

Current loops, phase transitions, and the Higgs mechanism in Josephson-coupled multicomponent superconductors

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The N -component London $U(1)$ superconductor is expressed in terms of integer-valued supercurrents. We show that the inclusion of interband Josephson couplings introduces monopoles in the current fields, which convert the phase transitions of the charge-neutral sector to crossovers. The monopoles only couple to the neutral sector, and leave the phase transition of the charged sector intact. The remnant noncritical fluctuations in the neutral sector influence the one remaining phase transition in the charged sector, and may alter this phase transition from a 3DXY inverted phase transition into a first-order phase transition depending on what the values of the gauge charge and the intercomponent Josephson coupling are. This preemptive effect becomes more pronounced with increasing number of components N , since the number of charge-neutral fluctuating modes that can influence the charged sector increases with N . We also calculate the gauge-field correlator, and by extension the Higgs mass, in terms of current-current correlators. We show that the onset of the Higgs mass of the photon (Meissner effect) is given in terms of a current loop blowout associated with going into the superconducting state as the temperature of the system is lowered.

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I. INTRODUCTION

Models with multiple $U(1)$ condensates coupled by a vector potential are relevant to a variety of condensed matter systems. The number of possible interactions between the individual condensates make the models much more complex than single-band systems. Multiple, individually conserved condensates are applicable to systems of low-temperature atoms, such as hydrogen under extreme pressures [1–6] and as effective models of easy-plane quantum antiferromagnets [7,8]. Superconductors with multiple superconducting bands, such as MgB_2 [9–11] and iron pnictides [12], may also be described by a model of multiple $U(1)$ condensates, but in these systems the individual condensates are not conserved. Interband Josephson couplings must always be included, as they cannot *a priori* be excluded on symmetry grounds.

Ginzburg-Landau models of N -component superconductors in the London limit host a rich variety of interesting phenomena [13–16]. Each condensate supports topological vortex line defects, which represent disorder in the condensate ordering field. When the condensates are coupled through a gauge field, the vortices carry magnetic flux quanta, and may be bound into composite vortices with $\pm 2\pi$ phase windings in multiple condensates [17]. It turns out that this gives rise to composite superfluid modes that do not couple to the gauge field, even though their constituent vortices interact via the gauge field. In addition to the superfluid modes, there will be a single charged mode which is coupled by the gauge field. This causes the N -component model without Josephson interactions to have $N - 1$ superfluid phase transitions and a single superconducting phase transition [17]. For certain values of the gauge charge these transitions will interfere in a nontrivial way, causing the transitions to merge in a single first-order transition [18,19].

The question of the nature of the phase transitions present in Josephson-coupled multiband superconductors is of considerable interest. Symmetry arguments dictate that the inclusion of the Josephson coupling breaks the $[U(1)]^N$

symmetry down to $U(1)$, at any strength. The Josephson term locks the superfluid modes so that the phase transition in the neutral sector is replaced by a crossover [17], while the phase transition in the gauge-coupled sector is expected to remain. If this transition remains continuous, it is expected to be in the inverted 3DXY universality class [17]. A recent study has observed a first-order transition in this model for weak Josephson coupling [20], suggesting a subtle interplay between the two length scales dictated by the Josephson length and the magnetic field penetration depth. A schematic phase diagram is shown in Fig. 1 for the two-component case. This is based on arguments provided in this work, and supports the numerical results obtained in recent numerical studies [20]. Also of note are multiband superconductors with frustrated interband couplings, which is $U(1) \times \mathbb{Z}_2$ symmetric. These systems have been shown to have a single first-order transition in three dimensions from a symmetric state into a state that breaks both $U(1)$ and \mathbb{Z}_2 symmetry for weak values of the gauge field coupling. For stronger values of the charge, the transitions split [21,22].

In this paper we present an alternate approach to the multiband superconductor which has certain advantages over standard formulations, allowing further analytical insights to be made. In particular, we are able to reconcile the different results for the character of the phase transition in the charged sector found in Refs. [17] and [20] in the presence of interband Josephson couplings. By applying a character expansion [23,24] to the action, we replace the phases of the order parameter with integer-current fields. These currents are the actual supercurrents of the model. Section II presents the details and basic properties of the multiband superconductor in the London limit. In Sec. III A we present the character expansion, apply it to the model with no Josephson coupling, and compare the resulting representation to the original model. We apply the character expansion to the multiband superconductor with Josephson couplings in Sec. III B and discuss it in the light of the current representation. In Sec. IV

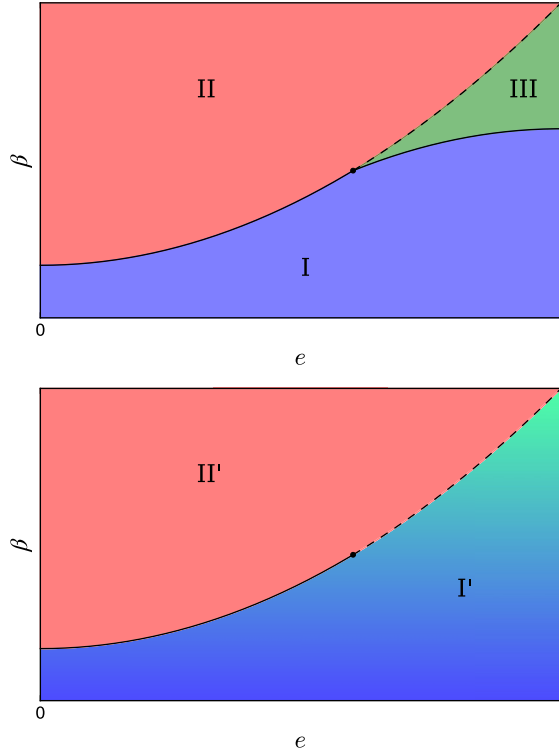


FIG. 1. A schematic phase diagram for the model with $N = 2$. The top panel shows the case $\lambda = 0$, while the lower panel shows the case with $\lambda > 0$. Top panel, $\lambda = 0$: Phase I is the fully symmetric normal phase with no superfluidity and no superconductivity. Phase III is the phase with no superconductivity, but nonzero superfluid stiffness in the neutral mode (metallic superfluid). Phase II is the low-temperature fully ordered state with finite Higgs mass in the charged sector and finite superfluid density in the neutral sector, a superconducting superfluid. The solid line separating phase I from phase II is a first-order phase transition line. The dotted line separating phase II from phase III is a critical line in the inverted 3DXY universality class. The solid line separating phase I from phase III is a critical line in the 3DXY universality class. Along the line $e = 0$, we recover two uncoupled 3DXY models, and the phase transition will be two superimposed independent phase transitions in the 3DXY universality class. Bottom panel, $\lambda > 0$: Phase I' is the high-temperature phase with no superconductivity. The entire phase is analytically connected with only a crossover regime separating the high-temperature phase from the lower-temperature phase. There is no spontaneous symmetry breaking in the neutral sector, since the Josephson coupling effectively acts as an explicit symmetry-breaking term in this sector, analogous to a magnetic field coupling linearly to XY spins. Phase II' is the low-temperature superconducting state. The solid part of the line separating phase I' from phase II' is a first-order phase transition line. The dotted part is a critical line in the inverted 3DXY universality class. For both $\lambda = 0$ and $\lambda > 0$, the line separating the superconducting states (II and II') from the nonsuperconducting state changes character from a first-order phase transition (solid line) to a second-order phase transition (dotted line) as via a tricritical point. The 3DXY critical line separating phase I from phase III for $\lambda = 0$ is converted to a crossover line in phase I' for $\lambda > 0$. Along the $e = 0$ line, the system is described by two neutral sectors coupled by an intercomponent Josephson coupling, such that the global $U(1) \times U(1)$ symmetry is reduced to a global $U(1)$ symmetry. Therefore, the phase transition reverts to a single 3DXY transition.

we present the calculation of the Higgs mass in terms of current correlators. We present our conclusions in Sec. V.

