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Decoherence of high-energy electrons in weakly disordered quantum Hall edge states

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We investigate theoretically the phase coherence of electron transport in edge states of the integer quantum Hall effect at filling factor $\nu = 2$, in the presence of disorder and inter edge state Coulomb interaction. Within a Fokker-Planck approach, we calculate analytically the visibility of the Aharonov-Bohm oscillations of the current through an electronic Mach-Zehnder interferometer. In agreement with recent experiments, we find that the visibility is independent of the energy of the current-carrying electrons injected high above the Fermi sea. Instead, it is the amount of disorder at the edge that sets the phase space available for inter edge state energy exchange and thereby controls the visibility suppression.

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Phase coherent electron transport at the edge of a two-dimensional electron gas (2DEG) is a fascinating topic in condensed matter physics, both because of its fundamental role in unveiling new correlated states of matter [1,2], as well as for its practical implications for electronic quantum information processing [3–6], and the emerging field of quantum coherent thermoelectrics [7,8]. Although among the oldest quasi-one-dimensional systems to have been discovered [9–13], edge states (ESs) in the integer quantum Hall regime are still not fully understood theoretically. In particular, despite intense activity [14–35], our understanding of the dominant decoherence mechanism in transport through ESs is incomplete. This is illustrated by the recent experiment of Tewari *et al.* [33], in which it was observed that decoherence of high-energy electrons sent through a Mach-Zehnder interferometer (MZI), formed with two copropagating ESs at filling factor $\nu = 2$, does not depend on the energy of the injected electrons. This contradicts theoretical predictions based on the Luttinger-liquid model for one-dimensional, translationally invariant systems [25–27,33,34]. Disorder, however, is conspicuous for its absence in these approaches. While macroscopic phenomena, such as the quantization of the Hall resistance, are robust to disorder, more subtle quantum effects, such as energy exchange and phase coherence between copropagating ESs, can be expected to be sensitive to even weak disorder at the edge of a high mobility 2DEG [36].

In this work, we show that by taking into account disorder, which breaks translation invariance along the edge, a gapless continuum of low-energy quasiparticle excitations emerges. Their dynamics provides a simple physical picture of interaction-induced decoherence, which in turn provides a natural explanation for the experimental findings of [33]. Our theory has previously also been successfully applied to energy relaxation in out-of-equilibrium ESs [23,37]. In particular, in [37], we showed that energy relaxation of electrons injected high above the Fermi sea into the outermost of two copropagating, interacting and *weakly disordered* ESs, can be described in terms of a drift-diffusion process of their energy distribution function: As the injected electrons propagate along the outer ES, they lose energy and their energy

