Spin-glass phase transition and behavior of nonlinear susceptibility in the Sherrington-Kirkpatrick model with random fields

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The behavior of the nonlinear susceptibility χ_3 and its relation to the spin-glass transition temperature T_f in the presence of random fields are investigated. To accomplish this task, the Sherrington-Kirkpatrick model is studied through the replica formalism, within a one-step replica-symmetry-breaking procedure. In addition, the dependence of the Almeida-Thouless eigenvalue $\lambda_{\rm AT}$ (replicon) on the random fields is analyzed. Particularly, in the absence of random fields, the temperature T_f can be traced by a divergence in the spin-glass susceptibility $\chi_{\rm SG}$, which presents a term inversely proportional to the replicon $\lambda_{\rm AT}$. As a result of a relation between $\chi_{\rm SG}$ and χ_3, the latter also presents a divergence at T_f, which comes as a direct consequence of $\lambda_{\rm AT} = 0$ at T_f. However, our results show that, in the presence of random fields, χ_3 presents a rounded maximum at a temperature T^* which does not coincide with the spin-glass transition temperature T_f (i.e., $T^* > T_f$ for a given applied random field). Thus, the maximum value of χ_3 at T^* reflects the effects of the random fields in the paramagnetic phase instead of the nontrivial ergodicity breaking associated with the spin-glass phase transition. It is also shown that χ_3 still maintains a dependence on the replicon $\lambda_{\rm AT}$, although in a more complicated way as compared with the case without random fields. These results are discussed in view of recent observations in the $\mathrm{LiHo}_x\mathrm{Y}_{1-x}\mathrm{F}_4$ compound.

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I. INTRODUCTION

The presence of disorder in spin systems represents a permanent source of challenging problems, due to the richness of physical properties that emerge from the interplay between disorder and many-spin interactions. Random-field (RF) and spin-glass (SG) models are important examples of such richness [1–3]. Furthermore, the combination of these two highly nontrivial manifestations of disorder leads to a fascinating area of research in spin systems, which is not only a theoretical possibility. Actually, they can be found in diluted Ising-like antiferromagnets, like $\mathrm{Fe}_x\mathrm{Zn}_{1-x}\mathrm{F}_2$ and $\mathrm{Fe}_x\mathrm{Mg}_{1-x}\mathrm{Cl}_2$ [4]. Additionally, recent investigations have suggested the diluted Ising-like dipolar ferromagnetic compound $\mathrm{LiHo}_x\mathrm{Y}_{1-x}\mathrm{F}_4$ as a new candidate presenting these two types of disorder, bringing novel interesting and controversial issues [5–13]. For instance, the $\mathrm{LiHo}_{0.167}\mathrm{Y}_{0.833}\mathrm{F}_4$ compound, in the absence of an applied transverse field B_t, displays χ_3 (the lowest term of the nonlinear susceptibility χ_{nl}) with a sharp peak at the freezing temperature T_f, which resembles a conventional second-order SG phase transition [14–16]. On the other hand, the sharp peak of χ_3 becomes increasingly rounded when B_t is enhanced, being located at the temperature T^*, which is lower than T_f obtained in the absence of B_t [17].

The suggestion that an effective longitudinal RF can be induced by the interplay of a transverse applied field B_t with the off-diagonal terms of the dipolar interactions in the $\mathrm{LiHo}_x\mathrm{Y}_{1-x}\mathrm{F}_4$ [5–8] brought a new push to clarify the controversies of the experimental behavior of χ_3 and, therefore, the meaning of T^*, i.e., whether or not it is a true SG transition temperature. In the droplet picture used in Refs. [5] and [6], the presence of a RF $h_i(B_t)$, induced by the uniform transverse field, suppresses the SG transition for the same reason that a uniform field does it in that picture [18]. On the other hand, within the mean-field Parisi's framework [19,20], Tabei and collaborators [7], using a quantum version of the Sherrington-Kirkpatrick (SK) model [21] with additional off-diagonal interactions, longitudinal RF $h_i(B_t)$, and a transverse field $\Gamma(B_t)$, succeeded in reproducing the χ_3 experimental behavior. Indeed, this result is a strong evidence that the RF plays an important role in the $\mathrm{LiHo}_x\mathrm{Y}_{1-x}\mathrm{F}_4$ compound. These authors also suggested that the SG quantum criticality is unlikely in this transverse field, induced longitudinal RF scenario; additionally, susceptibility measurements presented evidence of a canonical SG behavior [10,11]. From that point of view, one can raise the question of what happens with the SG criticality in a regime where thermal fluctuations should be dominant as compared with the quantum ones. One possible consequence of the transverse field, induced longitudinal RF, is that the Almeida-Thouless (AT) line [22] can be suppressed, as suggested by numerical simulations in short-range-interaction SGs [23]. However, previous studies using a mean-field Parisi's framework have shown that the SK model with a RF does preserve the AT line [24–28]. Consequently, assuming that Parisi's mean-field theory is a