II. STANDARD REPRESENTATION OF THE MODEL

We consider a model of N bosonic complex matter fields in three dimensions. The matter fields are given by $\psi_\alpha(\mathbf{r}) = |\psi_\alpha(\mathbf{r})| \exp i\theta_\alpha(\mathbf{r})$, interacting through the electromagnetic vector potential, $\mathbf{A}(\mathbf{r})$. We also allow interband Josephson couplings of the matter fields. In the general case, this is described by a partition function

$$\mathcal{Z} = \int \mathcal{D}A \left(\prod_\alpha \int \mathcal{D}\psi_\alpha \right) e^{-S}, \quad (1)$$

where the action is

$$\begin{aligned}
 S = \beta \int d^3r \left\{ \frac{1}{2} \sum_\alpha [|\nabla - ie\mathbf{A}(\mathbf{r})]\psi_\alpha(\mathbf{r})|^2 \right. \\
 + V(|\psi_\alpha(\mathbf{r})|) + \frac{1}{2} [\nabla \times \mathbf{A}(\mathbf{r})]^2 \\
 \left. - \sum_{\alpha < \beta} \lambda_{\alpha,\beta} |\psi_\alpha(\mathbf{r})| |\psi_\beta(\mathbf{r})| \cos[\theta_\alpha(\mathbf{r}) - \theta_\beta(\mathbf{r})] \right\}. \quad (2)
 \end{aligned}$$

The potential V contains terms that are powers of $|\psi_\alpha|$. At this point we employ the phase-only, or London, approximation and choose all bare stiffnesses, $|\psi_\alpha|$, equal to unity. Hence, V is an unimportant constant. We will also focus on equal couplings between all bands, i.e., $\lambda_{\alpha,\beta} = \lambda \forall \alpha, \beta$. The action is then given by

$$\begin{aligned}
 S = \beta \int d^3r \left\{ \frac{1}{2} \sum_\alpha [\nabla\theta_\alpha(\mathbf{r}) - e\mathbf{A}(\mathbf{r})]^2 + \frac{1}{2} [\nabla \times \mathbf{A}(\mathbf{r})]^2 \right. \\
 \left. - \lambda \sum_{\alpha < \beta} \cos[\theta_\alpha(\mathbf{r}) - \theta_\beta(\mathbf{r})] \right\}. \quad (3)
 \end{aligned}$$

We regularize this action on a cubic lattice of size L^3 by defining the fields on a discrete set of coordinates $r_\mu \in (1, \dots, L)$, that is, $\theta_\alpha(\mathbf{r}) \rightarrow \theta_{r,\alpha}$ and $\mathbf{A}(\mathbf{r}) \rightarrow \mathbf{A}_r$. On the lattice, the action reads

$$\begin{aligned}
 S = \beta \sum_r \left\{ - \sum_{\mu,\alpha} \cos(\Delta_\mu \theta_{r,\alpha} - e A_{r,\mu}) + \frac{1}{2} (\Delta \times \mathbf{A}_r)^2 \right. \\
 \left. - \lambda \sum_{\alpha < \beta} \cos(\theta_{r,\alpha} - \theta_{r,\beta}) \right\}. \quad (4)
 \end{aligned}$$

Here, we use the cosine function to represent the kinetic term of the continuum Hamiltonian in a way that preserves the periodic nature of the phases. Alternatively, one may arrive at Eq. (4) by directly replacing the derivatives in Eq. (2) with the gauge-invariant forward difference,

$$[\nabla - ie\mathbf{A}(\mathbf{r})]\psi_\alpha(\mathbf{r}) \rightarrow \psi_{r+\hat{\mu},\alpha} e^{-ieA_r} - \psi_{r,\alpha}, \quad (5)$$

and then taking the London limit as described above. We discuss the two-dimensional case in Appendix B.

In the formulation of Eq. (4) with $\lambda = 0$, the model is known [17,19] to have one phase transition from a normal state to a superconducting state in one composite degree of freedom, and $N - 1$ phase transitions from a normal fluid to a superfluid in the remaining degrees of freedom. The reason for this division into one superconducting and $N - 1$ superfluid degrees of freedom becomes apparent when one correctly identifies the relevant combinations of the phase fields. The part of the continuum action describing the coupling between the phases and the gauge field is

$$S' = \beta \int d\mathbf{r} \left\{ \frac{1}{2} \sum_{\alpha} [\nabla\theta_{\alpha}(\mathbf{r}) - e\mathbf{A}(\mathbf{r})]^2 \right\}. \quad (6)$$

This can be rewritten into [17]

$$S' = \beta \int d\mathbf{r} \left\{ \frac{1}{2N} \left(\sum_{\alpha} \nabla\theta_{\alpha}(\mathbf{r}) - Ne\mathbf{A}(\mathbf{r}) \right)^2 + \frac{1}{2N} \sum_{\alpha < \beta} [\nabla(\theta_{\alpha} - \theta_{\beta})]^2 \right\}. \quad (7)$$

Hence, the phase combination $\sum_{\alpha} \theta_{\alpha}$ will couple to the gauge field, and is identified as the single charged mode, while all other combinations $\theta_{\alpha} - \theta_{\beta}$ do not couple, and are neutral. Note that for $N = 1$ only the charged mode remains. Two important points need to be emphasized. First, the composite variables are not compact in the same sense that the individual phases are. This means that the composite variables do not support topological defects by themselves, only composite topological defects. Second, the last term in the action of Eq. (7) has $N(N - 1)/2$ terms. Therefore, one may not interpret the phase differences $\theta_{\alpha} - \theta_{\beta}$ as independent degrees of freedom. This is because of the multiple connectedness of the physical space; fluctuations in a single individual phase induce fluctuations in $N - 1$ composite neutral modes, as well as in the charged mode.

In the present form, with $\lambda = 0$ and e sufficiently large, this model is known to have one phase transition in the inverted 3DXY universality class, and $N - 1$ transitions in the 3DXY universality class at a higher temperature [17,19]. These transitions correspond to proliferations of the composite charged mode and the composite neutral modes, respectively. If the charge is lowered, the charged and neutral transitions will approach each other in temperature. When they merge, the proliferation of neutral vortices will trigger proliferation of the charged mode. Consequently, the N phase transitions collapse into a single first-order transition. This interplay between the charged and neutral sector has been coined a *preemptive* phase transition [25], and has been verified numerically in two-component systems in the absence of intercomponent Josephson coupling in several detailed large-scale Monte Carlo simulations [18,19,25].

In the following section, we reformulate the model in terms of integer-valued current fields, considering first the case with zero Josephson coupling and then moving on to include Josephson coupling. The first case is useful to consider in connecting the results of previous works mentioned above to the current formulation.

III. CURRENT REPRESENTATION OF THE MODEL

A. Zero intercomponent Josephson coupling

The basis of the expansion used is a character expansion [23,24].

$$e^{\beta \cos \gamma} = \sum_{b=-\infty}^{\infty} I_b(\beta) e^{ib\gamma}, \quad (8)$$

where $I_b(\beta)$ are the modified Bessel functions of integer order. We apply this to the terms $\exp \beta \cos(\Delta_{\mu} \theta_{r,\alpha} - e\mathbf{A}_{\mathbf{r}})$ for each value of \mathbf{r} , μ , and α . This introduces integer vector fields $\mathbf{b}_{r,\alpha}$, representing supercurrents. In fact, the integer vector fields will be the actual physical supercurrents of the system [24]. The low-temperature phase is characterized by a state with proliferated current loops on all length scales, while the high-temperature phase only features small current loops.

By applying Eq. (8) to the partition function with Eq. (4) as the action, and integrating out the phases and the gauge field, details of which may be found in Appendix A, we arrive at the partition function

$$\mathcal{Z} = \sum_{\{b,m\}} \prod_{r,\alpha} \delta_{\Delta \cdot \mathbf{b}_{r,\alpha}, 0} \prod_{r,\mu,\alpha} I_{b_{r,\mu,\alpha}}(\beta) \times \prod_{r,r'} e^{-\frac{e^2}{2\beta} \sum_{\alpha,\beta} \mathbf{b}_{r,\alpha} \cdot \mathbf{b}_{r',\beta} D(\mathbf{r}-\mathbf{r}')}. \quad (9)$$

This is a model of N current fields, with contact intracomponent interactions parametrized by the Bessel functions, and long-range intra- and intercomponent interactions originating with the gauge-field fluctuations, $D(\mathbf{r} - \mathbf{r}')$. The constraint $\Delta \cdot \mathbf{b}_{r,\alpha} = 0$ forces the currents $\mathbf{b}_{r,\alpha}$ to form closed loops, and implies a nonanalytical behavior of each individual component, and an associated phase transition.

In the current language, the interpretation of the phase transitions explained in the previous section is as follows. Consider first a single-component model. In the high-temperature state, only the lowest term in the Bessel-function expansion will contribute, and only small loops of supercurrents will be present in the system. As the temperature is *lowered* all orders of the expansion contribute, and the integer currents will proliferate, filling the system with loops of supercurrent. In the low-temperature state all b fields have proliferated. As temperature is increased, the proliferated current loops in the charged sector will collapse. Only the neutral superfluid currents fill the system, and the state is therefore a metallic superfluid [15]. As temperature is raised further the superfluid currents collapse as well, and the system is in the normal-metallic state.

