

Quantization of conductance minimum and index theorem

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We discuss the minimum value of the zero-bias differential conductance G_{\min} in a junction consisting of a normal metal and a nodal superconductor preserving time-reversal symmetry. Using the quasiclassical Green function method, we show that G_{\min} is quantized at $(4e^2/h)N_{\text{ZES}}$ in the limit of strong impurity scatterings in the normal metal at the zero temperature. The integer N_{ZES} represents the number of perfect transmission channels through the junction. An analysis of the chiral symmetry of the Hamiltonian indicates that N_{ZES} corresponds to the Atiyah-Singer index in mathematics.

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I. INTRODUCTION

The quantization of an observable value in physics is closely related some of the time to an invariant in mathematics. A good example may be the quantized Hall conductivity in condensed matter physics. Although the quantization of the Hall conductivity itself occurs for physical reasons, the quantized value is proportional to a Chern invariant in a two-dimensional manifold [1]. Another example is the number of gapless states at the surface of a topologically nontrivial material characterized by a topological invariant Z . The integer Z depends on the spatial dimensionality and the symmetry class of the Hamiltonian [2,3]. The conductance in a junction consisting of such a topologically nontrivial superconductor is quantized at $(2e^2/h)Z$ with $Z = 1$ for a one-dimensional class D superconductor [4–8]. A similar phenomenon has been discussed as regards superconductors in class BDI [9–12] with Z being an integer number.

The Atiyah-Singer theorem relates a topological invariant to an invariant defined in terms of solutions of a differential equation. The index theorem provides the mathematical background to the quantum anomaly in particle physics. In condensed matter physics, the index theorem describes the number of gapless modes at a boundary between two chiral superfluids [13]. When a quantized physical value is described by a mathematical invariant, the quantization should be robust under various perturbations preserving the invariant. In this paper, we show a relationship between the minimum value of the conductance in a superconducting junction and the Atiyah-Singer index.

We discuss the zero-bias differential conductance G_{NS} in a normal-metal/superconductor (NS) junction in two dimensions, where the normal metal contains a number of random impurities and its normal resistance is R_{N} . The superconductor is characterized by unconventional time-reversal pairing symmetries such as p_x -, d_{xy} -, and f -wave symmetry. The analytical expression of the conductance is obtained by solving the quasiclassical Usadel equation [14–16] in a normal metal. We find that G_{NS} decreases to the quantized value of $(4e^2/h)|N_{\text{ZES}}|$ with increasing in R_{N} . The results suggest that $|N_{\text{ZES}}|$ is the number of perfect transmission channels in a dirty NS junction. The analysis in terms of the chiral

symmetry of the Hamiltonian enables us to understand the relationship between N_{ZES} and the index in the Atiyah-Singer theorem. To confirm the theoretical prediction by experiments, temperature T must be low so that the thermal coherence length $\xi_T = \sqrt{\hbar D/2\pi k_B T}$ is longer than the length of a dirty normal metal L , where D is the diffusion constant in the normal metal. This is the only condition that we assume in this paper.

II. CONDUCTANCE MINIMUM

Let us consider an NS junction described by a 2×2 Bogoliubov-de Gennes (BdG) Hamiltonian,

$$\hat{H}_{\text{BdG}} = \begin{bmatrix} \xi_r + V(\mathbf{r}) & \Delta(\mathbf{r})\Theta(x) \\ \Delta(\mathbf{r})\Theta(x) & -\xi_r - V(\mathbf{r}) \end{bmatrix}, \quad (1)$$

$$V(\mathbf{r}) = V_{\text{imp}}(\mathbf{r})\Theta(-x)\Theta(x+L) + v_0\delta(x), \quad (2)$$

$$\Delta(\mathbf{r}) = \begin{cases} \Delta, & s \\ -2\Delta \partial_x \partial_y / k_F^2, & d_{xy} \\ -i\Delta \partial_x / k_F, & p_x \\ -i\Delta \partial_x (k_F^2 + 2\partial_y^2) / k_F^3, & f, \end{cases} \quad (3)$$

where $\xi_r = -\hbar^2 \nabla^2 / (2m) - \mu_F$, m is the mass of an electron, μ_F is the chemical potential, k_F is the Fermi wave number, and $\Theta(x)$ is the step function. We introduce the random impurity potential V_{imp} in the normal metal ($-L < x < 0$) as shown in Fig. 1(a) and consider the barrier potential v_0 at the NS interface. In the y direction, we apply a periodic boundary condition with W denoting junction width.