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valid framework to describe the SG problem with a transverse field, induced longitudinal RF, one can also raise the question of how the behavior of χ_3 can be related with the AT line when a RF is present in the SK model. One can expect that the answer to this question may also help to clarify the meaning of the temperature T^*.

Therefore, in this work we present a detailed investigation of the role of a RF in the behavior of χ_3 and its relation to the AT line in the SK model within the mean-field Parisi's framework. In order to relate with experimental verifications on the $LiHo_xY_{1-x}F_4$ compound, we assume that B_t is sufficiently small to assure that quantum fluctuations are negligible but enough to guarantee that the effective field-induced RF $h_i(B_t)$ is still appreciable. When $h_i = 0$ ($B_t = 0$), it is known that χ_3 is related with the SG susceptibility,

$$\chi_{SG} = (\beta/N)\sum_{i,j}[(\langle S_i S_j\rangle - \langle S_i\rangle\langle S_j\rangle)^2]_{av}, \tag{1}$$

where, as usual, $\langle..\rangle$ and $[..]_{av}$ denote, respectively, thermal averages and an average over the disorder. The SG susceptibility has a term inversely proportional to the AT eigenvalue λ_{AT}, the so-called replicon [29]. Therefore, the diverging behavior, $\chi_3 \propto (T - T_f)^{-\gamma}$, in the SG transition is directly related with $\lambda_{AT} = 0$ at T_f, corresponding to the onset of replica symmetry breaking (RSB).

However, the situation changes considerably when $h_i \neq 0$ (i.e., $B_t \neq 0$). For instance, the RF can induce directly the SG order parameter, but only in the replica-symmetric (RS) approximation, since the RF is unable to produce any RSB. This result was demonstrated not only for infinite-ranged spin interactions [30], but also for the Bethe lattice [31]. As a consequence, the smooth behavior of the SG RS order parameter q is not appropriate for identifying a SG transition of the SK model in the presence of a random field; however, such a transition may be related with the onset of RSB, associated with the replicon $\lambda_{AT} = 0$ [24]. In spite of this, the derivative of q with respect to the temperature increases as one approaches T_f from above; such an increase is responsible for the rounded maximum in χ_3 at a temperature T^*, which does not coincide with the SG transition temperature T_f (i.e., $T^* > T_f$ for a given applied random field). Thus, the maximum value of χ_3 at T^* should reflect the effects of the RF inside the paramagnetic (PM) phase instead of the nontrivial ergodicity breaking of the SG phase transition. Our results also suggest that χ_3 still maintains a dependence on the replicon λ_{AT}, although in a much more complicated way as compared with the case without the RF.

This paper is structured as follows. In the next section we define the model and the analytical procedure to be used; then, we calculate λ_{AT}, the order parameters within the one-step replica-symmetry-breaking (1S-RSB) scheme, the susceptibilities χ_1, χ_3, as well as the temperature T^* in the presence of RFs, following both Gaussian and bimodal distributions. In Sec. III we discuss the numerical solutions of the saddle-point equations for the order parameters and susceptibilities. Finally, the last section is reserved to the conclusions.