B. Nonzero intercomponent Josephson couplings

The expansion of Eq. (8) may also be applied to the Josephson term. The expansion is only valid when the argument of the cosine is expanded around zero; the present formulation is therefore not valid for any ground state which does not fulfill this requirement. In particular, if the Josephson coupling is negative and sufficiently strong, the phase differences will be locked to nonzero values [21,22]. For $N = 2$ the phases are locked to π , while for $N = 3$ the ground state of the three phases may form a star pattern with an accompanying

\mathbb{Z}_2 symmetry associated with the two possible chiralities of the star [21,22]. These cases are not covered by the current loop formulation derived from the character expansion Eq. (8). While the above arguments do not constrain us to only consider all Josephson couplings equal, we may limit our considerations to the case $\lambda_{\alpha\beta} = \lambda > 0$ without loss of generality in the present discussion. Having universal $\lambda_{\alpha\beta}$ will not allow for any additional physics than simply having unequal strength of the individual phase lockings, when they are constrained to be all positive.

Applying the expansion introduces an additional $N(N-1)/2$ integer fields $m_{r,\alpha,\beta}$. After expanding both the kinetic terms and the Josephson terms, the partition function reads

$$\begin{aligned} \mathcal{Z} = & \int \mathcal{D}\mathbf{A} \left(\prod_{\alpha} \int \mathcal{D}\theta_{\alpha} \right) \\ & \times \prod_{r,\mu,\alpha} \sum_{b_{r,\mu,\alpha}=-\infty}^{\infty} I_{b_{r,\mu,\alpha}}(\beta) e^{ib_{r,\mu,\alpha}(\Delta_{\mu}\theta_{r,\alpha} - eA_{r,\mu})} \\ & \times \prod_{r,\alpha < \beta} \sum_{m_{r,\alpha,\beta}=-\infty}^{\infty} I_{m_{r,\alpha,\beta}}(\beta\lambda) e^{im_{r,\alpha,\beta}(\theta_{r,\alpha} - \theta_{r,\beta})} \\ & \times \prod_r e^{-\frac{\beta}{2}(\Delta \times A_r)^2}. \end{aligned} \quad (10)$$

The effect of the Josephson coupling becomes apparent when we integrate out the phase fields. The divergences of the \mathbf{b} fields will no longer be constrained to zero, but may take any finite integer value, determined by the value of the m fields. The new constraints read

$$\Delta \cdot \mathbf{b}_{r,\alpha} = \sum_{\beta \neq \alpha} m_{r,\alpha,\beta} \quad \forall \alpha, r, \quad (11)$$

where we have defined $m_{r,\alpha,\beta} = -m_{r,\beta,\alpha}$. The gauge term is not coupled directly to the m fields, and we may integrate it out in the same fashion as before. The resulting partition function is

$$\begin{aligned} \mathcal{Z} = & \sum_{\{b,m\}} \prod_{r,\alpha} \delta_{\Delta \cdot \mathbf{b}_{r,\alpha}, \sum_{\beta \neq \alpha} m_{r,\alpha,\beta}} \\ & \times \prod_{r,\mu,\alpha} I_{b_{r,\mu,\alpha}}(\beta) \prod_{r,\alpha < \beta} I_{m_{r,\alpha,\beta}}(\beta\lambda) \\ & \times \prod_{r,r'} e^{-\frac{e^2}{2\beta} \sum_{\alpha,\beta} \mathbf{b}_{r,\alpha} \cdot \mathbf{b}_{r',\beta} D(\mathbf{r}-\mathbf{r}')}. \end{aligned} \quad (12)$$

C. Monopoles and phase transitions

The effect of the m fields is to introduce monopoles into the closed loops of \mathbf{b} currents. A current of a particular component (color) may now terminate at any site. However, this termination must always be accompanied by a current of another color originating at the same site. Termination of a current of one component, and the appearance of a current of another component at the same site, represents an excitation of ± 1 in m . An important observation is that if one adds the constraints, we have

$$\sum_{\alpha} \Delta \cdot \mathbf{b}_{r,\alpha} = 0 \quad \forall r. \quad (13)$$

This reflects the color-changing event stated above; the total current when summing over all colors is conserved at all sites. It also shows that there is a particular combination of currents, the sum of all components, which will be divergence-free. The net effect of the Josephson coupling, pictorially, is to chop up the closed currents of the individual components and glue them together into closed loops that may change color on any site.

We may expand the partition function first in terms of m fields, and then in terms of λ , by using the Bessel-function representation

$$I_{\nu}(z) = \left(\frac{z}{2}\right)^{\nu} \sum_{k=0}^{\infty} \frac{\left(\frac{z}{2}\right)^{2k}}{k!(\nu+k)!}. \quad (14)$$

This demonstrates that the partition function consists of a single term with zero divergence on all sites, which we know has one or more phase transitions from a superconducting superfluid state into a non-superconducting normal fluid, and many terms where the divergence of $\mathbf{b}_{r,\alpha}$ is finite on any number of sites.

Let us now consider two limits, and assume e is large, so that there is no preemptive effect for $\lambda = 0$. For $\lambda = 0$, it is evident that only $m = 0$ will contribute, and we are left with only divergenceless terms, and hence the behavior described previously. The other limit is $\lambda \rightarrow \infty$. In this case we must examine the asymptotic form of the Bessel functions, which to leading order in the argument is

$$I_m(z) \sim \frac{e^z}{\sqrt{2\pi z}}, \quad (15)$$

i.e., independent of m , and the monopole field will fluctuate strongly, causing the zero-divergence constraint on each component to be removed. The only remaining constraint on the current fields pertains to the composite current $\sum_{\alpha} \mathbf{b}_{\alpha}$, which is divergence-free. The interpretation of this is that the phase transitions in the $N-1$ superfluid modes are converted to crossovers by the Josephson coupling, while the single superconducting mode still undergoes a genuine phase transition. The neutral crossover will be far removed from the charged phase transition in this limit, and the remaining fluctuations in the neutral sector will be almost completely suppressed. There is no possibility of any interference between the sectors, and therefore no preemptive phase transition. The phase transition in the charged sector will therefore be in the universality class of the inverted 3DXY phase transition.

For intermediate and small values of λ , the effect of the Josephson coupling on the interplay between the charged and neutral sectors is quite subtle in the present formulation, and will be discussed in the following section.

D. Charged and neutral currents

We start with the action where the phase sum and phase differences have been separated, Eq. (7), with a Josephson coupling included. To simplify the notation, we introduce composite fields $\Theta \equiv \sum_{\alpha} \theta_{\alpha}$ and $\vartheta_{\alpha\beta} \equiv \theta_{\alpha} - \theta_{\beta}$. The lattice

action then reads

$$S = \beta \sum_r \left\{ - \sum_\mu \cos(\Delta_\mu \Theta_r - NeA_{r,\mu}) - \sum_{\mu, \alpha < \beta} \cos(\Delta_\mu \vartheta_{r,\alpha\beta}) - \lambda \sum_{\alpha < \beta} \cos(\vartheta_{r,\alpha\beta}) + \frac{1}{2} (\Delta \times A_r)^2 \right\}. \quad (16)$$

One may arrive at this form by defining the composite fields in Eq. (7), then use the Villain approximation on the original action of Eq. (4), rewrite the resulting action into one with the composite fields, then reverse the Villain approximation.

In Eq. (16), there is one charged mode and $N(N-1)/2$ neutral modes, while the original theory has N degrees of freedom. There is therefore an excess of $(N-1)(N-2)/2$ degrees of freedom. (Note that there are no redundant modes for $N=1$ and $N=2$.) Therefore, not all of the phase differences are independent when $N > 2$. Consider the case $N=3$, where one may form the phase differences $\theta_1 - \theta_2$, $\theta_2 - \theta_3$, and $\theta_1 - \theta_3$, but $\theta_1 - \theta_3 = (\theta_1 - \theta_2) + (\theta_2 - \theta_3)$. It suffices to include the phase differences ϑ_{12} and ϑ_{23} .