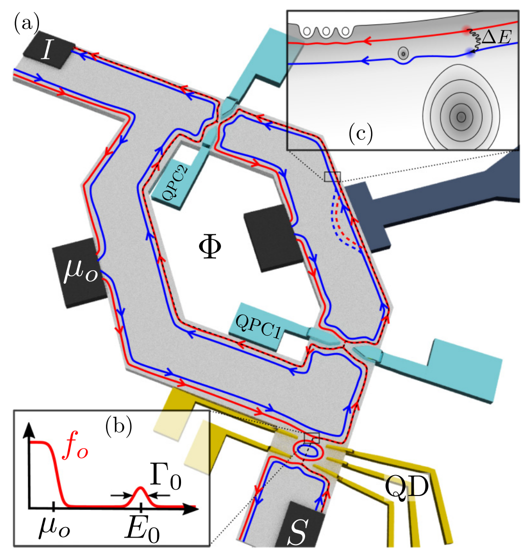

FIG. 1. Schematics of a Mach-Zehnder interferomter realized in [33] with ESs of the integer quantum Hall effect at filling factor $\nu = 2$. (a) The outer (inner) chiral ES is shown by a solid red (blue) line following the edge of the patterned 2DEG structure (light-gray area). The arrows indicate the propagation direction which is determined by the orientation of the magnetic field perpendicular to the 2DEG (not shown). A quantum dot (bottom right) is used for energy-resolved injection of high-energy electrons, with mean energy E_0, into the outer ES by filtering the electrons emitted at the source marked with S. This creates a nonequilibrium energy distribution in the outer ES composed of a Fermi sea part and a narrow bump around E_0 as shown in (b). An injected electron is scattered at two QPCs and can follow two possible paths, marked by dashed (black) lines, before exiting the interferometer at the top left corner where the current is measured. Interference between the current amplitudes corresponding to these two paths can be modulated either by threading a magnetic flux Φ through the loop created by the two paths or, as in the experiment [33], by applying a local gate voltage along one arm, in order to modify the path length difference ΔL (dark-blue side gate). The visibility of the current oscillations in either ΔL or Φ, is suppressed by inelastic scattering between electrons in the inner and outer ESs, which follow the equipotential lines of the disordered confinement potential [36] (c). In the case of unequal arm lengths, additional dephasing takes place due to the initial energy spread Γ_0 of the injected electrons (b).

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distribution moves towards the Fermi sea with a *constant* energy drift velocity and broadens at a position-dependent rate. The latter is determined by the induced heating of the inner ES, which absorbs the energy lost by the injected electrons and subsequently redistributes part of this energy to the Fermi sea of the outer ES. The central new idea of the present work is that, given a relation between the energy and the phase of a propagating electron, knowing the dynamics of the energy distribution function enables us to calculate the statistics of the interaction-induced phase fluctuations. In the absence of extrinsic dephasing mechanisms, the latter fully determines the coherence of electron transport.

Explicitly, we find that the interaction-induced suppression of the visibility of the current interference fringes through an electronic MZI [see Fig. 1(a)] is determined by the temperature of the electronic system, the drift velocity of the energy distribution of the electrons injected into the outer ES, and the heating of the Fermi sea of the inner ES. Importantly, none of these quantities depend on the injection energy of the electrons, resulting in dephasing that is independent of the injection energy, in line with the experiment of Tewari *et al.* [33]. Rather, the amount of dephasing is governed by the amount of disorder, which sets the available phase space for inelastic, non momentum conserving, electron-electron scattering. This result suggests that disorder along the edges of a patterned 2DEG plays a more important role, with regards to energy relaxation and decoherence of ESs, than hitherto assumed.

The system we consider is that of [33] and is depicted schematically in Fig. 1. It consists of two copropagating chiral ESs, one of which is split via two quantum point contacts (QPCs) such as to form a MZI. Furthermore, a quantum dot (QD) side-coupled to the sample edge at the input of the interferometer is used for energy-resolved injection of electrons into the outer ES with an average energy E_0 much larger than the Fermi energy μ_o of the outer ES as compared with the initial energy spread Γ_0 of the injected electrons, i.e., $E_0 - \mu_o \gg \Gamma_0$ [see Fig. 1(b)]. If all contacts, with the exception of the source, are kept at the same voltage, the dc current $I(\Phi,\Delta L)$ measured at the output port of the interferometer will stem exclusively from electrons injected into the outer ES via the QD energy filter. The flux dependence of this current is thus a sensitive probe of phase coherence along the outer ES. Instead of varying a magnetic flux Φ through the loop formed by the two arms of the interferometer, one may alternatively, as in the experiment [33], vary the path length difference $\Delta L = L_1 - L_2$, e.g., by applying a gate voltage to one of the arms. A similar setup, albeit with a simpler topology, has been used previously to investigate energy relaxation in out-of-equilibrium ESs [20,21].

