Electric-field-induced spin resonance in antiferromagnetic insulators: Inverse process of the dynamical chiral magnetic effect

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We propose a realization of the electric-field-induced antiferromagnetic resonance. We consider threedimensional antiferromagnetic insulators with spin-orbit coupling characterized by the existence of a topological term called the θ term. By solving the Landau-Lifshitz-Gilbert equation in the presence of the θ term, we show that, in contrast to conventional methods using ac magnetic fields, the antiferromagnetic resonance state is realized by ac electric fields along with static magnetic fields. This mechanism can be understood as the inverse process of the dynamical chiral magnetic effect, an alternating current generation by magnetic fields. In other words, we propose a way to electrically induce the dynamical axion field in condensed matter. We discuss a possible experiment to observe our proposal, which utilizes the spin pumping from the antiferromagnetic insulator into a heavy metal contact.

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Introduction. Possible applications of materials in which the low-energy effective models are described by relativistic Dirac fermions have been studied intensively and extensively. The representative examples of Dirac fermion systems are graphene [1] and topological insulators [2–4]. Studies on topological phases of matter are now extended to various gapless topological phases such as Weyl semimetals [5-9] and Dirac semimetals [10–13]. Unlike graphene, topological phases are realized in spin-orbit coupled systems. It has been revealed that strong spin-orbit coupling (SOC) is a key element in unconventional phenomena. For example, the spin-momentum locking, which is realized in the metallic surface states of three-dimensional (3D) topological insulators, arises as a consequence of SOC and results in a persistent pure spin current on the surface that is robust against disorder [2-4]. Recently, possible ways to manipulate and utilize such topological surface states have been investigated experimentally in spintronics [14–17]. Another interesting example is the chiral magnetic effect, which was originally proposed in high-energy theory as a direct current generation by static magnetic fields [18]. Its possibility has been investigated theoretically in Weyl semimetals [19-24], and the dynamical realization of the chiral magnetic effect, i.e., an alternating current generation by magnetic fields, has also been proposed in spin-orbit coupled insulators [25].

One of the most important themes in spintronics is the generation and control of spin currents. Spin pumping is a powerful technique to generate a pure spin current [26–28]. While a magnetization is precessing, the spin angular momentum in a magnet is injected across the interface into a neighboring material through the exchange interaction. The spin current injected into a heavy metal (HM) such as Pt can be detected electrically via the inverse spin Hall effect [27,29,30]. So far, ferromagnets have been used as a source material in which magnetization precession is caused for spin pumping. On the other hand, unlike ferromagnets, antiferromagnets had not been considered to be of practical use due to zero net

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magnetization. However, recent studies in spintronics is now extended to active use of antiferromagnets [31–35]. It has been suggested that antiferromagnets can complement or replace ferromagnets as active elements of a memory [36] or logic device [37], e.g., because antiferromagnets do not generate unwanted stray fields.

In this Rapid Communication, we propose a realization of the electrically driven antiferromagnetic (AF) resonance in 3D AF insulators with SOC. This is in sharp contrast to conventional methods using magnetic fields. In spintronics, electrical manipulation of magnetism is one of the most important subjects in the pursuit of energy-saving and higherdensity information storage. However, preceding studies have been based on electric-current-induced methods that require such high-density currents as $\sim 10^{10}$ A/m² [38]. Namely, there is a large energy dissipation due to Joule heat. In contrast, since the system we consider is insulating, there is no energy dissipation due to Joule heat in the presence of electric fields. Moreover, whereas the electric-field-induced ferromagnetic resonance has been realized experimentally in a ferromagnetic metal [39], the electric-field-induced AF resonance has not yet been realized. The key ingredient for its realization in our study is a topological term called the θ term which arises due to strong SOC. The existence of the θ term leads to a coupling of electric fields and the Néel field. By solving the Landau-Lifshitz-Gilbert equation in the presence of the θ term, we show that the AF resonance state is realized by ac electric fields. We also show that the resonance state can be detected as a usual spin-pumping-induced voltage signal. We argue that the mechanism of the electric-field-induced AF resonance in this study can be understood as the inverse process of the dynamical chiral magnetic effect [25].

Low-energy effective model. We study a class of 3D AF insulators that can be realized in systems with electron correlations and SOC, such as 5*d* transition-metal oxides [40,41]. As a theoretical model, we adopt a tight-binding model called the Fu-Kane-Mele-Hubbard model on a diamond lattice [25,42], in which the nearest-neighbor electron hopping, spin-dependent next-nearest-neighbor electron hopping (i.e., SOC), and on-site electron-electron repulsive interaction are taken into account. In this model, an AF insulator phase

develops when on-site interactions are strong. Here, the mean-field AF order parameter is parametrized between the two sublattices as $(n_0 \sin \theta \cos \varphi, n_0 \sin \theta \sin \varphi, n_0 \cos \theta)$, with n_0 the magnitude of the order parameter, and angles θ and φ obtained from the coordinate of a diamond lattice [42].

