Disorder-induced enhancement and critical scaling of spontaneous magnetization in random-field quantum spin systems

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(Received 23 September 2015; revised manuscript received 13 June 2016; published 18 July 2016)

We investigate the effect of a unidirectional quenched random field on the anisotropic quantum spin-1/2 XY model, which magnetizes spontaneously in the absence of the random field. We adopt a mean-field approach for this analysis. In general, the models considered have Ising symmetry, and as such they exhibit ferromagnetic order in two and three dimensions in the presence of not too large disorder. Even in the special case when the model without disorder has U(1) symmetry, a small U(1)-symmetry-breaking random field induces ferromagnetic long-range order in two dimensions. The mean-field approach, consequently, provides a rather good qualitative and even quantitative description when applied not too close to the criticality. We show that spontaneous magnetization persists even in the presence of the random field, but the magnitude of magnetization gets suppressed due to disorder, and the system magnetizes in the directions parallel and transverse to the random field. Our results are obtained via analytical calculations within a perturbative framework and by numerical simulations. Interestingly, we show that it is possible to enhance a component of magnetization in the presence of the disorder field provided that we apply an additional constant field in the XY plane. Moreover, we derive generalized expressions for the critical temperature and the scalings of the magnetization near the critical point for the XY spin system with arbitrary fixed quantum spin angular momentum.

DOI: 10.1103/PhysRevB.94.014421

I. INTRODUCTION

Disorder is ubiquitous in solid-state materials. The effects of disorder are often nonintuitive, and hence they have generated a lot of attention in condensed-matter physics [1–5]. Disorder can currently also be engineered in a controlled way via ultracold atoms trapped in optical lattices subjected to an additional, e.g., optical speckle potential [4,5]. Disordered systems are often endowed with nontrivial properties, dramatically different from those of their homogeneous counterparts. The novel quantum phases [6,7] and unique phenomena, such as Anderson localization [8], dynamical many-body localization [9], the presence/absence of thermalization [10], and high-T_c superconductivity [11], are some of the prominent examples. Quenched disorder in type II superconductors has been investigated using Ginzburg-Landau theory to uncover "glassy" properties in the system [12]. In particular, considerable efforts have been dedicated to understanding the effects of disorder in spin models, both classical and quantum [13–17].

In classical systems with continuous symmetry, it has been shown that a random field with the symmetry of the system may cause significant changes in its properties [18,19]. A small random magnetic field of this kind can destroy magnetization in a classical spin system at any temperature, including zero temperature. For example, two-dimensional XY and Heisenberg models do not magnetize in the presence of random fields with SO(2) and SO(3) symmetries, respectively [14,18] (see also [19] for the analogous effect in systems with discrete symmetry). The effect prevails in higher dimensions as long as the random field exhibits the corresponding symmetry [14]. However, in the absence of appropriate symmetry, these systems exhibit spontaneous magnetization [15,16]. Interestingly, Ref. [15] demonstrated a counterintuitive phenomenon, namely random-field-induced order, where disorder favors ordering in certain spin models that magnetize at a higher temperature in the presence of an arbitrarily small symmetry-breaking random field in comparison to the disorder-free systems. Disorder-induced ordering has also been reported in several other contexts [20].

Despite the difficulty in dealing with spontaneous magnetization and other system characteristics in quantum disordered systems, many important results have been obtained. A quantum Hall nematic phase has been predicted in a zero-temperature two-dimensional electron system that is unstable to weak disorder [21,22]. Collective properties of magnetic impurities on a topological surface were studied both theoretically [23] and experimentally [24]. Experimental and theoretical investigations in solid-state systems have revealed that alloy disorder can reduce the Curie temperature in the system [25]. Arbitrarily weak interparticle interactions were shown to destabilize the surface states of topological superconductors in the presence of nonmagnetic disorder [26]. The critical behavior and effective exponents in ferromagnetic quantum phase transitions of disordered systems were derived in Ref. [27].

Recently, a mean-field classical spin model with SO(n) symmetry was considered, and its spontaneous magnetization was investigated in the presence of unidirectional random fields [16]. The natural question is how a symmetry-breaking random field affects these systems in the quantum limit. As already mentioned, this question is particularly relevant as the disordered quantum spin models are currently accessible to the experimental community [4]. If we restrict ourselves to the XY

model with a random field, and to one dimension, then such a quantum system of a moderately large size can be investigated using the Jordan-Wigner technique [28]. However, higher dimensional quantum spin models remain intractable due to the lack of analytical and numerical techniques, even for ordered systems. We work within the mean-field (MF) approximation, which is often effective in capturing the system's properties qualitatively. Numerical schemes such as exact diagonalization are usually inefficient for even moderately large systems due to the exponential growth of the dimension of the system's Hilbert space. Metastability effects and slow relaxation rates, usually present in disordered systems, make other numerical simulation techniques such as the density-matrix renormalization group and the Monte Carlo approach difficult to apply, particularly in higher-dimensional lattice systems with higher-dimensional spins [29]. The mean-field approach essentially liberates one from these immediate challenges, allowing for a detailed analysis that enables us to answer some of the key questions. Of course, the price to pay is that the mean-field approach is not expected to describe the details of the critical behavior precisely, except in high dimensions.

Before turning to the main body of the paper, let us explain why the use of mean-field theory is interesting in the context of the problem considered in this paper. It may be expected that the interplay of short-range interactions and random fields leads to situations in which mean-field approaches fail. For instance, Ising ferromagnetism turns out to be unstable to infinitesimal random fields in dimensions lower than 2. Strictly speaking, mean-field theory is exact in infinite dimension or in systems of infinite connectivity (an infinite number of next neighbors).

Let us mention here that the mean-field approximation should be understood as a variational method in which minimization of energy of a lattice system corresponding to the ground state at zero temperature is assumed to have a product form for each lattice site, and the minimization of the free energy corresponding to the density matrix at nonzero temperature also has a form of a product over the sites. Such a mean-field treatment works well whenever ferromagnetic order is present far from quantum criticality, e.g., Ising systems without disorder in lattice dimension $d \geqslant 2$, and XY as well as Heisenberg systems in $d \geqslant 3$ at temperature $T > 0$. For $d \geqslant 4$, the mean field starts to work well even at quantum criticality.

We consider here anisotropic XY systems with Ising symmetry. In the absence of disorder, the system orders for $d \geqslant 2$, whereas it exhibits Kosterlitz-Thouless-Berezinskii transitions in the special isotropic U(1) symmetric case in two dimensions. In the presence of the parallel [U(1) symmetry breaking] random field, the system orders ferromagnetically even in two dimensions (as proven in Refs. [15,34]). For this reason, far from quantum criticality, the mean field provides a quantitatively reasonable approximation that allows us to estimate very well the critical temperature and the value of the order parameters in two, three, and higher dimensions. On the contrary, mean-field results do not work in one dimension, where there is no long-range order.

This work considers a quantum spin-1/2 XY model with anisotropic interaction in the presence of a unidirectional quenched random field. The purely isotropic case, i.e., the quantum XY model with vanishing anisotropy parameter and vanishing disorder, exhibits a spontaneous magnetization that has circular symmetry. The continuous symmetry is broken due to the presence of anisotropy even for the pure system. The pure spin-1/2 XY system magnetizes below a certain critical temperature. This system still magnetizes when a random field is introduced at a critical temperature that is higher than in the system without disorder. We show by means of numerical as well as perturbative analysis that the system now magnetizes in specific directions, which is either along the parallel or the perpendicular directions to the random field. The critical temperature in both cases decreases with the increase in the random field strength. We find that the critical temperature corresponding to the transverse and the parallel directions shows opposite behavior with respect to the anisotropy in the system. Specifically, with the increase of anisotropy, the critical temperature corresponding to the transverse magnetization increases, while the opposite happens for parallel magnetization. It is important to mention that for a vanishing anisotropy parameter, the continuous symmetry of the pure XY system is broken by the introduction of an arbitrarily small random field. We also present general expressions of the scalings of critical temperature of magnetization for the quantum XY spin systems with arbitrary half-integer and integer spins.