The quasiclassical Usadel equation in the normal metal is represented by using θ parameterization [14–16],

$$\hbar D \frac{\partial^2 \theta(x, \epsilon)}{\partial x^2} + 2i \epsilon \sin \theta(x, \epsilon) = 0, \quad (4)$$

where ϵ is the energy of a quasiparticle measured from the Fermi level (zero energy). The quasiclassical Green function in a normal metal is represented in terms of θ in Eq. (4). The normal Green function $g = \cos \theta$ represents the quasiparticle excitation, whereas the anomalous Green function $f = \sin \theta$ describes the pairing correlations. They are linked to each

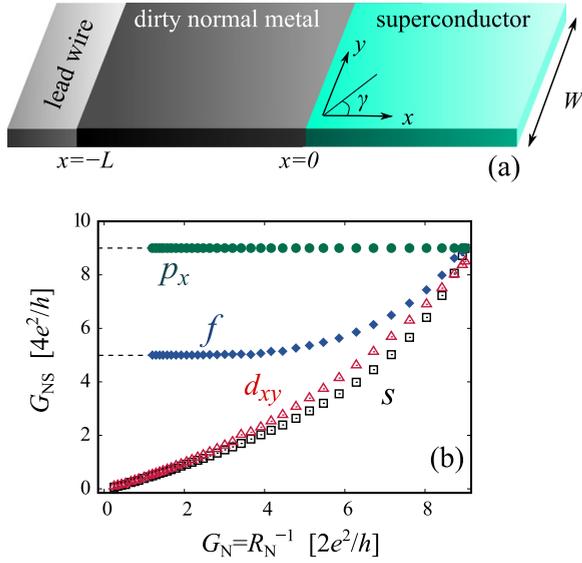


FIG. 1. (a) Schematic picture of a normal-metal/superconductor junction. (b) Numerical results on a two-dimensional tight-binding model. The index $|N_{\text{ZES}}|$ is 9 and 5 for p_x - and f -wave symmetry, respectively.

other through the normalization condition $g^2 + f^2 = 1$. The quasiclassical approximation is justified for $\Delta/\mu_F \ll 1$ which is satisfied in most superconductors. The Usadel equation is supplemented by two boundary conditions [17–19],

$$\theta(x = -L, \epsilon) = 0, \quad \frac{L}{G_Q R_N} \left(\frac{\partial \theta}{\partial x} \right) \Big|_{x=0} = 2I_F. \quad (5)$$

$$I_F = \sum_{k_y} F_{bc}(k_y). \quad (6)$$

The first boundary condition in Eq. (5) represents the absence of Cooper pairs in the lead wire because of $f = 0$ at $\theta = 0$. The influence of a superconductor on a normal metal is considered through the second boundary condition which represents the current conservation law. The wave number on the Fermi surface is described by $k_x = k_F \cos \gamma$ and $k_y = k_F \sin \gamma$, where γ is the angle measured from the x axis as shown in Fig. 1(a). In Eq. (6), \sum_{k_y} means the summation over all the propagating channels whose number is calculated as $N_c = [Wk_F/\pi]_G$, where $[\dots]_G$ is the Gauss symbol giving the integer part of the argument. (See also the Appendix.) The pair potentials in Eq. (3) are represented as $\Delta \sin(2\gamma)$, $\Delta \cos \gamma$, and $\Delta \cos \gamma(1 - 2 \sin^2 \gamma)$ for d_{xy} -, p_x -, and f -wave symmetries, respectively. The Andreev reflection at the NS interface is described by two pair potentials: $\Delta_+ = \Delta(\gamma)$ and $\Delta_- = \Delta(\pi - \gamma)$. When Δ_+ is the pair potential for a quasiparticle incoming into the NS interface at γ , Δ_- is the pair potential for an outgoing quasiparticle specularly reflected by the NS interface. In Eq. (6), F_{bc} is defined by using the two pair potentials, and its expression is given in the Appendix. To calculate the zero-bias resistance at the zero temperature, the solution of the Usadel equation at $\epsilon = 0$ is necessary and is described by $\theta(x) = \theta_0(1 + x/L)$, where θ_0 is determined by the second boundary condition in Eq. (5).

