Magnetic properties of a two-dimensional electron gas strongly coupled to light

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Considering the quantum dynamics of two-dimensional electron gas (2DEG) exposed to both a stationary magnetic field and an intense high-frequency electromagnetic wave, we found that the wave decreases the scattering-induced broadening of Landau levels. Therefore, various magnetoelectronic properties of two-dimensional nanostructures (density of electronic states at Landau levels, magnetotransport, etc.) are sensitive to irradiation by light. Thus, the elaborated theory paves the way for optically controlling the magnetic properties of 2DEG.

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I. INTRODUCTION

The study of two-dimensional electron gas (2DEG) exposed to a high-frequency electromagnetic field is one of the most exciting areas in the modern physics of nanostructures. The continual interest in this topic originates from the rich fundamental and applied capabilities of two-dimensional electron systems (see, e.g., Refs. [1–3]). In particular, the magnetoelectronic properties of 2DEG subjected to microwave irradiation have been studied actively in recent years [4–16]. However, the largest amount of attention in the subject has been paid to the simplest case of a weak electromagnetic field that does not change electron states. The only effect of the weak field are the field-induced electron transitions between the unperturbed states. In contrast, a strong electromagnetic field can mix the electron states substantially. As a result of this mixing, the composite electron-field object referred to as an "electron dressed by a field" (dressed electron) appears [17,18]. The light-induced renormalization of the physical properties of dressed electrons has been studied in various atomic systems [17–19] and condensed-matter structures, including bulk semiconductors [20–22], quantum wells [23–28], quantum rings [29–31], graphene [32–40], etc. In the present research, we develop a theory to describe the magnetic properties of dressed 2DEG, and we demonstrate that these properties can be modified substantially by the dressing field.

The paper is organized as follows. In the second section, we solve the Schrödinger equation for 2DEG subjected to both a stationary magnetic field and a high-frequency dressing field. In the third section, the found solutions of the Schrödinger problem are used to analyze various magnetoelectronic characteristics of dressed 2DEG, including the density of electron states and magnetotransport. The final section contains our conclusions.

II. SCHRÖDINGER PROBLEM FOR LANDAU LEVELS IN DRESSED 2DEG

Let us consider two-dimensional electron gas (2DEG) confined in the (x,y) plane, which is subjected to both a stationary magnetic field $\mathbf{B}=(0,0,B)$ directed along the z axis and a linearly polarized electromagnetic wave (dressing field) propagating along the same axis z (see Fig. 1). The Hamiltonian of 2DEG reads

$$\hat{\mathcal{H}}_e=\frac{1}{2m_e}[\hat{\mathbf{p}}-e(\mathbf{A}_0+\mathbf{A}_t)]^2, \tag{1}$$

where m_e is the effective electron mass, e is the electron charge, $\mathbf{A}_0=(-By,0,0)$ is the stationary vector potential of the magnetic field, $\mathbf{A}_t=(0,[E/\omega]\cos\omega t,0)$ is the time-dependent vector potential of the electromagnetic wave, E is the amplitude of the electric field of the wave, ω is the wave frequency, and $\hat{\mathbf{p}}=(\hat{p}_x,\hat{p}_y,0)$ is the operator of two-dimensional electron momentum, $p_{x,y}$. Solutions of the nonstationary Schrödinger problem with the Hamiltonian (1) should be sought in the form

$$\psi(\mathbf{r},t)=\frac{1}{\sqrt{L_x}}\exp\left[i\frac{p_x x}{\hbar}+i\frac{eE(y-y_0)}{\hbar\omega}\cos\omega t\right]\times\phi(y-y_0,t), \tag{2}$$

where $L_{x,y}$ are dimensions of the 2DEG plane, $\mathbf{r}=(x,y,0)$ is the radius vector of an electron in the 2DEG plane, and $y_0=-p_x/eB$ is the center of the cyclotron orbit along the y axis. Substituting the wave function (2) into the Schrödinger equation with the Hamiltonian (1), $i\hbar\partial\psi/\partial t=\hat{\mathcal{H}}_e\psi$, we arrive at the equation for the driven quantum oscillator,

$$\left[\frac{m_e\omega_0^2y^2}{2}-eEy\sin\omega t-\frac{\hbar^2}{2m_e}\frac{\partial^2}{\partial y^2}-i\hbar\frac{\partial}{\partial t}\right]\phi(y,t)=0,$$