We focus on Coulomb mediated energy exchange between the inner and outer copropagating ESs, without particle exchange. This is reasonable in the absence of magnetic impurities, since the two edge states have opposite spins and therefore particle exchange would require a spin flip. In analogy with the noninteracting scattering theory [38,39], the contribution from the outer ES to the current at the output port of the MZI can then be written as

$$I(\Phi,\Delta L) = \frac{e}{h}\int dE\, b_0(E)\, \langle |r_1 r_2 + t_1 t_2 e^{i\phi_E}|^2\rangle\,. \quad (1)$$

Here $b_0(E)$ denotes the energy distribution function of the electrons injected via the QD and centered at E_0. r_i (t_i) is the real reflection (transmission) probability amplitude at the ith QPC, and ϕ_E denotes the relative phase acquired by an electron injected with energy E but traversing different arms of the MZI. Crucially, the phase ϕ_E is a random variable that depends on the injection energy E, and on the random energy exchange events between injection and detection. The brackets $\langle\cdot\rangle$ denote averaging over all possible realizations of scattering events. Assuming that the energy dependence of the transmission and reflection amplitudes through the QPCs around the injection energy is negligible [18,33], it follows from Eq. (1) that the coherent part of the current is given by

$$I_\varphi = \frac{e}{h}(r_1 r_2 t_1 t_2)\int dE\, b_0(E)\, \langle e^{i\phi_E} + e^{-i\phi_E}\rangle\,. \quad (2)$$

Hence, assuming $b_0(E)$ is known, our task is reduced to computing the average of $\exp(i\phi_E)$ over scattering events.

For the case of a linear dispersion considered here, the phase acquired by an electron propagating in the outer ES along one of the arms of the interferometer (say $l = 1$ for the upper and $l = 2$ for the lower arm according to Fig. 1) is simply given by

$$\phi_E^{(l)}(x) = \frac{1}{\hbar v_o}\int_0^x dy\, E(y), \quad (3)$$

where v_o is the velocity of the electron in the outer ES and $E(y)$ denotes the energy of the electron at position y in arm l, given the initial energy $E(0) = E$. The relative phase at the detector is then simply given by

$$\phi_E = \phi_E^{(1)}(L_1) - \phi_E^{(2)}(L_2) + 2\pi\Phi/\Phi_0, \quad (4)$$

where L_l is the length of interferometer arm l, possibly including a gate-induced path length variation. Because electrons on different arms do not interact, owing to screening and a sufficiently large spatial separation, the average over scattering events factorizes

$$\langle e^{i\phi_E}\rangle = \big\langle \exp\big[i\phi_E^{(1)}(L_1)\big]\big\rangle\big\langle \exp\big[-i\phi_E^{(2)}(L_2)\big]\big\rangle e^{2\pi i\Phi/\Phi_0}, \quad (5)$$

and it is sufficient to evaluate the interaction-induced coherence suppression factor $\mathcal{F}_E^{(l)}(x) \equiv \langle \exp(i\phi_E^{(l)}(x))\rangle$ for one arm. From now on, we thus suppress the arm label l.

Our starting point for evaluating $\mathcal{F}_E(x)$ is the kinetic Boltzmann equation for the energy distribution functions f_α of the inner ($\alpha = i$) and outer ($\alpha = o$) ESs

$$v_\alpha \partial_x f_\alpha(E,x) = \mathcal{I}_{Ex\alpha}[f_\alpha, f_{\bar\alpha}]. \quad (6)$$

The term on the right-hand side is the difference of in-scattering ($\{E'\} \to E$) and out-scattering ($E \to \{E'\}$) energy exchange processes between the inner and outer ESs [40]. Here and below, we use the shorthand notation $\overline{\alpha} = \delta_{\alpha i} o + \delta_{\alpha o} i$. If both energy and momentum are conserved, then two-body collisions cannot change the distribution function in one dimension, as long as $v_i \neq v_o$ [41]. However, disorder along the edge breaks translation invariance such that inelastic electron-electron scattering without momentum conservation becomes possible. Thereby, an effective interaction is induced and the phase space for energy exchange between electrons in the inner and outer ESs opens up. In contrast to collective excitations in a finite length system [42], these excitations are gapless. As shown in [23,37,43], this situation is described by Eq. (6) with the collision integral

$$\mathcal{I}_{Ex\alpha}[f_\alpha, f_{\bar\alpha}] = v_\alpha \gamma \int d\omega\, e^{-(\omega/\Delta E)^2} \times \{f_\alpha(E+\omega,x)[1-f_\alpha(E,x)]D_{\bar\alpha}(\omega,x) - f_\alpha(E,x)[1-f_\alpha(E+\omega,x)]D_{\bar\alpha}(-\omega,x)\}, \quad (7)$$

where γ is the effective inter-ES interaction strength, ΔE is the energy scale for the amount of energy exchanged per non momentum conserving collision [43], and

$$D_\alpha(\omega,x) = \int dE\, f_\alpha(E-\omega,x)[1-f_\alpha(E,x)]. \quad (8)$$