This may be generalized to arbitrary N . Identify all $\theta_{\alpha\beta}$ where

$$\{(\alpha, \beta) | \alpha \in (1, \dots, N-1) \wedge \beta = \alpha + 1\}. \quad (17)$$

Then, all $\theta_{\alpha\beta}$ where

$$\{(\alpha, \beta) | \alpha \in (1, \dots, N-2) \wedge \beta \in (\alpha + 2, \dots, N)\} \quad (18)$$

may be constructed by adding up the intermediate phase differences, that is, $\vartheta_{\alpha\beta} = \vartheta_{\alpha, \alpha+1} + \vartheta_{\alpha+1, \alpha+2} + \dots + \vartheta_{\beta-1, \beta}$. With this in mind, we may write out the partition function in terms of the charged and neutral modes

$$\mathcal{Z} = \int \mathcal{D}\Theta \left(\prod_{\alpha < \beta} \int \mathcal{D}\vartheta_{\alpha\beta} \right) \times \left[\prod_{\alpha=1}^{N-1} \prod_{\beta=\alpha+2}^N \delta \left(\vartheta_{\alpha\beta} - \sum_{\eta=\alpha}^{\beta-1} \vartheta_{\eta, \eta+1} \right) \right] e^S, \quad (19)$$

where S is the action of Eq. (16).

As an illustration, we perform the character expansion on the model where the charged and neutral sectors have been separated, for the special cases $N=2$ and $N=3$.

For $N=2$ there are no redundant variables, and we have the two composite variables $\Theta \equiv \theta_1 + \theta_2$ and $\vartheta \equiv \theta_1 - \theta_2$. Using the identity Eq. (8), and integrating out the phases and gauge field, we obtain

$$\mathcal{Z} = \sum_{\{\mathcal{B}, \mathcal{B}_r, m\}} \prod_r \delta_{\Delta \cdot \mathcal{B}_r, 0} \delta_{\Delta \cdot \mathcal{B}_r, m_r} \times \prod_{r, \mu} I_{\mathcal{B}_r, \mu}(\beta) I_{\mathcal{B}_r, \mu}(\beta) \prod_r I_{m_r}(\beta\lambda) \times \prod_{r, r'} \exp \left\{ - \frac{(Ne)^2}{2\beta} \mathcal{B}_r \cdot \mathcal{B}_{r'} D(r - r') \right\}. \quad (20)$$

Here, \mathcal{B} is the charged current field associated with Θ , while \mathcal{B} is the neutral current field associated with ϑ .

In this formulation, it is immediately clear that the model features two integer vector-field degrees of freedom, one which has long-range interactions mediated by the gauge field, and one with contact interactions. The neutral current field has its constraint removed by the m field, while the charged field is still constrained to be divergenceless. Hence, the model will feature a single phase transition in the charged sector driven by the collapse of closed loops of charged currents, while the transition of the neutral sector is converted to a crossover by the complete removal of constraints on \mathcal{B} .

Let us consider this in a bit more detail. In Eq. (20), we may perform the summation over the fields $m \in \mathbb{Z}$. Since we have that $\Delta \cdot \mathcal{B}_r \in \mathbb{Z}$ as well, the summation over the m 's will guarantee that the constraint is satisfied for some value of m , such that the summation over m effectively removes the constraints on $\Delta \cdot \mathcal{B}_r$. Hence, we have $\sum_{\{m\}} \delta_{\Delta \cdot \mathcal{B}_r, m_r} \prod_r I_{m_r}(\beta\lambda) = \prod_r I_{\Delta \cdot \mathcal{B}_r}(\beta\lambda)$, with no constraints on $\Delta \cdot \mathcal{B}_r$. We may thus perform the now unconstrained summation of the field \mathcal{B}_r , namely

$$\sum_{\{\mathcal{B}\}} \left(\prod_{r, \mu} I_{\mathcal{B}_r, \mu}(\beta) \right) \left(\prod_r I_{\Delta \cdot \mathcal{B}_r}(\beta\lambda) \right) = F(\beta, \lambda), \quad (21)$$

where F is an analytic function of its arguments. This may be seen by mapping the left-hand side of Eq. (21) to a Villain model, using the approximation [23]

$$\frac{I_b(x)}{I_0(x)} \approx \frac{1}{|b|!} e^{\log(\beta/2)|b|}. \quad (22)$$

This may be rewritten as a Gaussian provided β is sufficiently small so that contributions $|b| > 1$ are small,

$$\frac{I_b(x)}{I_0(x)} \approx e^{-\frac{b^2}{2\beta'}}, \quad (23)$$

where β' is a renormalized coupling constant, and we find

$$F(\beta, \lambda) = \sum_{\{\mathcal{B}\}} \left(\prod_{r, \mu} \exp \frac{-\mathcal{B}_{r, \mu}^2}{2\beta'} \right) \left(\prod_r \exp \frac{-(\Delta \cdot \mathcal{B}_r)^2}{2\lambda\beta'} \right). \quad (24)$$

Since there are no constraints \mathcal{B} , this demonstrates that Eq. (21) essentially is a discrete Gaussian theory, and the neutral sector therefore does not suffer any phase transition. This point may be further corroborated by going back to the formulation of Eq. (16). The neutral sector of the action is seen to be identical to that of an XY spin model in an external magnetic field, with field strength λ . Any $\lambda \neq 0$ converts the phase transition, from a low-temperature ferromagnetic state to a high-temperature paramagnetic state, into a crossover from an ordered to a disordered system. Note also that in the limit $\lambda = 0$, the Bessel function will revert to $I_{\Delta \cdot \mathcal{B}}(0) = \delta_{\Delta \cdot \mathcal{B}, 0}$, and the nonanalytical constraint is reintroduced.

We emphasize that although the above argument utilized a Villain approximation to the Bessel functions, the conclusion that the phase transition is wiped out in the neutral sector by introducing monopoles (Josephson coupling) does not depend on this approximation. At any rate, a Villain-approximation to the XY model does not change the symmetry of the

problem or the character of phase transitions. What is crucial is the introduction of monopoles and the ensuing removal of constraints on the neutral currents.

The total partition function for the entire system is thus given by

$$\mathcal{Z} = F(\beta, \lambda) \sum_{\{\mathbf{B}\}} \prod_{\mathbf{r}} \delta_{\Delta \cdot \mathbf{B}_{\mathbf{r}}, 0} \prod_{\mathbf{r}, \mu} I_{B_{\mathbf{r}, \mu}}(\beta) \times \prod_{\mathbf{r}, \mathbf{r}'} \exp \left\{ -\frac{(Ne)^2}{2\beta} \mathbf{B}_{\mathbf{r}} \cdot \mathbf{B}_{\mathbf{r}'} D(\mathbf{r} - \mathbf{r}') \right\}. \quad (25)$$

The phase transition in the neutral sector is converted to a crossover, and there are no longer any *critical* fluctuations associated with disordering the neutral sector, unlike the case $\lambda = 0$. This occurs as soon as λ is finite, however small. However, even without a phase transition and associated critical fluctuations, there will still be a crossover with associated fluctuations in its vicinity. Hence, the preemptive first-order phase transition in the charged sector, which occurs for $\lambda = 0$, may still take place provided λ is sufficiently small.

The argument is as follows. In the preemptive scenario for $\lambda = 0$, fluctuations in the neutral and charged sectors increase as T is increased from below in the fully ordered state. The charged sector influences the fluctuations in the neutral sector and vice versa, such that the putative continuous transitions in these sectors are preempted by a common first-order phase transition [17, 19]. The important point to realize is that neither of the sectors actually reaches criticality, since there are no critical fluctuations at the preemptive first-order phase transition.

We may have the same scenario occurring with finite but small λ . A necessary requirement is that the gauge charge e is not too large, such that gauge-field fluctuations are not so large as to separate the phase transitions in the charged and the neutral sector too much [17, 19]. The key point is that the inclusion of Josephson couplings converts the phase transition in the neutral sector to a crossover in exactly the same way that the ferromagnetic-paramagnetic phase transition in the 3DXY model is converted to a crossover by the inclusion of a magnetic field coupling linearly to the XY spins; cf. Eq. (16). This leaves only a phase transition in the charged sector, but it does not completely suppress fluctuations in the neutral sector. It merely cuts the fluctuations off on a length scale given by the Josephson length $1/\lambda$, thereby preventing them from becoming critical. As temperature is increased, the neutral sector approaches its crossover region, with increasingly large fluctuations. At the same time, the charged sector approaches its putative inverted 3DXY fixed point. Provided that the crossover region of the neutral sector and the fixed point of the charged sector are sufficiently close, the fluctuations in both sectors may still strongly influence each other, and a first-order preemptive phase transition may still occur in the charged sector. This is consistent with recent numerical work [20], which observed a first-order phase transition in multiband superconductors with weak Josephson coupling in Monte Carlo simulations using the original U(1) phases.

