2} c_{k\downarrow}$. By defining

$$\mathbf{s}_{kk'} = \frac{1}{2} \sum_{ss'} c_{k's}^\dagger \boldsymbol{\tau}_{ss'} c_{ks'} \quad \text{and} \quad \mathbf{S} = \frac{1}{2} \sum_{ss'} d_s^\dagger \boldsymbol{\tau}_{ss'} d_{s'}, \tag{B3}$$

with $\boldsymbol{\tau}$ being the Pauli matrices including the identity τ^0, we can finally write

$$H_{\rm DM} = -2i \sum_{kk'} \Gamma_{kk'} (\mathbf{s}_{k'k} \times \mathbf{S}) \cdot \hat{\mathbf{y}}, \tag{B4}$$

which is of the usual form of the Dzyaloshinskii-Moriya interaction, and

$$H_{\rm EY} = 2 \sum_{kk'} \big[\Gamma_{kk'}^{(1)} s_{k'k}^0 S^y + \Gamma_{kk'}^{(2)} S^0 s_{k'k}^y\big]. \tag{B5}$$

This expression is similar to the Elliott-Yafet scattering term studied in spin relaxation processes [29,30]. Note, for instance, that the second term contains spin-flip scattering of the conduction electrons without changing the spin of the impurity.

APPENDIX C: POOR-MAN'S SCALING ANALYSIS

In the spirit of Anderson's perturbative renormalization group, the renormalization procedure consists of progressively reducing the bandwidth of the conduction electrons (D) step by step from its initial value D towards $D = 0$. Within this idea, if at a given step the conduction band lies within the interval $[-\Lambda, \Lambda]$ (where $0 < \Lambda \leqslant D$), it is reduced to $[-(\Lambda + \delta\Lambda), (\Lambda + \delta\Lambda)]$ (with $\delta\Lambda < 0$). The part of the Hamiltonian lying within the edges of the conduction band are integrated out while their effects are taken into account perturbatively up to second order in the Hamiltonian couplings. Using the T-matrix formalism, we search for scattering processes involving the edges of the conduction bands that renormalize the Hamiltonian, leaving it invariant [31]. Then, if H_0 in the unperturbed Hamiltonian and V is the interaction, up to second order in the couplings, we can write the renormalized interaction by

$$\tilde{V} = V + V \frac{1}{E - H_0} V = V + \Delta T, \tag{C1}$$

which has the same form of V. Note that ΔT corresponds to the change in the T matrix due to all the processes involving the edge of the conduction band.

Explicitly, we can write

$$\Delta T = \sum_{kk'} \sum_{\substack{q\,|\,\Lambda-\delta\Lambda<\varepsilon_q<\Lambda \\ q'\,|\,\Lambda-\delta\Lambda<\varepsilon_{q'}<\Lambda}} V_{k'q'} \frac{1}{E - H_0} V_{qk} + \sum_{kk'} \sum_{\substack{q\,|\,-\Lambda<\varepsilon_q<-\Lambda+\delta\Lambda \\ q'\,|\,-\Lambda<\varepsilon_{q'}<-\Lambda+\delta\Lambda}} V_{qk} \frac{1}{E - H_0} V_{k'q'}. \tag{C2}$$

Note that in the sum above, q represents momentum such that ε_q lies within the edge of the conduction bands. The first term is associated with particle states and the second with hole states, removed, respectively, from the top and bottom of conduction band. Even though we follow the standard procedure found in many textbooks, for the sake of completeness, let us illustrate the how the term $J_{\parallel B}$ is renormalized by integrating out the degrees of freedom "living" at the edge of the conduction band. Using expression (C2) we see that it rather simple because is not renormalized by the SO terms but only by the Kondo coupling terms of the Hamiltonian. To shown an example of among the many contributions for Eq. (C2), let us calculate product

$$H_{\perp F}^K \frac{1}{E - H_0} H_{\perp B}^K, \tag{C3}$$

where H_0 is given by (9) and $H_{\perp \rm F}^{\rm K}$ and $H_{\perp \rm B}^{\rm K}$ represent the third and fourth terms of the Hamiltonian (31). Although this term involves only the Kondo coupling, it is instructive to show how we deal with the various Kondo couplings split into backward and forward scatterings. For the particlelike scattering processes [first term of Eq. (C2)] we have

$$H_{\perp F}^{K}\frac{1}{E-H_0}H_{\perp B}^{K}=J_{\perp F}J_{\perp B}\left[\sum_{\substack{q'k'>0\\q'k'<0}}(c_{k'+}^{\dagger}c_{q'-}S_-+c_{k'-}^{\dagger}c_{q'+}S_+)\sum_{\substack{k>0,q<0\\k<0,q>0}}\frac{1}{E-H_0}(c_{q+}^{\dagger}c_{k-}S_-+c_{q-}^{\dagger}c_{k+}S_+)\right]. \tag{C4}$$

Here, we have dropped the constraints for q and q', but recall that q and q' run for all momenta such that ε_q and $\varepsilon_{q'}$ lie within the top edge of the conduction band. Since for a $S=1/2$, S_-^2 and S_+^2 acting on any impurity state vanish, we can write

$$H_{\perp F}^{K}\frac{1}{E-H_0}H_{\perp B}^{K}=J_{\perp F}J_{\perp B}\sum_{\substack{q'k'>0\\q'k'<0}}\sum_{\substack{k>0,q<0\\k<0,q>0}}\left(c_{k'+}^{\dagger}c_{q'-}\frac{1}{E-H_0}c_{q-}^{\dagger}c_{k+}S_-S_++c_{k'-}^{\dagger}c_{q'+}\frac{1}{E-H_0}c_{q+}^{\dagger}c_{k-}S_+S_-\right). \tag{C5}$$

