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Signatures of topological phase transitions in Josephson current-phase discontinuities

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Topological superconductors differ from topologically trivial ones due to the presence of topologically protected zero-energy modes. To date, experimental evidence of topological superconductivity in nanostructures has been mainly obtained by measuring the zero-bias conductance peak via tunneling spectroscopy. Here, we propose an alternative and complementary experimental recipe to detect topological phase transitions in these systems. We show in fact that, for a finite-sized system with broken time-reversal symmetry, discontinuities in the Josephson current-phase relation correspond to the presence of zero-energy modes and to a change in the fermion parity of the ground state. Such discontinuities can be experimentally revealed by a characteristic temperature dependence of the current, and can be related to a finite anomalous current at zero phase in systems with broken phase-inversion symmetry.

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Introduction. The recent discovery of topological materials has deeply impacted condensed matter research [1]. These materials exhibit a number of exceptional properties which are related to the presence of topologically protected states localized at their edges. In topological superconductors (TSs), for instance, Majorana edge states [2] are characterized by a distinctive non-Abelian statistics, which makes them ideal candidates for fault-tolerant quantum computation [3]. Theoretically, a TS can be realized at the interface between a conventional superconductor and a topological insulator [4], or in semiconductor-superconductor heterostructures with spin-orbit coupling (SOC) in a magnetic field [5].

However, revealing signatures of topological nontrivial phases in TSs is not straightforward [6]. This is mainly because, unlike more conventional continuous phase transitions, topological phase transitions [7] do not break any symmetry nor exhibit any critical behavior, but are instead identified by a change of the corresponding topological invariant [8] and of the edge properties in continuous systems [9]. To date, experimental evidence of nontrivial superconductivity comprises the measure of the 4π-periodic Josephson current in TS rings [10], or the zero-bias conductance peak via tunneling spectroscopy [11] or spatially resolved spectroscopic imaging [12].

In this Rapid Communication we propose a direct method to probe variations of the topological invariant, i.e., the fermion parity of the ground state [13], in Josephson junctions [14,15]. It is known that the Josephson current-phase relation (CPR) may exhibit discontinuities at low temperatures corresponding to Andreev level crossings [16], in systems as different as quantum dots [17], nanowires [18], Josephson junction arrays [19], and Weyl semimetals [20]. Here, we show that in finite-sized noninteracting s-wave TSs with broken time-reversal symmetry, such discontinuities are related to the presence of zero-energy modes and to variations of the fermion parity, which define the topological phase transitions of this system. These discontinuities can be experimentally revealed by a characteristic temperature dependence and are moreover related, in systems with broken phase-inversion symmetry, to an anomalous current at zero phase.

The model. A TS can be realized by a nanostructure embedded into a superconducting ring [21] with SOC and a magnetic field [5] (see Fig. 1). Here, we consider the case of finite-sized systems, e.g., a quantum dot, wire, or planar well [see Supplemental Material (SM) Sec. A [22]], described by a tight-binding Bogoliubov–de Gennes (BdG) Hamiltonian [23] with linear dimensions smaller than the coherence length $\xi = \hbar v_F/\Delta$, where v_F is the Fermi velocity and Δ the superconducting gap. This system is the zero-dimensional (0D) limit of a one-dimensional (1D) Majorana chain [2], in the sense that its Hamiltonian does not depend on any momentumlike continuous parameter. The Andreev spectrum of this system is a discrete set of particle-hole energy levels $E_i(\varphi)$ which depend on the gauge-invariant phase difference φ of the superconducting order parameter between the two leads, induced by a magnetic flux Φ. If $L \ll \xi$, where L is the distance between the leads, the CPR is given by [24]

$$I(\varphi) = \frac{e}{\hbar}\sum_i f\left[\frac{E_i(\varphi)}{k_B T}\right]\partial_\varphi E_i(\varphi), \tag{1}$$

where $f(x) = 1/(e^x+1)$ is the Fermi-Dirac distribution. Note that the low-energy Andreev spectrum does not depend on the superconducting ring length, in the limit of short junctions [25] (see SM Sec. A [22]). If the lowest-energy (LE) levels close the gap with linear phase dispersion, the CPR exhibits a discontinuity at zero temperature $T = 0$. In fact, in this case the Fermi-Dirac distribution in Eq. (1) converges to a step function, and thus the only contribution to the current is given by energy levels $E \leqslant 0$, i.e., $I(\varphi) = (e/\hbar)\sum_{E_i\leqslant 0}\partial_\varphi E_i(\varphi)$. This mandates a discontinuous drop $\Delta I(\varphi^*) = -(e/\hbar)\sum_{E_j(\varphi^*)=0}|\partial_\varphi E_j(\varphi^*)|$ at any gapless point φ^* where zero-energy levels $E_j(\varphi^*) = 0$ have a linear phase dispersion $\partial_\varphi E_j(\varphi^*) \neq 0$. Since Andreev levels are

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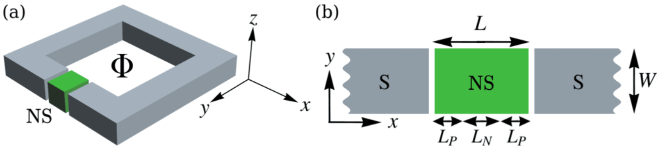

FIG. 1. A TS (a) realized by a quantum nanostructure embedded into a superconducting ring with Rashba SOC $\propto (\boldsymbol{\sigma}\times\mathbf{p})_z$ along the z axis. The nanostructure (b) of length L, width W, and proximity length L_P ($L_N = L - 2L_P$), connected to two superconducting leads, can be either a quantum dot ($L = W = 1$ lattice site), wire ($L > W = 1$), or planar well ($L, W > 1$).

