Transmission of terahertz waves through layered superconductors controlled by a dc magnetic field

S. S. Apostolov,[1,2] Z. A. Maizelis,[1,2] N. M. Makarov,[3] F. Pérez-Rodríguez,[3]
T. N. Rokhmanova,[1] and V. A. Yampol'skii[1,2,3]
[1]*A.Ya. Usikov Institute for Radiophysics and Electronics NASU, 61085 Kharkov, Ukraine*
[2]*V.N. Karazin Kharkov National University, 61077 Kharkov, Ukraine*
[3]*Benemérita Universidad Autónoma de Puebla, Puebla, Pue. 72000, Mexico*
(Received 13 December 2015; revised manuscript received 30 June 2016; published 15 July 2016)

The transmission of THz electromagnetic waves via a slab of layered superconductor in the presence of dc magnetic field H_0 is theoretically studied. We demonstrate that the external dc field turns the layered superconductor into nonuniform medium with spatially and frequency-dependent dielectric permittivity. Even a relatively weak dc magnetic field, when the superconductor is in the Meissner state, significantly affects the transmittance of the layered superconductor. Moreover, the proper choice of H_0 can provide the perfect transparency of the slab. In addition, the dc magnetic field changes the dependence of the transmittance on the slab thickness, the frequency, and the incident angle of the wave. Thus, it can serve as an effective tool to control the transmissivity of layered superconductors.

DOI: 10.1103/PhysRevB.94.024513

I. INTRODUCTION

Layered superconductors are either natural high-T_c superconductors, such as $Bi_2Sr_2CaCu_2O_{8+\delta}$, or artificially grown stacks of Josephson junctions, e.g., Nb/Al-AlO$_x$/Nb. These materials consist of thin superconducting layers separated by thicker dielectric layers. The experimental studies for the **c**-axis conductivity (see, e.g., Refs. [1,2]) proved that the superconducting layers are electrodynamically coupled due to the intrinsic Josephson effect. Owing to the layered structure, these materials are strongly anisotropic media. Indeed, the strong electric currents flowing along the layers are of the same nature as in bulk superconductors, while the weak current across the layers emerges due to the Josephson effect. This anisotropy gives rise to the existence of the specific electromagnetic excitations in layered superconductors, the Josephson plasma waves (JPWs) (see, e.g., Refs. [3,4] and references therein). The frequencies of JPWs belong to the terahertz (THz) range that makes layered superconductors to be promising materials for different applications (see, e.g., Ref. [5]).

Being of the similar origin as common plasma waves, the bulk JPWs can propagate in layered superconductors with frequencies above some threshold value ω_J. In addition, as was theoretically demonstrated in Refs. [6,7], the surface Josephson plasma waves (SJPWs) can propagate along interfaces between layered superconductors and vacuum, similarly to the surface plasmon polaritons in usual plasmas. The excitation of SJPWs leads to various resonant phenomena [7–9] similar to the Wood anomalies well known in optics (see, e.g., Refs. [10–12]). However, in contrast to usual plasmas, SJPWs can propagate with frequencies not only below the Josephson plasma frequency ω_J but also above it [7]. The strongly anisotropic plasma in layered superconductors can also exhibit properties inherent to the left-handed media: a negative refractive index for THz waves can be observed at the boundaries of layered superconductors [7,13]. As was shown in Ref. [14], the phenomena similar to the Anderson localization and the formation of a transparency window for THz waves can be observed in layered superconductors with randomly fluctuating value of the maximum Josephson current.

The electrodynamic equations for the layered superconductors are nonlinear. The nonlinearity originates from the nonlinear relation $J \propto \sin\varphi$ between the Josephson interlayer current J and the gauge-invariant interlayer phase difference φ of the order parameter. This can result in a number of nontrivial nonlinear effects accompanying the propagation of JPWs, e.g., slowing down of light [15], self-focusing of terahertz pulses [15,16], excitation of nonlinear waveguide modes [17], as well as self-induced transparency of the slabs of layered superconductors and hysteretic jumps in the dependence of the slab transmissivity on the wave amplitude [18]. The noticeable change in the transmissivity of the cuprate superconductors while increasing the wave amplitude was recently observed in Ref. [19], where the excitation of Josephson plasma solitons led to effective decrease of the Josephson resonance frequency.

It is important to underline the principal difference between the behavior of the usual plasmas and plasma in layered superconductors subjected to the external dc magnetic field. The dc magnetic field penetrates uniformly into nonmagnetic normal metals whereas, in layered superconductors, the excited Meissner currents result in appreciably nonuniform dc field distribution. Therefore, the external dc magnetic field creates a nonuniform background for the JPWs propagation, inherent to the plasma in layered superconductors. Thus, the dc magnetic field can be used to control the properties of the electromagnetic waves in layered superconductors. For example, a strong effect of the dc magnetic field on the dispersion characteristics of the surface Josephson plasma waves was studied in Refs. [20,21].

In the present paper, we focus our attention on the transmission of the transverse magnetic (TM) JPWs through a slab of layered superconductor in the presence of the external dc magnetic field. We show that the dc magnetic field can be used as a novel tool to control the transmission of the THz waves through the layered superconductors. We consider the case of sufficiently weak magnetic fields when the Josephson vortices do not penetrate into the superconductor. We obtain the explicit analytical expression for the transmission coefficient and study its dependence on the magnitude of the dc field and other problem parameters, such as the sample thickness, the angle of the wave incidence, and the wave frequency. We show that the slab can become completely transparent for definite magnitudes of the dc field. The dependence of the transmittance on the incident angle can be increasing or decreasing for different magnitudes of the dc field.

The paper is organized as follows. In Sec. II, we describe the geometry of the problem, discussing the theoretical model applied for the description of the dc and ac fields distribution in the layered superconductor. Section III contains the analytic expression for the transmission coefficient and its analysis as a function of the slab thickness, the incident angle, frequency, and the magnitude of the dc magnetic field. The last section summarizes the obtained results.

II. STATEMENT OF THE PROBLEM: ELECTROMAGNETIC FIELDS

In this section we describe the geometry of the problem, present basic equations, and calculate analytically the distribution of electromagnetic fields in the vacuum regions and in the slab of layered superconductor.