II. MODEL AND SUSCEPTIBILITIES

Herein we consider the infinite-range-interaction spin-glass model, defined by the following Hamiltonian:

$$H = -\sum_{(i,j)} J_{ij} S_i S_j - \sum_{i=1}^{N} h_i S_i - H_l \sum_{i=1}^{N} S_i, \tag{2}$$

where $S_i = \pm 1$, H_l represents a uniform field, and the sum $\sum_{(i,j)}$ applies to all distinct pairs of spins. The spin-spin couplings $\{J_{ij}\}$ and the magnetic random fields $\{h_i\}$ follow independent Gaussian probability distributions,

$$P(X) = \left[\frac{1}{2\pi\sigma^2}\right]^{1/2} \exp\left[-\frac{X^2 - C}{2\sigma^2}\right], \tag{3}$$

where X may represent either couplings or random fields; in the former case one has $\sigma = J/\sqrt{N}$ and $C = J_0/N$, whereas in the later, $\sigma = \Delta$ and $c = 0$. We also consider a bimodal probability distribution,

$$P(h_i) = p\,\delta(h_i - h_0) + (1 - p)\,\delta(h_i + h_0), \tag{4}$$

for the random fields $\{h_i\}$. We follow closely the procedure used in Ref. [24] to obtain the average free energy per spin, $f = -1/(\beta N)[\ln Z(\{J_{ij}\},\{h_i\})]_{J,h}$, where $Z(\{J_{ij}\},\{h_i\})$ represents the partition function for a given quenched distribution of random couplings and fields; moreover, $[..]_{J,h}$ denotes averages over these types of disorder, and $\beta = 1/T$. As usual, the replica method [1,2,29] is applied; thus,

$$-\beta f = \lim_{N\to\infty}\lim_{n\to 0}\frac{1}{Nn}([Z(\{J_{ij}\},\{h_i\})^n]_{J,h} - 1), \tag{5}$$

where Z^n corresponds to the replicated partition function. In the replica space, the average over the disorder may be evaluated and we adopt the 1S-RSB; this procedure leads to the following free energy [26]:

$$\beta f = \frac{(\beta J)^2}{4}x\left(q_1^2 - q_0^2\right) - \frac{(\beta J)^2}{4}(1 - q_1)^2 + \frac{\beta J_0}{2}m^2 - \frac{1}{x}\int Dz \ln\int Dv[2\cosh\Xi(z,v)]^x, \tag{6}$$

with $Dz \equiv \frac{1}{\sqrt{2\pi}}dz\exp(-z^2/2)$ and all integrals should be considered over the whole interval (from $-\infty$ to $+\infty$). The 1S-RSB internal field $\Xi(z,v)$ is given by

$$\Xi(z,v) = \beta J(\sqrt{q_0 + \Theta(1-n)}z + \sqrt{q_1 - q_0}v) + \beta J_0 m + \beta(H_l + n\Theta). \tag{7}$$

The equations above apply to both types of random fields, through the identifications $n = 0$ and $\Theta = (\Delta/J)^2$ (Gaussian RF), whereas $n = 1$ and $\Theta = h_0$ (bimodal RF). It should be mentioned that in the present work the results for the bimodal RF become independent of p, which may be seen by means of a change of variables $z \to -z$ [27]. Therefore, the analysis of the bimodal distribution becomes completely equivalent to that of a shifted uniform field given by $H_l + h_0$. Since the analysis of the SK model in the presence of a uniform field has been carried in the literature by many authors [26,27,32,33], from now on we focus our analysis to the Gaussian random field, for which the internal field of Eq. (7) becomes

$$\Xi(z,v) = \beta J(\sqrt{q_0 + (\Delta/J)^2}\, z + \sqrt{q_1 - q_0}\, v) + \beta J_0 m + \beta H_l. \tag{8}$$

The 1S-RSB parameters q_0, q_1, and x should extremize the free energy of Eq. (6), from which the RS solution is recovered when $q = q_0 = q_1$ [32,33]. The linear susceptibility $\chi_1 = \frac{\partial m}{\partial H_l}|_{H_l \to 0}$ is given by $\chi_1 = \beta[1 - q_1 + x(q_1 - q_0)]$ [20] when $J_0 = 0$. The nonlinear susceptibility χ_3 can be obtained from $\chi_3 = -\frac{1}{3!}\frac{\partial^3 m}{\partial H_l^3}|_{H_l \to 0}$. Moreover, important effects on χ_3 appear already inside the region where the RS solution is stable, more precisely, χ_3 presents a rounded maximum at a temperature T^*, above the SG transition. Particularly, we can expand q and m in powers of H_l, for $J_0 = 0$, as (following Wada [14])