The inner ES is initially in thermal equilibrium so that $f_i(E,0) = 1/\{1+\exp[(E-\mu_i)/k_B T]\}$. Furthermore, because we consider electrons injected high above the Fermi sea in the outer ES ($E_0 - \mu_o \gg \Gamma_0$), we can split the distribution function in the outer ES into two essentially nonoverlapping contributions

$$f_o(E,x) = \mathfrak{f}_o(E,x) + b(E,x), \quad (9)$$

where $\mathfrak{f}_o(E,x=0) = 1/\{1+\exp[(E-\mu_o)/k_B T]\}$ and $b(E,x)$ is the energy distribution of the injected electrons at position x with boundary condition $b(E,0) = b_0(E)$. If, as in the experiment [33], the transmission probability through the QD is small, then $b(E,x) \ll 1$. Consequently, we can neglect, in the collision integral, all terms of order $\mathcal{O}(b^2, b\mathfrak{f}_o)$. Finally, since we are interested in the limit of weak disorder, ΔE is taken to be the smallest energy scale, e.g., $\Delta E \ll k_B T, \Gamma_0$. These steps allow us to derive, from the kinetic equation, the following set of coupled Fokker-Planck equations [37]:

$$\partial_x b(E,x) = \eta\{\partial_E b(E,x) + D_i(0,x)\partial_E^2 b(E,x)\}, \quad (10a)$$

$$\partial_x f_i(E,x) = \eta \frac{N_b}{\rho_o}\partial_E^2 f_i(E,x) + \eta\{[1-2f_i(E,x)]\partial_E f_i(E,x) + \mathfrak{D}_o(0,x)\partial_E^2 f_i(E,x)\}, \quad (10b)$$

$$\partial_x \mathfrak{f}_o(E,x) = \eta\{[1-2\mathfrak{f}_o(E,x)]\partial_E \mathfrak{f}_o(E,x) + D_i(0,x)\partial_E^2 \mathfrak{f}_o(E,x)\}. \quad (10c)$$

Here $\eta = (\sqrt{\pi}/4)\gamma(\Delta E)^3$ is the energy drift velocity, ρ_o is the density of states in the outer ES, $N_b = \rho_o \int dE\, b_0(E)$ is the mean number of injected electrons, and $\mathfrak{D}_o(\omega,x) = \int dE\, \mathfrak{f}_o(E-\omega,x)[1-\mathfrak{f}_o(E,x)]$.

The Fokker-Planck equation (10a) is equivalent [44] to the Itô stochastic differential equation

$$dE = -\eta dx + g(x)\mathrm{d}W_x, \quad (11)$$

where $g(x) = \sqrt{2\eta D_i(0,x)}$ and $\mathrm{d}W_x$ is a Wiener process. The random energy of an electron injected at $x=0$ with energy E is obtained by integrating Eq. (11) and using the initial condition $E(x=0) = E$:

$$E(x) = E - \eta x + \int_0^x g(y)\mathrm{d}W_y. \quad (12)$$

The last term in Eq. (12) is a stochastic Itô integral. By applying the Itô calculus [$\langle \mathrm{d}W_x \mathrm{d}W_{x'}\rangle = \delta(x-x')dx$, $\langle \mathrm{d}W_x\rangle = 0$], we find the mean and variance ($\mathrm{Var}[\cdot] = \langle(\cdot)^2\rangle - \langle\cdot\rangle^2$) of the energy at position x as