In addition, adding a constant magnetic field along with the random field, we find that the component of the magnetization perpendicular to the random field gets enhanced due to the disorder, which has also been reported for several other models [17,20,30,31].

The rest of the paper is arranged as follows. In Sec. II, we introduce the spin-1/2 quantum XY model in the presence of a random field, and subsequently we derive critical scaling of the magnetization via a perturbative approach. We also discuss numerical results obtained within the MF approximation. In Sec. III, we demonstrate, both numerically and analytically, the disorder-induced enhancement under the influence of an additional constant field. In Sec. IV, we derive the generalized expressions for the case of arbitrary integer and half-integer spins. Finally, we conclude in Sec. V.

II. QUANTUM SPIN-1/2 XY MODEL IN A RANDOM FIELD

We consider the quantum spin-1/2 XY model in a random field. Our aim is to study the effect of the random field on the magnetization as a function of temperature, and to find the scaling of the magnetization around the critical temperature. In the following subsection, we introduce the system and the mean-field approximation.

A. The system and its mean-field treatment

The Hamiltonian of the ferromagnetic quantum XY model is given by

$$H_{XY} = H_{\text{int}} + H_{\text{ext}}, \tag{1}$$

where

$$H_{\text{int}} = -\sum_{|i-j|=1}^{N} \left[J_x \sigma_i^x \sigma_j^x + J_y \sigma_i^y \sigma_j^y \right]. \tag{2}$$

The coupling constants, J_α, are assumed to be positive. They can be further expressed in terms of an anisotropy parameter, γ, as $J_x = J(1+\gamma)$ and $J_y = J(1-\gamma)$. In the following discussion, we assume $\gamma \geqslant 0$, unless stated otherwise. The indices i and j denote the sites of an arbitrary d-dimensional lattice, and σ_i^α, $\alpha = x,y$ are the Pauli matrices on the ith site. The part of the Hamiltonian in Eq. (1) due to an inhomogeneous magnetic field, $H_{\rm ext}$, equals

$$H_{\rm ext} = -\epsilon \sum_i \vec{h}_i \cdot \vec{\sigma}_i, \tag{3}$$

where ϵ (>0), a dimensionless parameter that quantifies the strength of the randomness, is typically chosen to be small. The unidirectional random field, $\vec{h}_i$, is chosen to be $\vec{h}_i = \eta_i \cdot \hat{e}_y$, where η_i are independent and identically distributed quenched Gaussian random variables with zero mean and unit variance, and $\hat{e}_y$ is the unit vector in the y direction. Within the mean-field limit, as we show below, the pure systems governed by the Hamiltonian $H_{\rm int}$ magnetize even in low dimensions. This does not contradict the Mermin-Wagner-Hohenberg theorem [32], which predicts no spontaneous magnetization in one and two dimensions, at $\gamma = 0$, since the predictions made by the mean-field approximation only become accurate in higher dimensions [33] where the Mermin-Wagner-Hohenberg theorem is not valid. Interestingly, it has been shown that a uniaxial random field may help the system to magnetize even at two dimensions [15,34]. Note that had the random field, $\vec{h}_i$, been chosen to be invariant under rotations, the system would not magnetize at any nonzero temperature in any dimension $d \leqslant 4$ [14,18,19].

Within the mean-field approximation, each spin is regarded as reacting to an average field due to all the other spins in the system. Assuming N to be the total number of spins in the system, the effective interaction, replacing the nearest-neighbor interaction in $H_{\rm int}$, for large N, equals approximately $H_{\rm int} = -1/N(\sum_{\alpha=x,y}\sum_{j;j\neq i}^{N} \nu J'_\alpha \sigma_i^\alpha)\sigma_j^\alpha = -\sum_{\alpha=x,y} \nu J'_\alpha m_\alpha \sigma_i^\alpha$, where $m_\alpha = \frac{1}{N}\sum_{j=1}^{N} \sigma_j^\alpha$. J' is the coupling constant and ν is the coordination number, which depends on the geometry of the lattice. Note that the mean-field approximation provides close to the exact description for large dimensional lattice systems [35]. Within the mean-field approximation, the Hamiltonian, H_{XY}, is given by

$$H = -J(1+\gamma)m_x\sigma_x - J(1-\gamma)m_y\sigma_y - \epsilon\vec{\eta}\cdot\vec{\sigma}, \tag{4}$$

where the operators m_α are replaced by their average values denoted by the same symbol, in the canonical equilibrium state at absolute temperature T and $J = J'\nu$. Now in order to monitor the behavior of the magnetization as a function of temperature, one needs to calculate the expectation value of the spin operators, σ_α, $\alpha = x,y$. So in the mean-field regime, the magnetization of the system governed by the Hamiltonian, H [see Eq. (4)], is given by

$$m_\alpha = {\rm av}_\eta\left[\frac{{\rm Tr}[\sigma_\alpha \exp(-\beta H)]}{{\rm Tr}[\exp(-\beta H)]}\right], \tag{5}$$

where $\beta = 1/(k_B T)$, where k_B is the Boltzmann constant, and ${\rm av}_\eta[.]$ denotes the average over the realizations of randomness. From Eqs. (4) and (5), we obtain a coupled set of the following two equations:

$$m_\perp^{\epsilon,2} \equiv m_x = {\rm av}_\eta\left[\frac{J(1+\gamma)m_x}{k^\epsilon}\tanh(\beta k^\epsilon)\right] \tag{6}$$

and

$$m_\parallel^{\epsilon,2} \equiv m_y = {\rm av}_\eta\left[\frac{J(1-\gamma)m_y + \epsilon\eta}{k^\epsilon}\tanh(\beta k^\epsilon)\right], \tag{7}$$

where $k^\epsilon = \sqrt{[J(1+\gamma)m_x]^2 + [\epsilon\eta + J(1-\gamma)m_y]^2}$. Note that the subscripts $\perp$ and $\parallel$ classify two distinct cases, as would be apparent later. The superscripts ϵ and 2 keep track of the strength of the disorder and of the value of the spin (1/2), respectively.

B. Critical point and scaling of magnetization near criticality

The magnetization, $\vec{m}$, can be obtained by finding the common zeros of the following two functions obtained from Eqs. (6) and (7):

$$F_x^{\epsilon,2}(\vec{m}) = {\rm av}_\eta\left[\frac{J(1+\gamma)m_x}{k^\epsilon}\tanh(\beta k^\epsilon)\right] - m_x \tag{8}$$

and

$$F_y^{\epsilon,2}(\vec{m}) = {\rm av}_\eta\left[\frac{J(1-\gamma)m_y + \epsilon\eta}{k^\epsilon}\tanh(\beta k^\epsilon)\right] - m_y. \tag{9}$$

Let us set $m_x = m\cos\phi_1$, $m_y = m\sin\phi_1$, and $\vec{m} = (m_x, m_y)$.