$$q(H_l) = Q_0 + Q_2 H_l^2, \tag{9}$$

$$m(H_l) = \chi_1 H_l + \chi_3 H_l^3, \tag{10}$$

which results in

$$\chi_3(T) = \frac{\beta^3}{3}(3J^2 Q_2 + 1) I_0, \tag{11}$$

with $Q_0 = \int Dz \tanh^2 \Xi_0(z)$ and $Q_2 = \frac{\partial^2 q}{\partial H_l^2}|_{H_l \to 0}$, where the RS internal field is obtained from Eq. (8) by setting $H_l = 0$ and $q_0 = q_1 = q$, i.e., $\Xi_0(z) = \beta J(\sqrt{q + (\Delta/J)^2}\, z)$ and

$$I_0 = \int Dz[\mathrm{sech}^4 \Xi_0(z) - 2\, \mathrm{sech}^2 \Xi_0(z) \tanh^2 \Xi_0(z)]. \tag{12}$$

Moreover, Q_2 can be obtained as

$$Q_2 = \frac{1}{2!}\frac{\partial^2 q}{\partial H_l^2}|_{H_l \to 0} = \frac{\beta^2 I_0}{1 - (\beta J)^2 I_0}, \tag{13}$$

so that $\chi_3(T)$ becomes

$$\chi_3(T) = -\frac{\beta^3}{3}\left[\frac{3(\beta J)^2 I_0}{1 - (\beta J)^2 I_0} + 1\right] I_0. \tag{14}$$

These results hold when the RS solution is stable, given by a positive value of the eigenvalue λ_{AT} [24–28],

$$\lambda_{AT} = 1 - (\beta J)^2 \int Dz \mathrm{sech}^4 \Xi_0(z). \tag{15}$$

Particularly, $\chi_3(T)$ can be written in terms of λ_{AT},

$$\chi_3(T) = \frac{\beta^3}{3}\left[\frac{3}{\lambda_{AT} + (\beta J)^2 I_1} - 2\right] I_0, \tag{16}$$

where

$$I_1 = 2\int Dz\, \mathrm{sech}^2 \Xi_0(z) \tanh^2 \Xi_0(z). \tag{17}$$

In the absence of RFs, $I_1 = 0$ in the PM phase, implying on a divergence of χ_3 when $\lambda_{AT} = 0$, as expected [29]. Moreover, in the presence of RFs, one has that $I_1 > 0$, so that Eq. (16) leads to a rounded maximum at a temperature T^*.

III. RESULTS AND DISCUSSION

Numerical results are now presented. The effects of RFs on the SG order parameters q_0, q_1, $\delta \equiv q_1 - q_0$, susceptibilities χ_1 and χ_3, as well as the stability of the RS solution (i.e., λ_{AT}) are discussed. In particular, the onset of RSB (location of T_f) and how χ_3 behaves in the neighborhood of the SG phase transition are studied.

For instance, Fig. 1 shows that the SG order-parameter behavior, signaling RSB ($\delta > 0$), occurs at lower temperatures due to the presence of RFs, i.e., the increase of Δ/J moves T_f to lower temperatures. The freezing temperature T_f, which is located within the 1S-RSB scheme as the onset of the parameter δ, is shown herein to coincide with $\lambda_{AT} = 0$. As presented in the inset of Fig. 1, the RFs induce the order parameters q_0 and q_1 for $T > T_f$, where the RS solution is stable [$q = q_0 = q_1$, $\delta = 0$ and $\lambda_{AT} > 0$], characterizing the PM phase. In the cases $(\Delta/J) > 0$ one notices that $T^* > T_f$, with the arrows indicating the temperature T^* where χ_3 presents a rounded maximum.

As shown in Fig. 2, the magnetic susceptibility χ_1 exhibits a clear cusp at T_f in the absence of the RF, whereas in the presence of a RF, one notices a smooth behavior around T_f. Below this temperature, the 1S-RSB and RS solutions become distinct, with the former presenting higher values, being weakly dependent on the temperature.

