Exciton-polariton localized wave packets in a microcavity

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We investigate the possibility of creating X waves, or localized wave packets, in resonantly excited exciton-polariton superfluids. We demonstrate the existence of X-wave traveling solutions in the coupled exciton-photon system past the inflection point, where the effective mass of lower polaritons is negative in the direction perpendicular to the wave vector of the pumping beam. Contrary to the case of bright solitons, X waves do not require nonlinearity for sustaining their shape. Nevertheless, we show that nonlinearity is important for their dynamics, as it allows for their spontaneous formation from an initial Gaussian wave packet. Unique properties of exciton-polaritons may lead to applications of their X waves in long-distance signal propagation inside novel integrated optoelectronic circuits based on excitons.

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I. INTRODUCTION

Sending an optical signal without distortion necessitates control over diffraction and dispersion of the traveling pulse. This can be achieved by creation of strongly confined, propagation-invariant wave packets, which behave like particles rather than waves. Perhaps the most well-known example of such a wave packet is a soliton, which is a self-localized wave existing due to the balance between nonlinearity and dispersion. It is less known that a wide class of qualitatively different solutions, called localized waves, exists and can be more usable than solitons in real-life applications [1,2]. In particular, they exist even in the linear regime, where there is no self-localization mechanism as in the case of solitons. The most interesting localized waves, the X-shaped optical or acoustic pulses, have been applied in numerous practical situations, including large depth of field or high-frame-rate medical imaging, optical tomography, and high-capacity communications [1–5]. They are nonmonochromatic, yet nondispersive superpositions of nondiffracting Bessel beams [6] with characteristic biconical X shape.

Recently, bright exciton-polariton solitons [7,8] as well as gap solitons [9,10] were observed in a semiconductor microcavity, and the potential of traveling polariton pulses for applications in information processing has been pointed out [11–14]. The study of quantum coherent phenomena of exciton-polaritons in optical microcavities is nowadays a very active area of research [15–19].

Exciton-polaritons are quantum quasiparticles which are superpositions of cavity photons and excitons (electron-hole pairs) in a semiconductor [20]. Their composite nature gives rise to mixed properties characteristic to both light and matter. The strong nonlinear interactions between the excitons links the polaritons inherently with nonlinear quantum processes. The photonic part gives rise to an extremely light effective mass and allows for a direct detection of the system evolution on a picosecond time scale.

Here, we show that the creation of confined propagation-invariant exciton-polariton wave packets does not have to rely on the soliton concept [7,21–30]. It was pointed out [25,26] that fully two-dimensional bright polariton solitons can exist due to the peculiar dispersion relation with positive and negative effective masses in orthogonal directions, but only in a very narrow range of pumping powers. Instead of relying on the nonlinearity, we demonstrate that in the same dispersion regime localized X-wave pulses can be created and propagate without distortion over long distances even in the low-density linear regime. There is no necessity to tune the pumping power precisely; in fact the wave packet remains nondiffractive even in the absence of pumping, as we demonstrate in direct simulations of the mean-field Gross-Pitaevskii equations. The nonlinearity, however, plays an important role in the creation of X waves. We show that polariton nonlinear X waves develop spontaneously from a simple Gaussian input pulse in a four-wave mixing process [31]. We note that Bragg X-wave solutions were previously considered in a setup consisting of a large number of quantum wells [19]. In contrast, here we show that X waves can be created in a typical sample with one or several quantum wells.

Uniqueness of the exciton-polariton X waves is manifested by their potential for applications in exciton-based optoelectronic systems. A concept of polariton-based systems operating at 100 GHz–10 THz frequency range has been already proposed, filling a gap between electronics and photonics [32,33]. These operation speeds would be suitable for processing information at rates exceeding 1 Tbit/s, necessary in the contemporary world of information [34]. Transistors, spin-based switches, and even logic gates have been already demonstrated [14,35,36]. However, up to date, dispersion effects were detrimental to signal guidance within polaritonic circuits [37]. Localized X waves propagate over long distances, transferring information with high-speed and low-power requirements. They may overcome the recent slowdown of development in silicon-based chip architecture [38].