$$\langle E(x)\rangle = E - \eta x, \quad (13a)$$

$$\mathrm{Var}[E(x)] = 2\eta \int_0^x D_i(0,y)dy. \quad (13b)$$

Note that averaging Eq. (13a) over the injection energy using the probability density $(\rho_o/N_b)b_0(E)$ yields $\langle\langle E(x)\rangle\rangle_0 = E_0 - \eta x$, which explains why η is called the energy drift velocity of the energy distribution of the injected electrons. Because of Eq. (13b), we further call $2\eta D_i(0,x)$ the dynamic diffusion coefficient [37]. According to Eq. (3), the phase of the electron at position x is now given by

$$\phi_E(x) = \frac{1}{\hbar v_o}\left(Ex - \frac{1}{2}\eta x^2\right) + \frac{1}{\hbar v_o}\int_0^x \int_0^y g(z)\mathrm{d}W_z\, dy. \quad (14)$$

Using again the Itô calculus [43], the last integral can be rewritten as

$$\int_0^x \int_0^y g(z)\mathrm{d}W_z\, dy = \int_0^x (x-y)g(y)\mathrm{d}W_y, \quad (15)$$

from which it follows that the variance of the phase is

$$\delta\phi^2(x) \equiv \mathrm{Var}[\phi_E(x)] = \frac{2\eta}{(\hbar v_o)^2}\int_0^x (x-y)^2 D_i(0,y)dy. \quad (16)$$

Because the fluctuating part of the phase is itself a Gaussian random variable with zero mean, we can use the identity $\langle\exp(i\phi)\rangle = \exp(i\langle\phi\rangle)\exp(-\delta\phi^2/2)$, and the interaction-induced dephasing factor is therefore given by

$$\mathcal{F}_E(x) = \exp\left(\frac{i}{\hbar v_o}\left[Ex - \frac{1}{2}\eta x^2\right]\right)\exp\left(-\frac{\delta\phi^2(x)}{2}\right). \quad (17)$$

Equation (17) together with Eq. (16) are the main analytic results of this work. They link the interaction-induced phase coherence suppression factor of the outer ES to the relaxation-induced smearing of the energy distribution of the inner ES, quantified by $D_i(0,x)$ [see Eq. (8)]. Importantly, the latter is independent of the injection energy as shown below. Combining Eqs. (2), (4), (5), and (17), we obtain an explicit expression for the coherent current through the interferometer

$$I_\varphi(\Phi,\Delta L) = \frac{2e}{h}\frac{N_b}{\rho_o}(r_1 r_2 t_1 t_2)B_0(\Delta L)e^{-(1/2)[\delta\phi^2(L_1)+\delta\phi^2(L_2)]} \times \cos\left(\frac{E_0\Delta L}{\hbar v_o} - \frac{\eta(L_1^2 - L_2^2)}{2\hbar v_o} + \frac{2\pi\Phi}{\Phi_0}\right). \quad (18)$$

Here the factor $B_0(\Delta L) = \frac{\rho_o}{N_b}\int dE\, b_0(E+E_0)e^{i\frac{E\Delta L}{\hbar v_o}}$ characterizes the dephasing due to the initial energy spread of the injected electrons for finite path length difference, and the exponential factor quantifies the interaction-induced dephasing. For an initial Gaussian energy distribution of the form $b_0(E) = \frac{N_b}{\rho_o\sqrt{\pi}\Gamma_0}\exp[(E-E_0)^2/\Gamma_0^2]$, we have $B_0(\Delta L) = \exp[-(\frac{\Gamma_0\Delta L}{2\hbar v_o})^2]$. For an initial distribution of the form $b_0(E) = \frac{N_b}{4\Gamma_0\rho_o}\cosh^{-2}(\frac{E-E_0}{2\Gamma_0})$, which with $\Gamma_0 = k_B T$ is appropriate for injection through a thermally broadened QD level [33], we have $B_0(\Delta L) = \frac{\pi\Gamma_0\Delta L}{2\hbar v_o}\mathrm{csch}(\frac{\pi\Gamma_0\Delta L}{\hbar v_o})$. In both cases $B_0(\Delta L\to 0)\to 1$ as expected. The experimentally relevant visibility of the current interference $\mathcal{V}\equiv(I_{\max}-I_{\min})/(I_{\max}+I_{\min})$ is found by extremizing the cosine in Eq. (18) over either Φ or variations of the path length difference ΔL [45] and reads