which has the well-known exact solution (see, e.g., Refs. [41–43])

$$\phi(y,t)=\chi_N[y-\zeta(t)]\exp\left[-\frac{i\varepsilon_N t}{\hbar}+\frac{im_e\dot{\zeta}(t)[y-\zeta(t)]}{\hbar}+\frac{i}{\hbar}\int^t dt' L(t')\right], \tag{3}$$

where $\chi_N(y)$ is the eigenfunction of the quantum harmonic oscillator, $\varepsilon_N=\hbar\omega_0(N+1/2)$ is the energy spectrum of the oscillator, $N=0,1,2,\dots$ is the number of the Landau level,

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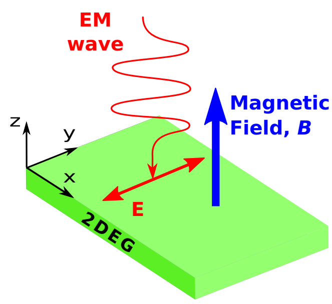

FIG. 1. Sketch of the system under consideration: Two-dimensional electron gas (2DEG) subjected to both a linearly polarized electromagnetic wave (EM) with the electric field amplitude, E, and a stationary magnetic field, $\mathbf{B}$, directed perpendicularly to the 2DEG plane.

$\omega_0 = |e|B/m_e$ is the cyclotron frequency,

$$\zeta(t) = \frac{eE\sin\omega t}{m_e(\omega_0^2 - \omega^2)}$$

is the trajectory of the driven classical oscillator, and

$$L(t) = \frac{m_e\dot{\zeta}^2(t)}{2} - \frac{m_e\omega_0^2\zeta^2(t)}{2} + eE\zeta(t)\sin\omega t$$

is the Lagrangian of the classical oscillator.

It should be noted that the field-induced terms in the wave functions (2) and (3) do not depend on the Landau level number, N. This means that the dressing field does not change the structure of Landau levels. However, the dressing field produces exponential phase shifts in the wave functions (2) and (3). In the absence of a magnetic field, similar phase shifts have a strong effect on the transport characteristics of dressed 2DEG via the renormalization of electron scattering [26,27]. Since the phase shifts in Eqs. (2) and (3) depend on both the dressing field and the magnetic field, one can expect that the magnetotransport properties of 2DEG will be renormalized by the dressing field as well. To describe this renormalization accurately, we have to solve the scattering problem for the dressed electron states (2) and (3).

Let an electron interact with scatterers in the presence of the same fields, $\mathbf{A}_0$ and $\mathbf{A}_t$. Then the wave function of the electron, $\Psi(\mathbf{r},t)$, satisfies the Schrödinger equation

$$i\hbar\frac{\partial\Psi(\mathbf{r},t)}{\partial t} = [\hat{\mathcal{H}}_e + U(\mathbf{r})]\Psi(\mathbf{r},t), \tag{4}$$

where $U(\mathbf{r})$ is the total scattering potential of 2DEG coming from a macroscopically large number of scatterers. Since the wave functions (2) at any time t coincide with the eigenfunctions of a quantum harmonic oscillator, they form the complete basis. Therefore, one can seek solutions to the Schrödinger equation (4) as an expansion,

$$\Psi(\mathbf{r},t) = \sum_j a_j(t)\psi_j(\mathbf{r},t), \tag{5}$$

where the different indices j correspond to the different sets of all quantum numbers (p_x and N) describing the electron states of the considered system. It should be stressed that Eqs. (2) and (3) describe exact wave functions of a dressed electron. Therefore, use of the complete basis (2) in the expansion (5) takes into account the interaction between the electron and the dressing field in full, i.e., nonperturbatively. As to the electron transition from a state j to a state j' due to the potential $U(\mathbf{r})$, we will describe this scattering process within conventional perturbation theory.