A. Geometry of the problem

We study the transmission of the electromagnetic wave through a slab of layered superconductor with thickness D. The coordinate system is chosen in such a way that the crystallographic **ab** plane of the layered superconductor coincides with the xy plane, and the **c** axis is directed along the z axis. The slab occupies the spatial region $0 < x < D$ (see Fig. 1). We assume the sample to be infinite along the y and z axes in order to neglect the corresponding boundary effects.

The TM-polarized wave of the unit amplitude, THz frequency ω, and wave-vector components $k_x = k\cos\theta$, $k_z = k\sin\theta$ ($k = \omega/c$) irradiates surface $x = 0$ of the superconductor with the incident angle θ. Such a geometry allows us to present the electromagnetic field as

$$\vec{E}(x,z,t) = \{E_x(x),0,E_z(x)\}\exp(ik_z z - i\omega t),$$
$$\vec{H}(x,z,t) = \{0,H_y(x),0\}\exp(ik_z z - i\omega t). \tag{1}$$

The incident wave partly reflects from the layered superconductor and partly transmits through it, as shown in Fig. 1. The external dc magnetic field $\vec{H}_0$ is applied along the y axis. We study the case of relatively weak dc magnetic fields when the Josephson vortices do not penetrate into the slab.

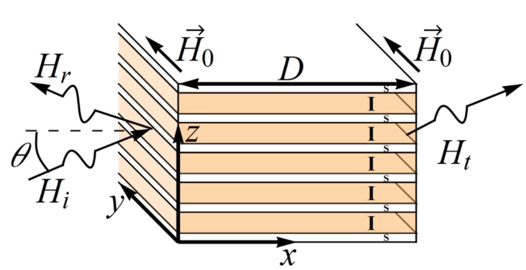

FIG. 1. Schematic geometry for the reflection and transmission of waves through a slab of layered superconductor. Here H_i, H_r, and H_t are the amplitudes of incident, reflected, and transmitted waves, respectively, θ is the incident angle, $\vec{H}_0$ is the external dc magnetic field, indices S and I stand for superconducting and insulator layers, respectively, D is the thickness of the slab. The slab is infinite along the y and z directions.

B. Electromagnetic field in the vacuum regions

The electromagnetic field in the vacuum regions to the right and to the left from the sample (see Fig. 1) is a superposition of the dc magnetic field and the fields of the incident, reflected, and transmitted waves. Using the Maxwell equations, one can readily derive the following expressions for the tangential components of the electric and magnetic fields in the left vacuum region:

$$H_y^{\text{left}}(x) = \exp(ik_x x) + H_r\exp(-ik_x x),$$
$$E_z^{\text{left}}(x) = -\frac{k_x}{k}[\exp(ik_x x) - H_r\exp(-ik_x x)], \tag{2}$$

where H_r is the amplitude of the reflected wave.

The tangential components of the magnetic and electric fields of transmitted wave in the right vacuum region are

$$H_y^{\text{right}}(x) = H_t\exp[ik_x(x-D)],$$
$$E_z^{\text{right}}(x) = -\frac{k_x}{k}H_t\exp[ik_x(x-D)], \tag{3}$$

where H_t is the amplitude of transmitted wave.

C. Electromagnetic field in the slab of layered superconductor

The electromagnetic field in the layered superconductor is defined by the distribution $\varphi(\vec{r},t)$ of interlayer gauge-invariant phase difference of the order parameter. This phase difference is governed by a set of coupled sine-Gordon equations [3,22–28]. This set describes properly the propagation of the electromagnetic waves in layered superconductors and allows important predictions. For instance, a way to produce the coherent terahertz radiation was proposed in Ref. [29] on the basis of the coupled sine-Gordon equations and then realized in the experiment [5].

Here we study the electromagnetic field in the continual limit where the spatial period d of the layered structure is much smaller than the wavelength k_z^{-1} in the z direction,

$$k_z d \ll 1. \tag{4}$$

In this case, the layered superconductor can be considered as uniform, however strongly anisotropic, medium. In the continual limit, the coupled sine-Gordon equations reduce to the following form:

$$\left(1 - \lambda_{ab}^2\frac{\partial^2}{\partial z^2}\right)\left(\sin\varphi + \frac{1}{\omega_J^2}\frac{\partial^2\varphi}{\partial t^2}\right) - \lambda_c^2\frac{\partial^2\varphi}{\partial x^2} = 0. \tag{5}$$

Here λ_{ab} and $\lambda_c = c/\omega_J\varepsilon^{1/2} \gg \lambda_{ab}$ are the London penetration depths across and along the layers, respectively, $\omega_J = (8\pi e J_c d/\hbar\varepsilon)^{1/2}$ is the Josephson plasma frequency,

J_c is the maximal Josephson current density, ε is the dielectric constant of insulator layers, and e is the elementary charge. Equation (5) does not take into account the relaxation terms. This assumption is correct for the case when the slab thickness D is much smaller than the decay length $l_{\rm dec}$ of JPWs. The condition $D \ll l_{\rm dec}$ is difficult for realization in high-T_c superconductors. Indeed, because of the d-wave pairing, the decay length $l_{\rm dec}$ in such materials is comparable with $20\lambda_c$ and can reduce the transparency of the slab even at low temperatures. Therefore, in this paper, we calculate the transmission coefficient with account for dissipation and show that it does not impact on our main result. Namely, we demonstrate that the small dc magnetic field can substantially affect the transmissivity of the superconducting slabs even in the case of account for dissipation.

The gauge-invariant phase difference $\varphi(\vec{r},t)$ of the order parameter defines components E_z^s and H_y^s of the electric and magnetic fields in the layered superconductor (see, e.g., Ref. [3] and references therein),

$$E_z^s = \frac{\mathcal{H}_0}{2\omega_J\sqrt{\varepsilon}}\frac{\partial\varphi}{\partial t}, \quad \frac{\partial H_y^s}{\partial x} = \frac{\mathcal{H}_0}{2\lambda_c}\left[\sin\varphi + \frac{1}{\omega_J^2}\frac{\partial^2\varphi}{\partial t^2}\right], \tag{6}$$

where $\mathcal{H}_0$ is the characteristic magnetic field within the Josephson vortices, which turns out to be associated with the flux quantum Φ_0,

$$\mathcal{H}_0 = \frac{\Phi_0}{\pi d\lambda_c}, \quad \Phi_0 = \frac{\pi\hbar c}{e}. \tag{7}$$

Note that the component E_z^s of the electric field causes the breakdown of electroneutrality of the superconducting layers and results in an additional, so-called capacitive, interlayer coupling. This coupling can play an important role in the properties of the longitudinal JPWs with wave vectors oriented across the layers. The dispersion equation for linear plane JPWs with account of the capacitive coupling was obtained in Ref. [30]. According to this dispersion equation, the capacitive coupling is important if the wave vector is oriented mainly along the z axis (across the superconducting layers). In our case, when the incident angle is not close to $\pi/2$, i.e., $k_z \sim k_x \sim k$, capacitive coupling can be safely neglected because of the smallness of the parameter $\alpha = R_D^2\varepsilon/sd$, where R_D is the Debye length for a charge in a superconductor.