II. EXCITON-POLARITONS

Typically, exciton-polaritons are created in planar microcavities, consisting of one or several quantum wells placed between two Bragg reflectors, and pumped by a laser beam.

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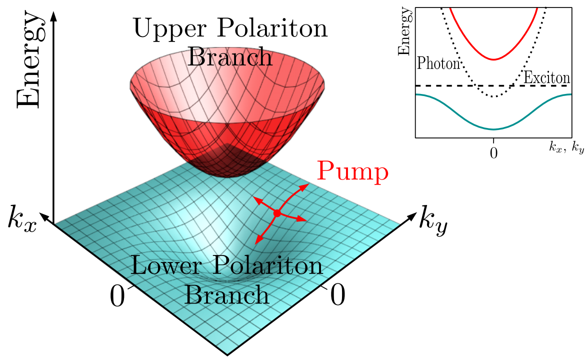

FIG. 1. Dispersion relation for exciton-polaritons in a planar microcavity. A saddle inflection point enables creation of polaritonic X waves. The inset compares the dispersion for photons, excitons, and polaritons in 1D.

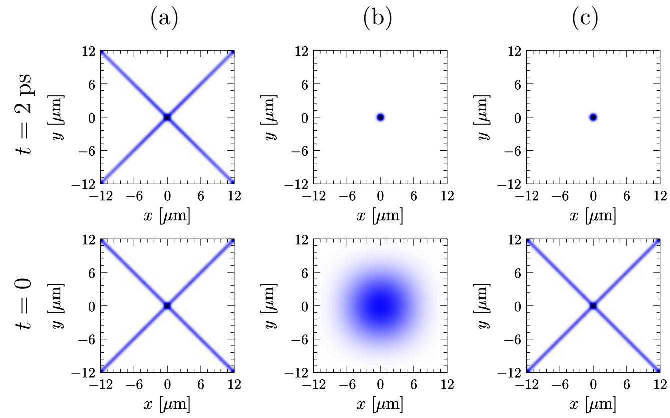

FIG. 2. Evolution of $\psi(x,y)$ governed by Eq. (1) of (a) X-shaped stationary state, (b) Gaussian state of initial polariton density 0.8 $(\mu\text{m})^{-2}$, which spreads in time. (c) In the presence of nonlinearity $g = 20\ \mu\text{eV}\ (\mu\text{m})^2$, the Gaussian spontaneously turns into the X-shaped state. Computation parameters are given in the text.

Excitons are formed in wells located at the antinodes of the photonic mode. Just like photons, they are strongly confined in the direction along the cavity axis, but are free to move in the transversal x-y plane. The cavity introduces strong coupling between excitons and photons, leading to formation of new bosonic quasiparticles—the exciton-polaritons—which are superpositions of the photonic and excitonic component with wave functions ψ_c and ψ_x, respectively. Their dispersion relation, shown in Fig. 1, results from crossing of dispersions for photons and excitons and consists of two branches: for the lower and the higher energy solution, the lower (LP) and upper polariton, respectively. We focus exclusively on the LPs. Their dispersion reveals a saddle inflection point $k_{\text{inflection}}$, highlighted in red, which is the crucial feature enabling generation of X waves. They possess a relatively light effective mass, determined by the photon dispersion, and a rather short lifetime, up to 100 ps, due to the leakage of cavity photons. Thus, external pumping is necessary for creating the population of polaritons at a level necessary for obtaining the superfluid phase. We assume this is done by a strong pump with momentum k_0, tuned to the saddle point in the dispersion (highlighted red in Fig. 1), and the LPs are created in a coherent fashion near this point.