Let an electron be in state j at time $t = 0$ and, correspondingly, $a_{j'}(0) = \delta_{j',j}$. Substituting the expansion (5) into the Schrödinger equation (4) and restricting the accuracy by the first order of the perturbation theory (the Born approximation), we can write the amplitude of scattering to state j' as

$$a_{j'}(t) = -\frac{i}{\hbar}\int_0^t dt\int_S d^2\mathbf{r}\ \psi^*_{j'}(\mathbf{r},t)U(\mathbf{r})\psi_j(\mathbf{r},t), \tag{6}$$

where the integration should be performed over the 2DEG area, $S = L_xL_y$. Applying the Jacobi-Anger expansion,

$$e^{iz\cos\theta} = \sum_{n=-\infty}^{\infty} i^n J_n(z)e^{in\theta},$$

to transform the time-dependent exponential terms in the wave functions (2) and (3), we go from the scattering amplitude (6) to the scattering probability

$$|a_{j'}(t)|^2 = \frac{|U_{j'j}|^2}{\hbar^2}\Bigg|\sum_{n=-\infty}^{\infty} i^n J_n\left(\frac{eE[y_0' - y_0]\omega_0^2}{\hbar\omega[\omega_0^2 - \omega^2]}\right) \times e^{i(\varepsilon_{j'}-\varepsilon_j+n\hbar\omega)t/2\hbar}\int_{-t/2}^{t/2} dt'\, e^{i(\varepsilon_{j'}-\varepsilon_j+n\hbar\omega)t'/\hbar}\Bigg|^2, \tag{7}$$

where

$$U_{j'j} = \langle\varphi_{j'}(\mathbf{r})|U(\mathbf{r})|\varphi_j(\mathbf{r})\rangle \tag{8}$$

is the matrix element of the scattering between the "bare" electron eigenstates,

$$\varphi_j(\mathbf{r}) = \frac{e^{ip_xx/\hbar}}{\sqrt{L_x}}\chi_N(y),$$

which satisfy the Schrödinger equation with the Hamiltonian (1) in the absence of the dressing field ($\mathbf{A}_t = 0$). Since the integral in Eq. (7) for long time $t \to \infty$ turns into the δ function, the scattering probability (7) can be rewritten as

$$|a_{\mathbf{k}'}(t)|^2 = 4\pi^2|U_{j'j}|^2\sum_{n=-\infty}^{\infty} J_n^2\left(\frac{eE[y_0' - y_0]\omega_0^2}{\hbar\omega[\omega_0^2 - \omega^2]}\right) \times \delta^2(\varepsilon_{j'} - \varepsilon_j + n\hbar\omega). \tag{9}$$

To transform the square δ functions in Eq. (9), we can apply the conventional procedure,

$$\delta^2(\varepsilon) = \delta(\varepsilon)\delta(0) = \frac{\delta(\varepsilon)}{2\pi\hbar}\lim_{t\to\infty}\int_{-t/2}^{t/2} e^{i0\times t'/\hbar}dt' = \frac{\delta(\varepsilon)t}{2\pi\hbar}.$$

Then the probability of electron scattering between states j and j' per unit time is

$$w_{j'j} = \frac{d|a_{j'}(t)|^2}{dt} = |U_{j'j}|^2\sum_{n=-\infty}^{\infty} J_n^2\left(\frac{eE[y_0' - y_0]\omega_0^2}{\hbar\omega[\omega_0^2 - \omega^2]}\right) \times \frac{2\pi}{\hbar}\delta(\varepsilon_{j'} - \varepsilon_j + n\hbar\omega). \tag{10}$$

It should be noted that the derivation of Eqs. (6)–(10) is done within the conventional time-dependent perturbation theory, which is extended to the case of the oscillating basis (2). Physically, this extension is similar to the scattering theory developed recently for dressed electron states in various conductors [26,27].

To avoid the energy exchange between a high-frequency field and electrons, the field should be purely dressing (nonabsorbable). In the considered electron system, there are two mechanisms of absorption of the field by electrons: (i) resonant absorption of the field, which corresponds to electron transitions between different Landau levels; and (ii) collisional absorption of the field, which corresponds to electron transitions between different states within the broadened Landau level. To exclude the first mechanism, the field frequency ω should be far from the resonant frequencies, $n\omega_0$ ($n = 1,2,3,\ldots$), corresponding to interlevel electron transitions. To exclude the second mechanism, the photon energy $\hbar\omega$ should be much more than the scattering-induced broadening of Landau levels, $\Gamma = \hbar/\tau$ (i.e., $\omega\tau \gg 1$). Physically, the terms with $n \neq 0$ in Eq. (10) describe the electron scattering accompanied by the absorption (emission) of n photons. It follows from the aforementioned that these terms can be neglected if the dressing field is both off-resonant and high-frequency. Therefore, the only effect of the dressing field on 2DEG is the renormalization of the probability of elastic electron scattering within the same Landau level ($\varepsilon_{j'} = \varepsilon_j$), which is described by the term with $n = 0$ in Eq. (10):