The normal resistance of a potential barrier at $x = 0$ is calculated as $R_B = (G_Q N_c T_B)^{-1}$ with $G_Q = 2e^2/h$ and the transmission probability of the barrier in the normal state T_B in the Appendix. The normal resistance of a normal metal is R_N which is the inverse of the normal conductance $G_N = G_Q 2\pi \hbar \rho_0 DW/L$ with ρ_0 being the density of states at the Fermi level in the normal state. The total resistance of an NS junction at a bias voltage V_{bias} is represented by a modified Ohm's law by using the Keldysh technique [17,19],

$$R_{\text{NS}} = \frac{1}{G_Q I_B} + \frac{R_N}{L} \int_{-L}^0 \frac{dx}{\cosh^2(\text{Im}(\theta(x, \epsilon)))}, \quad (7)$$

by putting $\epsilon \rightarrow eV_{\text{bias}}$ on the right-hand side. The first term represents the resistance due to a potential barrier, where I_B given in the Appendix describes how the Andreev reflection modifies the resistance at the NS interface. The second term in Eq. (7) is the resistance of a normal conductor, where the integral describes how the proximity effect modifies R_N . The proximity effect is a phase coherence phenomenon. Therefore, we assume that the thermal coherence length $\xi_T = \sqrt{\hbar D/2\pi k_B T}$ is larger than L at a finite temperature T . In this paper, we calculate R_{NS} at zero temperature in the zero-bias limit.

The unconventional pair potentials in Eq. (3) have nodes on the Fermi surface. Such nodal superconductors cannot be straightforwardly classified into the ten well known topological classes [2,3]. To characterize a nodal superconductor topologically, we consider the one-dimensional Brillouin zone by fixing k_y in the clean limit and define the one-dimensional winding number w_{1D} [20]. We find that

$$w_{1D}(k_y) = s_+ \frac{1 - s_+ s_-}{2}, \quad s_{\pm} = \Delta_{\pm}/|\Delta_{\pm}|. \quad (8)$$

The winding number is s_+ for a channel satisfying $s_+ s_- = -1$ and is zero for a channel satisfying $s_+ s_- = 1$. A channel with $|w_{1D}| = 1$ hosts a zero-energy state (ZES) at a surface of superconductor. The three unconventional pair potentials in Eq. (3) satisfy $s_+ s_- = -1$ for all the propagating channels. Therefore, such an unconventional superconductor hosts dispersionless ZESs at its clean surface [21–25].

The effects of the flat ZESs on the conductance depend on the parity of the pair potential. With spin-singlet even-parity superconductors, we can easily find that I_F in Eq. (6) and $\theta(x)$ are real numbers at $\epsilon = 0$ as shown in the Appendix. Since $\text{Im}[\theta(x)] = 0$, the second term in Eq. (7) becomes R_N [26]. Therefore, the zero-bias differential conductance $G_{\text{NS}} = R_{\text{NS}}^{-1}$ becomes

$$\lim_{R_N \rightarrow \infty} G_{\text{NS}} \rightarrow 0 \quad (9)$$

for all spin-singlet even-parity superconductors. By contrast, in the spin-triplet odd-parity superconductors, we find that $I_F = i N_{\text{ZES}}$ is a pure imaginary integer number with

$$N_{\text{ZES}} \equiv \sum_{k_y} w_{1D}(k_y) = N_+ - N_-. \quad (10)$$

The integer N_{ZES} is defined by the summation of one-dimensional winding number for all the propagation channels.