III. POLARITONIC X WAVES

The first sign of the existence of polaritonic X waves comes from the simplified description of LPs, in terms of a single wave function, disregarding their compound nature. In this approach the LP superfluid can be directly mapped to an atomic Bose-Einstein condensate (BEC) in a potential of an external lattice, the system where the matter X waves are recognized [39]. Thus, the dynamics of polaritons are well described in the mean-field approximation by the generic nonlinear Schrödinger equation, equivalent to the Gross-Pitaevskii (GP) equation [1]

$$i\hbar\frac{d\psi}{dt} + \frac{\hbar^2}{2m_x}\left(\partial_x^2 - \frac{m_x}{m_y}\,\partial_y^2\right)\psi - \frac{3g}{2}|\psi|^2\psi = 0. \quad (1)$$

Here ψ denotes the LP wave function which corresponds to the BEC one, t is time, $\hbar$ is the Planck constant, and g quantifies the polariton-polariton interaction and mimics the interaction in the BEC. The quasiparticle character of LPs is captured by a peculiar feature that their mass measured in the direction x and y is different: m_x and m_y, respectively. This allows us to complete the analogy to the BEC system by identifying, e.g., m_x with the atomic mass and $-m_y$ with the negative mass associated with the lattice (or vice versa). An important feature of (1) is its hyperbolic form—the signs in front of the second derivatives are opposite. This provides (1) with stationary X-wave solutions.

We will first seek for the family of stationary solutions of (1) for the linear case $g = 0$ in the form of plane waves $\psi(x,y) = \psi^0 e^{ik_x x + ik_y y - \omega t}$. In particular, we are interested in the family of solutions fulfilling the condition $k_y = \pm\sqrt{\frac{m_y}{m_x}}k_x$, i.e., $\omega = 0$. A general stationary solution can be easily constructed using its Fourier transform $\psi(x,y) = \frac{1}{2\pi}\int_{\mathbb{R}^2}\tilde{\psi}(k_x,k_y)e^{ik_x x + ik_y y}dk_x dk_y$, with $\tilde{\psi}(k_x,k_y)$ fulfilling the above relation between k_y and k_x. We take its simplest form being a convolution of an arbitrary envelope function, e.g., a Gaussian in k_x, and a characteristic function $\chi(k_x,k_y)$ on the set of (k_x,k_y), equal 1 if $\omega = 0$ and 0 otherwise, $\tilde{\psi}(k_x,k_y) \propto e^{-k_x^2}\chi(k_x,k_y)$. This state initially X-shaped both in the configuration and the Fourier space, when subjected to the evolution set by (1), indeed does not change in time as is shown in Fig. 2(a). This is in contrast to a Gaussian initial state which spreads quickly; see Fig. 2(b). The computations were performed for $m_x = m_y = 2\times10^{-5}m_e$.

Strong nonlinearity in (1) introduces a dramatic change in the behavior of the system; it allows creation of the X wave spontaneously from a Gaussian initial state, see Fig. 2(c) computed for $g = 20\ \mu\text{eV}\ (\mu\text{m})^2$ and initial Gaussian state of polariton density 0.8 $(\mu\text{m})^{-2}$. We note that similar behavior has been observed previously in the context of nonlinear optics, which explained the spontaneous localization of pulses in a medium with normal dispersion [40–42]. High-intensity X waves, unlike linear ones, can be formed spontaneously through a trigger mechanism of conical emission [43]. This is a great simplification in terms of the possibility of experimental observation of X waves, since they have a rather complex structure in the Fourier space. Nevertheless, the analysis of the linear case is very instructive for describing the shape of the X-shaped stationary state and for obtaining optimal parameters ensuring stable numerical simulations.

Repeating the steps analogous to the ones presented above makes it is possible to find the X-wave solutions for a set of coupled GP equations describing the polaritonic superfluid [25,26,44]

$$\begin{aligned} i\hbar\frac{d}{dt}\psi_c - \Omega_R\psi_x - \left(\frac{\hbar\gamma_c}{2i} - \frac{\hbar^2}{2m_c}\partial_x^2 - \frac{\hbar^2}{2m_c}\partial_y^2\right)\psi_c = 0, \\ i\hbar\frac{d}{dt}\psi_x - \Omega_R\psi_c - \left(\frac{\hbar\gamma_x}{2i} + g|\psi_x|^2\right)\psi_x = 0, \end{aligned} \tag{2}$$