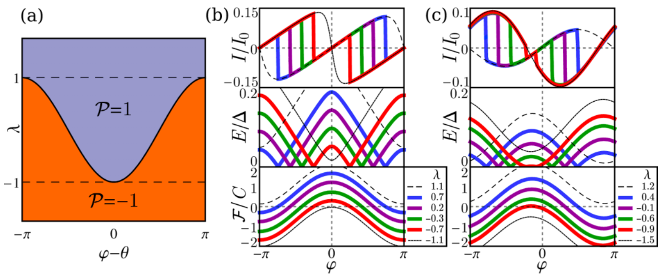

FIG. 2. Topological phase space (a) of a finite-sized TS as a function of φ, θ, and λ. Topological transitions between states with different fermion parities are possible for $|\lambda| < 1$, where $\mathcal{P}_\varphi = \pm 1$, respectively, for $|\varphi - \theta| \lessgtr \arccos(-\lambda)$. Zero-energy modes occur at $\cos(\varphi - \theta) = -\lambda$ (solid line). CPR in units of $I_0 = (e/\hbar)\Delta$, LE Andreev level, and Pfaffian $\mathcal{F}_\varphi/C$ for a quantum wire (b) and a planar well (c) with magnetic field $b = b_y$ perpendicular to the SOC and current directions. Different magnetic field directions give qualitatively similar results. For $|\lambda| < 1$ (continuous lines), CPR discontinuities correspond to zero-energy modes ($\mathcal{F}_{\varphi^*} = 0$) and to variations of the fermion parity $\mathcal{P}_\varphi = \operatorname{sgn}\mathcal{F}_\varphi$ at $\varphi^*_\pm = \theta \pm \arccos(-\lambda)$. The trivial $\mathcal{F}_\varphi > 0$ and nontrivial $\mathcal{F}_\varphi < 0$ branches of the CPR correspond, respectively, to $\lambda > 1$ (dashed lines) and $\lambda < -1$ (dotted lines). In a planar well with $b_y \neq 0$ and $\alpha \neq 0$ (c), the CPR and Pfaffian are no longer symmetric under phase inversion $\varphi \to -\varphi$ ($\theta \neq n\pi$ in this case), and an anomalous current $I(0) \neq 0$ is present for $\lambda = -0.9$ and -1.5.

continuously differentiable in finite-sized systems (see SM Sec. A [22] and Ref. [26]) these are the only points where the CPR can be discontinuous. Hence, discontinuities at zero temperature correspond to zero-energy modes closing the particle-hole gap. In general, the converse is not true, i.e., the gap may close without any discontinuity if $\partial_\varphi E_j(\varphi^*) = 0$. Hereafter we will show that such discontinuities correspond, if time-reversal symmetry is broken, to fermion parity transitions and thus can be related to a topological phase transition of the TS.

Superconductors exhibit particle-hole symmetry, i.e., their BdG Hamiltonian $\mathcal{H}(\varphi)$ is invariant under the antiunitary transformation $\Xi = \tau_x K$ with $\Xi^2 = 1$, where K is the complex conjugate operator and τ_x the Pauli matrix in particle-hole space. For a finite magnetic field $\mathbf{b} \neq \mathbf{0}$, time-reversal and chiral symmetries are broken, and hence the system is in the Altland-Zirnbauer [27] symmetry class D. This class is characterized both in 1D (Majorana chain, continuous spectrum) and in 0D (finite-sized system with discrete energy spectrum) by the $\mathbb{Z}_2$ topological invariant accordingly to the periodic table of topological phases [27]. Analogously to the 1D case (Majorana chain), the topological invariant in a 0D system is defined following Ref. [28] as the fermion parity of the ground state [2,13] $\mathcal{P}_\varphi = \operatorname{sgn}\mathcal{F}_\varphi$, where $\mathcal{F}_\varphi = \operatorname{pf}[\mathcal{H}(\varphi)\imath\tau_x]$ is the Pfaffian [29] of the matrix $\mathcal{H}(\varphi)\imath\tau_x$. The fermion parity labels the topological inequivalent phases, i.e., trivial $\mathcal{P}_\varphi = 1$ ($\mathcal{F}_\varphi > 0$) and nontrivial $\mathcal{P}_\varphi = -1$ ($\mathcal{F}_\varphi < 0$). Moreover, since $\mathcal{F}_\varphi^2 = \det[\mathcal{H}(\varphi)\imath\tau_x] = \det[\mathcal{H}(\varphi)] = \prod_i E_i(\varphi)$, the condition $\mathcal{F}_{\varphi^*} = 0$ corresponds to gapless points φ^* where zero-energy modes occur. The topological invariant in 0D (finite-sized system) and in 1D (continuous limit) are closely related: In the limit $L \to \infty$, in fact, the Majorana number [2] coincides with the fermion parity of the ground state, i.e., $\mathcal{M} = \operatorname{sgn}\{\operatorname{pf}[\mathcal{H}\imath\tau_x]\}$ [13,28].