$$\mathcal{V} = \frac{2r_1r_2t_1t_2}{(r_1r_2)^2+(t_1t_2)^2}B_0(\Delta L)e^{-(1/2)[\delta\phi^2(L_1)+\delta\phi^2(L_2)]}. \quad (19)$$

To obtain the variance of the phase fluctuations, we need to evaluate the function $D_i(0,x)$, i.e., solve Eqs. (10b) and (10c). A thorough discussion of these equations can be found in [37], where it was shown that an approximate solution takes the form of an effective temperature ansatz for $f_i(E,x)$ and $\mathfrak{f}_o(E,x)$:

$$f_i^F(E,x) = \frac{1}{1+\exp\left[\frac{E-\mu_i}{k_B T_i(x)}\right]}, \quad (20a)$$

$$\mathfrak{f}_o^F(E,x) = \frac{1}{1+\exp\left[\frac{E-\mu_o}{k_B T_o(x)}\right]}. \quad (20b)$$

From Eq. (8) it immediately follows that, within the effective temperature approximation, $D_i(0,x) = k_B T_i(x)$. The coupled Fokker-Planck equations (10b) and (10c) now reduce to coupled ordinary differential equations for $T_i(x)$ and $T_o(x)$:

$$k_B\partial_x T_i(x) = \eta\frac{3}{\pi^2}\left(\frac{N_b/\rho_o}{k_B T_i(x)}+\frac{T_o(x)}{T_i(x)}-1\right), \quad (21a)$$

$$k_B\partial_x T_o(x) = \eta\frac{3}{\pi^2}\left(\frac{T_i(x)}{T_o(x)}-1\right). \quad (21b)$$

In the case of interest here, the Fermi seas of the inner and outer ESs initially have the same temperature $T_i(0)=T_o(0)=T$. Moreover, since we are working in the limit of weak disorder where $\Delta E\ll k_B T$, it is reasonable to expect that the difference between the two effective temperatures $T_d(x)=[T_i(x)-T_o(x)]/2$ remains small compared with the sum of the temperatures $T_s(x)=[T_i(x)+T_o(x)]/2$ at all positions. From this assumption, one can then derive an approximate solution of (21) which yields [37]

$$k_B T_i(x)\simeq k_B T\sqrt{1+\frac{x}{x_s}}+\frac{N_b}{4\rho_o}\left(1-e^{\frac{4k_BT}{N_b/\rho_o}\left[1-\sqrt{1+\frac{x}{x_s}}\right]}\right), \quad (22)$$

with $x_s=(\pi k_B T)^2\rho_o/(3\eta N_b)$. At short distances $x\ll x_s$, we have $T_i(x)\simeq T(1+x/x_s)$, in which case the integral in (16) can be evaluated analytically, yielding

$$|\mathcal{F}_E(x)|^2\simeq\exp\left[-\frac{2}{3}\frac{\eta k_B T}{(\hbar v_o)^2}\left(x^3+\frac{x^4}{12x_s}\right)\right], \quad \text{for } x\ll x_s. \quad (23)$$

Hence, the smaller the propagation velocity v_o, the stronger the dephasing, a trend which was recently observed by Gurman *et al.* [35]. At large distances $x\gg x_s$, the exponential term in Eq. (22) vanishes and $T_i(x)\simeq k_B T\sqrt{1+x/x_s}+N_b/(4\rho_o)$.