where m_c is the effective mass of polaritons, Ω_R is the Rabi frequency coupling the excitonic and photonic modes, and γ_c and γ_x are decay rates for the photons and excitons, respectively. Now the task is more challenging since the stationary X-wave solution for (2) does not exist at $k = 0$. This is because the signs in front of the derivatives in the x and y directions are the same, which reflects the fact that in a microcavity there is no distinguished direction for the polaritons. Flipping one of the signs is necessary for the existence of the X wave. Therefore, we will construct an initial state past the inflection point, for which second derivatives have opposite signs. To this end, we will first look for analytical plane wave solutions of (2) for the ideal case ($\gamma_x = \gamma_c = 0$) and in the noninteracting (linear) regime $g = 0$. We will assume that the waves are moving with a constant velocity v in the y direction, $\psi_{x,c}(x,y,t) = \phi_{x,c}(x,y - vt)e^{-i\mu t/\hbar}$. Such solutions fulfill the relation $\frac{\partial\psi_{x,c}}{\partial t} = -v\frac{\partial\psi_{x,c}}{\partial y} - i\frac{\mu}{\hbar}\psi_{x,c}$, which in Fourier space translates to

$$\omega = vk_y + \frac{\mu}{\hbar}. \tag{3}$$

Some specific values of ω and μ obeying (3) provide a solution that has an X shape and is quasilocalized in space. We obtain them from the dispersion for LPs, which results from the form of the plane wave solutions $\psi_{x,c}(x,y,t) = \psi_{x,c}^0 \exp(ik_x x + ik_y y - i\omega t)$ and the GP equations (2)

$$\hbar\omega = \frac{1}{2}\left(\epsilon_k - \sqrt{\epsilon_k^2 + 4\Omega_R^2}\right), \quad \phi_x = \frac{\Omega_R}{\hbar\omega}\phi_c, \tag{4}$$

with $\epsilon_k = \frac{\hbar^2}{2m_c}(k_x^2 + k_y^2)$. Note that the superposition of plane waves linked with (3) amounts to a moving solution. Importantly, the dependence $\omega(k_x,k_y)$ in (4) ensures $d^2\omega/dk_y^2 < 0$ if $k_y > k_{\text{inflection}}$ while $d^2\omega/dk_x^2 > 0$ when $k_x < k_{\text{inflection}}$. In our simulations we take $\Omega_R = 4.4$ meV, $m_c = 2{\times}10^{-5}m_e$, and $k_x = 0$; thus, the inflection point lies at $k_{\text{inflection}} \approx 1.26\ \mu\text{m}^{-1}$.

We will now explicitly construct the X-wave solutions. We take $k_y = k_0 > k_{\text{inflection}}$ and $k_x = 0$. Expansion of (4) in the Taylor series around k_y gives the optimal values $\hbar v = \frac{\epsilon_{k_0}}{k_0} - \frac{\epsilon_{k_0}^2}{k_0\sqrt{\epsilon_{k_0}^2 + 4\Omega_R^2}}$, $\mu = \frac{1}{2}(\epsilon_{k_0} - \sqrt{\epsilon_{k_0}^2 + 4\Omega_R^2}) - \hbar v k_0$, where $\epsilon_{k_0} = \frac{\hbar^2}{2m_c}k_0^2$. We build the solution in its Fourier space, as before for (1), but for a different characteristic function $\chi(k_x,k_y)$, which now results from (3) and (4). We express these conditions, linking k_x and k_y, in the polar coordinates (r,θ)

$$k_x = r(\theta)\cos\theta, \quad k_y = k_0 + r(\theta)\sin\theta, \tag{5}$$

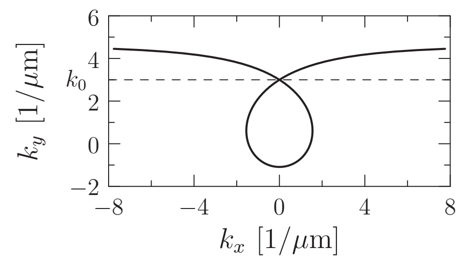

FIG. 3. The line depicts condition (5). Waves from the vicinity of k_0 are used to obtain stationary spatially localized state.