1. *Distribution of the dc magnetic field in the slab of layered superconductor*

At first, we describe how the dc magnetic field penetrates into the slab of layered superconductor. Here we assume that the slab is sufficiently thick,

$$D \gg \lambda_c \gg \lambda_{ab}. \tag{8}$$

In this case, we can neglect the interaction between two magnetic fluxes penetrated into the slab from its opposite sides $x = 0$ and $x = D$, since the dc field decays exponentially inside the slab over the distance of about λ_c from the slab boundaries. Using Eq. (5), we obtain the expressions for the static phase difference φ_0 in the vicinity of the boundaries $x = 0$ and $x = D$,

$$\begin{aligned}\varphi_0^{\rm left}(\xi) &= -4\arctan[\exp(-\xi - \xi_0)],\\ \varphi_0^{\rm right}(\xi) &= 4\arctan\{\exp[\xi - (\delta - \xi_0)]\}.\end{aligned} \tag{9}$$

Here we introduced the dimensionless coordinate ξ and the normalized thickness δ of the slab,

$$\xi = \frac{x}{\lambda_c}, \quad \delta = \frac{D}{\lambda_c} \gg 1. \tag{10}$$

The constant ξ_0 is defined by the magnitude H_0 of the external dc magnetic field,

$$\xi_0 = \mathrm{arccosh}\,\frac{1}{h_0}, \qquad h_0 = \frac{H_0}{\mathcal{H}_0}. \tag{11}$$

The value h_0 represents the external dc magnetic field normalized to the critical field $\mathcal{H}_0 = \Phi_0/\pi d\lambda_c$. The typical value of $\mathcal{H}_0$ for $Bi_2Sr_2CaCu_2O_{8+\delta}$ is about 100 Oe. If $H_0 > \mathcal{H}_0$, the Meissner state becomes unstable. Here we consider an opposite case when the dc magnetic field is weak,

$$h_0 \leqslant 1. \tag{12}$$

Under this condition, the magnetic flux inside the layered superconductor exists only in the form of the tails of two fictitious Josephson vortices whose centers are situated outside the slab at the points $\xi = -\xi_0$ and $\xi = \delta + \xi_0$. Equation (9) represents the well-known solutions to the sine-Gordon equation for the phase φ_0 of these fictitious vortices. As we show below, even the weak dc magnetic field has an essential effect upon the high-frequency properties of the layered superconductors and, in particular, upon the transmissivity of the superconducting slab.

2. *Electromagnetic field of the JPW in the slab of layered superconductor*

Here we discuss the ac fields associated with the Josephson plasma wave propagating inside the layered superconductor in the presence of the nonuniformly distributed dc magnetic field. This nonuniform dc field is defined by the static phase difference (9). We consider the case when the amplitude of the incident wave is much smaller than the magnitude of the dc magnetic field, and the gauge-invariant phase difference can be presented as a sum of three terms,

$$\varphi(\xi,z,t) = \varphi_0^{\rm left}(\xi) + \varphi_0^{\rm right}(\xi) + \varphi_w(\xi,z,t), \tag{13}$$

where first two terms given by Eqs. (9) describe the static part of φ. The last term is the small ac addition emerging due to JPW.

We linearize Eq. (5) in the small ac phase difference φ_w and, in accordance with Eq. (1), seek φ_w in the form

$$\varphi_w(\xi,z,t) = a(\xi)\exp[i(k_z z - \omega t)]. \tag{14}$$

Then, within the linear approximation, the equation for the factor $a(\xi)$ reads

$$\begin{aligned}\frac{d^2 a(\xi)}{d\xi^2} + \left(1 + k_z^2\lambda_{ab}^2\right)\bigg[&\frac{2}{\cosh^2(\xi + \xi_0)}\\ &+ \frac{2}{\cosh^2(\delta + \xi_0 - \xi)} + \Omega^2 - 1\bigg]a(\xi) = 0,\end{aligned} \tag{15}$$

where Ω is the normalized frequency,

$$\Omega = \frac{\omega}{\omega_J}. \tag{16}$$

It is remarkable that, due to relation (6), Eq. (15) gives rise to the well-known equation for the z component of the electric field in the wave with TM polarization (1),

$$\frac{d^2 E_z}{dx^2} + \left(k^2\varepsilon_{zz} - k_z^2\frac{\varepsilon_{zz}}{\varepsilon_{xx}}\right)E_z = 0, \tag{17}$$

however, with the certain components of permittivity tensor. With account for dissipation terms, these components can be written as,

$$\varepsilon_{xx}(\Omega) = \varepsilon_{yy}(\Omega) = \varepsilon\left[1 + \frac{i\nu_{ab}}{\Omega} - \frac{\lambda_c^2}{\lambda_{ab}^2}\frac{1}{\Omega^2}\right],$$
$$\varepsilon_{zz}(\xi,\Omega) = \varepsilon\left\{1 + \frac{i\nu_c}{\Omega} - \frac{1}{\Omega^2}\left[1 - \frac{2}{\cosh^2(\xi+\xi_0)} - \frac{2}{\cosh^2(\delta+\xi_0-\xi)}\right]\right\}. \tag{18}$$

Here the dimensionless relaxation frequencies $\nu_{ab} = 4\pi\sigma_{ab}/\varepsilon\omega_J$ and $\nu_c = 4\pi\sigma_c/\varepsilon\omega_J$ are related to the averaged quasiparticle conductivities along the layers, σ_{ab}, and across the layers, σ_c. In the following calculations we assume $\nu_{ab} \ll (\lambda_c/\lambda_{ab})^2$ and $\nu_c \ll 1$.