By performing perturbative analysis, we can study the magnetization for small ϵ. A Taylor series expansion of Eqs. (8) and (9) in ϵ around $\epsilon = 0$ gives

$$F_x^{\epsilon,2}(\vec{m}) = c_1 + \tfrac{1}{2}b_1\epsilon^2 + O(\epsilon^4) \tag{10}$$

and

$$F_y^{\epsilon,2}(\vec{m}) = c_2 + \tfrac{1}{2}b_2\epsilon^2 + O(\epsilon^4), \tag{11}$$

where

$$c_1 = m_x\left(-1 + \frac{J(1+\gamma)}{k}\tanh(\beta k)\right), \tag{12}$$

$$c_2 = m_y\left(-1 + \frac{J(1-\gamma)}{k}\tanh(\beta k)\right), \tag{13}$$

$$b_1 = \frac{-3J^3 m_x m_y^2\beta(1-\gamma)^2(1+\gamma)}{k^4}\frac{1}{\cosh(\beta k)^2} + \frac{Jm_x\beta(1+\gamma)}{k^2}\frac{1}{\cosh(\beta k)^2} + \frac{3J^3 m_x m_y^2(1-\gamma)^2(1+\gamma)\tanh(\beta k)}{k^5}$$
$$- \frac{Jm_x(1+\gamma)\tanh(\beta k)}{k^3} - \frac{2J^3 m_x m_y^2\beta^2(1-\gamma)^2(1+\gamma)\tanh(\beta k)}{k^3}\frac{1}{\cosh(\beta k)^2}, \tag{14}$$

$$b_2 = \frac{-3J^3m_y^3\beta(1-\gamma)^3}{k^4}\frac{1}{\cosh(\beta k)^2} + \frac{3Jm_y\beta(1-\gamma)}{k^2}\frac{1}{\cosh(\beta k)^2} + \frac{3J^3m_y^3(1-\gamma)^3\tanh(\beta k)}{k^5} - \frac{3Jm_y(1-\gamma)\tanh(\beta k)}{k^3} - \frac{2J^3m_y^3\beta^2(1-\gamma)^3\tanh(\beta k)}{k^3}\frac{1}{\cosh(\beta k)^2}. \quad (15)$$

Here $k = \sqrt{[J(1+\gamma)m_x]^2 + [J(1-\gamma)m_y]^2}$.

A contour analysis at this point becomes helpful to characterize the behavior of the system, particularly in finding the directions in which the system magnetizes (see Fig. 1). This amounts to identification of the zero-contour lines corresponding to Eqs. (10) and (11). The intersection points of the zero-contour lines are possible solutions of the magnetization. For any given set of parameters, one immediately finds that the roots of $F_x^{\epsilon,2}(\vec{m})$ and $F_y^{\epsilon,2}(\vec{m})$ exist only at $\phi_1 = 0$ or $\pi/2$. This implies that the magnetization is either transverse to the random field (case I) or parallel to the random field (case II). Note that for $\epsilon = 0$ and $\gamma = 0$, the zero contour lines for both equations would lie on top of each other due to the circular symmetry in the system. However, an arbitrary small random field is enough to break this symmetry. It follows from the contour analysis that above a certain temperature (the critical temperature), the zero-contour lines corresponding to Eqs. (10) and (11) intersect only if $m_x = m_y = 0$, which is a trivial solution.

To find the critical temperature and the scaling of magnetization near criticality, we perform another round of Taylor expansions in Eqs. (10) and (11) around $m = 0$ to obtain

$$F_x^{\epsilon,2}(\vec{m}) = -\frac{1}{3}[3 + J\beta(1+\gamma)(-3+\beta^2\epsilon^2)]m\cos\phi_1 - \frac{1}{3!}\frac{2}{5}J^3\beta^3(1+\gamma)^3(5 - 8\beta^2\epsilon^2 + 4\beta^2\epsilon^2\cos 2\phi_1)m^3\cos\phi_1 + O(m^5) \quad (16)$$

and

$$F_y^{\epsilon,2}(\vec{m}) = [-1 + J\beta(1-\gamma)(1-\beta^2\epsilon^2)]m\sin\phi_1 - \frac{1}{3!}\frac{2}{5}J^3\beta^3(1-\gamma)^3(5 - 16\beta^2\epsilon^2 + 4\beta^2\epsilon^2\cos 2\phi_1)m^3\sin\phi_1 + O(m^5). \quad (17)$$

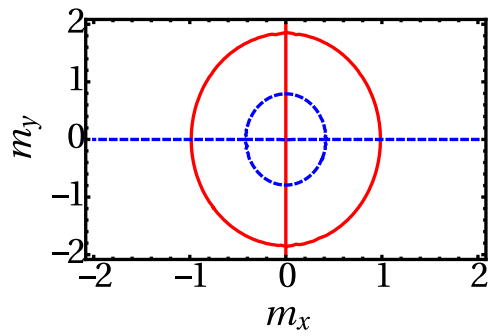

FIG. 1. The system magnetizes in directions parallel and transverse to the disorder field. Zero contour lines of the $F_x^{\epsilon,2}(m)$ and $F_y^{\epsilon,2}(m)$ in Eqs. (10) (solid red line) and (11) (dotted blue line) for $\gamma = 0.3$, $\epsilon/J = 0.1$, and $J\beta = 2$, respectively, as functions of m_x and m_y. All quantities are dimensionless.

The contour analysis implies that the allowed values of ϕ_1 are $\pi/2$ and 0. For transverse magnetization, i.e., for the case I with $\phi_1 = 0$, $F_y^{\epsilon,2}(\vec{m})$ vanishes [see Eq. (17)]. The nontrivial solutions, which solely appear from Eq. (16), are given by

$$m_\perp^{\epsilon,2} = \pm\sqrt{5}\sqrt{\frac{3 + J\beta(1+\gamma)(\epsilon^2\beta^2 - 3)}{(-5 + 4\epsilon^2\beta^2)J^3\beta^3(1+\gamma)^3}}. \quad (18)$$

The critical point is obtained by setting $m_\perp^{\epsilon,2} = 0$ in Eq. (18). We get

$$\beta_{c,\perp}^{\epsilon,2} = \frac{1}{J(1+\gamma)} + \frac{\epsilon^2}{3J^3(1+\gamma)^3}. \quad (19)$$

Here β is associated with the subscript $\perp$, following a similar convention in magnetization.

The magnetization values corresponding to case II are obtained by setting $\phi_1 = \pi/2$ in Eqs. (16) and (17). In this case, the function in Eq. (16) vanishes. The nontrivial solutions of Eq. (17) are given by

$$m_\parallel^{\epsilon,2} = \pm\sqrt{3}\sqrt{\frac{1 + J\beta(1-\gamma)(\epsilon^2\beta^2 - 1)}{J^3\beta^3(1-\gamma)^3(4\epsilon^2\beta^2 - 1)}}. \quad (20)$$

Subjecting Eq. (20) to the constraint $m_\parallel^{\epsilon,2} = 0$, we obtain the following expression for the critical temperature:

$$\beta_{c,\parallel}^{\epsilon,2} = \frac{1}{J(1-\gamma)} + \frac{\epsilon^2}{J^3(1-\gamma)^3}. \quad (21)$$

From the set of Eqs. (18), (19) and (20), (21), one can immediately infer that for all positive values of γ, the effect of disorder is more conspicuous if the system chooses to magnetize along the direction parallel to the random field, as compared to the other possibility with a transverse magnetization. On the contrary, the random field may cause a reverse scenario below a certain negative value of the anisotropy parameter. Interestingly, the findings are consistent with the pictures drawn within a classical limit [16] for the isotropic case, i.e., $\gamma = 0$. However, a quantitative comparison shows that the analysis with classical spins overestimates the effect of disorder on the critical scaling. Note that the disorder creates an ordering of the critical temperatures corresponding to the directions in which the system magnetizes, and which is interchanged at a certain disorder-strength-dependent transition anisotropy, γ_c. For example, for $\epsilon/J = 0.1$, $\gamma_c \approx -0.0033$. For $\gamma > \gamma_c$, $\beta_{c,\perp}^{\epsilon,2} > \beta_{c,\parallel}^{\epsilon,2}$, while the critical temperatures have the opposite ordering for $\gamma < \gamma_c$. This can be easily deduced from Eqs. (19) and (21), and it has also been verified numerically.