with $r(\theta) = \frac{1}{2a_0 k_0 \sin\theta} - (1 + a_0 k_0^2)k_0 \sin\theta$, $a_0 = \frac{\hbar^2}{2m_c\sqrt{\epsilon_{k_0}^2 + 4\Omega_R^2}}$, and $\epsilon_{k_0} = \frac{\hbar^2}{2m_c}k_0^2$. The Fourier transforms for the photonic and excitonic wave functions equal to

$$\tilde{\psi}_c(k_x,k_y) \propto e^{-(k_y-k_0)^2 - k_x^2}\chi(k_x,k_y), \tag{6}$$

$$\tilde{\psi}_x(k_x,k_y) \propto \frac{2\Omega_R}{\epsilon_k - \sqrt{\epsilon_k^2 + 4\Omega_R^2}}\tilde{\psi}_c(k_x,k_y), \tag{7}$$

where $\chi(k_x,k_y) = 1$ if (k_x,k_y) fulfill (5) and $\chi(k_x,k_y) = 0$ otherwise. Figure 3 depicts $\tilde{\psi}_c(k_x,k_y)$ computed for exemplary values of $k_0 = 3\ \mu\text{m}^{-1}$, $\Omega_R = 4.4$ meV, and $m_c = 2{\times}10^{-5}m_e$.

Having formulated the condition for the initial X-wave semistationary solution, Eqs. (6) and (7), we performed numerical simulations of the X-wave evolution using Eq. (2) for the realistic system parameters, including decay, but without nonlinearity. Solutions of Eq. (2) were found numerically by direct integration using the Runge-Kutta method of the fourth order. Figure 4(a) shows time evolution of an X-wave packet, computed for $k_0 = 3\ \mu\text{m}^{-1}$, $\Omega_R = 4.4$ meV,

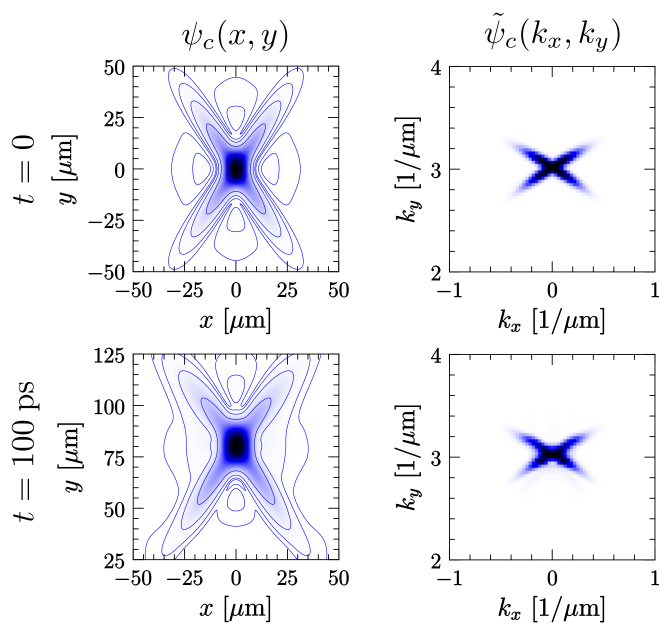

FIG. 4. Linear evolution of (2) for the X-shaped stationary state (6). The state does not spread, but moves with constant velocity. Computation parameters are given in the text.

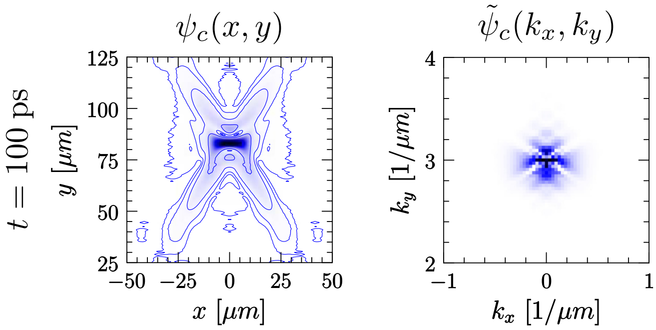

FIG. 5. As in Fig. 4, but with a Gaussian initial state and nonlinearity. The state spontaneously turns into an irregular X wave.