In Fig. 3 we present results for the nonlinear susceptibility χ_3, computed directly from the numerical derivatives $\chi_3 = -\frac{1}{3!}\frac{\partial^3 m}{\partial H_l^3}|_{H_l \to 0}$. As a check, for $T \geqslant T_f$, we verified that these results coincide with those obtained from Eq. (16). For the case without RFs, χ_3 shows a strong divergence at T_f (see inset in Fig. 3); however, the presence of a RF eliminates this

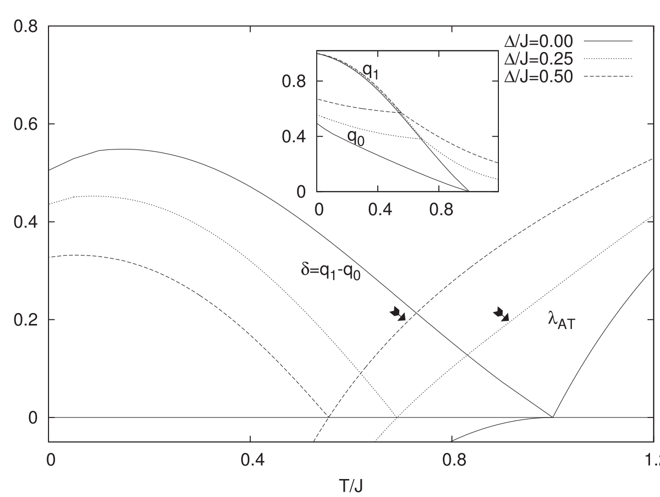

FIG. 1. The 1S-RSB parameter $\delta \equiv q_1 - q_0$ and the eigenvalue λ_{AT} are presented versus the dimensionless temperature T/J, for typical values of Δ/J. The inset shows the SG parameters q_1 and q_0 separately versus the dimensionless temperature. The freezing temperature T_f is identified with the onset of RSB, where $\lambda_{AT} = 0$, or equivalently, where the parameter δ becomes nonzero. The arrows indicate the temperature T^*, where χ_3 presents a rounded maximum, showing that $T^* > T_f$. Due to the usual numerical difficulties, the low-temperature results [typically $(T/J) < 0.05$)] correspond to smooth extrapolations from higher-temperature data.

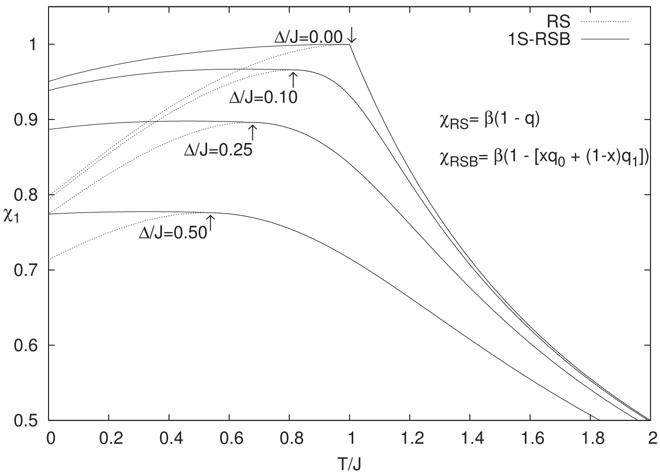

FIG. 2. Magnetic susceptibility χ_1 versus T/J for different values of Δ/J. The arrows indicate the onset of the RSB solution ($\lambda_{\rm AT} = 0$), defining the temperature T_f. Below T_f, solid and dotted lines indicate linear susceptibilities computed using 1S-RSB ($\chi_{\rm RSB}$) and RS ($\chi_{\rm RS}$) solutions, respectively. Due to the usual numerical difficulties, the low-temperature results [typically $(T/J) < 0.05$] correspond to smooth extrapolations from higher-temperature data.