Fermion parity phase dependence. The nontrivial phase of a Majorana chain (1D, continuous spectrum) requires an open particle-hole gap, which can be realized only with SOC [5] and for specific magnetic field directions [30]. Contrarily, in 0D finite-sized systems, the topological invariant $\mathcal{P}_\varphi$ is well defined even in the absence of SOC and for any magnetic field direction, as long as $\mathcal{F}_\varphi \neq 0$, and depends explicitly on the phase φ. Changes in the fermion parity $\mathcal{P}_\varphi$ define the topological phase transitions in this system. Independently from the details of the tight-binding Hamiltonian, the Pfaffian can be expanded as a Fourier series in the phase φ with coefficients $a_n \propto \Delta^{2n}$. If the superconducting gap is smaller than the bandwidth of the nanostructure (e.g., in conventional superconductors), higher harmonics become negligible and one obtains at the first order (see SM Sec. B [22])

$$\mathcal{F}_\varphi \approx C[\cos(\varphi - \theta) + \lambda], \tag{2}$$

where $\lambda = (\mathcal{F}_{\varphi'} + \mathcal{F}_{\varphi'+\pi})/(2C)$ (the sum does not depend on the choice of the angle φ'), $2C = \sqrt{(\mathcal{F}_0 - \mathcal{F}_\pi)^2 + (\mathcal{F}_{\frac{\pi}{2}} - \mathcal{F}_{-\frac{\pi}{2}})^2}$, and $\tan\theta = (\mathcal{F}_{\frac{\pi}{2}} - \mathcal{F}_{-\frac{\pi}{2}})/(\mathcal{F}_0 - \mathcal{F}_\pi)$. These parameters depend on the Hamiltonian, e.g., on the hopping parameter t, magnetic field $\mathbf{b}$, chemical potential μ, SOC α, and superconducting gap Δ, but not on the gauge-invariant phase φ. The topological phase space is thus completely characterized by the phase φ and the parameter $\lambda = \lambda(t,\alpha,\mu,\mathbf{b},\Delta)$, as shown in Fig. 2(a). If $|\lambda| \geqslant 1$, the system is either in the trivial $\mathcal{F}_\varphi > 0$ for $\lambda \geqslant 1$ or nontrivial phase $\mathcal{F}_\varphi < 0$ for $\lambda \leqslant -1$, with the exception of a single gapless point at $\varphi^* = \theta + \pi$ or θ, respectively, for $\lambda = \pm 1$. If $|\lambda| < 1$ instead, topological transitions occur at the gapless points $\varphi^*_\pm \approx \theta \pm \arccos(-\lambda)$. Hence, the closing of the particle-hole gap $\mathcal{F}_\varphi = 0$ defines the boundaries between trivial and nontrivial phases.

Furthermore, if SOC or the magnetic field vanish, the Andreev spectrum, CPR, and fermion parity are invariant under phase inversion, which mandates $\theta = n\pi$ with n integer in Eq. (2). This invariance is related to the magnetic mirror symmetry [31] $M_{xz}\Theta = K$, where $M_{xz} = \imath\sigma_y$ is the spin mirror reflection across the xz plane and $\Theta = -\imath\sigma_y K$ the time-reversal operator. If $\alpha = 0$ or $\mathbf{b} = \mathbf{0}$ in fact, the quantization axis can be arbitrarily chosen such that the only complex terms in the BdG Hamiltonian are those in the phase φ (see SM Sec. B [22]). Therefore, $\mathcal{H}(\varphi)^* = \mathcal{H}(-\varphi)$, and consequently $I(\varphi) = -I(-\varphi)$, $\mathcal{P}_\varphi = \mathcal{P}_{-\varphi}$, and $\theta = n\pi$. However, if the magnetic mirror symmetry is broken, i.e., if $\mathcal{H}(\varphi)^* \neq \mathcal{H}(-\varphi)$, the CPR and the fermion parity may no longer be symmetric under phase inversion, i.e., $I(\varphi) \neq -I(-\varphi)$, $\mathcal{P}_\varphi \neq \mathcal{P}_{-\varphi}$, and $\theta \neq n\pi$. In this case, the magnetic mirror symmetry corresponds to the inversion of the Pfaffian phase shift $\theta \to -\theta$ (see SM Sec. B [22]), and the fermion parity is still invariant under the more general transformation $\varphi \to 2\theta - \varphi$, i.e., $\mathcal{P}_\varphi = \mathcal{P}_{2\theta-\varphi}$ [cf. Eq. (2)]. A Pfaffian phase shift $\theta \neq n\pi$ is thus a signature of the broken phase-inversion and magnetic mirror symmetries, and can result in an anomalous current [32–34] at zero phase (see below).