Thus, one can conclude that the plasma in layered superconductors in the external dc magnetic field represents frequency dispersive nonuniform medium specified by the uniaxial diagonal permittivity tensor (18). It is noteworthy that the nonuniformity arising in ε_{zz}, is originated from and controlled by the external magnetic field. With no dc magnetic field, the dielectric functions (18) are reduced to those written down in, e.g., Ref. [13].

Note that due to the strong anisotropy, $\lambda_{ab} \ll \lambda_c$, the parameter $k_z\lambda_{ab}$ is always negligibly small,

$$k_z\lambda_{ab} = \frac{\lambda_{ab}}{\lambda_c}\frac{\Omega}{\sqrt{\varepsilon}}\cos\theta \ll 1. \tag{19}$$

Therefore, in Eq. (15), the parentheses $(1 + k_z^2\lambda_{ab}^2)$ can be undoubtedly omitted. Then, within the accuracy in the exponentially small parameter $\exp(-\delta) = \exp(-D/\lambda_c) \ll 1$, the asymptotically exact general solution to Eq. (15) turns out to be found analytically,

$$a(\xi) = C_1 e^{i\tilde{\Omega}\xi}\left[p a_0(\xi) + p^{-1}a_0(\delta-\xi) + \sqrt{\tilde{\Omega}^2+1}\right] + C_2 e^{-i\tilde{\Omega}\xi}\left[p^{-1}a_0(\xi) + p a_0(\delta-\xi) + \sqrt{\tilde{\Omega}^2+1}\right], \tag{20}$$

with C_1, C_2 being integration constants, and

$$a_0(\xi) = \tanh(\xi_0+\xi) - 1, \qquad p = \frac{1+i\tilde{\Omega}}{\sqrt{\tilde{\Omega}^2+1}}. \tag{21}$$

Here $\tilde{\Omega} = \sqrt{\Omega^2 + i\nu_c\Omega - 1}$.

Now, using Eqs. (6), (13), (14), and (20), we can find the electromagnetic field in the slab of layered superconductor. The tangential components of the electric and magnetic fields are:

$$E_z^s(x) = -\frac{i\Omega}{\sqrt{\varepsilon}}\frac{\mathcal{H}_0}{2}a(x/\lambda_c),$$
$$H_y^s(x) = \frac{\mathcal{H}_0}{2}a'(x/\lambda_c). \tag{22}$$

Here the prime stands for the first derivative of a function with respect to its argument.

Matching tangential components of the electric and magnetic fields (2), (3) in the vacuum regions with the fields (20), (22) inside the slab, we find out the unknown constants H_r, H_t, C_1, and C_2. As a result, one can obtain the following relation for the transmitted amplitude:

$$H_t = \frac{4\Omega\tilde{\Omega}(\tilde{\Omega}^2+1)^2\sqrt{\varepsilon}\cos\theta}{e^{-i\delta\tilde{\Omega}}(i-\tilde{\Omega})^2M_+^2 - e^{i\delta\tilde{\Omega}}(i+\tilde{\Omega})^2M_-^2}, \tag{23}$$

where

$$M_\pm = \sqrt{\varepsilon}\cos\theta\left(h_0^2 \pm i\tilde{h}_0\tilde{\Omega} + \tilde{\Omega}^2\right) + \Omega(i\tilde{h}_0 \pm \tilde{\Omega}). \tag{24}$$

III. TRANSMISSION COEFFICIENT

Now we are in a position to obtain the analytical expression for the transmittance $T = |H_t|^2$. At first, in Sec. III A, we analyze the transmission coefficient as a function of the slab thickness, the incident angle, frequency, and the magnitude of the dc magnetic field disregarding the relaxation terms and then, in Sec. III B, discuss the role of the dissipation.

A. Transmission coefficient in the absence of dissipation

At $\nu_c = \nu_{ab} = 0$, Eq. (23) gives the following expression for the transmission coefficient $T = |H_t|^2$:

$$T = \frac{1}{1 + \sin^2(\tilde{\Omega}\delta - \phi)\left\{\left[\frac{1}{4\Theta} + \left(\frac{h_0^4\tilde{h}_0^2}{\Omega^4\tilde{\Omega}^2} + 1\right)\Theta\right]^2 - 1\right\}}, \tag{25}$$

where

$$\tilde{h}_0 = \sqrt{1-h_0^2}, \qquad \Theta = \frac{\Omega\,\tilde{\Omega}\sqrt{\varepsilon}}{2\left(\tilde{h}_0^2+\tilde{\Omega}^2\right)}\cos\theta,$$
$$\phi = \frac{\pi}{2} - \arctan\left(\frac{1-\tilde{\Omega}^2}{2\tilde{\Omega}} + \frac{\Omega^4\tilde{h}_0}{2\,\tilde{\Omega}\,h_0^2}P\right), \tag{26}$$
$$P = \left[\frac{\Omega^2\varepsilon\cos^2\theta}{\Omega^2 + \left(\tilde{h}_0^2 - \Omega^2\right)\varepsilon\cos^2\theta} - \frac{\tilde{h}_0 + \tilde{\Omega}^2}{\tilde{h}_0 + 1}\right]^{-1}.$$

In the absence of dc magnetic field ($h_0 = 0$), the general expression (25) for the transmittance gets a simple form,

$$T(h_0 = 0) = \left[1 + \left(\frac{1}{4\Theta_0} - \Theta_0\right)^2\sin^2(\tilde{\Omega}\delta)\right]^{-1},$$
$$\Theta_0 = \frac{\tilde{\Omega}}{\Omega}\frac{\sqrt{\varepsilon}}{2}\cos\theta. \tag{27}$$

One can see that the slab of layered superconductor becomes completely transparent ($T = 1$) for definite incident angle, when the equality $\cos\theta = \Omega/\tilde{\Omega}\sqrt{\varepsilon}$ holds true, regardless of the slab thickness D. This case can be realized at high enough frequencies meeting the condition $\Omega > (1 - \varepsilon^{-1})^{-1/2}$.

At the critical value of the dc magnetic field, $h_0 = 1$, the transmittance reads

$$T(h_0 = 1) = \left[1 + \left(\frac{1}{4\Theta_1} - \Theta_1\right)^2 \sin^2(\tilde{\Omega}\delta - \phi_1)\right]^{-1}, \tag{28}$$
$$\phi_1 = 2\arctan\tilde{\Omega}, \quad \Theta_1 = \frac{\Omega}{\tilde{\Omega}}\frac{\sqrt{\varepsilon}}{2}\cos\theta.$$

In this case, the total transparency can also be observed, however for the other incident angle when $\cos\theta = \tilde{\Omega}/\Omega\sqrt{\varepsilon}$. This condition can be satisfied for any frequency $\Omega > 1$.