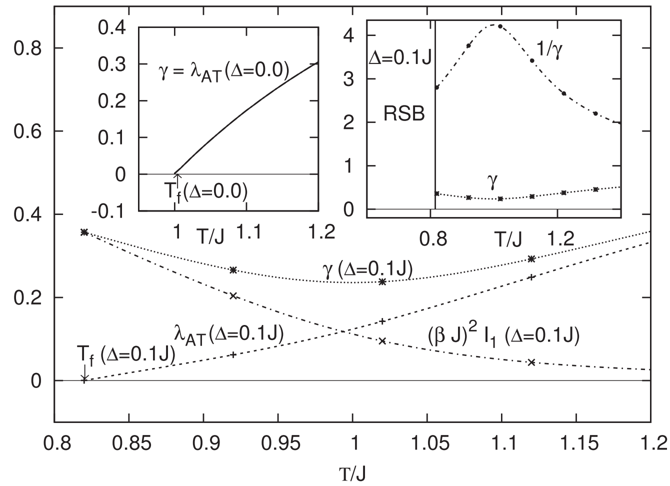

FIG. 4. The quantities appearing in the denominator of Eq. (16), $\gamma = \lambda_{\rm AT} + (\beta J)^2 I_1$, are presented versus T/J for $(\Delta/J) = 0.1$. The arrows locate the freezing temperature T_f. The inset on the right shows in detail the behaviors of γ and $1/\gamma$, for $(\Delta/J) = 0.1$, in the region where the RS solution is stable (to the left of this region one should use RSB); the quantity $1/\gamma$ presents a rounded maximum, which is directly related with that found in χ_3. The inset on the left shows in detail the behaviors of γ and $\lambda_{\rm AT}$, for $\Delta/J = 0.0$, which are responsible for the divergence of χ_3 in the absence of RFs.

divergence, and rounded maxima appear in the χ_3 curves, defining the temperature T^* for each value of Δ/J. It is important to remark that T^* is always higher than T_f. Furthermore, the T^* and χ_3 values decrease for increasing values of Δ/J. Within the RSB region, similarly to what was shown for the linear susceptibility χ_1 (cf. Fig. 2), χ_3 also presents a split between the results with RS and 1S-RSB solutions. However, differently from χ_1, the nonlinear susceptibility χ_3 displays an evident discontinuity at T_f when the 1S-RSB solution is adopted.

An important quantity in Eq. (16) is the denominator, $\gamma = \lambda_{\rm AT} + (\beta J)^2 I_1$, which is illustrated in detail in Fig. 4 versus T/J for the typical value $(\Delta/J) = 0.1$. As a comparison, the inset on the left shows the behavior of γ and $\lambda_{\rm AT}$, for $(\Delta/J) = 0.0$; in this case, $\gamma = 0$ leads to the divergence of χ_3 in the absence of RFs. When $(\Delta/J) > 0$, one has that the contribution $(\beta J)^2 I_1 > 0$, so that now $\gamma > 0$. The two contributions, $\lambda_{\rm AT}$ (that increases for increasing values of T/J) and $(\beta J)^2 I_1$ (that decreases for increasing values of T/J), are presented separately, leading to a minimum value for γ, which is found to occur very close to the temperature T^*. The inset on the right shows the maximum attained by $1/\gamma$, appearing inside the region where RS is stable; to the left of this region, one should analyze these quantities within RSB. This maximum is directly related with the one presented in Fig. 3, at the temperature T^*, and since this temperature is found in the RS region, we consider the rounded maximum to occur in the paramagnetic phase. One should remember the role played by the RF on the replicon, leading to a shift in the freezing temperature towards lower temperatures, i.e., $T_f(\Delta > 0) < T_f(\Delta = 0)$ [24,26,27]. Hence, in Fig. 4 one notices that in the temperature range $T_f(\Delta > 0) < T < T_f(\Delta = 0)$ the behavior of the denominator γ changes completely from decreasing to increasing. This inversion yields the minimum of γ, which is the ultimate mechanism leading to the rounded maximum of χ_3 at T^*.

In Fig. 5 we present the phase diagram of the model, showing the paramagnetic and SG phases. The SG phase is associated with the onset of RSB, being signaled by the zero of the replicon of Eq. (15) (i.e., $\lambda_{\rm AT} = 0$), which defines the freezing temperature T_f. The temperature T_f is lowered due to the RFs; in fact, such a decrease in T_f can be verified analytically for $(\Delta/J) \ll 1$, in which case an expansion can