Note that one can immediately deduce the scaling expressions for the magnetizations and critical temperature of the isotropic ordered systems by setting $\gamma = \epsilon = 0$ in Eqs. (20) and (21) [or equivalently, in Eqs. (18) and (19)]. In this case, the solutions form a circle in the XY plane. This can be easily understood by following the set of Eqs. (6) and (7),

which become identical for the isotropic ordered systems. The symmetry is broken in the presence of the random field, and then the system prefers a specific direction of magnetization.

Moreover, it is clear from Eqs. (19) and (21) that the critical temperature at which the system magnetizes increases with the coordination number, i.e., with the dimension of the system, for any value of γ.

C. Numerical results

In the previous section, we derived the expressions for magnetization near the critical point. However, away from the critical point, where the perturbative approach is no longer valid, one has to rely on a numerical simulation to find the roots of the coupled set of equations, given in Eqs. (6) and (7). We use the classical Monte Carlo technique for performing a configurational averaging over η. It requires a few thousand random realizations in order to obtain converged values. Our numerical searches show the presence of two kinds of solutions, either $m_{\perp}^{\epsilon,2} \neq 0, m_{\parallel}^{\epsilon,2} = 0$ (i.e., case I) or $m_{\perp}^{\epsilon,2} = 0, m_{\parallel}^{\epsilon,2} \neq 0$ (i.e., case II), which is in accordance with previous discussions in the context of analytical perturbative analysis.

Figure 2(a) exhibits the results obtained by numerical analysis for the transverse magnetization, i.e., case I, with vanishing y component and nonzero x component, for $\epsilon/J = 0.1$ and $\gamma = 0.1$. When the temperature is high enough, the system does not magnetize, similar to the case of an ordered system, i.e., $\epsilon/J = 0$. However, below the critical temperature (i.e., if $\beta > \beta_{c,\perp}^{\epsilon,2}$), the system magnetizes in the direction transverse to the applied random field. We see that the critical temperature decreases in the presence of the disorder, i.e., that the critical point, $\beta_{c,\perp}^{\epsilon,2}$, shifts toward the right in the presence of the random field. We also find excellent agreement between the exact numerical results and the approximate analytical expression of the transverse magnetization derived using a perturbative approach given in Eq. (18) [see the inset of Fig. 2(a)].

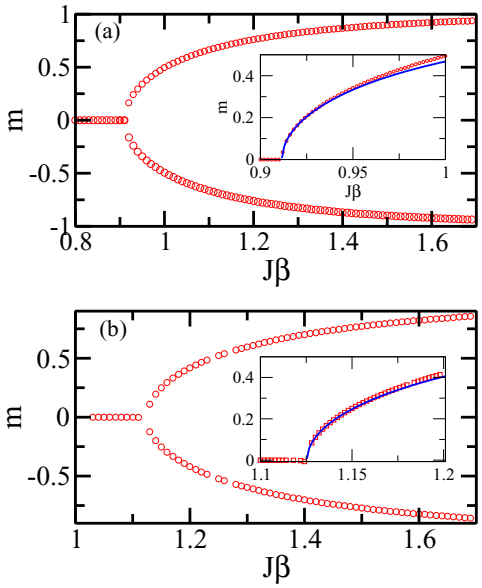

FIG. 2. Numerical and analytical results exhibit persistence of spontaneous magnetization in specific directions even after insertion of disorder. Numerical results for the magnetization as a function of $J\beta$, in the directions (a) transverse and (b) parallel to the disordered field. Red circles correspond to the roots of Eqs. (6) and (7) with $\epsilon/J = 0.1$ and $\gamma = 0.1$. Insets: The blue solid lines correspond to the analytic solutions derived for small m given in Eqs. (18) and (20) for the same set of parameters. The red circles are the numerical results. We find that the numerical and analytical results agree in the small-m regime. All quantities are dimensionless.

The numerical results for case II with a vanishing x component and a nonzero y component are shown in Fig. 2(b) for $\epsilon/J = 0.1$ and $\gamma = 0.1$. The features of parallel magnetization are qualitatively similar to that of transverse magnetization. However, we find that the critical point, $\beta_{c,\parallel}^{\epsilon,2}$, as may be expected by now given the analytical results, shifts toward an even higher value compared to the case of transverse magnetization. A closer examination of Figs. 2(a) and 2(b) shows that the effect of disorder is more prominent in the parallel magnetization than in the transverse one. This is confirmed by the expressions derived in the small-m regime [see Eqs. (18) and (20)].

The behavior of the transverse and the parallel magnetizations for a given ϵ and selectively chosen values of the anisotropy constant, γ, is demonstrated in Fig. 3. We find that the inverse critical temperature, $\beta_{c,\perp}^{\epsilon}$, decreases with increasing γ for the case when the system magnetizes in the direction that is transverse to the applied random field. The opposite happens when the system magnetizes in the direction that is parallel to the random field. The insets of Figs. 3(a) and 3(b) show the critical temperatures β_c's as functions of γ. The trends suggest that for highly anisotropic systems, the parallel magnetization would occur only at sufficiently low temperatures. High anisotropy favors transverse magnetization, i.e., the system starts magnetizing in the transverse direction at comparatively higher temperatures.

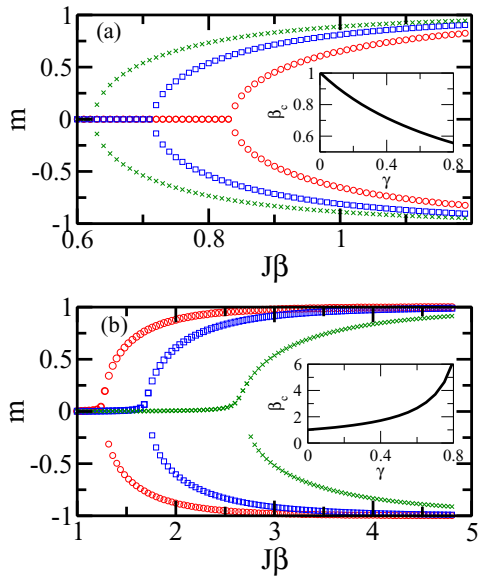

FIG. 3. Magnetization as a function of $J\beta$ for different choices of the anisotropy constant, γ, in the directions (a) transverse and (b) parallel to the random field. Circles, squares, and crosses correspond to the magnetization of the system with $\epsilon/J = 0.1$ and $\gamma = 0.2, 0.4$, and 0.6, respectively. Insets show the inverse critical temperatures as functions of γ for $\epsilon/J = 0.1$. All quantities are dimensionless.

For $\epsilon = 0$ and $\gamma = 0$, Hamiltonian (4) has U(1) symmetry. As seen from Eq. (4), positive values of the parameter γ destroy this symmetry, leading to an anisotropic system that magnetizes due to a $\boldsymbol{Z}^2$-symmetry breaking mechanism, both in the direction transverse to the random field and in the direction parallel to it. In particular, the Mermin-Wagner-Hohenberg result no longer applies, so that the system magnetizes in two dimensions, unlike in the $\gamma = 0$ case. Magnetization persists in the presence of a (small) random field directed along the y axis. Figure 3 presents the values of the magnetization in both directions for three different values of γ. The critical temperature for the onset of the transverse magnetization is higher than that for the onset of the parallel magnetization, and the latter is smaller than the former. One also sees from Fig. 3(a) that the critical temperature for the onset of the transverse magnetization grows with γ (β_c decreases with γ). Figure 3(b) demonstrates the reversal of this tendency for the parallel magnetization.