Now we proceed to the general case of the nonzero dc magnetic field. We will discuss the dependence of the transmittance on the slab thickness $\delta = D/\lambda_c$, incident angle θ, and frequency $\Omega = \omega/\omega_J$, and study the effect of the dc magnetic field $h_0 = H_0/\mathcal{H}_0$ on this dependence.

1. Dependence $T(\delta)$

The general expression (25) for the transmittance depends on the slab thickness $D = \lambda_c\delta$ via the argument of the sine only. If $\sin(\tilde{\Omega}\delta - \phi) = 0$, the slab is completely transparent. In the absence of external dc magnetic field, the phase $\phi = 0$. In this case, $T = 1$ when the slab thickness equals an integer multiple of the half wavelengths,

$$D = \frac{\pi n c}{\sqrt{\varepsilon}}\frac{1}{\sqrt{\omega^2 - \omega_J^2}}, \qquad n = 1,2,3,\ldots. \tag{29}$$

As a result, the dependence $T(D)$ contains oscillations of the Fabry-Perot type, modified by the hyperbolic dispersion law for the JPWs in the layered superconductor.

If the dc magnetic field H_0 is turned on, the H_0-dependent phase shift ϕ appears in the argument of the sine in Eq. (25). Thus, the dc magnetic field moves the $T(D)$ curve to the left, while the periodicity of the function $T(D)$ remains the same. In addition, the amplitude of the Fabry-Perot resonance increases at $H_0 = \mathcal{H}_0$ in comparison with the case $H_0 = 0$. However, more detailed study reveals that the H_0 dependence of the amplitude of the Fabry-Perot resonance turns out to be nonmonotonic if the incident angles θ satisfy the inequalities

$$\frac{\Omega}{\sqrt{\varepsilon(\Omega^2+1)}} < \cos\theta < \frac{\Omega}{\sqrt{\varepsilon(\Omega^2-1)}}, 1. \tag{30}$$

Figure 2 demonstrates the dependence of the transmittance T on the dimensionless slab thickness δ for $H_0 = 0$ and $H_0 = \mathcal{H}_0$. The corresponding phase shift is shown by the arrow.

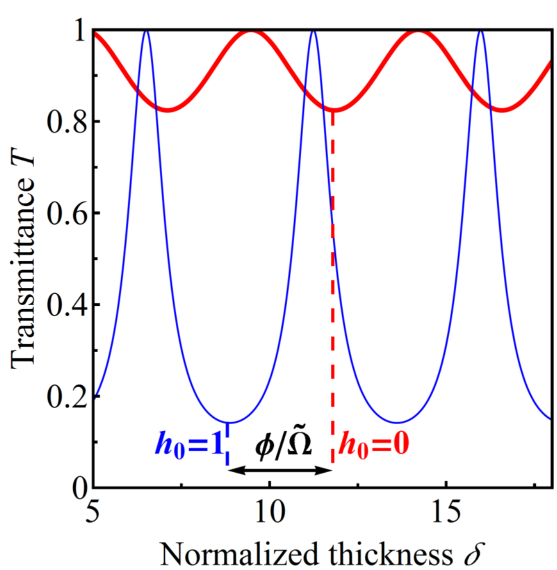

FIG. 2. Transmittance T vs normalized slab thickness $\delta = D/\lambda_c$ in the absence of dc magnetic field (thick red curve, $h_0 = 0$) and for the critical field $h_0 = 1$ (thin blue curve). The arrow shows shift of the minimum position when applying the dc magnetic field. Other parameters are: $\Omega = 1.2$, $\theta = \pi/4$, $\lambda_c = 4\times10^{-3}$ cm, $\lambda_{ab} = 2\times10^{-5}$ cm, $\omega_J/2\pi = 0.3$ THz, and $\varepsilon = 16$.

2. Dependence $T(\theta)$

The dependence of the transmittance T on the incident angle θ is more complicated. Fig. 3 shows the value of T by the color grade as a function of the incident angle and external dc magnetic field at frequencies $\Omega = 1.1, 1.15, 1.2, 1.28$. Although this dependence changes quantitatively with increase of Ω, one can notice a qualitative repetition of its behavior. Indeed, when $\tilde{\Omega}$ changes by π/δ, the argument of sine in Eq. (25) changes approximately by π (see, e.g., panels

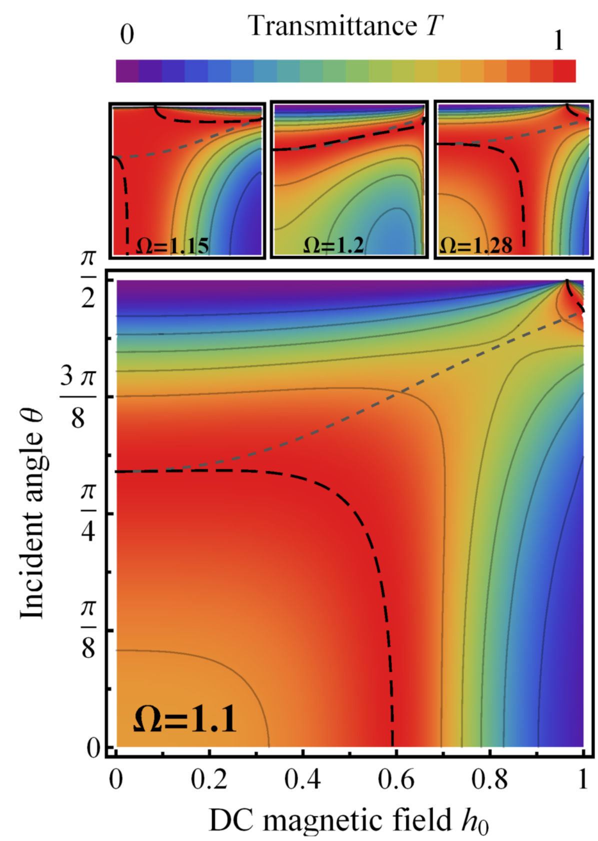

FIG. 3. The dependence of transmittance T (shown by the color grade) on the incident angle θ and external dc magnetic field h_0 for different frequencies: $\Omega = 1.1$ (main panel) and $\Omega = 1.15$, $\Omega = 1.2$, $\Omega = 1.28$ (top panels). The dashed black lines correspond to $\sin(\tilde{\Omega}\delta - \phi) = 0$, and the dashed gray lines show the points where factor in the curly brackets in Eq. (25) reaches the minimum as a function of θ. The parameters are: $\delta = 11$, $\lambda_c = 4\times10^{-3}$ cm, $\lambda_{ab} = 2\times10^{-5}$ cm, $\omega_J/2\pi = 0.3$ THz, and $\varepsilon = 16$.

with $\Omega = 1.28$ and $\Omega = 1.1$ in Fig. 3). The transmission coefficient T is equal to 1 when $\sin(\tilde{\Omega}\delta - \phi) = 0$. This condition corresponds to the dashed black lines in Fig. 3. Additionally, we show the points where the factor in the circle brackets in Eq. (25) reaches the minimum as a function of θ by the dashed gray line.