$$w_{j'j} = J_0^2\left(\frac{eE[y_0' - y_0]\omega_0^2}{\hbar\omega\left[\omega_0^2 - \omega^2\right]}\right)w_{j'j}^{(0)}, \tag{11}$$

where

$$w_{j'j}^{(0)} = \frac{2\pi}{\hbar}|U_{j'j}|^2\delta(\varepsilon_{j'} - \varepsilon_j) \tag{12}$$

is the probability of scattering of a "bare" electron. As expected, the probabilities (11) and (12) are identical in the absence of the dressing field ($E = 0$). The formal difference between the scattering probability of dressed electron (11) and the scattering probability of "bare" electron (12) consists in the Bessel-function factor depending on both the dressing field and the stationary magnetic field. Just this factor is responsible for all the effects discussed below. In particular, the lifetime of the dressed electron at the Landau level, τ, is renormalized by the Bessel function as

$$\frac{1}{\tau} = \sum_{j'} w_{j'j} = \sum_{j'} J_0^2\left(\frac{eE[y_0' - y_0]\omega_0^2}{\hbar\omega\left[\omega_0^2 - \omega^2\right]}\right)w_{j'j}^{(0)}. \tag{13}$$

To calculate the lifetime (13), let us rewrite the δ function, $\delta(\varepsilon_{j'} - \varepsilon_j)$, using the well-known representation

$$\delta(\varepsilon) = \frac{1}{\pi}\lim_{\Gamma\to 0}\frac{\Gamma}{\Gamma^2 + \varepsilon^2}. \tag{14}$$

In the context of the discussed problem, the parameter $\Gamma = \hbar/\tau$ has the physical meaning of scattering-induced broadening of the Landau level. For the considered case of elastic scattering within the same Landau level, we can write the δ function (14) as $\delta(\varepsilon_{j'} - \varepsilon_j) \approx 1/(\pi\Gamma)$, and, therefore, Eq. (13) takes the form

$$\frac{1}{\tau} = \left[\frac{2}{\hbar^2}\sum_{j'} J_0^2\left(\frac{eE[y_0' - y_0]\omega_0^2}{\hbar\omega\left[\omega_0^2 - \omega^2\right]}\right)\left|U_{j'j}\right|^2\right]^{1/2}, \tag{15}$$

where the summation is performed over electron states j' within the same Landau level. To calculate the lifetime (15), let us approximate the scattering potential using the model of δ-function scatterers,

$$U(\mathbf{r}) = \sum_{i=1}^{N_s} U_0\delta(\mathbf{r} - \mathbf{r}_i),$$

which is commonly used to describe electronic transport in various two-dimensional systems [44–47]. Assuming that the scatterers are distributed randomly and the total number of scatterers, N_s, is macroscopically large, we can obtain from Eq. (15) the final expression for the electron lifetime at the Nth Landau level,

$$\frac{1}{\tau} = \sqrt{\frac{n_s U_0^2}{\pi l_0^2\hbar^2}}\left[\iint_{-\infty}^{\infty}\chi_N^2(y')\chi_N^2(y + y')J_0^2\left(\frac{eEy\omega_0^2}{\hbar\omega\left[\omega_0^2 - \omega^2\right]}\right)dy\,dy'\right]^{1/2}, \tag{16}$$

where $n_s = N_s/S$ is the density of scatterers per unit area of 2DEG, and $l_0 = \sqrt{\hbar/|e|B}$ is the magnetic length. The argument of the Bessel function in the integrand of Eq. (16) is the dimensionless parameter, which describes the ratio of the characteristic energy of the electron-field interaction and the photon energy. Physically, it describes the strength of electron-photon coupling in the considered electron-field system. Since the dressing field, E, leads to a decreasing Bessel function, the scattering time, τ, increases due to the field. Magnetoelectronic effects following from this increase are discussed below.