The integer N_{\pm} corresponds to the number of ZESs characterized by $w_{1D} = s_{\pm} = \pm 1$. A solution from the boundary condition

$$\theta(x) = i\beta_0(1 + x/L), \quad \beta_0 = 2G_Q N_{ZES} R_N, \quad (11)$$

enables us to conclude that

$$\lim_{R_N \rightarrow \infty} G_{NS} \rightarrow \frac{4e^2}{h} |N_{ZES}| \quad (12)$$

for spin-triplet odd-parity superconductors. The derivation is given in the Appendix. The minimum value of the zero-bias conductance is quantized at a nonzero value, which suggests that $|N_{ZES}|$ is the number of perfect transmission channels through an NS junction. This is the first conclusion of this paper.

III. ATIYAH-SINGER INDEX

Next we analyze the physical meaning of N_{ZES} by using the chiral symmetry of the Hamiltonian. The BdG Hamiltonian in Eq. (1) satisfies

$$\{\hat{H}_{BdG}, \hat{\Lambda}\}_+ = 0, \quad \hat{\Lambda} = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}, \quad (13)$$

which represents the chiral symmetry of the Hamiltonian. The eigenvalue of $\hat{\Lambda}$ is either $\lambda = 1$ or $\lambda = -1$. The eigenstates of \hat{H}_{BdG} have a characteristic property summarized as follows [20].

(i) A zero-energy state of \hat{H}_{BdG} is simultaneously an eigenstate of $\hat{\Lambda}$. Namely, $\hat{\Lambda} \chi_{\pm} = \pm \chi_{\pm}$ holds for χ_{\pm} satisfying $\hat{H}_{BdG} \chi_{\pm} = 0$.

(ii) On the other hand, nonzero-energy states are described by the linear combination of two states: one belongs to $\lambda = 1$ and the other belongs to $\lambda = -1$. Namely $\chi_{E \neq 0} = a_+ \chi_+ + a_- \chi_-$. Moreover the relation $|a_+| = |a_-|$ always holds [27].

In what follows, we first discuss a mathematical aspect of N_{ZES} briefly and show that $|N_{ZES}|$ is the number of ZESs in a dirty normal metal while taking these properties into account. By deleting the normal segment $x < 0$ in Fig. 1(a), we focus on a surface of a clean semi-infinite superconductor. The wave function of a ZES localized at the surface can be represented for each propagating channel,

$$\phi_{k_y}(\mathbf{r}) = A_{k_y} \begin{bmatrix} i \\ s_+ \end{bmatrix} \sin(k_x x) e^{-x/\xi_y} e^{ik_y y} \frac{1 - s_+ s_-}{2}, \quad (14)$$

where A_{k_y} is the normalization constant and $\xi_y = \hbar v_F \cos \gamma / |\Delta_+|$ depends on the pair potential in Eq. (3) and $v_F = \hbar k_F / m$ is the Fermi velocity. As suggested by property (i), ϕ_{k_y} is the eigenstate of $\hat{\Lambda}$ belonging to its eigenvalue $\lambda = s_+$. Since $w_{1D} = s_+$ and $s_+ = \lambda$, the chiral eigenvalue λ and the one-dimensional winding number w_{1D} are identical to each other. Therefore, N_{\pm} in Eq. (10) is exactly equal to the number of ZESs that belong to $\lambda = \pm 1$. In Eq. (10), N_{ZES} is initially defined in terms of a topological invariant. Here N_{ZES} is characterized simultaneously by an invariant in terms of the solutions of the BdG equation. Mathematically, such an integer number is called the Atiyah-Singer index.

At a clean surface, the degree of degeneracy at zero energy is N_c for three unconventional pair potentials in Eq. (3). The translational symmetry in the y direction protects such

TABLE I. The key integer numbers are summarized, where $A = Wk_F/\pi$, $N_c = [A]_G$ and $[\dots]_G$ means the Gauss symbol providing the integer part of the argument.