CPR discontinuities. In a superconductor with broken time-reversal symmetry, any zero-energy mode is at least doubly degenerate due to particle-hole symmetry. Hence the Pfaffian can be expanded near any gapless point as $\mathcal{F}_\varphi \propto (\varphi - \varphi^*)^d$, where the order d is half the total multiplicity $2d = \sum_j m_j$, with m_j the multiplicities of zero-energy modes $E_j(\varphi) \propto (\varphi - \varphi^*)^{m_j}$. Thus, if the Pfaffian first derivative $\mathcal{F}'_\varphi = \partial_\varphi \mathcal{F}_\varphi$ is nonzero at the gapless point φ^*, there exist only two doubly degenerate LE levels $E_\pm(\varphi^*) = 0$ with linear phase dispersion $E_\pm(\varphi) \propto (\varphi - \varphi^*)$. Therefore, one obtains $\partial_\varphi E_\pm(\varphi^*) = \pm \mathcal{F}'_{\varphi^*}/\chi_{\varphi^*}$, where $\chi_{\varphi^*} = \prod_{E_i>0} E_i(\varphi^*) > 0$ (see SM Sec. C [22]). Hence, the LE level contribution changes its sign passing through the gapless point, and the total current exhibits a discontinuous drop given by

$$\Delta I(\varphi^*) = -\frac{2e}{\hbar}\frac{|\mathcal{F}'_{\varphi^*}|}{\chi_{\varphi^*}}, \tag{3}$$

where $|\mathcal{F}'_{\varphi^*}| = C|\sin(\varphi^* - \theta)| = C\sqrt{1-\lambda^2}$ [cf. Eq. (2)]. Eq. (3) relates CPR discontinuities at zero temperature with the variations of the fermion parity $\mathcal{P}_\varphi = \operatorname{sgn} \mathcal{F}_\varphi$ in superconductors with broken time-reversal symmetry, and is valid for $\mathcal{F}'_{\varphi^*} \neq 0$. If $|\lambda| < 1$ in fact, the Pfaffian changes its sign at $\varphi^*_\pm = \theta \pm \arccos(-\lambda)$, where $\mathcal{F}'_{\varphi^*_\pm} \neq 0$ according to Eq. (2). Here, the CPR has two discontinuities $\Delta I(\varphi^*_\pm) \neq 0$ which correspond to transitions between topological phases with even and odd fermion parity, where zero-energy modes appear. If $|\lambda| > 1$ instead, no zero-energy mode or CPR discontinuity occur. In the limit cases $\lambda = \pm 1$, the Pfaffian vanishes at the gapless point without any sign change ($\mathcal{F}'_{\varphi^*} = 0$), while the CPR may exhibit at most one discontinuity, corresponding to a zero-energy mode with $m_\pm = 1$. Hence, the presence of two distinct discontinuities in the CPR of a TS with broken time-reversal symmetry defines the boundaries between inequivalent topological phases. Moreover, these discontinuities are a direct signature of zero-energy modes, and coincide with a sign change of the LE level contribution to the current. These zero-energy modes signal the topological transition at φ^* between phases with different fermion parity, and are described locally as a linear superposition of particle and hole states or, equivalently, of two orthogonal Majorana states (see SM Sec. E [22]). The results presented here hold for *any* discrete 0D Hamiltonian in the Altland-Zirnbauer class D (particle-hole symmetry and broken time-reversal symmetry). Note that, if time-reversal symmetry is unbroken ($\mathbf{b} = \mathbf{0}$ and $\theta = n\pi$), the total multiplicity is $2d \geqslant 4$ due to spin and particle-hole degeneracy, and therefore $\mathcal{F}'_{\varphi^*} = 0$ at any gapless point. Hence, no fermion parity transition occurs ($|\lambda| > 1$) and the CPR may exhibit at most one discontinuity at $\varphi^* = n\pi$ if $|\lambda| = 1$, e.g., in point contact junctions [24]. Indeed, a time-reversal invariant s-wave superconductor is topologically trivial. Since CPR discontinuities correspond to an abrupt jump to the LE state, they can be measured at the equilibrium (DC Josephson current) and are not affected by quasiparticle poisoning [35].