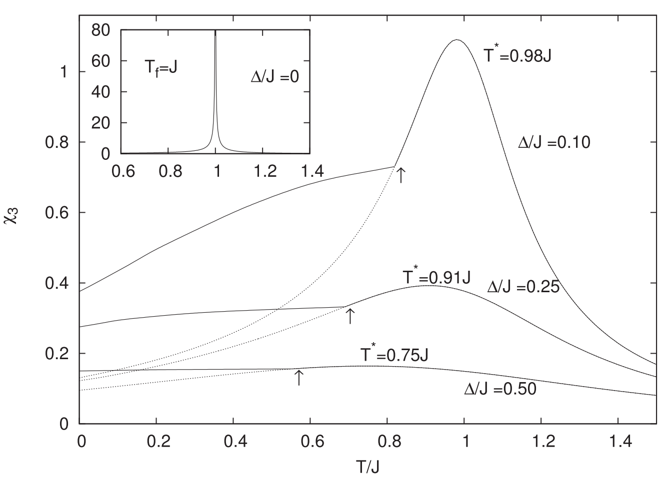

FIG. 3. The susceptibility χ_3 as a function of T/J for different values of Δ/J. The arrows indicate the onset of the RSB solution ($\lambda_{\rm AT} = 0$), defining the temperature T_f. Below T_f, solid and dotted lines indicate 1S-RSB and RS solutions, respectively. The temperature T^*, where χ_3 presents a rounded maximum, is estimated in each case shown. In the inset we exhibit the χ_3 behavior without the RF.

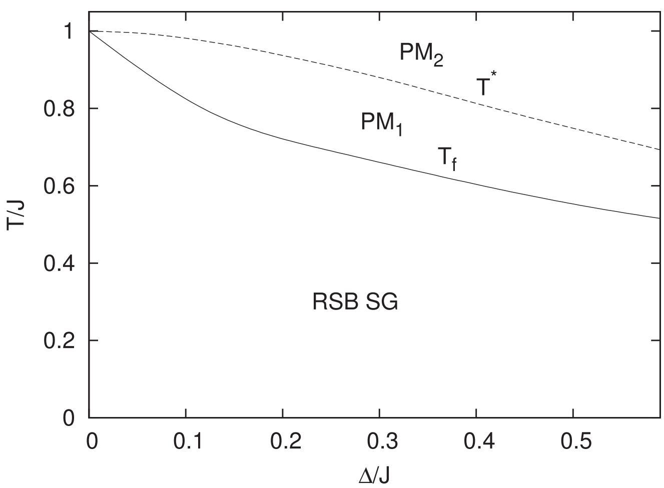

FIG. 5. Phase diagram T/J versus Δ/J showing the paramagnetic and SG phases. The freezing temperature T_f, signaling the onset of RSB, defines the SG phase for $T < T_f$. For completeness, we also present the line associated with the maximum of χ_3, defining the temperature T^* (dashed line). The possibility of two paramagnetic phases (PM_1 and PM_2) is discussed in the text.

be obtained from Eq. (15) [24,27]:

$$\frac{T_f}{J} \approx 1 - \left(\frac{3}{4}\right)^{1/3} \left(\frac{\Delta}{J}\right)^{2/3}. \quad (18)$$

The dashed line in Fig. 5 represents the temperature T^* ($T^* > T_f$), characterizing the maximum of χ_3, which exists for any $(\Delta/J) > 0$. For $T > T^*$ the phase PM_2 occurs, along which one has weak correlations and consequently, the usual paramagnetic type of behavior. However, close to T^*, and particularly for temperatures in the range $T_f < T < T^*$, one expects a rather nontrivial behavior in real systems, as happens with the compound $LiHo_xY_{1-x}F_4$, resulting in very controversial interpretations [5–11]. Due to such aspects, herein we call the temperature region $T_f < T < T^*$ of PM_1. The line PM_1–PM_2 may not characterize a real phase transition, in the sense of a diverging χ_3, but the region PM_1 is certainly characterized by a rather nontrivial dynamics. As shown in Fig. 4, the region PM_2 presents small values for the quantity I_1 [cf. the denominator of Eq. (16)], whereas along PM_1 the couplings between RFs and spins become dominant, as compared with thermal fluctuations, and I_1 increases significantly. As a possible relation, one should have a growth of free-energy barriers in the region PM_1, leading to a slow dynamics, whereas only below T_f does the nontrivial ergodicity breaking appear, typical of RSB in SG systems. It is important to remember also that Griffiths singularities are found currently in disordered magnetic systems, like for site-diluted ferromagnets [34], as well for a ferromagnet in a random field [35]. Whether the region PM_1 in the present problem may be related to this latter type of behavior is a matter for further investigation.