	N_+	N_-	N_{ZES}
s	0	0	0
d_{xy}	$[A/2]_G$	$[A/2]_G$	0
p_x	N_c	0	N_c
f	$[A/\sqrt{2}]_G$	$[A(1 - 1/\sqrt{2})]_G$	$[A(\sqrt{2} - 1)]_G$

a high degeneracy at zero energy. When we attach a dirty normal metal to form an NS junction, however, the potential disorder would lift the degeneracy at zero energy. We first count N_{\pm} at a clean surface of the three unconventional superconductors as listed in Table I. Then we discuss how the potential disorder lifts the degeneracy at zero energy. With a p_x wave, we find $N_+ = N_c = N_{ZES}$ and $N_- = 0$ because $\lambda = 1$ for all the propagating channels. Such pure chiral states cannot form nonzero-energy states according to property (ii) because ZESs belonging to $\lambda = -1$ are absent [27]. This fact explains the anomalous proximity effect [18,28–30]. With a d_{xy} wave, we find $N_{ZES} = 0$ because the ZESs for $k_y > 0$ ($k_y < 0$) belong to $\lambda = 1$ ($\lambda = -1$). The impurity potential completely eliminates the degeneracy at zero energy because two ZESs with $\lambda = 1$ and $\lambda = -1$ couple one-by-one and form two nonzero-energy states according to property (ii). This explains the absence of the proximity effect in a d_{xy} -wave NS junction [17,31]. The conclusion $N_{ZES} = 0$ is valid for all spin-singlet even-parity superconductors. Actually I_F in Eq. (5) is real for all spin-singlet superconductors, whereas N_{ZES} is defined by the imaginary part of I_F . Finally, with an f wave, we obtain $N_{ZES} = [(Wk_F/\pi)(\sqrt{2} - 1)]_G$. The integer $|N_{ZES}|$ physically means the number of ZESs in the presence of potential disorder. Mathematically, N_{ZES} is an invariant as far as \hat{H}_{BdG} preserves the chiral symmetry. This is the second conclusion of this paper.

The integer $|N_{ZES}|$ indicates also the number of ZESs penetrating into a dirty normal metal. The local density of states (LDOS) given by $\rho(x, \epsilon) = \rho_0 \text{Re}[\cos \theta(x, \epsilon)]$ justifies this statement. For a spin-triplet NS junction, $\theta(x)$ in Eq. (11) results in

$$\rho(x)/\rho_0 = \cosh[2G_Q N_{ZES} R_N (1 + x/L)] \gg 1, \quad (15)$$

at $\epsilon = 0$. The LDOS shows a drastic enhancement at zero energy, which directly indicates the penetration of the ZESs into a normal metal [18,28,29]. The analytical results in Eq. (12) suggest that the penetrating ZESs form the resonant transmission channels. Such degenerate ZESs in a spin-triplet junction are called Majorana flat band in recent literature [32–37].

We check the validity of the above argument by employing a numerical simulation on the two-dimensional single-band tight-binding model. We choose $L = 30a_0$, $W = 25a_0$, $\mu_F/t = 1.0$, $\Delta/t = 0.01$ with a_0 and t being the lattice constant and the hopping integral, respectively. This parameter choice leads to $N_c = 9$. In the presence of the random potential, we plot the G_{NS} versus $G_N = R_N^{-1}$ in Fig. 1(b), where the two conductances are calculated independently by using

the recursive Green function method [38,39]. As predicted in Eq. (12), G_{NS} is quantized at $2G_Q N_{\text{ZES}}$ for spin-triplet junctions. For a p_x -wave symmetry, $N_+ = N_c = 9$ and $N_- = 0$ are obtained numerically, which results in $N_{\text{ZES}} = 9$. For an f -wave symmetry, we find $N_{\text{ZES}} = 5$ because $N_+ = 7$ and $N_- = 2$. The results also show that G_{NS} goes to zero with a decrease in G_{N} for spin-singlet s - and d_{xy} -wave cases as predicted in Eq. (9). It is well known that the self-consistently calculated pair potential is suppressed near the interface as $\Delta(x) = \Delta \tanh(x/\xi_0)$ [22,40–42] in p_x -, d_{xy} -, and f cases, where $\xi_0 = \hbar v_F / \pi \Delta$ is the coherence length in a superconductor. We have numerically confirmed that the conductance under such suppressed pair potential shows the same limiting behaviors as those in Fig. 1(b). The results are reasonable because the suppression of the pair potential near the interface does not break the chiral symmetry. The numerical results justify our conclusions. Although we assume a two-dimensional junction in this paper, the conclusions are valid also for a realistic three-dimensional junction. The winding number in the reduced Brillouin zone in Eq. (8) and the index in Eq. (10) are well defined in the three-dimensional junction by replacing k_y by k_{\parallel} , where k_{\parallel} is the wave vector in the parallel direction to the NS interface.