Numerical results. Figures 2(b) and 2(c) show the CPR at zero temperature, the LE Andreev level, and the Pfaffian of a quantum wire ($W = 1$) and a planar well ($W > 1$) as a function of the phase and magnetic field $b = b_y$ perpendicular to the SOC [$\propto (\boldsymbol{\sigma} \times \mathbf{p})_z$] and current directions, calculated directly from the BdG Hamiltonian $\mathcal{H}(\varphi)$ (see SM Sec. A [22] for details). For $|\lambda| < 1$, the gap closes with linear phase dispersion and thus the CPR has two discontinuities at $\varphi^*_\pm$. In a planar well with $b_y \neq 0$ and $\alpha \neq 0$ [Fig. 2(c)], the CPR and the Pfaffian are no longer symmetric under phase inversion $\varphi \to -\varphi$ ($\theta \neq n\pi$ in this case). Note that fermion parity transitions and CPR discontinuities are present for any magnetic field direction. In a quantum wire with $L \to \infty$ (1D, continuous spectrum), we verified numerically that Eqs. (2) and (3) reproduce the well-known results of Ref. [5]. In particular, the trivial ($\mathcal{M} = 1$) and nontrivial ($\mathcal{M} = -1$) phases correspond, respectively, to $\lambda > 1$ and $\lambda = -1$ in Eq. (2). We have also verified that Majorana bound states localize at the wire edges, corresponding to a discontinuity in the CPR at $\varphi^* = \pi$ if $\mathbf{b} \perp y$.

Anomalous current. When the magnetic mirror symmetry is broken ($\theta \neq n\pi$), the current may no longer be symmetric under phase inversion, which can result in a finite anomalous current [32–34] at $\varphi = 0$, as shown in Fig. 2(c). This can be realized, e.g., in planar quantum wells with SOC and magnetic field, where a finite anomalous current has been related to the presence of chiral edge states [33] or to a SOC-induced Lorentz force [34]. In these systems, a topological phase transition can be revealed by a discontinuity of the current at zero phase $\Delta I(\varphi = 0, \nu)$ with respect to the parameter ν (magnetic field, chemical potential, or SOC) which drives the system through the topological transition. In this case one can always find a value $\nu = \nu^*$ such that the gap closes at $\varphi^* = 0$, i.e., $\lambda(\nu^*) = -\cos\theta$ [cf. Eq. (2) and Fig. 2(a)]. At this gapless point the fermion parity changes and the current at $\varphi = 0$ has a discontinuous drop with respect to the parameter ν given by

$$\Delta I(\varphi^* = 0, \nu = \nu^*) = -\frac{2e}{\hbar}\frac{C}{\chi_0}|\sin\theta|. \tag{4}$$

The value ν^* corresponds to a crossover between the trivial $\mathcal{F}_\varphi > 0$ ($\lambda > 1$) and nontrivial $\mathcal{F}_\varphi < 0$ ($\lambda < -1$) branches of the CPR, as shown in Fig. 2(c). This discontinuity mandates a finite anomalous current $I(0,\nu) \neq 0$ near $\nu = \nu^*$ either in the trivial or nontrivial phases, or in both. Numerical calculations indicate that it is nonzero only in the nontrivial phase [cf. Fig. 2(c)]. Hence, a discontinuity of the anomalous current at zero phase with respect to any system parameter (e.g., magnetic field) is also a signature of a topological phase transition.

Experimental proposal. At finite temperatures, CPR discontinuities are smoothed out by the thermal spreading of the Fermi-Dirac distribution. For $T \ll T_d = \delta_d/k_B$, where δ_d is the gap between the first and second Andreev levels at φ^*, the current can be expanded as a sum of two contributions $I_{he}(\varphi)$ and $I_{le}(\varphi)$ coming, respectively, from higher-energy

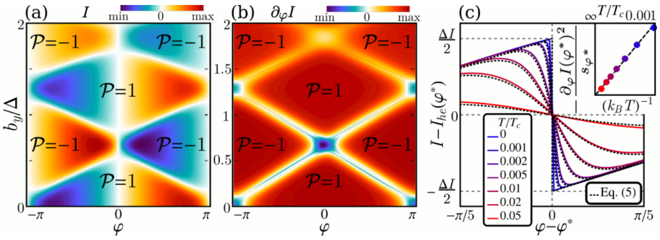

FIG. 3. Fermion parity $\mathcal{P}_\varphi$, CPR (a), and phase derivative $\partial_\varphi I(\varphi)$ of the current (b) at $T = 0.02T_c$ of a quantum wire as a function of the magnetic field along the y axis. Spikes in the phase derivative correspond to topological phase transitions. Josephson current (c) near the gapless point and phase derivative $\partial_\varphi I(\varphi^*)$ (inset). Dotted lines correspond to Eq. (5), continuous lines to numerical calculations. The two minima (b) at $\varphi = 0$ ($b_y/\Delta \approx 0.5$ and ≈ 2, respectively) are constant in temperature, i.e., do not correspond to any topological phase transition.

levels ($E > \delta_d$) and from the LE level. The latter contribution depends strongly on the temperature and can be obtained from Eqs. (1) and (3) (see SM Sec. D [22]), which yield

$$I(\varphi) \approx I_{he}(\varphi) + \frac{\Delta I(\varphi^*)}{2} \tanh\left[-\frac{\hbar}{e}\frac{\Delta I(\varphi^*)(\varphi - \varphi^*)}{4k_B T}\right]. \quad (5)$$