As already mentioned and addressed in several works [14–16], in the absence of RFs, the SG phase transition is given by the divergence of χ_3 at T_f. In Parisi's mean-field theory this divergence is directly related with the onset of RSB, signaled by a zero of the replicon $\lambda_{AT} = 0$ [29]. However, the presence of RFs induces the SG order parameter q in the PM phase, within the RS solution. Moreover, χ_3 no longer diverges at the SG transition temperature but instead, presents a rounded maximum at T^*, which becomes smoother as Δ/J increases. Such a difference with respect to the case without RFs can be understood from Eqs. (16) and (17). In fact, the term I_1 in Eq. (16), which is responsible for these effects, can be rewritten as

$$I_1 = 2(q - r) \quad (19)$$

with

$$\langle S^\alpha S^\beta \rangle \equiv q = \int Dz \tanh^2 \Xi_0(z), \quad (20)$$

and

$$\langle S^\alpha S^\beta S^\gamma S^\delta \rangle \equiv r = \int Dz \tanh^4 \Xi_0(z). \quad (21)$$

These equations lead to $\gamma = \lambda_{AT} + (\beta J)^2 I_1 = 1 - (\beta J)^2(1 - 4q + 3r)$, which is precisely the longitudinal eigenvalue of the RS stability analysis [22,29]. This longitudinal eigenvalue is related with the magnitude of the fluctuations of the of RS SG order parameter q. Hence, the maximum of χ_3 at T^* becomes completely unrelated with the SG phase transition when Δ departs from zero, being directly associated with the longitudinal eigenvalue.

IV. CONCLUSIONS

The role of random fields on the spin-glass freezing temperature, as well as on the nonlinear susceptibility, was analyzed. For that, we have investigated the Sherrington-Kirkpatrick model in the presence of random fields, following a Gaussian distribution characterized by a width Δ, within a one-step replica-symmetry-breaking procedure. We have shown that the divergence in χ_3 only occurs in the absence of random fields and that χ_3 exhibits a broad maximum at a temperature T^* for $\Delta > 0$. The freezing temperature T_f is associated with the onset of replica symmetry breaking, signaled by the zero of the Almeida-Thouless (replicon) eigenvalue, occurring at lower temperatures, i.e., $T^* > T_f$ for a given value of Δ.

The splitting between T_f and T^*, for $\Delta > 0$, was studied by analyzing the contribution due to the random fields in the replica-symmetry spin-glass order parameter. Particularly, we have shown that the behavior of χ_3 is not regulated only by the spin-spin correlations associated to the Almeida-Thouless line, but also to correlations coming from the longitudinal eigenvalue. These correlations play an important role inside the paramagnetic phase, when the random fields are applied, being responsible for the maximum in χ_3, although they are not directly associated with the spin-glass phase transition.

Although the present results refer a specific model, we expect they could shed some light in the theoretical and experimental description of disordered magnetic systems like, for instance, the compound $LiHo_xY_{1-x}F_4$. Considering recent observations in $LiHo_xY_{1-x}F_4$, we follow the proposal that an applied transverse field B_t induces longitudinal random fields [7], and thus, we assume herein $\Delta = \Delta(B_t)$. In this way, one can interpret the present results, e.g., the temperatures T^* and T_f, as manifestations of the transverse field. Based on this, we point out below two possibilities which may contribute to elucidate the recent controversies on this system [5–11]: (i) The temperature T^* associated with the rounded maximum in the nonlinear susceptibility does not signal any phase transition, being an effect of random fields inside the paramagnetic phase, although it is related to a minimum of the longitudinal eigenvalue, and hence, to large fluctuations in the replica-symmetric spin-glass order parameter. A true spin-glass phase transition, indicated through the Sherrington-Kirkpatrick model, by means of the Almeida-Thouless line, should occur at the lower temperature T_f. (ii) There is no spin-glass phase transition in $LiHo_xY_{1-x}F_4$ in the presence of a transverse field, implying that the replica-symmetry-breaking procedure does not apply to this compound. Certainly, these two points require meticulous experimental observations for temperatures around T^*, which has been the most investigated temperature region so far, as well as below T^*, representing a challenge for experiments.

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