Using $S_-S_+=1/2-S_z$ and $S_+S_-=1/2+S_z$ and performing the commutations of $c_{k'+}^{\dagger}$ and $c_{q'-}$ with $(E-H_0)^{-1}$, we obtain

$$H_{\perp F}^{K}\frac{1}{E-H_0}H_{\perp B}^{K}=J_{\perp F}J_{\perp B}\sum_{\substack{q'k'>0\\q'k'<0}}\sum_{\substack{k>0,q<0\\k<0,q>0}}\left(-\frac{c_{k'+}^{\dagger}c_{q'-}c_{q-}^{\dagger}c_{k+}}{E+\varepsilon_{k'+}-\varepsilon_{q'-}}+\frac{c_{k'-}^{\dagger}c_{q'+}c_{q+}^{\dagger}c_{k-}}{E+\varepsilon_{k'-}-\varepsilon_{q'+}}\right)S_z. \tag{C6}$$

In the expression above we have neglected the potential scattering term generated by the commutations and then set H_0 to zero. Now, for the top edge (particlelike scattering) we assume $c_{qs}c_{q's'}^{\dagger}=\delta_{ss'}\delta_{qq'}$, with $s=\pm$. Therefore,

$$H_{\perp F}^{K}\frac{1}{E-H_0}H_{\perp B}^{K}=J_{\perp F}J_{\perp B}\sum_{\substack{k<0,\,k',q>0\\k>0,\,k',q<0}}\left(-\frac{c_{k'+}^{\dagger}c_{k+}}{E+\varepsilon_{k'+}-\varepsilon_{q-}}+\frac{c_{k'-}^{\dagger}c_{k-}}{E+\varepsilon_{k'-}-\varepsilon_{q+}}\right)S_z. \tag{C7}$$

Now, since ε_{qs} lies within a very narrow energy interval near the edge of the reduced conduction band, we can make $\varepsilon_{q+}\sim\varepsilon_{q-}\sim\Lambda$ to obtain

$$H_{\perp F}^{K}\frac{1}{E-H_0}H_{\perp B}^{K}=-J_{\perp F}J_{\perp B}\sum_{\substack{k<0,\,k',q>0\\k>0,\,k',q<0}}\frac{1}{E+\varepsilon_{k'+}-\Lambda}(c_{k'+}^{\dagger}c_{k+}-c_{k'-}^{\dagger}c_{k-})S_z. \tag{C8}$$

We now convert the sum in q into an integral and assume a constant density of states ρ for the conduction electrons. Noticing that the sum in q is constrained by the sign of k', we can write

$$H_{\perp F}^{K}\frac{1}{E-H_0}H_{\perp B}^{K}=-J_{\perp F}J_{\perp B}\frac{\rho|\delta\Lambda|}{2}\sum_{\substack{k<0,\,k'>0\\k>0,\,k'<0}}\frac{1}{E+\varepsilon_{k'+}-\Lambda}(c_{k'+}^{\dagger}c_{k+}-c_{k'-}^{\dagger}c_{k-})S_z. \tag{C9}$$

For processes near the Fermi level we can neglect E and ε_{k+} in the expression, obtaining

$$H_{\perp F}^{K}\frac{1}{E-H_0}H_{\perp B}^{K}=J_{\perp F}J_{\perp B}\frac{\rho|\delta\Lambda|}{2\Lambda}\sum_{\substack{k<0,\,k'>0\\k>0,\,k'<0}}(c_{k'+}^{\dagger}c_{k+}-c_{k'-}^{\dagger}c_{k-})S_z. \tag{C10}$$

Comparing the operators in this expression with those in Eq. (31) we see that this is in fact similar to the second term of Eq. (31). Therefore, it contributes to a renormalization of $J_{\parallel B}$. Another identical contribution is provided by interchanging $H_{\perp F}^{K}$ and $H_{\perp B}^{K}$. Performing the same analysis for the holelike term in Eq. (C2), one finds an equal contribution. Therefore, the total contribution is given by

$$\delta J_{\parallel B}=2J_{\perp F}J_{\perp B}\frac{\rho|\delta\Lambda|}{\Lambda}=-2J_{\perp F}J_{\perp B}\delta\ln\Lambda. \tag{C11}$$

The minor sign in the last step came because $\delta\Lambda<0$. In the limit $|\delta\Lambda|\to 0$ we finally obtain the traditional form

$$\dot{J}_{\parallel B}=-2J_{\perp F}J_{\perp B}. \tag{C12}$$

In the calculation above we have considered only two terms of the Kondo Hamiltonian (31). Interestingly, after checking all the calculations we see that for $J_{\parallel B}$ this is the only contribution. Terms involving the SO interaction will renormalize the other Kondo couplings. For example,

$$\dot{J}_{\perp B}=-\rho J_{\parallel F}J_{\perp B}-\rho J_{\parallel B}J_{\perp F}+\rho\Gamma\Gamma_1-\rho\Gamma\Gamma_2. \tag{C13}$$

The calculation of all the remaining contributions to the set of differential (32) is lengthy but straightforward.

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