For $N = 3$ we must consider carefully the redundant variable, $\vartheta_{13} = \vartheta_{12} + \vartheta_{23}$. The partition function, prior to

integration of the phases and the gauge field, reads

$$\begin{aligned} \mathcal{Z} = & \int \mathcal{D}\Theta \left(\prod_{\alpha < \beta} \int \mathcal{D}\vartheta_{\alpha\beta} \right) \delta(\vartheta_{13} - \vartheta_{12} - \vartheta_{23}) \\ & \times \prod_{\mathbf{r}, \mu} \sum_{B_{\mathbf{r}, \mu} = -\infty}^{\infty} I_{B_{\mathbf{r}, \mu}}(\beta) e^{i B_{\mathbf{r}, \mu} (\Delta_{\mu} \Theta_{\mathbf{r}} - N e A_{\mathbf{r}, \mu})} \\ & \times \prod_{\mathbf{r}, \mu} \sum_{\substack{B_{\mathbf{r}, \mu, \alpha\beta} = -\infty \\ \alpha < \beta}}^{\infty} I_{B_{\mathbf{r}, \mu, \alpha\beta}}(\beta) e^{i B_{\mathbf{r}, \mu, \alpha\beta} \Delta_{\mu} \vartheta_{\alpha\beta}} \\ & \times \prod_{\mathbf{r}, \alpha < \beta} \sum_{m_{\mathbf{r}, \alpha\beta} = -\infty}^{\infty} I_{m_{\mathbf{r}, \alpha\beta}}(\beta \lambda) e^{i m_{\mathbf{r}, \alpha\beta} \vartheta_{\alpha\beta}} \\ & \times \prod_{\mathbf{r}} e^{-\frac{\beta}{2} (\Delta \times A_{\mathbf{r}})^2}. \end{aligned} \quad (26)$$

Again, \mathbf{B} is the charged current associated with Θ , while $\mathcal{B}_{\alpha\beta}$ are the neutral currents associated with $\vartheta_{\alpha\beta}$. The δ function is included to account for the redundancy of the composite phase representation.

We now proceed with the integration of phases and gauge field, taking care to integrate out the redundant phase first. The partition function may then be written as

$$\begin{aligned} \mathcal{Z} = & \sum_{\{\mathbf{B}, \mathcal{B}, m\}} \prod_{\mathbf{r}} \delta_{\Delta \cdot \mathbf{B}_{\mathbf{r}}, 0} \prod_{\mathbf{r}, \mu} I_{B_{\mathbf{r}, \mu}}(\beta) \\ & \times \prod_{\mathbf{r}} \delta_{\Delta \cdot \mathcal{B}_{\mathbf{r}, 12} + \Delta \cdot \mathcal{B}_{\mathbf{r}, 13}, m_{\mathbf{r}, 12} + m_{\mathbf{r}, 13}} \\ & \times \prod_{\mathbf{r}} \delta_{\Delta \cdot \mathcal{B}_{\mathbf{r}, 23} + \Delta \cdot \mathcal{B}_{\mathbf{r}, 13}, m_{\mathbf{r}, 23} + m_{\mathbf{r}, 13}} \\ & \times \prod_{\substack{\mathbf{r}, \mu \\ \alpha < \beta}} I_{B_{\mathbf{r}, \mu, \alpha\beta}}(\beta) \prod_{\mathbf{r}} \prod_{\alpha < \beta} I_{m_{\mathbf{r}, \alpha\beta}}(\beta \lambda) \\ & \times \prod_{\mathbf{r}, \mathbf{r}'} \exp \left\{ -\frac{(Ne)^2}{2\beta} \mathbf{B}_{\mathbf{r}} \cdot \mathbf{B}_{\mathbf{r}'} D(\mathbf{r} - \mathbf{r}') \right\}. \end{aligned} \quad (27)$$

This is a model of a single gauge coupled supercurrent \mathbf{B} which is constrained to form closed loops, and three superfluid currents \mathcal{B}_{12} , \mathcal{B}_{23} , and \mathcal{B}_{13} which are not constrained to form closed loops. The three superfluid currents are not independent, as is seen from the two constraints on them. As in the case $N = 2$, the summation over the m fields may be performed, eliminating the constraints on the fields $\mathcal{B}_{\mathbf{r}, \mu, \alpha\beta}$, after which the *unconstrained* summation over these fields may be performed. As for $N = 2$, this yields multiplicative analytic factors in the partition function, and the phase transitions in the neutral sectors will be converted to crossovers. Given that the crossovers in the neutral sectors and the charged fixed point have sufficient overlap, the system may still feature a single preemptive first-order phase transition arising from the interplay between the charged and neutral modes. Furthermore, the inclusion of the additional degree of freedom enhances the combined fluctuations of the neutral mode at a given Josephson coupling, λ , and therefore strengthens the

preemptive first-order transition. This is consistent with the results of recent numerical work [20].

E. Preemptive effect and current loop interactions

In this subsection, we discuss further the preemptive scenario discussed above, interpreting it in terms of renormalizations of current-current interactions. This provides a dual picture to the physical picture of the first-order phase transition presented in Ref. [20].

The preemptive phase transition may be understood in the current loop picture by considering the effect of the monopoles on the neutral counter-flowing current sector (facilitated by the presence of monopoles, i.e., Josephson coupling), and how this in turn influences the interaction between the charged co-flowing currents which interact via the fluctuating gauge field.

Consider first the current loop excitations allowed by Eq. (12) for the case $N = 2$. The lowest order configurations in the individual fields are closed loops of a single color. On top of these one may add monopoles, such that one has closed loops that change color twice before completing a closed loop. The presence of the Josephson coupling also allows for small dumbbells of counter-flowing currents with a monopole at one end and an anti-monopole at the other end. The gauge field will bind loops of co-flowing currents together, creating small loops of both colors flowing in the same direction. At high temperatures, the co-flowing currents only form small closed loops, and the system is non-superconducting. Barring any influence from the neutral sector, they will proliferate in an inverted 3DXY transition [26] at some critical temperature. If the charge, or the Josephson coupling, is sufficiently strong, there will be no significant fluctuations in the neutral sector that may influence this. The co-flowing current loops simply proliferate in a background of *only* tightly bound counter-flowing currents, with which they do not interact at all. The only way they can interact is if a counter-flowing composite current locally dissociates into individual currents on length scales below the Josephson length, which needs to be large enough. This will not happen if either the Josephson coupling is sufficiently strong, or if the charge is sufficiently large so that the charged transition is separated sufficiently from the neutral crossover.

Figures 2 and 3 show simple representations of current configurations as the transition occurs in the two scenarios. For simplicity the illustration is given in two spatial dimensions. In Fig. 2, we show the case of having a sufficiently strong Josephson coupling. A generic snapshot of a single loop of charged current is shown, represented by two co-flowing red and green lines, surrounding a gas of tightly bound pieces of counter-flowing neutral currents. As there are no individual red or green lines, there will be no interactions between the loop of composite charged current and the small pieces of composite neutral current, and hence no renormalization of the interactions in the charged sector. The loops of charged current will therefore proliferate in an inverted 3DXY transition [26] as the temperature is lowered. In Fig. 3, the situation is different. Here, the Josephson coupling is sufficiently low, or alternatively the Josephson length is sufficiently large, so that the individual pieces of current may undergo local

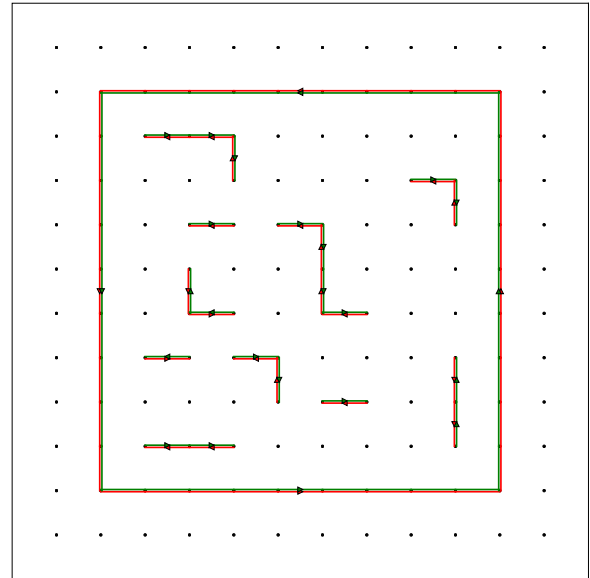


FIG. 2. Example of a current loop configuration in the case of strong interband Josephson coupling, when there is no screening of the charged-current interaction. Red and green lines represent currents of the individual fields b_i flowing in the direction indicated by the arrows. Charged and neutral currents are therefore represented by overlapping red and green lines flowing either in the same or the opposite direction, respectively. The configuration shown represents a snapshot close to the charged transition, where a closed loop of charged current encircles pieces of a tightly bound composite neutral current. As there is no interaction between pieces of charged and neutral current, the inverted 3DXY transition of the charged sector is not influenced by the tightly bound composite neutral currents.

dissociations of the tightly bound counter-flowing configurations. These individual pieces of currents, represented by only red or green lines, will interact with the loop of charged current, and may therefore influence the proliferation of composite charged current loops.