From the data in [33], we can estimate that in the experiment, $N_b/\rho_o\approx 1.6\ \mu$eV [43]. The only remaining free parameter η can then be determined by fitting Eq. (19) to the measured visibility, using the experimentally determined values for the other parameters [33]: $k_B T\approx 31$ mK, $v_o\approx 5\times10^4$ ms^{-1} and $r_1=r_2=t_1=t_2\approx 1/\sqrt{2}$, as well as $L_1=L_2=L\approx 7.2\ \mu$m. This yields [43] an energy drift velocity of $\eta\approx 2.8\ \mu$eV/μm. This value further justifies our perturbative analysis for weak momentum conservation breaking of the experiment [33], where a visibility independent of the injection energy is observed in the range $(E_0-\mu_o)\in[30,130]\ \mu\text{eV}>\eta L\approx 28\ \mu$eV. The regime where $\eta L>E_0-\mu_o$ is outside of the Fokker-Planck regime, since the distribution in the outer ES can no longer be separated into two nonoverlapping contributions. Experimentally, dephasing in this regime is observed to depend on the injection energy [33]. Using the above estimates, we plot the visibility according to Eq. (19), with $\Delta L=0$, as a function of the interferometer length in Fig. 2.

To further validate our analytic results, we compare them with the results from a Monte Carlo simulation of the Fokker-Planck dynamics of the kinetic equation, for different values of the injection energy. In this simulation, we discretize the stochastic energy exchange process for a given injection energy. At each step, we determine the scattering rate and the distribution of scattering energies from Eq. (7). We then use these to update the energy and accumulated phase of an

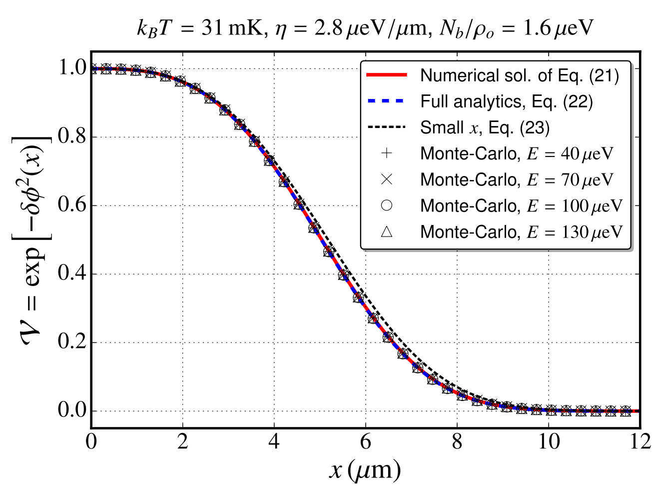

FIG. 2. Visibility of the current interference fringes (19) as a function of the interferometer arm length for $\Delta L=0$, $r_1=r_2=t_1=t_2=1/\sqrt{2}$, and $v_o=5\times10^4$ m/s. The solid (red) curve shows the result obtained by numerically integrating the differential equations (21). The dashed (blue) curve shows the analytic result obtained using Eq. (22). The thin dashed (black) curve shows Eq. (23), obtained from the short distance limit of (22), when $x\ll x_s\approx 5.2\ \mu$m. The symbols show results from Monte Carlo simulations of the kinetic equation for different injection energies $E\in\{40,70,100,130\}\ \mu$eV [43].

electron as it propagates along the edge. The phase suppression factor is estimated by averaging over many such "trajectories": $\mathcal{F}_E(x) = (1/M)\sum_{m=1}^{M} \exp[i\phi_{E,m}(x)]$. Further details on our implementation are given in [43]. The results confirm our analytic predictions (see Fig. 2).

In conclusion, we have shown how the interplay of disorder and Coulomb interaction leads to the loss of phase coherence of the current through a MZI formed with two copropagating ESs of the integer quantum Hall effect. Crucially we find that dephasing does not depend on the injection energy, in agreement with recent experiments [33]. Furthermore, our theory makes quantitative predictions for the length dependence of the dephasing [see Fig. 2 and Eqs. (16), (17), and (22)], which could easily be tested by adapting existing experimental systems.

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[46] See http://scicore.unibas.ch/.