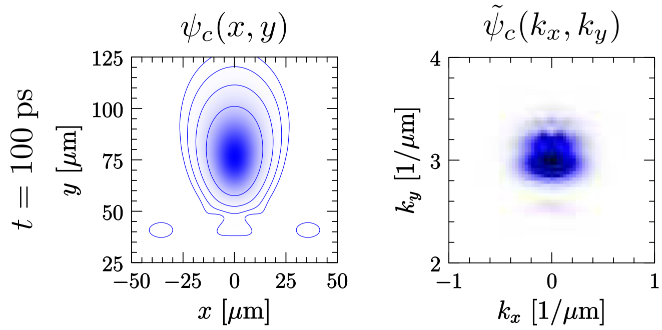

FIG. 6. As in Fig. 5, but in the linear regime. The state decays quickly.

$m_c = 2\times10^{-5}m_e, \gamma_c = 5\times10^{-3}$ ps^{-1}, $\gamma_x = 0.02$ ps^{-1}, $g = 0$, within time interval 100 ps. These parameters correspond to state-of-the-art samples reported in the literature; see, e.g., [49]. The X wave moves with a constant velocity v covering the distance of around 75 μm. Any significant change in the shape of the X wave is notable only for very late times of evolution; e.g., in Fig. 4(b) this is 100 ps, which by far exceeds the average lifetime of polaritons. Since the excitonic wave function (7) is just a rescaled photonic one, its evolution looks qualitatively the same.

In the presence of a nonlinearity, the quasistationary solution of (2) is very similar to (6) and (7) and the position of the inflection point does not change. However, this solution behaves as an attractor and together with the nonlinearity allows for an X-shaped wave packet to be created spontaneously from a Gaussian initial state. The phase matching condition for the four-wave mixing process in energy-momentum space corresponds exactly to the X-wave shape of our solution, as follows from our mathematical construction [31]. This process corresponds to the formation of a 8-shaped state of parametric scattering in the reciprocal space, when the pump is placed in the inflection point [45,46].

Figure 5 depicts the evolution within $t = 100$ ps evaluated for a Gaussian initial state of polariton density 0.8 $(\mu\text{m})^{-2}$, $k_0 = 3\ \mu\text{m}^{-1}$, $\Omega_R = 4.4$ meV, $m_c = 2\times10^{-5}m_e$, $\gamma_c = 5\times10^{-3}$ ps^{-1}, $\gamma_x = 0.02$ ps^{-1}, and nonlinearity $g = 20\ \mu\text{eV}\ (\mu\text{m})^2$. The value of the nonlinearity is related to the order of magnitude predicted theoretically [47,48] and observed in an experiment [49]. A 1% random noise was added in order to test the stability of the results; it seeds all potentially unstable wave vectors in the system, including the ones close to the $k = 0$ mode. Shortly after the beginning of the evolution the characteristic tails of the X-wave develop. Note that the shape of the developed wave packet is not completely the same as in the case of Fig. 4, but some irregularity is present both in real and Fourier space. This is in analogy to the observations made in optical Kerr media [40–42]. X-shaped quasistationary wave packets appeared spontaneously in the normal dispersion regime, but their internal dynamics was rather complex.

To illustrate the strength of the X-wave localization induced by the nonlinearity, we show the result of analogous simulations starting from a Gaussian initial state, but in the linear regime with $g = 0$ (Fig. 6). In this case, due to the dispersion, the initial Gaussian state quickly spreads and decays anisotropically. Note that the distance traveled by the wave packet is similar to that in the nonlinear X-wave case, as this is dictated by the group velocity at the dispersion point which is seeded by the excitation pulse. For the same reason, the X wave cannot be turned into the Gaussian state since it is not a quasistationary solution.

IV. CONCLUSIONS

We have demonstrated the possibility to create localized X-wave solutions in resonantly excited exciton-polariton superfluids. We have constructed stationary X-wave packets by appropriate superposition of plane waves and demonstrated their undistorted movement over large distances. We have shown that X waves can be created with a Gaussian-shaped initial pulse in the case of interacting superfluid. Contrary to bright solitons, X waves can propagate even in the linear regime; hence they are more robust against inherent polariton decay. They do not require constant laser pumping. These properties allow X waves to be used as transmission channels between stations without distortion, contributing to the development of spin-based integrated circuits.

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