The current loops are dual objects to vortex loops. It is known that there is a precise correspondence between the sign of vortex interactions and the character of the phase transition in superconductors. Namely, attractive interactions between vortices leads to a first-order phase transition, while repulsive vortex interactions lead to second-order phase transitions [20,27]. Therefore, an alternative natural way of interpreting the preemptive first-order phase transition in the dual picture is that neutral counter-flowing currents on the co-flowing charged currents screen or overscreen the interactions between the latter, effectively changing the sign of the interactions between charged current segments.

With reference to Fig. 3, we elaborate briefly on how the configurations depicted there may cause attractive interactions between composite charged current segments. Note that the screening is accounted for entirely by removing all tightly bound counter-flowing currents, leaving only the closed color-changing loops. The relevant screening fluctuations are therefore complicated collective phase fluctuations amounting to inserting closed color-changing loops in the problem. Loops which interact attractively with the composite charged current segments will have a larger Boltzmann weight in

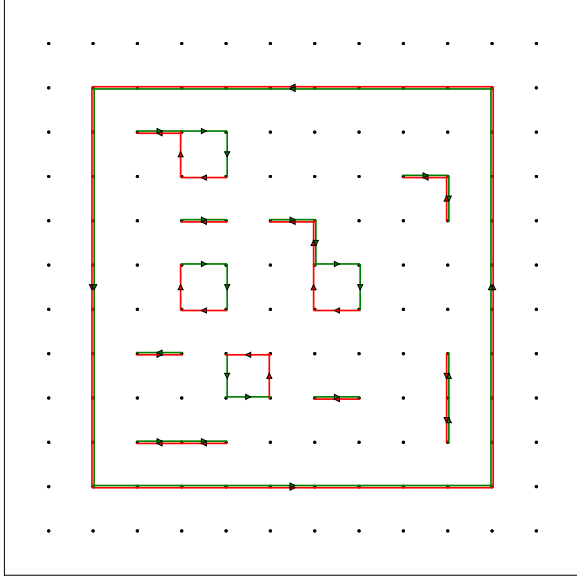


FIG. 3. Example of a current loop configuration in the case of weak, but nonzero, interband Josephson coupling. Red and green lines represent currents of the individual fields \mathbf{b}_i flowing in the direction indicated by the arrows. Charged and neutral currents are therefore represented by overlapping red and green lines flowing either in the same or the opposite direction, respectively. The present configuration shows the same loop of charged current encircling pieces of neutral current, as shown in Fig. 2. However, in the case of weak interband Josephson coupling, the individual currents will fluctuate away from the neutral-current configuration slightly close to the neutral crossover. The screening of the interaction between the segments of the outer charged composite current is accounted for by removing all tightly bound counterflowing currents in the interior, leaving only closed loops that change color an even number of times as the loops are traversed. These closed loops screen the charged-current interaction and may effectively change the sign of the interaction between the segments of charged currents, as explained in the text. This in turn may cause the transition of the charged sector to turn first order.

the dual action than those that attract repulsively, and they will therefore dominate the configurations where many closed color-changing loops (originating with tightly bound counterflowing currents) are present. This attraction may cause an effective attraction between the charged composite current segments, via the attraction to the closed current-changing loops. An identical physical picture holds when working with the dual objects to the currents, namely vortices.

To summarize, the basic mechanism causing a first-order phase transition is the influence of partial decomposition of composite neutral currents on the interaction between charged composite currents, equivalently the influence of partial decomposition of composite neutral vortices on the interaction between composite charged vortices. These pictures are particular dual manifestations of the general concept of a preemptive first-order phase transition. In such a transition, a putative second-order phase transition associated with proliferation of topological defects in a given order parameter is converted to a first-order phase transition preemptively by strong fluctuations (not necessarily critical) in some other field.

IV. CURRENT CORRELATIONS AND THE HIGGS MECHANISM

The defining characteristic of the inverted 3DXY transition in the charged sector is a spontaneous U(1) gauge-symmetry breaking associated with the gauge field \mathbf{A} becoming massive as the system crosses the transition point of the metallic state into the superconducting state. In this section, we investigate how the onset of the mass m_A of the photon (the Higgs mass), which is equivalent to the Meissner effect of the superconductor, comes about as result of a nonanalytic change in the infrared properties of the current correlations of the system. m_A is found from the limiting form of the gauge-field correlation function

$$\langle A_q^\mu A_{-q}^\nu \rangle \sim \frac{1}{q^2 + m_A^2}. \quad (28)$$

To calculate $\langle A_q^\mu A_{-q}^\nu \rangle$, we consider the action of the charged sector given on the form Eq. (A7) before integrating out the gauge field, and insert source terms source \mathbf{J}_q ,

$$S_J = \sum_q \left[\frac{ie}{2} \sum_\alpha \mathbf{b}_{q,\alpha} \cdot \mathbf{A}_{-q} + \frac{ie}{2} \sum_\alpha \mathbf{b}_{-q,\alpha} \cdot \mathbf{A}_q + \frac{\beta}{2} |\mathbf{Q}_q|^2 \mathbf{A}_q \cdot \mathbf{A}_{-q} + \frac{1}{2} (\mathbf{J}_q \cdot \mathbf{A}_{-q} + \mathbf{J}_{-q} \cdot \mathbf{A}_q) \right], \quad (29)$$

which in turn may be written in the form

$$S_J = \sum_q \left\{ \left[\mathbf{A}_q + \frac{1}{2} \left(\mathbf{J}_q + ie \sum_\alpha \mathbf{b}_{q,\alpha} \right) D_q^{-1} \right] D_q \times \left[\mathbf{A}_{-q} + \frac{1}{2} \left(\mathbf{J}_{-q} + ie \sum_\alpha \mathbf{b}_{-q,\alpha} \right) D_q^{-1} \right] - \frac{1}{4} \left(\mathbf{J}_q + ie \sum_\alpha \mathbf{b}_{q,\alpha} \right) D_q^{-1} \times \left(\mathbf{J}_{-q} + ie \sum_\beta \mathbf{b}_{-q,\beta} \right) \right\}. \quad (30)$$

Here, $D_q = \beta |\mathbf{Q}_q|^2 / 2$ as before. After shifting and integrating the gauge field, we have

$$S_J = - \sum_q \left\{ \frac{1}{2\beta |\mathbf{Q}_q|^2} \left[\mathbf{J}_q^\mu P_T^{\mu\nu} \mathbf{J}_{-q}^\nu - e^2 \sum_{\alpha\beta} b_{q,\alpha}^\mu b_{-q}^\mu + ie \sum_\alpha (J_{-q}^\mu b_{q,\alpha}^\mu + J_q^\mu b_{-q,\alpha}^\mu) \right] \right\}, \quad (31)$$

where repeated indices are summed over, and $P_T^{\mu\nu}$ is the transverse projection operator

$$P_T^{\mu\nu} = \delta^{\mu\nu} - \frac{Q_q^\mu Q_{-q}^\nu}{|\mathbf{Q}_q|^2}. \quad (32)$$

The gauge-field correlator is then given by

$$\begin{aligned} \langle A_q^\mu A_{-q}^\nu \rangle &= \frac{1}{\mathcal{Z}_0} \frac{\delta^2 \mathcal{Z}_J}{\delta J_{-q,\mu} \delta J_{q,\nu}} \Big|_{J=0} \\ &= \frac{1}{\mathcal{Z}_0} \sum_{\{b,m\}} \prod_{r,\alpha} \delta_{\mathbf{\Delta} \cdot \mathbf{b}_{r,\alpha}} \sum_{\beta \neq \alpha} \epsilon_{\alpha\beta} m_{r,\alpha,\beta} \prod_{r,\mu,\alpha} I_{b_{r,\alpha,\mu}}(\beta) \\ &\quad \times \prod_{r,\alpha < \beta} I_{m_{r,\alpha,\beta}}(\beta\lambda) \\ &\quad \times \left(-\frac{\delta^2 S_J}{\delta \mathbf{J}_{-q}^\mu \delta \mathbf{J}_q^\nu} - \frac{\delta S_J}{\delta \mathbf{J}_{-q}^\mu} \frac{\delta S_J}{\delta \mathbf{J}_q^\nu} \right) e^{-S_J} \Big|_{J=0}. \end{aligned} \quad (33)$$