As seen from Eqs. (19) and (21), the presence of the random field lowers the critical temperatures at which both magnetizations appear, but the effect is more pronounced in the case of parallel magnetization. This is consistent with the fact that for the two-dimensional system at $\gamma = 0$, it is the very presence of the disorder that makes the system magnetize in the transverse direction, i.e., a small random field actually enhances the value of the transverse magnetization.

It is worth mentioning here that we work within the equilibrium scenario, where the properties of the system depend only on the given choice of the system parameters. Our results tell us that within this treatment, there are three regions on the temperature scale with respect to spontaneous magnetizability. Region 1: the system does not magnetize in any direction; region 2: for a window of temperature below region 1, it magnetizes in a single direction; region 3: for still lower temperatures, it magnetizes in two orthogonal directions. The methods needed to reach these phases, from a possibly high temperature, are an important aspect, but they are beyond the scope of the present study. We would like to mention, however, that the spontaneous magnetization requires an infinitesimal external field to manifest itself. Therefore, in principle, one can imagine that the material is cooled through regions 1 and 2 where the system remains unmagnetized, and a symmetry-breaking infinitesimal field is applied only when we are inside region 3, causing the system to magnetize accordingly (depending on the symmetry-breaking field).

III. DISORDER-INDUCED ENHANCEMENT: RANDOM FIELD QUANTUM *XY* MODEL IN THE PRESENCE OF AN ADDITIONAL UNIFORM FIELD

Until now, we have seen that the spontaneous magnetization in the system persists, albeit only in a restricted set of directions, even in the presence of a disordered field. Is this still true when there is an additional constant field? In this section, we consider this question and show that not only does the spontaneous magnetization persist, but disorder can now help one of the components of the magnetization to achieve an enhanced value compared to the ordered system.

We first consider the case in which the ordered XY model is subject to a constant magnetic field, $\vec{h}$. The mean-field Hamiltonian, H_h, governing the system in this case is given by

$$H_h = -(J[(1+\gamma)m_x\sigma_x + (1-\gamma)m_y\sigma_y] + \vec{h}\cdot\vec{\sigma}). \quad (22)$$

The constant field $\vec{h}$ lies in the XY plane, i.e., $\vec{h} = (h_x, h_y) = (h\cos\theta, h\sin\theta)$ with magnitude h, where $0 < h \leqslant 1$, and phase θ, with $-\pi/2 \leqslant \theta \leqslant \pi/2$. In the presence of the constant field, the mean-field equation for the magnetization is obtained replacing H by H_h in Eq. (5). The system now has no critical temperature, as there is always a unique solution at any value of β.

Let us now investigate the effect of a random field, $\epsilon\vec{\eta}$, on the system. The mean-field Hamiltonian $H_{h,\epsilon}$ can be written as

$$H_{h,\epsilon} = -(J[(1+\gamma)m_x\sigma_x + (1-\gamma)m_y\sigma_y] + \vec{h}\cdot\vec{\sigma} + \epsilon\eta\sigma_y), \quad (23)$$

where we assume the random field to be directed along the y axis. Replacing again H by $H_{h,\epsilon}$ in Eq. (5), we obtain two coupled equations, which we solve to find $\vec{m}$. As may be expected, the solution for the magnetization is again unique.

Our numerical calculations show that the magnetization m and the y component of the magnetization vector m_y are

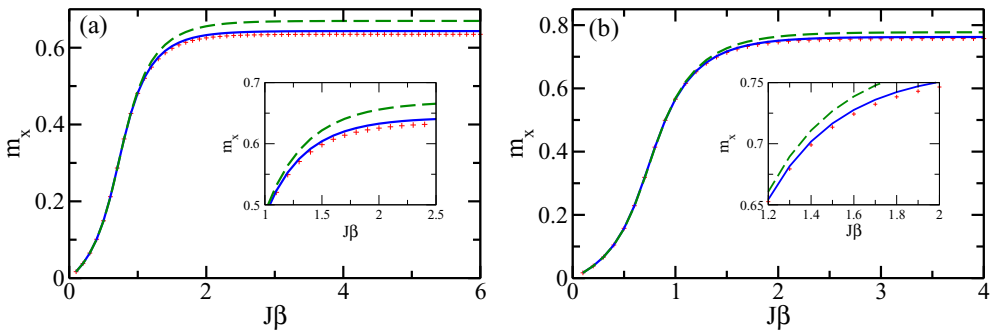

FIG. 4. Disorder-induced enhancement. Effect of the random field in the presence of a constant field. The X component of magnetization, m_x, as a function of $J\beta$ for (a) $\gamma = 0.05$ and (b) $\gamma = 0.1$. Red pluses show m_x for the case when the XY model is subjected to a constant field $\vec{h}$ with $h/J = 0.3$ and $\theta = \pi/3$. The blue solid and green dashed lines represent m_x when the system is treated with a random field of strength $\epsilon/J = 0.1$ and 0.2, respectively, along with the constant field [the corresponding Hamiltonian is given in Eq. (23)]. The insets show blowups of the same for a smaller range of $J\beta$. The enhancement of m_x in the presence of the disorder field uncovers a "random field induced order." Comparing panels (a) and (b), we observe disorder-induced enhancement as the anisotropy parameter is cranked down for fixed h/J and θ. θ is in radians. All other quantities are dimensionless.

reduced in length in the presence of disorder, i.e., when the system is governed by $H_{h,\epsilon}$ as compared to the ordered system described by H_h. However, the x component m_x behaves in a very different manner. Depending upon the system parameters, m_x can be both higher and lower than its value in the ordered system. In Fig. 4, we exhibit the results in the particular example for the system with $h/J = 0.3$, $\theta = \pi/3$, which demonstrate the random field-induced enhancement of m_x in the presence of disorder for two different values of γ. Our numerical observations are further supported by results obtained analytically via a perturbative approach at low temperature. The details are discussed below.

Perturbative analysis of low-temperature magnetization

The mean-field equations in Eq. (5) can be alternatively presented as

$$m_x = \frac{1}{\beta J(1+\gamma)}\frac{\partial \Gamma}{\partial m_x} \tag{24}$$

and

$$m_y = \frac{1}{\beta J(1-\gamma)}\frac{\partial \Gamma}{\partial m_y}, \tag{25}$$

where

$$\Gamma \equiv \mathrm{av}_\eta[\log_e \mathrm{Tr}\exp(-\beta H_h)], \tag{26}$$

or in the disordered case,

$$\Gamma \equiv \mathrm{av}_\eta[\log_e \mathrm{Tr}\exp(-\beta H_{h,\epsilon})], \tag{27}$$

depending on whether the governing Hamiltonian is H_h or $H_{h,\epsilon}$. The symmetry of the Gaussian distribution of η ensures that $\vec{m}$ is an even function of ϵ. As a result, $\frac{dm_x}{d\epsilon}$ and $\frac{dm_y}{d\epsilon}$ vanish at $\epsilon = 0$. Starting with Eqs. (24) and (25), straightforward algebra leads to the following set of coupled equations:

$$\frac{d^2m_x}{d\epsilon^2}\left[1 - \frac{1}{\beta J(1+\gamma)}\frac{\partial^2\Gamma}{\partial m_x^2}\right] = \frac{1}{\beta J(1+\gamma)}\left[\frac{\partial^3\Gamma}{\partial^2\epsilon\partial m_x} + \frac{\partial^2\Gamma}{\partial m_y\partial m_x}\frac{d^2m_y}{d\epsilon^2}\right] \tag{28}$$