The current-phase derivative diverges as $\partial_\varphi I(\varphi^*) \approx -\hbar/(8e)\Delta I(\varphi^*)^2/(k_B T)$ for $T \to 0$. Hence, the scaling factor $s_{\varphi^*} = -\partial_\varphi I(\varphi^*) k_B T$ is a direct measure of the discontinuous drop, since $\Delta I(\varphi^*) \approx -\sqrt{(8e/\hbar) s_{\varphi^*}}$. Since the separation between energy levels increases as the system size decreases, the temperature T_d, below which CPR discontinuities are measurable, is maximized for small linear dimensions. For parameters considered in Figs. 2 and 3, we obtain $T_d \approx 0.2T_c$.

In light of this, we propose to measure the low-temperature CPR [24,36] through a nanostructure with broken time-reversal symmetry. Figures 3(a) and 3(b) show the CPR of a quantum wire ($L = 200$ sites) at $T > 0$ and its phase derivative as a function of the magnetic field. Besides, Eq. 3(c) shows the effect of temperature on current discontinuities, which is described by Eq. (5). Spikes in the phase derivative which exhibit the characteristic temperature scaling of Eq. 3(c) identify the boundaries between inequivalent topological phases. These effects should be measurable in clean Josephson weak links of the order of 50–150 nm in InAs or InSb nanowires ($L = 100$–300 lattice sites) proximized by a conventional superconductor [21] (e.g., Nb or Al). Note that the Josephson critical current has been recently measured in an InAs nanowire in the relevant regime [37], at very low temperatures ($\sim$10 mK) comparable with the temperatures considered here.

Conclusions. We have shown that discontinuities in the Josephson CPR correspond, in TSs with broken time-reversal symmetry, to topological phase transitions between states with different fermion parities, where zero-energy modes occur. These current discontinuities are not affected by quasiparticle poisoning and can be revealed by spikes in the current-phase derivative and by their characteristic temperature dependence. Moreover, in systems with broken phase-inversion symmetry, topological phase transitions correspond to discontinuities of the anomalous current at zero phase. Such features in the CPR provide an experimental tool to probe the fermion parity and to resolve the topological phase space of TSs.

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[1] M. Z. Hasan and C. L. Kane, Colloquium: Topological insulators, Rev. Mod. Phys. **82**, 3045 (2010); X.-L. Qi and S.-C. Zhang, Topological insulators and superconductors, *ibid.* **83**, 1057 (2011); T. Wehling, A. Black-Schaffer, and A. Balatsky, Dirac materials, Adv. Phys. **63**, 1 (2014).

[2] A. Y. Kitaev, Unpaired Majorana fermions in quantum wires, Phys. Usp. **44**, 131 (2001).

[3] A. Yu. Kitaev, Fault-tolerant quantum computation by anyons, Ann. Phys. **303**, 2 (2003); C. Nayak, S. H. Simon, A. Stern, M. Freedman, and S. Das Sarma, Non-Abelian anyons and topological quantum computation, Rev. Mod. Phys. **80**, 1083 (2008).

[4] L. Fu and C. L. Kane, Superconducting Proximity Effect and Majorana Fermions at the Surface of a Topological Insulator, Phys. Rev. Lett. **100**, 096407 (2008).

[5] R. M. Lutchyn, J. D. Sau, and S. Das Sarma, Majorana Fermions and a Topological Phase Transition in Semiconductor/Superconductor Heterostructures, Phys. Rev. Lett. **105**, 077001 (2010); Y. Oreg, G. Refael, and F. von Oppen, Helical Liquids and Majorana Bound States in Quantum Wires, *ibid.* **105**, 177002 (2010); J. Alicea, New directions in the pursuit of Majorana fermions in solid state systems, Rep. Prog. Phys. **75**, 076501 (2012); D. Rainis, A. Saha, J. Klinovaja, L. Trifunovic, and D. Loss, Transport Signatures of Fractional Fermions in Rashba Nanowires, Phys. Rev. Lett. **112**, 196803 (2014).

[6] C. W. J. Beenakker, Search for Majorana fermions in superconductors, Annu. Rev. Condens. Matter Phys. **4**, 113 (2013).

[7] D. Thouless, *Topological Quantum Numbers in Nonrelativistic Physics* (World Scientific, Singapore, 1998).

[8] D. J. Thouless, M. Kohmoto, M. P. Nightingale, and M. den Nijs, Quantized Hall Conductance in a Two-Dimensional Periodic Potential, Phys. Rev. Lett. **49**, 405 (1982); S. Tewari and J. D. Sau, Topological Invariants for Spin/Orbit Coupled Superconductor Nanowires, *ibid.* **109**, 150408 (2012).

[9] B. Zhou, H.-Z. Lu, R.-L. Chu, S.-Q. Shen, and Q. Niu, Finite Size Effects on Helical Edge States in a Quantum Spin/Hall System, Phys. Rev. Lett. **101**, 246807 (2008).