IV. DISCUSSION

The nonzero conductance minimum is a character of odd-parity superconducting states that have been proposed in an artificial thin film [33] and exotic materials such as (TMTSF)₂X ($X = \text{PF}_6, \text{ClO}_4$, etc.) [43–45] and $\text{Na}_x\text{CoO}_2\text{yH}_2\text{O}$ [46,47]. Thus the contents of this paper have a strong connection to the physics of Majorana fermion [32]. Actually, the anomalous conductance observed in experiments [6–8] in a nanowire superconductor [4,5] is an example of the present theory. The Majorana fermions in a spin-triplet superconducting junction always accompany odd-frequency Cooper pairs [48–50]. The conductance quantization discussed in this paper is a part of the physics of odd-frequency Cooper pairs [29,51–53].

The topological classification of superconductors [3] cannot be applied to real superconductors straightforwardly because a number of unconventional superconductors have a nodal gap on the Fermi surface. In many cases, the nodes are necessary to realize the sign change of the pair potential on the Fermi surface which is an important factor for topologically nontrivial superconducting states. The topological characterization of such nodal superconductors is still possible in terms of a topological number defined in a restricted Brillouin zone. The dimensional reduction in the presence of the translational symmetry is the simplest method. Mirror reflection, inversion, and point group symmetries also enable us to classify symmetry-protected topologically nontrivial states [54]. A topologically nontrivial superconductor in this context has the Andreev bound states at its clean surface. According to the bulk-boundary correspondence, the degree of the degeneracy of surface bound states at zero energy should be identical to the summation of topological numbers over all the restricted Brillouin zones. Such degenerate ZESs at a surface or a junction interface cause the anomalous low energy transport which can be observed in experiments as a sign of topologically nontrivial superconducting phase. In a real superconductor,

however, the potential disorder near the surface is inevitable and it may lift the degeneracy of the ZESs. This is because the potential disorder breaks the symmetries that protect the topologically nontrivial superconducting phase. In this situation, a method to measure the robustness of the degenerate ZESs is desired. In this paper, we show that the Atiyah-Singer index represents the degree of the degeneracy in the presence of potential disorder. Our method can be applied to all time-reversal superconductors because the chiral symmetry is constructed by the combination of particle-hole symmetry and time-reversal symmetry.

V. CONCLUSION

In summary, we have discussed the zero-bias differential conductance G_{NS} in a normal-metal/superconductor junction consisting of a nodal superconductor preserving time-reversal symmetry. The minimum value of G_{NS} is quantized at $(4e^2/h)N_{\text{ZES}}$. The analysis in terms of the chiral symmetry of the Hamiltonian indicates that the integer N_{ZES} is the Atiyah-Singer index in mathematics.

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APPENDIX A: BOUNDARY CONDITION

To calculate the right-hand side of the second boundary condition in Eq. (5) in the text, the expression for I_F is necessary. The Andreev reflection by an unconventional superconductor is described by two pair potentials,

$$\Delta_+ = \Delta(\gamma), \quad \Delta_- = \Delta(\pi - \gamma). \quad (\text{A1})$$

The Green functions in a superconductor depend on the two pair potentials as

$$g_{\pm} = \frac{\epsilon}{\sqrt{\epsilon^2 - \Delta_{\pm}^2}}, \quad f_{\pm} = \frac{i\Delta_{\pm}}{\sqrt{\epsilon^2 - \Delta_{\pm}^2}}. \quad (\text{A2})$$

By using these Green functions, we define g_s, f_s, \bar{f}_s , and z_s by

$$g_s = g_+ + g_-, \quad z_s = 1 + g_+g_- + f_+f_-, \quad (\text{A3})$$

$$f_s = \begin{cases} f_+ + f_- & \text{singlet} \\ i(f_+g_- - f_-g_+) & \text{triplet,} \end{cases} \quad (\text{A4})$$

$$\bar{f}_s = \begin{cases} i(f_+g_- - f_-g_+) & \text{singlet} \\ f_+ + f_- & \text{triplet.} \end{cases} \quad (\text{A5})$$