and

$$\frac{d^2m_y}{d\epsilon^2}\left[1 - \frac{1}{\beta J(1-\gamma)}\frac{\partial^2\Gamma}{\partial m_y^2}\right] = \frac{1}{\beta J(1-\gamma)}\left[\frac{\partial^3\Gamma}{\partial^2\epsilon\partial m_y} + \frac{\partial^2\Gamma}{\partial m_y\partial m_x}\frac{d^2m_x}{d\epsilon^2}\right], \tag{29}$$

where the total and partial derivatives are evaluated at $\epsilon = 0$. To evaluate $\frac{d^2m_x}{d\epsilon^2}$ and $\frac{d^2m_y}{d\epsilon^2}$, at $\epsilon = 0$, we need to calculate the partial derivatives at $\epsilon = 0$. For example, the expression for $\frac{1}{\beta J(1+\gamma)}\frac{\partial^2}{\partial\epsilon^2}\frac{\partial\Gamma}{\partial m_x}$ is

$$\frac{1}{\beta J(1+\gamma)}\frac{\partial\Gamma}{\partial m_x} = m_x = \frac{Jm_x(1+\gamma)+h_x}{k_\epsilon{}'}\tanh(\beta k_\epsilon{}'). \tag{30}$$

It follows that

$$\frac{1}{\beta J(1+\gamma)}\frac{\partial^2}{\partial\epsilon^2}\frac{\partial\Gamma}{\partial m_x} = \mathrm{av}_\eta\Bigg[[Jm_x(1+\gamma)+h_x]\Bigg(\frac{-3\beta\eta^2[h_y+Jm_y(1-\gamma)+\epsilon\eta]^2}{k_\epsilon{}'^4\cosh^2(\beta k_\epsilon{}')} + \frac{\beta\eta^2}{k_\epsilon{}'^2[\cosh^2(\beta k_\epsilon{}')]} - \frac{\eta^2\tanh(\beta k_\epsilon{}')}{k_\epsilon{}'^3}$$
$$+\frac{3\eta^2[h_y+Jm_y(1-\gamma)+\epsilon\eta]^2\tanh(\beta k_\epsilon{}')}{k_\epsilon{}'^5} - \frac{2\beta^2\eta^2[h_y+Jm_y(1-\gamma)+\epsilon\eta]^2\tanh(\beta k_\epsilon{}')}{k_\epsilon{}'^3\cosh^2(\beta k_\epsilon{}')}\Bigg)\Bigg]. \tag{31}$$

Here $k_\epsilon{}' = \sqrt{\{[J(1+\gamma)m_x]+h_x\}^2 + \{[J(1-\gamma)m_y]+\epsilon\eta+h_y\}^2}$. Next, using the asymptotic expansion of the hyperbolic function $\tanh(\beta k_\epsilon{}') \approx 1\text{–}2\exp(-2\beta k_\epsilon{}')$, we obtain for the partial derivatives at $\epsilon = 0$,

$$\left.\frac{d^2m_x}{d\epsilon^2}\right|_{\epsilon=0} = \frac{1}{h^2}P\left(\theta,\frac{J}{h}\right) + O(e^{-\beta}) \tag{32}$$

and

$$\left.\frac{d^2m_y}{d\epsilon^2}\right|_{\epsilon=0} = \frac{1}{h^2}Q\left(\theta,\frac{J}{h}\right) + O(e^{-\beta}), \tag{33}$$

where the functions P and Q are given by (for $j = J/h$)

$$P(\theta,j) = \frac{AE+BC}{DE-CC'} \tag{34}$$

and

$$Q(\theta,j) = \frac{AC'+BD}{DE-CC'}. \tag{35}$$

Here,

$$A(\theta,j) = a\cos x\left(\frac{3b^2\sin^2\theta}{k'^5} - \frac{1}{k'^3}\right), \tag{36}$$

$$B(\theta,j) = b\sin x\left(\frac{3b^2\sin^2\theta}{k'^5} - \frac{3}{k'^3}\right), \tag{37}$$

$$C(\theta,j) = -\frac{Jab\cos\theta\sin\theta}{k'^3}(1-\gamma), \tag{38}$$

$$C'(\theta,j) = -\frac{Jab\cos\theta\sin\theta}{k'^3}(1+\gamma), \tag{39}$$

$$D(\theta,j) = 1 - \frac{J(1+\gamma)}{k'}\left(1 - \frac{a^2\cos\theta}{k'^2}\right), \tag{40}$$

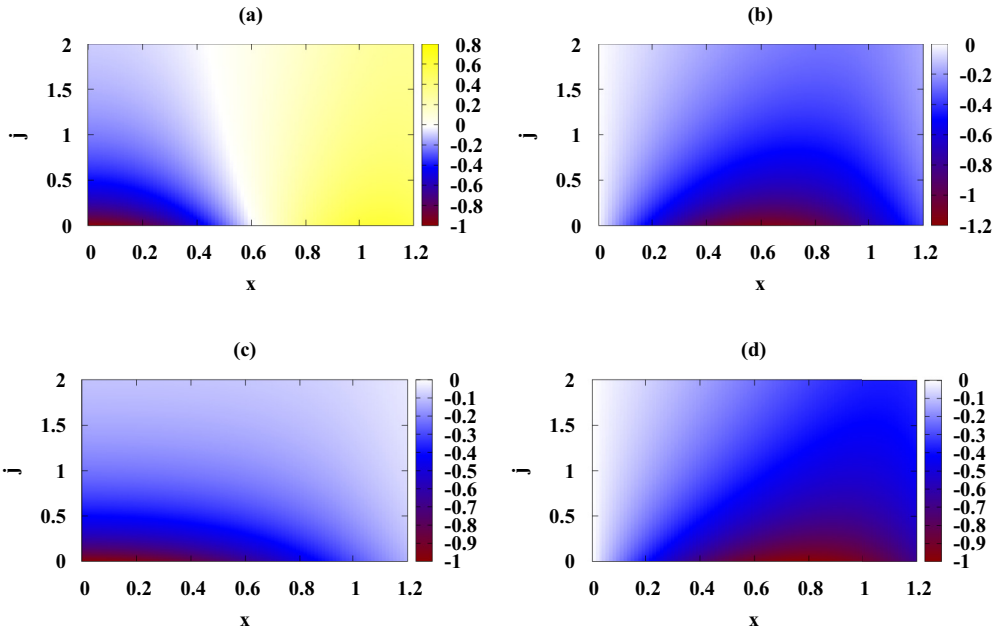

FIG. 5. The span of disorder-induced enhancement in parameter space. Plots of the functions (a) $P(\theta,j)$, (b) $Q(\theta,j)$, (c) $R(\theta,j)$, and (d) $S(\theta,j)$ with respect to θ and $j=J/h$. Note that there are ranges of the (θ,j), for which the function $P(\theta,j)$ is positive, signaling disorder-induced enhancement in the system described by the Hamiltonian $H_{h,\epsilon}$. However, this is not true for $Q(\theta,j)$ and $R(\theta,j)$, which are negative for the entire range of θ and j. θ is in radians. The fact that $S(\theta,j)$ is also negative in the entire range implies that in the presence of disorder, the magnetization vector moves away from the direction of the applied random field. All other quantities are dimensionless. Here, $\gamma=0.1$.

and

$$E(\theta,j)=1-\frac{J(1-\gamma)}{k'}\left(1-\frac{b^2\sin\theta}{k'^2}\right), \tag{41}$$

with $a=1+j(1+\gamma)$, $b=1+j(1-\gamma)$, and $k'=\sqrt{a^2\cos^2\theta+b^2\sin^2\theta}$. The positivity of $P(\theta,j)$ implies that disorder-induced enhancement occurs.