Here, \mathcal{Z}_0 is the partition function with the sources set to zero. The functional derivatives of the action are given by

$$-\frac{\delta S_J}{\delta \mathbf{J}_q^\nu} = \frac{1}{\beta |\mathbf{Q}_q|^2} \left(J_{-q}^\nu P_T^{\mu\nu} + i e \sum_\alpha b_{-q,\alpha}^\nu \right) \quad (34)$$

and

$$-\frac{\delta^2 S_J}{\delta \mathbf{J}_{-q}^\mu \delta \mathbf{J}_q^\nu} = \frac{1}{\beta |\mathbf{Q}_q|^2} P_T^{\mu\nu}. \quad (35)$$

Inserting this into Eq. (33) and setting the currents to zero, we have

$$\langle A_q^\mu A_{-q}^\nu \rangle = \frac{P_T^{\mu\nu}}{\beta |\mathbf{Q}_q|^2} - \frac{e^2}{\beta^2 |\mathbf{Q}_q|^4} \left\langle \sum_{\alpha\beta} b_{q,\alpha}^\mu b_{-q,\beta}^\nu \right\rangle. \quad (36)$$

Setting $\nu = \mu$ and summing over μ yields the relevant correlator

$$\langle \mathbf{A}_q \cdot \mathbf{A}_{-q} \rangle = \frac{1}{\beta |\mathbf{Q}_q|^2} \left(2 - \frac{e^2}{\beta |\mathbf{Q}_q|^2} \langle \mathbf{B}_q \cdot \mathbf{B}_{-q} \rangle \right), \quad (37)$$

where we have defined $\langle \mathbf{B}_q \cdot \mathbf{B}_{-q} \rangle = \langle \sum_{\alpha\beta} \mathbf{b}_{q,\alpha} \cdot \mathbf{b}_{-q,\beta} \rangle$.

The effective gauge field mass is given by the zero-momentum limit of the inverse propagator,

$$m_A^2 = \lim_{q \rightarrow 0} \frac{2}{\beta \langle \mathbf{A}_q \mathbf{A}_{-q} \rangle}. \quad (38)$$

As is seen from Eq. (37), the relevant combination of current-field correlators is the superconducting current, while charge-neutral currents do not appear in the expression. The current-correlator may be interpreted as the helicity modulus, which at a charged fixed point has a nonanalytic behavior of the term proportional to q^2 . We expect the leading behavior to be [28]

$$\lim_{q \rightarrow 0} \frac{e^2}{2\beta} \langle \mathbf{B}_q \cdot \mathbf{B}_{-q} \rangle \sim \begin{cases} [1 - C_2(T)]q^2, & T > T_C, \\ q^2 - C_3(T)q^{2+\eta_A}, & T = T_C, \\ q^2 - C_4(T)q^4, & T < T_C. \end{cases} \quad (39)$$

The result given above is dual to an expression for the gauge mass in terms of correlation function of topological defects of the superconducting order, i.e., vortices [17,28,29], since vortices are dual objects to the currents \mathbf{b} . In 3D, it is known that the dual of a superfluid is a superconductor, and vice versa [17,24,28,29]. Therefore, the above result for the current correlator of a superconductor features the same behavior as the vortex-vortex correlator at a *neutral* fixed point, since a

neutral fixed point in the original theory is a charged fixed point in the dual theory. Here, C_2 is the helicity modulus of the system, C_3 is a critical amplitude, and C_4 is essentially the inverse mass of the gauge field. The physical interpretation of $\lim_{q \rightarrow 0} \frac{e^2}{2\beta} \langle \mathbf{B}_q \cdot \mathbf{B}_{-q} \rangle$ is that when this quantity is zero, there are no long-range correlations of current loops in the system; i.e., there are no supercurrents threading the entire system which is therefore normal metallic. Conversely, when $T < T_C$ this correlator is nonzero. There are supercurrents threading the entire system, which is therefore superconducting. When $T > T_C$, the gauge mass will be zero in the long-wavelength limit. When $T < T_C$, however, the factors of q^2 will cancel, and the gauge correlator obtains a finite expectation value, and hence a mass. The Higgs mechanism (Meissner effect) in an N -component superconductor is therefore a result of a blowout of closed loops of charged currents as the temperature is lowered through the phase transition. Conversely, the transition to the normal state is driven by a collapse of closed current loops, which is dual to a blowout of closed vortex loops. In either way of looking at the problem, the Higgs mechanism is fluctuation driven.

Note that the above result is valid for any number of components $N \geq 1$, and any value of the Josephson coupling $\lambda \geq 0$.

The preemptive scenario described in the previous section impacts the temperature dependence of the Higgs mass at the transition from the superconducting to the normal-metallic state. The mass vanishes continuously in an inverted 3DXY phase transition if the value of the gauge charge is large enough for the preemptive scenario to be ruled out for any λ , including $\lambda = 0$. For small enough gauge-charge, such that fluctuations in the neutral sector strongly affect fluctuations in the charged sector, and vice versa, the preemptive effect comes into play. In that case, the Higgs mass vanishes discontinuously at the phase transition.

V. CONCLUSION

We have formulated an N -component London superconductor with intercomponent Josephson couplings as a model of N integer-current fields \mathbf{b}_α and $N(N-1)/2$ monopole fields, $m_{\alpha,\beta}$. These monopoles allow supercurrents of a particular condensate component to be converted to a supercurrent of a different component; i.e., currents may change ‘‘color’’ at any site. For zero Josephson coupling, λ , only configurations where all the monopole fields are zero contribute, and the model reverts to an N -component gauge-coupled 3DXY model. This model is known to have either (i) $N-1$ transitions in the XY universality class and a single phase transition in the inverted XY universality class, or (ii) a single preemptive first-order phase transition for intermediate values of the charge. For any $\lambda > 0$, the $N-1$ phase transitions in the neutral sector are converted to crossovers. In the limit $\lambda \rightarrow \infty$, all orders of monopole excitations will contribute. This effectively removes the constraints $\mathbf{\Delta} \cdot \mathbf{b}_\alpha = 0$ on each individual component. There is only one particular composite mode, $\sum_\alpha \mathbf{b}_\alpha$, which is still divergenceless, and which thus features a phase transition. This transition is known to be in the inverted 3DXY universality class for $\lambda = 0$. For small but finite λ , fluctuations in the neutral sector are still substantial although the phase transitions are

all converted to crossovers. These charge-neutral noncritical fluctuations nonetheless substantially influence the putative critical fluctuations in the charged sector, particularly at temperatures close to the $\lambda = 0$ 3DXY critical point. This converts the inverted 3DXY critical point into a first-order phase transition via a preemptive effect. The degree to which the charge-neutral fluctuations influence the fluctuations in the charged sector for small λ increases with the number of composite charge-neutral fluctuating modes. In the parameter regime (e, λ) where one may have a preemptive effect, the first-order character of the superconductor–normal metal phase transition will therefore be more pronounced with increasing N .

As a by-product of our analysis, we have recast the onset of the photon Higgs mass in the superconductor (Meissner effect) in terms of a blowout of current loops associated with the onset of superconductivity. This analysis goes beyond mean-field theory and takes all critical fluctuations of the theory into account. The description giving the onset of the Higgs mass of the photon in terms of a current loop blowout going into the superconducting state as temperature is lowered is dual to the description of the vanishing of the Higgs mass of the photon in terms of vortex-loop blowout going into the normal-metallic state as the temperature is increased.