For small ϵ , we obtain the following expression

$$g_s = -i\epsilon A_\Delta, \quad z_s = 1 + s_+s_- - \frac{\epsilon^2}{|\Delta_+||\Delta_-|}, \quad (\text{A6})$$

$$f_s = \begin{cases} s_+ + s_- & \text{singlet} \\ \epsilon s_+ B_\Delta & \text{triplet,} \end{cases} \quad (\text{A7})$$

$$\bar{f}_s = \begin{cases} \epsilon s_+ B_\Delta & \text{singlet} \\ s_+ + s_- & \text{triplet.} \end{cases} \quad (\text{A8})$$

$$A_\Delta = \frac{|\Delta_+| + |\Delta_-|}{|\Delta_+||\Delta_-|}, \quad B_\Delta = \frac{|\Delta_+| - s_+s_-|\Delta_-|}{|\Delta_+||\Delta_-|}. \quad (\text{A9})$$

The second boundary condition in Eq. (5) in the text for $\theta(x) = \theta_0(1 + x/L)$ results in

$$\frac{\theta_0}{G_Q R_N} = 2I_F. \quad (\text{A10})$$

The expression of I_F in Eq. (6) in the text is given by

$$I_F = \sum_{k_y} F_{bc}, \quad (\text{A11})$$

$$F_{bc} = \frac{|t_b|^2(f_s \cos \theta_0 - g_s \sin \theta_0)}{\Xi}, \quad (\text{A12})$$

$$\Xi = (2 - |t_b|^2)z_s + |t_b|^2(g_s \cos \theta_0 + f_s \sin \theta_0), \quad (\text{A13})$$

and F_{bc} depends significantly on the relative sign between the two pair potentials,

$$s_\pm = \Delta_\pm / |\Delta_\pm|. \quad (\text{A14})$$

The summation over propagating channel is evaluated as

$$\sum_{k_y} \rightarrow \frac{W}{2\pi} \int_{-k_F}^{k_F} dk_y = \frac{Wk_F}{2\pi} \int_{-\pi/2}^{\pi/2} d\gamma \cos \gamma = \frac{Wk_F}{\pi}. \quad (\text{A15})$$

Thus $[Wk_F/\pi]_G$ is the number of propagating channels N_c . In Eq. (A12), F_{bc} depends on the normal transmission coefficient of a potential barrier,

$$t_b = \frac{\cos \gamma}{\cos \gamma + iz_0}, \quad z_0 = v_0/(\hbar v_F), \quad (\text{A16})$$

where $v_F = \hbar k_F/m$ is the Fermi velocity. The summation of $|t_b|^2$ over the propagation channels gives the inverse of the normal resistance of the potential barrier $R_B = (G_Q N_c T_B)^{-1}$,

$$\sum_{k_y} |t_b|^2 = \frac{Wk_F}{\pi} T_B, \quad T_B = \int_0^{\pi/2} d\gamma \cos \gamma |t_b|^2, \quad (\text{A17})$$

where T_B is the normal transmission probability of the potential barrier.

In a spin-singlet junction, we obtain

$$I_F = \sum_{k_y} \left[\frac{s_+ |t_b|^2 \cos \theta_0}{2 - |t_b|^2(1 - s_+ \sin \theta_0)} \frac{1 + s_+s_-}{2} - \tan \theta_0 \frac{1 - s_+s_-}{2} \right], \quad (\text{A18})$$

at $\epsilon = 0$. In an s -wave case, it is possible to obtain an analytical expression of the boundary condition in Eq. (A10) for $T_B \ll 1$.

The relation $s_+ = s_- = 1$ for all the propagation channels leads to

$$\frac{R_B}{2R_N} \theta_0 = \cos \theta_0. \quad (\text{A19})$$

In the limit of $R_N \rightarrow \infty$, we find $\theta_0 \approx \pi/2$. In a d_{xy} -wave junction, the boundary condition in Eq. (A10) becomes

$$\frac{1}{G_Q R_N} \theta_0 = -2N_c \tan \theta_0 \quad (\text{A20})$$

at $\epsilon = 0$. The solution of this equation is $\theta_0 = 0$.