[10] L. Fu and C. L. Kane, Josephson current and noise at a superconductor/quantum-spin-Hall-insulator/superconductor junction, Phys. Rev. B **79**, 161408 (2009); B. van Heck,

F. Hassler, A. R. Akhmerov, and C. W. J. Beenakker, Coulomb stability of the 4π-periodic Josephson effect of Majorana fermions, *ibid.* **84**, 180502 (2011); L. P. Rokhinson, X. Liu, and J. K. Furdyna, The fractional a.c. Josephson effect in a semiconductor-superconductor nanowire as a signature of Majorana particles, Nat. Phys. **8**, 795 (2012); J. Wiedenmann, E. Bocquillon, R. S. Deacon, S. Hartinger, O. Herrmann, T. M. Klapwijk, L. Maier, C. Ames, C. Brüne, C. Gould, A. Oiwa, K. Ishibashi, S. Tarucha, H. Buhmann, and L. W. Molenkamp, 4π-periodic Josephson supercurrent in HgTe-based topological Josephson junctions, Nat. Commun. **7**, 10303 (2016).

[11] V. Mourik, K. Zuo, S. M. Frolov, S. R. Plissard, E. P. A. M. Bakkers, and L. P. Kouwenhoven, Signatures of Majorana fermions in hybrid superconductor/semiconductor nanowire devices, Science **336**, 1003 (2012).

[12] S. Nadj-Perge, I. K. Drozdov, J. Li, H. Chen, S. Jeon, J. Seo, A. H. MacDonald, B. A. Bernevig, and A. Yazdani, Observation of Majorana fermions in ferromagnetic atomic chains on a superconductor, Science **346**, 602 (2014).

[13] B. Zocher, M. Horsdal, and B. Rosenow, Robustness of topological order in semiconductor/superconductor nanowires in the Coulomb blockade regime, New J. Phys. **15**, 085003 (2013); J. C. Budich and E. Ardonne, Equivalent topological invariants for one-dimensional Majorana wires in symmetry class *D*, Phys. Rev. B **88**, 075419 (2013).

[14] C. W. J. Beenakker, D. I. Pikulin, T. Hyart, H. Schomerus, and J. P. Dahlhaus, Fermion-Parity Anomaly of the Critical Supercurrent in the Quantum Spin-Hall Effect, Phys. Rev. Lett. **110**, 017003 (2013).

[15] M. Diez, I. C. Fulga, D. I. Pikulin, M. Wimmer, A. R. Akhmerov, and C. W. J. Beenakker, Phase-locked magnetoconductance oscillations as a probe of Majorana edge states, Phys. Rev. B **87**, 125406 (2013); S. Barbarino, R. Fazio, M. Sassetti, and F. Taddei, Parity dependent Josephson current through a helical Luttinger liquid, New J. Phys. **15**, 085025 (2013); P. San-Jose, E. Prada, and R. Aguado, Mapping the Topological Phase Diagram of Multiband Semiconductors with Supercurrents, Phys. Rev. Lett. **112**, 137001 (2014).

[16] A. V. Balatsky, I. Vekhter, and J.-X. Zhu, Impurity-induced states in conventional and unconventional superconductors, Rev. Mod. Phys. **78**, 373 (2006).

[17] S.-G. Cheng, Y. X. Xing, X. C. Xie, and Q.-F. Sun, Supercurrent and its Fano effect in a Josephson Aharonov-Bohm ring, Eur. Phys. J. B **67**, 551 (2009); A. Brunetti, A. Zazunov, A. Kundu, and R. Egger, Anomalous Josephson current, incipient time-reversal symmetry breaking, and Majorana bound states in interacting multilevel dots, Phys. Rev. B **88**, 144515 (2013); G. Kiršanskas, M. Goldstein, K. Flensberg, L. I. Glazman, and J. Paaske, Yu-Shiba-Rusinov states in phase-biased superconductor—quantum dot—superconductor junctions, *ibid.* **92**, 235422 (2015).

[18] T. Yokoyama, M. Eto, and Y. V. Nazarov, Josephson current through semiconductor nanowire with spin-orbit interaction in magnetic field, J. Phys. Soc. Jpn. **82**, 054703 (2013).

[19] J. Koch and K. Le Hur, Discontinuous Current-Phase Relations in Small One-Dimensional Josephson Junction Arrays, Phys. Rev. Lett. **101**, 097007 (2008).

[20] U. Khanna, D. K. Mukherjee, A. Kundu, and S. Rao, Chiral nodes and oscillations in the Josephson current in Weyl semimetals, Phys. Rev. B **93**, 121409(R) (2016).

[21] F. Giazotto, P. Spathis, S. Roddaro, S. Biswas, F. Taddei, M. Governale, and L. Sorba, A Josephson quantum electron pump, Nat. Phys. **7**, 857 (2011).

[22] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevB.93.220507 for details and additional information.

[23] P. de Gennes, *Superconductivity of Metals and Alloys* (Westview Press, Boulder, CO, 1999).