As is seen in the main panel in Fig. 3, the dc field can drastically change the behavior of the function $T(\theta)$. Specifically, for $0 \leqslant h_0 \lesssim 0.2$ this function is nonmonotonic. As the incident angle θ increases, the transmittance initially increases and then decreases down to zero. Within the interval $0.2 \lesssim h_0 \lesssim 0.5$, function $T(\theta)$ monotonically decreases from $T(\theta = 0) = 1$ to $T(\theta = \pi/2) = 0$. For $0.5 \lesssim h_0 \lesssim 0.8$, the variation of θ does not result in a noticeable change of the transmittance $T(\theta)$. Finally, when the dimensionless dc field is higher than ≈ 0.8, the dependence $T(\theta)$ becomes again nonmonotonic.

It should be noted that, with a proper choice of the incident angle θ, one can attain almost perfect transmission of the superconducting slab for almost any values of the external dc field H_0 and frequency ω. To recognize this fact, we found from Eq. (25) the maximal values $T_{\max}$ of the function $T(\theta)$ for different dc magnetic fields and frequencies. These maximal values are depicted on the phase plane (h_0, Ω) in Fig. 4. One can see that the perfect transmission, $T_{\max} = 1$, can be observed in a wide area of this plane (light gray regions bordered by the dashed lines). The values of θ in this regions are chosen to nullify the sine in Eq. (25). The perfect transmission cannot be attained, $T_{\max} < 1$, for h_0 and Ω in the bluer regions.

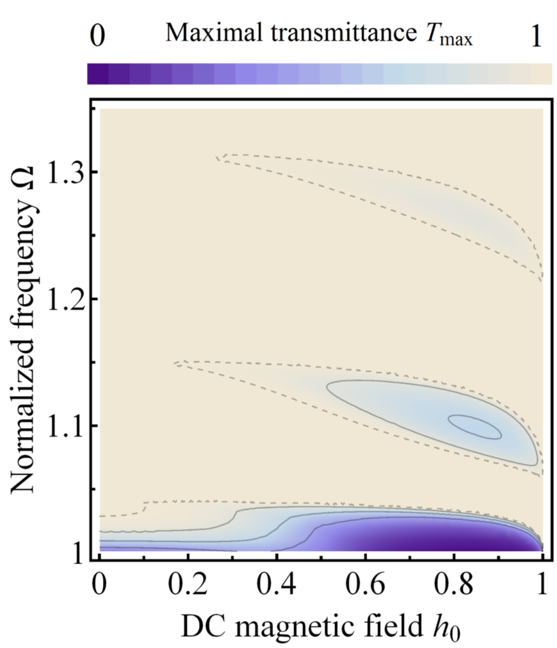

FIG. 4. The maximum $T_{\max}$ of the function $T(\theta)$ (shown by the color grade) vs normalized frequency Ω and dimensionless external dc magnetic field h_0. The dashed lines separate regions where the perfect transmission can be observed (regions with $T_{\max} = 1$, light gray color) from the regions with $T_{\max} < 1$ (darker bluish color). The parameters are: $\delta = 11$, $\lambda_c = 4 \times 10^{-3}$ cm, $\lambda_{ab} = 2 \times 10^{-5}$ cm, $\omega_J/2\pi = 0.3$ THz, and $\varepsilon = 16$.

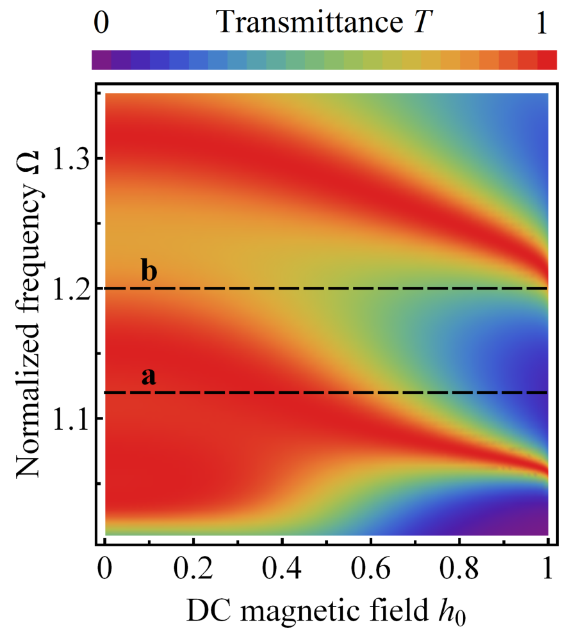

FIG. 5. The dependence of transmittance T (shown by the color grade) on normalized frequency Ω and external dc magnetic field h_0. The dashed horizontal lines **a** and **b** correspond to the dashed vertical lines **a** and **b** in Fig. 7. These lines show that, for given frequency, the change of dc field can affect the transmittance either significantly (**a**) or weakly (**b**). The parameters are: $\theta = \pi/4$, $\delta = 11$, $\lambda_c = 4 \times 10^{-3}$ cm, $\lambda_{ab} = 2 \times 10^{-5}$ cm, $\omega_J/2\pi = 0.3$ THz, $\varepsilon = 16$.