III. MAGNETOELECTRONIC CHARACTERISTICS OF DRESSED 2DEG

Since the scattering time (16) depends on the dressing field, the scattering-induced broadening of Landau levels, $\Gamma = \hbar/\tau$, is also affected by the field. To describe the broadening accurately, it is convenient to rewrite Eq. (16) in dimensionless form,

$$\frac{\Gamma^{(N)}}{\Gamma_0} = \left[\iint_{-\infty}^{\infty}\chi_N^2(y')\chi_N^2(y + y')J_0^2\left(\frac{eEy\omega_0^2}{\hbar\omega\left[\omega_0^2 - \omega^2\right]}\right)dy\,dy'\right]^{1/2}, \tag{17}$$

where $\Gamma^{(N)} = \hbar/\tau$ is the broadening for the Landau level with the number $N = 0,1,2,\ldots$, and Γ_0 is the broadening of Landau levels in the absence of the dressing field (natural broadening). It should be noted that Eq. (17) does not depend on the density of scatterers, n_s, and the strength of scatterers, U_0. Therefore, Eq. (17) describes the dependence of the broadening of Landau levels on the dressing field in the most general form, where the broadening of "bare" Landau levels,

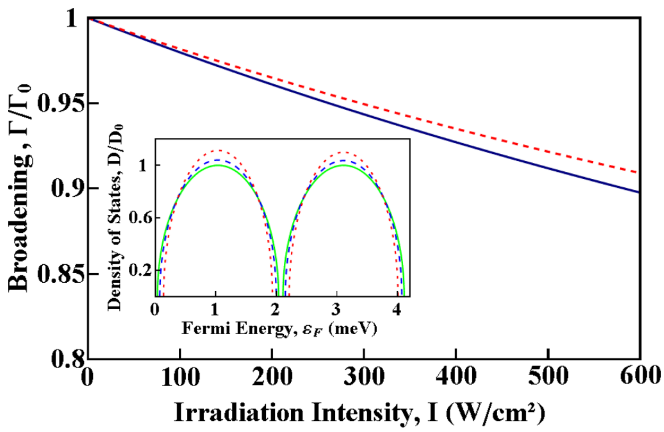

FIG. 2. The dependence of the Landau-level broadening, Γ, on the irradiation intensity, I, for the lowest two Landau levels with the numbers $N = 0$ (solid line) and $N = 1$ (dashed line) in a GaAs-based quantum well at a magnetic field $B = 1.2$ T, irradiation frequency $\omega = 2 \times 10^{12}$ rad/s, and natural broadening $\Gamma_0 = 1$ meV. The inset shows the density of electron states, D, in the absence of irradiation (solid line) and for irradiation intensities $I = 200$ W/cm^2 (dashed line) and $I = 600$ W/cm^2 (dotted line).

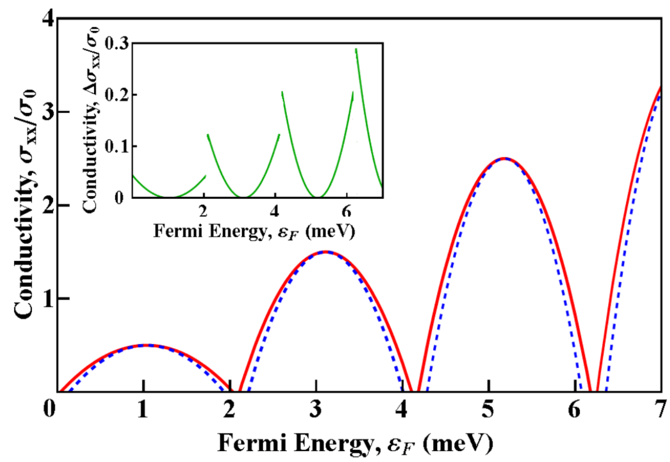

FIG. 3. The dependence of the longitudinal conductivity, σ_{xx}, on the Fermi energy, ε_F, in a GaAs-based quantum well at a magnetic field $B = 1.2$ T, irradiation frequency $\omega = 2 \times 10^{12}$ rad/s, and natural broadening $\Gamma_0 = 1$ meV. The solid line describes the conductivity of unirradiated 2DEG, whereas the dotted line corresponds to the conductivity at the irradiation intensity $I = 600$ W/cm^2. The inset shows the difference between these two conductivities, $\Delta\sigma_{xx}$.