As is clear from Fig. 5(a), there exists a region in the parameter space (θ,j), for which $P(\theta,j)>0$, which confirms that the quenched averaged X component m_x of the magnetization is enhanced by the presence of disorder. This does not hold for the quenched averaged Y component, m_y, which is reduced in length in the presence of disorder [see Fig. 5(b)].

To further investigate the effect of disorder on the length m and phase ϕ_1 of the magnetization, we expand $\tan\phi_1=\frac{m_y}{m_x}$ as

$$\tan\phi_1=\frac{m_y}{m_x}\bigg|_{\epsilon=0}+\epsilon^2\frac{d^2}{d\epsilon^2}\left(\frac{m_y}{m_x}\right)\bigg|_{\epsilon=0}+O(\epsilon^4), \tag{42}$$

with

$$\begin{aligned}\frac{d^2}{d\epsilon^2}\left(\frac{m_y}{m_x}\right)\bigg|_{\epsilon=0}&=\frac{m_x\frac{d^2m_y}{d\epsilon^2}-m_y\frac{d^2m_x}{d\epsilon^2}}{m_x^2}\bigg|_{\epsilon=0}\\&=\frac{1}{m_x^2|_{\epsilon=0}}\frac{1}{h^2}S(x,j)+O(e^{-\beta}),\end{aligned} \tag{43}$$

where

$$S(\theta,j)=Q(\theta,j)\cos x-P(\theta,j)\sin\theta. \tag{44}$$

$S(\theta,j)$ is negative for all θ and J/h [see Fig. 5(d)], implying that the phase always shifts toward the X axis in the presence of the random field.

The square of the length of the magnetization, when similarly expanded, is given by

$$m_x^2+m_y^2=\left(m_x^2+m_y^2\right)\big|_{\epsilon=0}+2\epsilon^2[R(\theta,j)+O(e^{-\beta})], \tag{45}$$

where

$$R(\theta,j)=(P\cos\theta+Q\sin\theta)\big|_{\epsilon=0}. \tag{46}$$

As seen in Fig. 5(c), $R(\theta,j)$ is negative regardless of the choice of parameters, i.e., the length of the magnetization decreases in the presence of the disorder. Note that the analytical results are in agreement with the numerical evidence presented above. It is worth mentioning here that the analytical results are valid for small ϵ and large β. The difference between the magnetization in the disordered system and the ordered system, as obtained analytically, is of the order of ϵ^2. Comparison of these analytical results with the numerical ones is valid only when the same difference, obtained numerically, has precision of order ϵ^2.

IV. GENERALIZATION TO ARBITRARY SPINS AND SCALING OF CRITICAL TEMPERATURE

In this section, our aim is to investigate d-dimensional lattices where the occupant of each lattice site is a quantum spin with arbitrary spin angular momentum. Here we restrict ourselves to the XX model. For our purposes, it is necessary to treat the half-integer and integer spins separately. In the following subsections, we derive the generalized expressions for the scaling of the magnetization and critical temperature for both cases.

A. Half-integer spins

The mean-field equations for a general half-integer spin $\frac{n+1}{2}$ ($n = 1,3,\dots$) are

$$m_x = \mathrm{av}_\eta\left[\frac{Jm_x}{k}\frac{\sum_{p=0}^{n}(2p+1)\sinh[(2p+1)\beta k]}{\sum_{p=0}^{n}\cosh[(2p+1)\beta k]}\right], \tag{47}$$

$$m_y = \mathrm{av}_\eta\left[\frac{Jm_y+\epsilon\eta}{k}\frac{\sum_{p=0}^{n}(2p+1)\sinh[(2p+1)\beta k]}{\sum_{p=0}^{n}\cosh[(2p+1)\beta k]}\right], \tag{48}$$

where $k = \sqrt{J^2m_x^2 + (Jm_y+\epsilon\eta)^2}$.

Finding the magnetization $\vec{m}$ requires simultaneous solution of the coupled set of Eqs. (47) and (48), i.e., finding the common zeros of the following two functions:

$$F_x^{\epsilon,n}(\vec{m}) = \mathrm{av}_\eta\left[\frac{Jm_x}{k}\frac{\sum_{p=0}^{n}(2p+1)\sinh[(2p+1)\beta k]}{\sum_{p=0}^{n}\cosh[(2p+1)\beta k]}\right] - m_x, \tag{49}$$

$$F_y^{\epsilon,n}(\vec{m}) = \mathrm{av}_\eta\left[\frac{Jm_y+\epsilon\eta}{k}\frac{\sum_{p=0}^{n}(2p+1)\sinh[(2p+1)\beta k]}{\sum_{p=0}^{n}\cosh[(2p+1)\beta k]}\right] - m_y. \tag{50}$$

The Taylor expansion in ϵ, followed by the expansion in m, of the functions given in Eqs. (49) and (50), around $\epsilon = 0$ and $m = 0$, gives

$$\begin{aligned}F_x^{\epsilon,s}(\vec{m}) = {}& \frac{1}{45}[-45 + 60J\beta s(s+1) - 8Js(s+1)(2s^2+2s+1)\beta^3\epsilon^2]m\cos\phi_1 \\ & + \frac{1}{3!}\frac{16}{315}\beta^3J^3\{s(s+1)[-21(2s^2+2s+1) + 2\beta^2\epsilon^2(4s^2+2s+1)(4s^2+6s+3)]\}m^3\cos\phi_1 + O(m^5)\end{aligned} \tag{51}$$

and

$$\begin{aligned}F_y^{\epsilon,s}(\vec{m}) = {}& \frac{1}{15}[-15 + 20J\beta s(s+1) - 8Js(s+1)(2s^2+2s+1)\beta^3\epsilon^2]m\sin\phi_1 \\ & + \frac{1}{3!}\frac{16}{315}\beta^3J^3\{s(s+1)[-21(2s^2+2s+1) + 10\beta^2\epsilon^2(4s^2+2s+1)(4s^2+6s+3)]\}m^3\sin\phi_1 + O(m^5),\end{aligned} \tag{52}$$

where $s = n + 1/2$ with $n = 0,1,2,\dots,\phi_1$ has two allowed values: $\pi/2$ (the system magnetizes in the direction parallel to the disordered field) and 0 (the system magnetizes in the direction transverse to the disordered field). For transverse magnetization, $F_y^{\epsilon,n}(\vec{m})$ vanishes, and two nontrivial solutions solely come from Eq. (51) as

$$m_\perp^{\epsilon,s} = \pm\sqrt{\frac{21}{8}}\sqrt{\frac{45 - 60J\beta s(s+1) + 8Js(s+1)(2s^2+2s+1)\beta^3\epsilon^2}{J^3\beta^3s(s+1)[-21(2s^2+2s+1) + 2\beta^2\epsilon^2(4s^2+2s+1)(4s^2+6s+3)]}}. \tag{53}$$

The critical point can now be easily obtained by setting $m_\perp^{\epsilon,s} = 0$ in Eq. (53). We get

$$8s(s+1)(2s^2+2s+1)J\beta^3\epsilon^2 - 60s(s+1)J\beta + 45 = 0, \tag{54}$$

which gives

$$\beta_{c,\perp}^{\epsilon,s} = \frac{3}{4Js(s+1)} + \frac{9}{160}\frac{(2s^2+2s+1)}{J^3s^3(s+1)^3}\epsilon^2. \tag{55}$$

The critical temperature decreases with the increase in the number of spins. The shift in critical temperature is of the order of ϵ^2 for all spins. Note that the generalized expressions for the scaling and for the critical temperature for the pure system with a transverse magnetization can be obtained simply by putting $\epsilon = 0$ in Eqs. (53) and (55), respectively.