3. Dependence $T(\Omega)$

The transmittance T of the layered superconductor versus the frequency ω and dc field H_0 is shown by the color grade in Fig. 5. The regions of perfect transmission are marked in red color. While increasing the frequency ω at any fixed H_0, the transmittance oscillates and takes on the value one at the frequencies of the Fabry-Perot resonances [when the sine in

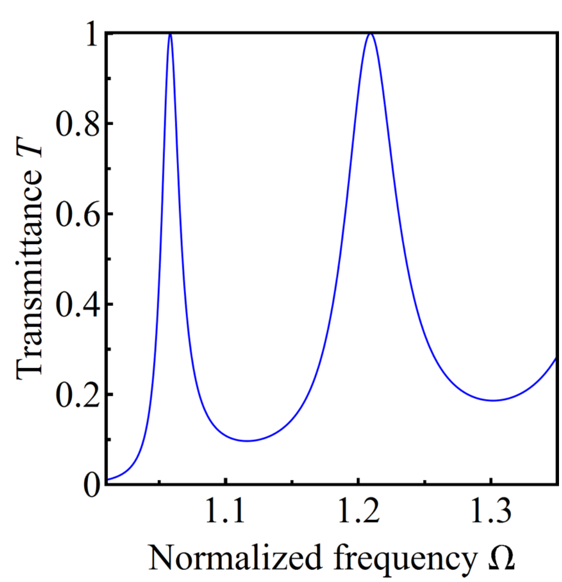

FIG. 6. Transmittance T vs normalized frequency Ω for $h_0 = 1$. Other parameters are: $\theta = \pi/4$, $\delta = 11$, $\lambda_c = 4 \times 10^{-3}$ cm, $\lambda_{ab} = 2 \times 10^{-5}$ cm, $\omega_J/2\pi = 0.3$ THz, $\varepsilon = 16$.

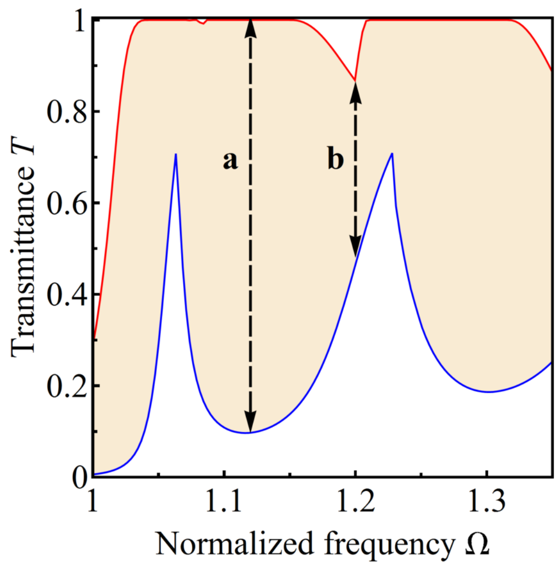

FIG. 7. Variation range (gray area) of function $T(h_0)$ vs normalized frequency Ω. The upper red and bottom blue curves show the maximum and minimum values of the transmittance. The dashed vertical straight lines, **a** and **b**, correspond to dashed horizontal lines, **a** and **b**, in Fig. 5. The parameters are: $\theta = \pi/4$, $\delta = 11$, $\lambda_c = 4 \times 10^{-3}$ cm, $\lambda_{ab} = 2 \times 10^{-5}$ cm, $\omega_J/2\pi = 0.3$ THz, and $\varepsilon = 16$.

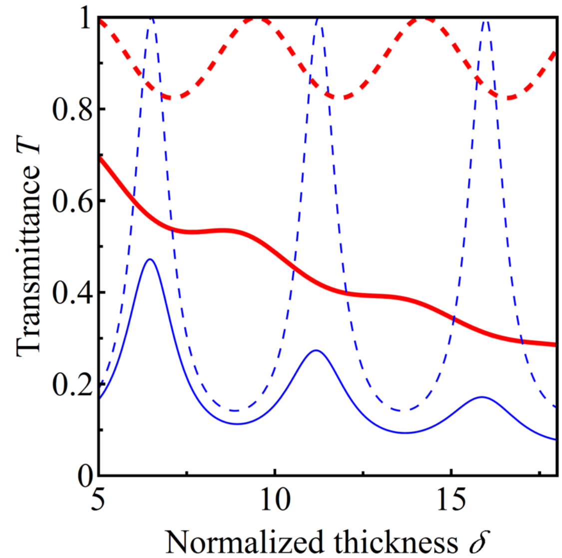

FIG. 8. Transmittance T vs normalized slab thickness $\delta = D/\lambda_c$ for $\nu_{ab} = \nu_c = 0.03\,\Omega$ (solid curves) and $\nu_{ab} = \nu_c = 0$ (dashed curves). Thick red curves correspond to the case of absence of dc magnetic filed, $h_0 = 0$, whereas thin blue curves are obtained for $h_0 = 1$. Other parameters are: $\Omega = 1.2, \theta = \pi/4, \lambda_c = 4 \times 10^{-3}$ cm, $\lambda_{ab} = 2 \times 10^{-5}$ cm, $\omega_J/2\pi = 0.3$ THz, and $\varepsilon = 16$.

Eq. (25) vanishes], see Fig. 6. According to Fig. 5, the dc field shifts the maximums of the transmittance and makes the resonances sharper.

Remarkably, the Fabry-Perot resonances can be realized by changing the dc magnetic field H_0, however, only for special fixed intervals of the frequency ω. To recognize the range of transmittance variation as the dc magnetic field is changed, we plotted two boundary curves in Fig. 7. Specifically, the upper red and lower blue curves represent, respectively, the transmittance maximized and minimized over the dc field, as a functions of the dimensionless frequency Ω. The gray intermediate region between the curves shows the range of transmittance variation. One can see from Fig. 7 that this range depends significantly on the choice of the frequency. For example, we can compare the ranges marked by dashed lines **a** and **b** in Figs. 5 and 7. For the frequency corresponding to line **a**, one can vary the transmittance from nearly zero to one, whereas the range of the transmittance variation is much smaller along line **b**. Evidently, the perfect transparency cannot be observed in the last case.