In a spin-singlet junction, the boundary condition in Eq. (A10) with Eq. (A18) always gives a real solution of θ_0 at $\epsilon = 0$. The proximity effect modifies the normal resistance as shown in the second term in Eq. (7) in the text. Since $\text{Im}(\theta(x)) = 0$, the second term in Eq. (7) is always equal to R_N at zero bias.

In a spin-triplet junction, we find

$$I_F = i \sum_{k_y} s_+ \frac{1 - s_+s_-}{2} = i \sum_{k_y} w_{1D}(k_y) \quad (\text{A21})$$

at $\epsilon = 0$. From the boundary condition in Eq. (A10), we obtain

$$\theta_0 = i\beta_0, \quad \beta_0 = 2G_Q N_{ZES} R_N. \quad (\text{A22})$$

APPENDIX B: RESISTANCE IN A SPIN-TRIPLET NS JUNCTION

The expression of the resistance in an NS junction in Eq. (7) requires a coefficient I_B which is defined by

$$I_B = \sum_{k_y} \frac{|t_b|^2 B}{|\Xi|^2}, \quad (\text{B1})$$

$$B = |t_b|^2(1 + |\cos \theta_0|^2 + |\sin \theta_0|^2) \times (|z_s|^2 + |g_s|^2 + |f_s|^2 + |\bar{f}_s|^2)/2 + 2(2 - |t_b|^2) \times [\text{Re}(g_s z_s^*) \text{Re}(\cos \theta_0) + \text{Re}(f_s z_s^*) \text{Re}(\sin \theta_0)] + 2|t_b|^2 \text{Im}(\cos \theta_0 \sin^* \theta_0) \text{Im}(f_s g_s^*). \quad (\text{B2})$$

In a spin-triplet junction, we find

$$I_B = 2 \sum_{k_y} \left[\frac{|t_b|^4 \cosh^2 \beta_0}{(2 - |t_b|^2)^2} \frac{1 + s_+s_-}{2} + (\cosh^2 \beta_0 - s_+ \cosh \beta_0 \sinh \beta_0) \frac{1 - s_+s_-}{2} \right], \quad (\text{B3})$$

$$= J_1 \cosh^2(\beta_0) - N_{ZES} \sinh(2\beta_0), \quad (\text{B4})$$

$$J_1 = 2 \sum_{k_y} \left[\frac{|t_b|^4}{(2 - |t_b|^2)^2} \frac{1 + s_+s_-}{2} + \frac{1 - s_+s_-}{2} \right] \quad (\text{B5})$$

at $\epsilon = 0$. By using Eq. (A22) and

$$\frac{1}{L} \int_{-L}^0 \frac{dx}{\cosh^2[\beta_0(1 + x/L)]} = \frac{\tanh \beta_0}{\beta_0}, \quad (\text{B6})$$

the resistance in Eq. (7) in the text results in

$$R_{NS} = \frac{1}{G_Q I_B} + \frac{1}{2G_Q N_{ZES}} \tanh \beta_0. \quad (\text{B7})$$

When we consider the limit of $R_N \rightarrow \infty$, the first term vanishes and $\tanh \beta_0 \rightarrow \text{sgn}(N_{\text{ZES}})$. Thus we conclude that

$$\lim_{R_N \rightarrow \infty} G_{\text{NS}} \rightarrow \frac{4e^2}{h} |N_{\text{ZES}}| \quad (\text{B8})$$

for a spin-triplet odd-parity superconductor.

In the absence of the pair potential (i.e., $\Delta = 0$), a superconductor becomes an ideal lead wire. The relations $g_{\pm} = 1$ and $f_{\pm} = 0$ results in $\theta_0 = 0$ from the boundary condition at any ϵ . As a result, we obtain $I_B = N_c T_B$ and $R_{\text{NS}} = R_B + R_N$. The last equation represents the usual Ohm's law in the normal state.

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