To find the expressions for the parallel magnetization, we put $\phi_1 = \pi/2$ in Eqs. (51) and (52). In this case also, the right-hand side of Eq. (51) vanishes to leading order, while Eq. (52) has two nontrivial solutions, given by

$$m_\parallel^{\epsilon,s} = \pm\sqrt{\frac{63}{8}}\sqrt{\frac{15 - 20Js(s+1)\beta + 8Js(s+1)(2s^2+2s+1)\beta^3\epsilon^2}{J^3\beta^3s(s+1)[-21(2s^2+2s+1) + 10\beta^2\epsilon^2(4s^2+2s+1)(4s^2+6s+3)]}}. \tag{56}$$

The critical point can be obtained by considering $m_\parallel^{\epsilon,s} = 0$ in Eq. (56), and we obtain

$$\beta_{c,\parallel}^{\epsilon,s} = \frac{3}{4Js(s+1)} + \frac{27}{160}\frac{(2s^2+2s+1)}{J^3s^3(s+1)^3}\epsilon^2. \tag{57}$$

The generalized expressions of the scaling and the critical temperature for the pure system with a parallel magnetization can again be obtained by putting $\epsilon = 0$ in Eqs. (56) and (57), respectively. However, the shift in the critical temperature due to the random field is bigger in this case than in the transverse case and hence the effect of the disorder is more prominent in the parallel case, similarly to what was seen in Sec. II B without a constant field.

B. Integer spins

The generalized mean-field equations for the system with integer spin $\frac{n}{2}$, n even, are given by

$$m_x = \mathrm{av}_\eta \left[\frac{J m_x}{k} \frac{\sum_{p=1}^{n} 2p(e^{2p\beta k} - e^{-2p\beta k})}{1 + \sum_{p=1}^{n}(e^{2p\beta k} + e^{-2p\beta k})} \right], \tag{58}$$

$$m_y = \mathrm{av}_\eta \left[\frac{J m_y + \epsilon\eta}{k} \frac{\sum_{p=1}^{n} 2p(e^{2p\beta k} - e^{-2p\beta k})}{1 + \sum_{p=1}^{n}(e^{2p\beta k} + e^{-2p\beta k})} \right], \tag{59}$$

where $k = \sqrt{J^2 m_x^2 + (J m_y + \epsilon\eta)^2}$. Now in order to find the magnetization $\vec{m}$, we have to solve the coupled set of Eqs. (58) and (59). As one can expect from the previous discussions, there are two different kinds of magnetizations—the transverse magnetization, $m_\perp^{\epsilon,s}$, and the parallel magnetization, $m_\parallel^{\epsilon,s}$. To derive the critical scaling for this case, we follow a Taylor expansion method, similar to the one used for the half-integer spin case. The final expressions for $m_\perp^{\epsilon,s}$, $m_\parallel^{\epsilon,s}$, and the associated critical temperatures are given by the set of Eqs. (53), (55), (56), and (57) with $s = n$, where $n = 1,2,3,\ldots$.

Therefore, we again obtain corrections of order ϵ^2 to the critical temperature for all the integer spin systems. Again, the effect of disorder is more pronounced in the parallel magnetization case than in the transverse case.

C. Critical temperature versus spin quantum number

To study the effect of disorder as a function of s, we define the dimensionless quantity δ_β, given by

$$\delta_\beta = \frac{\beta_c^{\epsilon,s} - \beta_c^{0,s}}{\beta_c^{0,s}}. \tag{60}$$

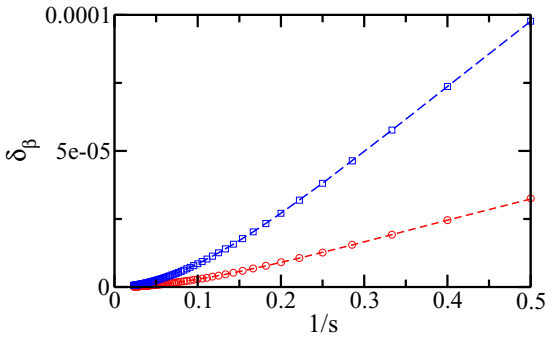

FIG. 6. δ_β as function of $1/s$ for the transverse (red circles) and parallel (blue squares) magnetizations for $\epsilon/J = 0.05$. The lines serve as guides to the eye. All quantities are dimensionless.

δ_β is shown as a function of $1/s$ in Fig. 6 for $\epsilon/J = 0.05$. We find that the shift in the critical temperature caused by the random field decreases with increasing spin quantum number.

V. CONCLUSIONS

We considered the quantum spin-1/2 XY model in the presence of a unidirectional quenched disordered field. Various experimentally feasible proposals to realize the quantum spin systems considered in this paper, and related models, have been discussed in several physical systems [4,5,16,31,36,37], which include Bose-Bose mixtures in optical lattices, the Raman coupled Bose-Einstein condensate (BEC), and Fermi-Bose mixtures. Perhaps the most interesting case is of the isotropic XY model [with U(1) symmetry], which may be realized using a gas of hardcore bosons in an optical lattice embedded in a diluted BEC of the same atoms in a different internal state. The hardcore bosons themselves realize a spin-1/2 XY model with U(1) symmetry, where the spin-up (-down) states at a given site are encoded into the presence (absence) of an atom. Parallel random fields may be realized by random Raman coupling with a fixed phase to the atoms in the BEC. Condensate creation and annihilation (field) operators for large BEC can be replaced by c numbers so that the coupling becomes proportional to the sum of hard boson creation and annihilation operators with random coefficients having a fixed phase. Taking the phase to be zero, the coupling corresponds to the desired random parallel field.

In this work, we treated the disordered system within the mean-field approximation and showed that the spontaneous magnetization persists in the system with the introduction of a unidirectional quenched disordered field, albeit it is smaller than in the pure system. Below a certain critical temperature, the magnetization occurs in specific directions, either parallel or transverse to the disordered field. The critical temperatures and the magnitude of the magnetization decrease with increasing strength of the disorder. We found perturbative expressions for scaling of the magnetization and the expressions for the scaling of the critical temperatures at which the system magnetizes. We also performed numerical simulations to obtain the behavior of magnetization for various values of the temperature, the disorder strength, and the anisotropy parameter, which match with the perturbative calculations for small disorder values. Moreover, we extended our analysis to arbitrary values of (half-integer or integer) spin. We found that the decrease in the length of the magnetization due to the random field is of the order of the square of the strength of the disorder for all values of spin. The system requires a lower temperature to magnetize when the spin quantum number increases. The normalized shift in critical temperature due to the disorder (δ_β) also decreases with the increase in the spin quantum number. In addition, we studied the random field quantum spin-1/2 XY model with an additional constant field, for which we showed a random-field-induced ordering in the component of magnetization transverse to the disordered field.

ACKNOWLEDGMENTS

A.B. acknowledges the support of the Department of Science and Technology (DST), Government of India, through

the award of an INSPIRE fellowship. M.L. acknowledges Spanish MINECO (Severo Ochoa Grant No. SEV-2015-0522 and FOQUS FIS2013-46768), Catalan AGAUR SGR 874, Fundació Cellex, ERC AdG OSYRIS, EU IP SIQS, EU STREP EQuaM, and EU FETPROACT QUIC. J.W. was partially supported by NSF Grant No. DMS 131271. A part of his work on this article was supported by U.S. NSF Grant No. DMS-1440140 while he was in residence at the Mathematical Sciences Research Institute in Berkeley during the Fall 2015 semester.

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