B. Transmission coefficient with account for dissipation

All results obtained in Sec. III A are valid for the case of the negligible dissipation. However, even in the case of not very small dissipation, the small dc magnetic field can substantially affect the transmissivity of the superconducting slabs. To demonstrate this we, using Eq. (23), have plotted the transmittance T vs normalized slab thickness $\delta = D/\lambda_c$ for the case of zero dissipation, $\nu_{ab} = \nu_c = 0$, and for $\nu_{ab} = \nu_c = 0.03\,\Omega$ (see Fig. 8). The dashed curves obtained for $\nu_{ab} = \nu_c = 0$ coincide with the curves in Fig. 2. They show the Fabry-Perot oscillations in the absence of dc magnetic filed (thick red dashed curve) and for $h_0 = 1$ (thin blue dashed curve). The solid curves in Fig. 8 are plotted for the same conditions as dashed lines but for $\nu_{ab} = \nu_c = 0.03\,\Omega$. One can see that the Fabry-Perot oscillations persist even in the case of small (but realistic) dissipation. Moreover, Fig. 2 shows that the dc magnetic field changes the transmissivity in a wide range even in the case of nonzero dissipation.

IV. CONCLUSIONS

In this paper, we have studied theoretically the transmission of THz electromagnetic waves through a slab of layered superconductor in the presence of external dc magnetic field. We have shown that the dc field changes significantly conditions for the propagation of the Josephson plasma waves. Specifically, the dc field turns a layered superconductor into spatially and frequency dispersive medium with dielectric permittivity $\varepsilon(x,\omega)$ described by Eq. (18). With tuning the dc magnetic field, one can easily control the spatial distribution of $\varepsilon(x,\omega)$ and, thus, the electrodynamic properties of the plasma in layered superconductors. As a result, we have an effective tool to manipulate the transmissivity of layered superconductors. We have demonstrated that applying even relatively weak dc magnetic field one can significantly change the transmittance of the layered superconductor. Moreover, the proper choice of the dc field magnitude can provide the perfect transparency of the slab.

ACKNOWLEDGMENT

We acknowledge partial support from the CONACYT (México) under Grant No. CB-2011-01-166382.

[1] R. Kleiner, F. Steinmeyer, G. Kunkel, and P. Müller, Phys. Rev. Lett. **68**, 2394 (1992).
[2] R. Kleiner and P. Müller, Phys. Rev. B **49**, 1327 (1994).
[3] S. Savel'ev, V. A. Yampol'skii, A. L. Rakhmanov, and F. Nori, Rep. Prog. Phys. **73**, 026501 (2010).
[4] X. Hu and S.-Z. Lin, Supercond. Sci. Technol. **23**, 053001 (2010).
[5] L. Ozyuzer, A. E. Koshelev, C. Kurter, N. Gopalsami, Q. Li, M. Tachiki, K. Kadowaki, T. Yamamoto, H. Minami, H. Yamaguchi, T. Tachiki, K. E. Gray, W.-K. Kwok, and U. Welp, Science **318**, 1291 (2007).
[6] S. Savel'ev, V. Yampol'skii, and F. Nori, Phys. Rev. Lett. **95**, 187002 (2005).
[7] V. A. Golick, D. V. Kadygrob, V. A. Yampolskii, A. L. Rakhmanov, B. A. Ivanov, and F. Nori, Phys. Rev. Lett. **104**, 187003 (2010).
[8] V. A. Yampol'skii, A. V. Kats, M. L. Nesterov, A. Yu. Nikitin, T. M. Slipchenko, S. Savel'ev, and F. Nori, Phys. Rev. B **76**, 224504 (2007).
[9] D. V. Kadygrob, N. M. Makarov, F. Perez-Rodriguez, T. M. Slipchenko, and V. A. Yampol'skii, New J. Phys. **15**, 023040 (2013).
[10] V. M. Agranovich and D. L. Mills (eds.), *Surface Polaritons* (North-Holland, Amsterdam, 1982).
[11] H. Raether, *Surface Plasmons* (Springer-Verlag, New York, 1988).
[12] R. Petit, *Electromagnetic Theory of Gratings* (Springer, Berlin, 1980).
[13] A. L. Rakhmanov, V. A. Yampol'skii, J. A. Fan, F. Capasso, and F. Nori, Phys. Rev. B **81**, 075101 (2010).
[14] V. A. Yampolskii, S. Savelev, O. V. Usatenko, S. S. Melnik, F. V. Kusmartsev, A. A. Krokhin, and F. Nori, Phys. Rev. B **75**, 014527 (2007).
[15] S. Savel'ev, A. L. Rakhmanov, V. A. Yampol'skii, and F. Nori, Nat. Phys. **2**, 521 (2006).
[16] V. A. Yampol'skii, S. Savel'ev, A. L. Rakhmanov, and F. Nori, Phys. Rev. B **78**, 024511 (2008).
[17] S. Savel'ev, V. A. Yampol'skii, A. L. Rakhmanov, and F. Nori, Phys. Rev. B **75**, 184503 (2007).
[18] S. S. Apostolov, Z. A. Maizelis, M. A. Sorokina, V. A. Yampol'skii, and F. Nori, Phys. Rev. B **82**, 144521 (2010).
[19] A. Dienst, E. Casandruc, D. Fausti, L. Zhang, M. Eckstein, M. Hoffmann, V. Khanna, N. Dean, M. Gensch, S. Winnerl, W. Seidel, S. Pyon, T. Takayama, H. Takagi, and A. Cavalleri, Nat. Mater. **12**, 535 (2013).
[20] V. A. Yampol'skii, D. R. Gulevich, S. Savel'ev, and F. Nori, Phys. Rev. B **78**, 054502 (2008).
[21] D. V. Kadygrob, V. A. Golick, V. A. Yampol'skii, T. M. Slipchenko, D. R. Gulevich, and S. Savel'ev, Phys. Rev. B **80**, 184512 (2009).
[22] S. Sakai, P. Bodin, and N. F. Pedersen, J. Appl. Phys. **73**, 2411 (1993).
[23] L. N. Bulaevskii, M. Zamora, D. Baeriswyl, H. Beck, and J. R. Clem, Phys. Rev. B **50**, 12831 (1994).
[24] T. Koyama and M. Tachiki, Phys. Rev. B **54**, 16183 (1996).
[25] S. N. Artemenko and S. V. Remizov, JETP Lett. **66**, 853 (1997).
[26] S. N. Artemenko and S. V. Remizov, Physica C **362**, 200 (2001).
[27] Ch. Helm, J. Keller, Ch. Peris, and A. Sergeev, Physica C **362**, 43 (2001).
[28] Ju. H. Kim and J. Pokharel, Physica C **384**, 425 (2003).
[29] L. N. Bulaevskii and A. E. Koshelev, Phys. Rev. Lett. **99**, 057002 (2007).
[30] Ch. Helm and L. N. Bulaevskii, Phys. Rev. B **66**, 094514 (2002).