[24] A. A. Golubov, M. Y. Kupriyanov, and E. Il'ichev, The current-phase relation in Josephson junctions, Rev. Mod. Phys. **76**, 411 (2004); Y. Nazarov and Y. Blanter, *Quantum Transport: Introduction to Nanoscience* (Cambridge University Press, Cambridge, U.K., 2009).

[25] I. Affleck, J.-S. Caux, and A. M. Zagoskin, Andreev scattering and Josephson current in a one-dimensional electron liquid, Phys. Rev. B **62**, 1433 (2000).

[26] M. Bronshtein, Smoothness of roots of polynomials depending on parameters, Sib. Math. J. **20**, 347 (1979); F. Colombini, N. Orrù, and L. Pernazza, On the regularity of the roots of hyperbolic polynomials, Isr. J. Math. **191**, 923 (2012).

[27] A. Altland and M. R. Zirnbauer, Nonstandard symmetry classes in mesoscopic normal-superconducting hybrid structures, Phys. Rev. B **55**, 1142 (1997); A. P. Schnyder, S. Ryu, A. Furusaki, and A. W. W. Ludwig, Classification of topological insulators and superconductors, in *Advances in Theoretical Physics: Landau Memorial Conference*, edited by V. Lebedev and M. Feigel'man, AIP Conf. Proc. No. 1134 (AIP, Melville, NY, 2009), p. 10; A. Kitaev, Periodic table for topological insulators and superconductors, *ibid.*, AIP Conf. Proc. No. 1134 (AIP, Melville, NY, 2009), p. 22.

[28] T. A. Loring, K-theory and pseudospectra for topological insulators, Ann. Phys. **356**, 383 (2015).

[29] M. Wimmer, Algorithm 923: Efficient numerical computation of the Pfaffian for dense and banded skew-symmetric matrices, ACM Trans. Math. Software **38**, 30 (2012).

[30] C.-H. Lin, J. D. Sau, and S. Das Sarma, Zero-bias conductance peak in Majorana wires made of semiconductor/superconductor hybrid structures, Phys. Rev. B **86**, 224511 (2012); J. Osca, D. Ruiz, and L. Serra, Effects of tilting the magnetic field in one-dimensional Majorana nanowires, *ibid.* **89**, 245405 (2014); S. Rex and A. Sudbø, Tilting of the magnetic field in Majorana nanowires: Critical angle and zero-energy differential conductance, *ibid.* **90**, 115429 (2014).

[31] B. Lu, K. Yada, M. Sato, and Y. Tanaka, Crossed Surface Flat Bands of Weyl Semimetal Superconductors, Phys. Rev. Lett. **114**, 096804 (2015).

[32] I. Kulagina and J. Linder, Spin supercurrent, magnetization dynamics, and φ-state in spin-textured Josephson junctions, Phys. Rev. B **90**, 054504 (2014); T. Yokoyama, M. Eto, and Y. V. Nazarov, Anomalous Josephson effect induced by spin/orbit interaction and Zeeman effect in semiconductor nanowires, *ibid.* **89**, 195407 (2014); G. Campagnano, P. Lucignano, D. Giuliano, and A. Tagliacozzo, Spin-orbit coupling and anomalous Josephson effect in nanowires, J. Phys.: Condens. Matter **27**, 205301 (2015).

[33] F. Dolcini, M. Houzet, and J. S. Meyer, Topological Josephson ϕ_0 junctions, Phys. Rev. B **92**, 035428 (2015).

[34] F. S. Bergeret and I. V. Tokatly, Theory of diffusive φ_0 Josephson junctions in the presence of spin-orbit coupling, Europhys. Lett. **110**, 57005 (2015).

[35] D. Rainis and D. Loss, Majorana qubit decoherence by quasi-particle poisoning, Phys. Rev. B **85**, 174533 (2012).

[36] S. M. Frolov, D. J. Van Harlingen, V. A. Oboznov, V. V. Bolginov, and V. V. Ryazanov, Measurement of the current-phase relation of superconductor/ferromagnet/superconductor π Josephson junctions, Phys. Rev. B **70**, 144505 (2004); I. Sochnikov, A. J. Bestwick, J. R. Williams, T. M. Lippman, I. R. Fisher, D. Goldhaber-Gordon, J. R. Kirtley, and K. A. Moler, Direct measurement of current-phase relations in superconductor/topological insulator/superconductor junctions, Nano Lett. **13**, 3086 (2013); D. B. Szombati, S. Nadj-Perge, D. Car, S. R. Plissard, E.P. A. M. Bakkers, and L. P. Kouwenhoven, Josephson φ_0-junction in nanowire quantum dots, Nat. Phys. **12**, 568 (2016).

[37] J. Paajaste, E. Strambini, M. Amado, S. Roddaro, P. San-Jose, R. Aguado, F. S. Bergeret, D. Ercolani, L. Sorba, and F. Giazotto, Magnetically-driven colossal supercurrent enhancement in InAs nanowire Josephson junctions, arXiv:1601.02955.