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APPENDIX A: THE CHARACTER EXPANSION

We apply the expansion

$$e^{\beta \cos \gamma} = \sum_{b=-\infty}^{\infty} I_b(\beta) e^{ib\gamma} \quad (\text{A1})$$

to the cosine terms of Eq. (4), with $\lambda = 0$. This gives the action

$$\begin{aligned} \mathcal{Z} &= \int \mathcal{D}\mathbf{A} \left(\prod_{\alpha} \int \mathcal{D}\theta_{\alpha} \right) \\ &\times \prod_{r, \mu, \alpha} \sum_{b_{r, \mu, \alpha} = -\infty}^{\infty} I_{b_{r, \mu, \alpha}}(\beta) e^{ib_{r, \mu, \alpha} (\Delta_{\mu} \theta_{r, \alpha} - e A_{r, \mu})} \\ &\times \prod_r e^{-\frac{\beta}{2} (\Delta \times \mathbf{A}_r)^2}. \end{aligned} \quad (\text{A2})$$

By performing a partial integration of each phase component, $\theta_{r, \alpha}$, we move the lattice derivative from the phase to the integer field \mathbf{b} in the first term. Then we factorize the terms dependent on the phases on each lattice site, which may then be integrated separately,

$$\mathcal{Z}_{\theta} = \prod_{r, \alpha} \int_0^{2\pi} d\theta_{r, \alpha} e^{-i\theta_{r, \alpha} (\sum_{\mu} \Delta_{\mu} b_{r, \mu, \alpha})}. \quad (\text{A3})$$

This constrains the \mathbf{b} fields to have zero divergence,

$$\Delta \cdot \mathbf{b}_{r, \alpha} = 0 \quad \forall r, \alpha. \quad (\text{A4})$$

The partition function then reads

$$\begin{aligned} \mathcal{Z} &= \int \mathcal{D}(\mathbf{A}) \sum_{\{\mathbf{b}\}} \prod_{r, \alpha} \delta_{\Delta \cdot \mathbf{b}_{r, \alpha}, 0} \prod_{r, \mu, \alpha} I_{b_{r, \mu, \alpha}}(\beta) \\ &\times \prod_r e^{-[ie \sum_{\alpha} \mathbf{b}_{r, \alpha} \cdot \mathbf{A}_r + \frac{\beta}{2} (\Delta \times \mathbf{A}_r)^2]}. \end{aligned} \quad (\text{A5})$$

This represents N integer-current fields which must form closed loops individually, coupled by a single gauge field, \mathbf{A} .

The next step is to integrate out the gauge degrees of freedom. To this end we Fourier-transform the action

$$S = \sum_r \left[ie \sum_{\alpha} \mathbf{b}_{r, \alpha} \cdot \mathbf{A}_r + \frac{\beta}{2} (\Delta \times \mathbf{A}_r)^2 \right] \quad (\text{A6})$$

into

$$\begin{aligned} S &= \sum_q \left[\frac{ie}{2} \sum_{\alpha} \mathbf{b}_{q, \alpha} \cdot \mathbf{A}_{-q} + \frac{ie}{2} \sum_{\alpha} \mathbf{b}_{-q, \alpha} \cdot \mathbf{A}_q \right. \\ &\left. + \frac{\beta}{2} (\mathbf{Q}_q \times \mathbf{A}_q)(\mathbf{Q}_{-q} \times \mathbf{A}_{-q}) \right]. \end{aligned} \quad (\text{A7})$$

Here, we have symmetrized the $\mathbf{b} \cdot \mathbf{A}$ term, and \mathbf{Q}_q is the Fourier representation of the lattice differential operator, Δ . We can further simplify the expression by choosing the gauge $\Delta \cdot \mathbf{A}_r = 0$, which translates to $\mathbf{Q}_q \cdot \mathbf{A}_q = 0$ in Fourier space. This reduces the last term to $\beta |\mathbf{Q}_q|^2 \mathbf{A}_q \cdot \mathbf{A}_{-q} / 2$, where $|\mathbf{Q}_q|^2 = \sum_{\mu} (2 \sin q_{\mu} / 2)^2$. Now we complete the squares in \mathbf{A}_q , to facilitate the Gaussian integration

$$\begin{aligned} S &= \sum_q \left[\left(\mathbf{A}_q + \frac{ie}{2} \sum_{\alpha} \mathbf{b}_{q, \alpha} D_q^{-1} \right) D_q \right. \\ &\times \left(\mathbf{A}_{-q} + \frac{ie}{2} \sum_{\alpha} \mathbf{b}_{-q, \alpha} D_q^{-1} \right) \\ &\left. + \frac{e^2}{4} \left(\sum_{\alpha} \mathbf{b}_{q, \alpha} \right) D_q^{-1} \left(\sum_{\beta} \mathbf{b}_{-q, \beta} \right) \right], \end{aligned} \quad (\text{A8})$$

where $D_q = \beta |\mathbf{Q}_q|^2 / 2$. Now we can shift and integrate out the gauge field, \mathbf{A}_q , which leaves us with

$$S = \sum_q \frac{e^2}{2\beta |\mathbf{Q}_q|^2} \left(\sum_{\alpha} \mathbf{b}_{q, \alpha} \right) \cdot \left(\sum_{\beta} \mathbf{b}_{-q, \beta} \right), \quad (\text{A9})$$

or in real space

$$S = \sum_{r, r'} \frac{e^2}{2\beta} \left(\sum_{\alpha} \mathbf{b}_{r, \alpha} \right) \cdot \left(\sum_{\beta} \mathbf{b}_{r', \beta} \right) D(\mathbf{r} - \mathbf{r}'). \quad (\text{A10})$$

Here, $D(\mathbf{r} - \mathbf{r}')$ is the Fourier transform of $1/|\mathbf{Q}_q|^2$. Inserting this into the action, we arrive at

$$\mathcal{Z} = \sum_{\{\mathbf{b}, m\}} \prod_{\mathbf{r}, \alpha} \delta_{\Delta \cdot \mathbf{b}_{\mathbf{r}, \alpha}, 0} \prod_{\mathbf{r}, \mu, \alpha} I_{b_{\mathbf{r}, \alpha, \mu}}(\beta) \times \prod_{\mathbf{r}, \mathbf{r}'} e^{-\frac{e^2}{2\beta} \sum_{\alpha, \beta} \mathbf{b}_{\mathbf{r}, \alpha} \cdot \mathbf{b}_{\mathbf{r}', \beta} D(\mathbf{r} - \mathbf{r}')}, \quad (\text{A11})$$

which is Eq. (9).

APPENDIX B: TWO-DIMENSIONAL MULTIBAND SUPERCONDUCTORS

In a thin-film superconductor, the effective magnetic penetration depth is inversely proportional to the film thickness. Hence, in a two-dimensional system, the magnetic penetration depth becomes infinite, and the effective charge of the charge carriers becomes zero. This effectively freezes out the gauge-field fluctuations of the interior of the film, in turn eliminating the long-range gauge-field-mediated vortex-vortex interactions. In this case the relevant lattice action will be

$$S = -\beta \sum_{\mathbf{r}} \sum_{\mu, \alpha} \cos(\Delta_{\mu} \theta_{\mathbf{r}, \alpha}) - \beta \lambda \sum_{\mathbf{r}} \sum_{\alpha < \beta} \cos(\theta_{\mathbf{r}, \alpha} - \theta_{\mathbf{r}, \beta}). \quad (\text{B1})$$

That is, it is effectively a neutral condensate.

We may apply the character expansion of Eq. (8) to Eq. (B1), which results in the partition function

$$\mathcal{Z} = \sum_{\{\mathbf{b}, m\}} \prod_{\mathbf{r}, \alpha} \delta_{\Delta \cdot \mathbf{b}_{\mathbf{r}, \alpha}, \sum_{\beta \neq \alpha} m_{\mathbf{r}, \alpha, \beta}} \times \prod_{\mathbf{r}, \mu, \alpha} I_{b_{\mathbf{r}, \alpha, \mu}}(\beta) \prod_{\mathbf{r}, \alpha < \beta} I_{m_{\mathbf{r}, \alpha, \beta}}(\beta \lambda). \quad (\text{B2})$$

This is of course very similar to Eq. (12), with the differences being as follows. The integer-current field, $\mathbf{b}_{\mathbf{r}}$, is now a two-component vector, as is naturally the position vector, \mathbf{r} , and the gauge-field-mediated interaction has disappeared.

We may apply the same reasoning to Eq. (B2) as we did in the main text. There will be a single mode, $\sum_{\alpha} \mathbf{b}_{\mathbf{r}}$, which is divergenceless, and $N(N-1)/2$ modes with finite divergence. The only difference now in the two-dimensional case is the lack of gauge-field-mediated interactions in the divergenceless mode. Hence, the single remaining phase transition is expected to be a Kosterlitz-Thouless transition from a two-dimensional superfluid to a normal fluid. This prediction could be verified in Monte Carlo simulations, as the partition function of Eq. (B2) is particularly well suited for worm-type algorithms.

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