Γ_0, should be treated as a phenomenological parameter that can be found from experiments. In the absence of the dressing field ($E = 0$), the broadening (17) is the same for all Landau levels, $\Gamma = \Gamma_0 \propto \sqrt{B}$, in complete agreement with the conventional theory of magnetoelectronic properties of 2DEG [44,45]. In contrast, the dressing field leads to different broadening (17) for different Landau levels (see Fig. 2). As to the density of electron states, it is described by the expression [44,45]

$$D(\varepsilon) = D_0 \sum_N \frac{\Gamma_0}{\Gamma^{(N)}} \left[1 - \left(\frac{\varepsilon - \varepsilon_N}{\Gamma^{(N)}}\right)^2\right]^{1/2}, \qquad (18)$$

where $D_0 = 1/(\pi^2 l_0^2 \Gamma_0)$. Substituting the broadening (17) into Eq. (18), one can calculate the density of states in dressed 2DEG (see the inset in Fig. 2). Since the dressing field decreases the broadening of Landau levels (17), this results in increasing the density of states at Landau-level energies, $\varepsilon = \varepsilon_N$. As a consequence, all phenomena sensitive to the density of electronic states (magnetotransport, magneto-optical effects, etc.) are affected by the dressing field. In particular, the longitudinal magnetoconductivity of 2DEG at the temperature $T = 0$ is described by the conventional expression [44,45]

$$\sigma_{xx} \approx \sigma_0 \left(N + \frac{1}{2}\right) \left[1 - \left(\frac{\varepsilon - \varepsilon_N}{\Gamma^{(N)}}\right)^2\right], \qquad (19)$$

where $\sigma_0 = e^2/\pi^2\hbar$ is the elementary conductivity and N is the number of the Landau level at the Fermi energy. Substituting the broadening (17) into Eq. (19), one can calculate the dependence of the conductivity on the dressing field. Experimentally, one can change the Fermi energy of 2DEG, ε_F, with a gate voltage. Then we arrive from Eq. (19) at the oscillating behavior of the conductivity (the Shubnikov–de Haas oscillations) plotted in Fig. 3.

It should be stressed that there is a crucial difference between the considered high-frequency dressing field and the low-frequency case. Since 2DEG absorbs a low-frequency field, the multiphoton-assisted scattering of electrons can increase the longitudinal conductivity [7]. In particular, this effect was proposed to explain the phenomenon of "zero resistance states" in 2DEG subjected to both a magnetic field and low-frequency (microwave) irradiation [4,13]. On the contrary, the considered high-frequency field cannot be absorbed by 2DEG. The only effect of the field is the suppression of electron scattering, which results in decreasing both the broadening of Landau levels and the longitudinal conductivity (see Figs. 2 and 3). Thus, high-frequency and low-frequency irradiation lead to different behaviors of the magnetoelectronic properties of 2DEG.

It should also be noted that the magnetoelectronic effects induced by a dressing field depend strongly on the kind of electron dispersion. In Dirac materials with linear electron dispersion, a dressing field changes the energy distance between Landau levels and, therefore, modifies all phenomena depending on the cyclotron frequency [40]. On the contrary, in the considered case of 2DEG with parabolic electron dispersion, a dressing field does not change the cyclotron frequency but influences the electron scattering within Landau levels.

As to the experimental observability of the discussed phenomena, all dressing effects increase with increasing the intensity of the dressing field. In particular, the strong dressing field can turn the Bessel function in Eq. (16) into zero, which corresponds physically to the field-induced suppression of electron scattering [26]. However, intense irradiation can fluidize a semiconductor quantum well. To avoid this fluidizing, it is reasonable to use narrow pulses of a strong dressing field. This well-known methodology was elaborated upon long ago, and it is commonly used to observe various dressing effects—in particular, modifications of the energy spectrum of dressed electrons arising from the optical Stark effect—in semiconductor structures (see, e.g., Refs. [48–50]). Within this approach, giant dressing fields (up to $\mathrm{GW/cm^2}$) can be applied to semiconductor structures.

IV. CONCLUSIONS

We can conclude that a strong high-frequency electromagnetic field (dressing field) decreases electron scattering between different cyclotron orbits within the same Landau level. As a consequence, the field decreases the scattering-induced broadening of Landau levels in 2DEG. This results in the field-induced modification of various magnetoelectronic properties, depending on the density of electron states (in particular, magnetotransport characteristics of 2DEG). Therefore, a dressing field can be considered as a perspective tool to manipulate the magnetoelectronic properties of various two-dimensional nanostructures. Since such nanostructures serve as a basis for nanoelectronic devices, the developed theory paves the way for optical control of their magnetoelectronic characteristics.

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