The mean-field low-energy effective action of the system in the presence of an external electromagnetic field is given by a massive Dirac fermion model of the form [25,42]

$$S_{\rm AF} = \int dt \, d^3r \sum_{f=1,2,3} \bar{\psi}_f [i\gamma^{\mu} D_{\mu} - M_0 + i\gamma^5 M_{5f}] \psi_f, \tag{1}$$

where t is real time, $\psi_f(\mathbf{r},t)$ is a four-component spinor in the basis of the sublattice degrees of freedom of the diamond lattice and electrons' spin degrees of freedom, $\bar{\psi}_f = \psi_f^{\dagger} \gamma^0$, $D_{\mu} = \partial_{\mu} + ieA_{\mu}$ with A_{μ} an electromagnetic potential. The 4×4 matrices γ^{μ} with $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ are the Dirac gamma matrices. The subscript f denotes the valley degrees of freedom. M_0 is a mass term (band gap) induced by strong SOC, and preserves time-reversal and inversion symmetries. $i\gamma^5 M_{5f}$ is a mean-field mass term induced by the AF ordering of itinerant electrons, and breaks both time-reversal and inversion symmetries. M_{5f} are given explicitly by $M_{5,1}$ = Un_0n_1 , $M_{5,2} = Un_0n_2$, and $M_{5,3} = Un_0n_3$, where U is the strength of on-site electron-electron repulsive interactions, and $(n_1, n_2, n_3) = (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta)$ is the Néel field. Effective actions similar to Eq. (1) have been obtained in the AF insulator phases of the magnetically doped Bi₂Se₃ family [43] and transition-metal oxides with the corundum structure [44].

Integrating out the fermionic field ψ_f , we obtain the effective action in terms of the Néel field and an electromagnetic field up to the relevant lowest order in n_f as $S_{AF} = S_0 + S_{\theta}$ [25]. Here, S_0 is the action of the Néel field (i.e., the nonlinear sigma model [45])

$$S_0 = \frac{1}{g} \int dt \, d^3 r \big[(\partial_\mu \boldsymbol{n}) \cdot (\partial^\mu \boldsymbol{n}) + \Delta_0^2 \boldsymbol{n}^2 \big], \qquad (2)$$

and S_{θ} is a topological term called the θ term [46]

$$S_{\theta} = \frac{e^2}{2\pi h} \int dt \, d^3 r \, \theta(\boldsymbol{r}, t) \boldsymbol{E} \cdot \boldsymbol{B}, \qquad (3)$$

where g is a constant, Δ_0 is the spin-wave gap, E(B) is an external electric (magnetic) field, and $\theta(\mathbf{r},t) = \frac{\pi}{2}[1 + \text{sgn}(M_0)] - (Un_0/M_0) \sum_{f=1,2,3} n_f(\mathbf{r},t)$. Here, $\theta(\mathbf{r},t)$ is known as the dynamical axion field [43]. The θ term results in the topological magnetoelectric effect in the bulk such that $\mathbf{P} = (e^2/2\pi h)\theta \mathbf{B}$ and $\mathbf{M} = (e^2/2\pi h)\theta \mathbf{E}$ with \mathbf{P} the electric polarization and \mathbf{B} the magnetization [2-4,46]. In the presence of time-reversal symmetry, $\theta = \pi \pmod{2\pi}$ in 3D topological insulators and $\theta = 0$ in normal insulators. However, the value of θ can be arbitrary in such systems with broken time-reversal and inversion symmetries as AF insulators described by Eq. (1) [47].

Let us implement a little more realistic condition in the above model. We take into account a small net magnetization m satisfying the constraint $n \cdot m = 0$ with |n| = 1 and $|m| \ll 1$. Furthermore, we assume the case of AF insulators with easy-axis anisotropy. Note that the magnetic anisotropy direction in Eq. (2) cannot be determined from Eq. (1), since the Néel field

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is isotropic in Eq. (1). Then a modification of Eq. (2) gives the free energy of the trivial part as [48,49]

$$F_0 = \int d^3r \left[\frac{a}{2} \boldsymbol{m}^2 + \frac{A}{2} \sum_{i=x,y,z} (\partial_i \boldsymbol{n})^2 - \frac{K}{2} n_z^2 - \boldsymbol{H} \cdot \boldsymbol{m} \right],$$
(4)

where *a* and *A* are the homogeneous and inhomogeneous exchange constants, respectively, and *K* is the easy-axis anisotropy along the *z* direction. The fourth term is the Zeeman coupling with $H = g\mu_B B$ being an external magnetic field. In the following, we define a laboratory frame in which the *z* direction is set to be the easy-axis direction. On the other hand, the free energy of the topological part is given by

$$F_{\theta} = -\frac{e^2}{2\pi h} \frac{\sqrt{3}Un_0}{M_0} \int d^3 r(\boldsymbol{n} \cdot \boldsymbol{e}_{[111]}) \boldsymbol{E} \cdot \boldsymbol{B}, \qquad (5)$$

where we have used the fact that $\sum_{f=1,2,3} n_f = (n_1 e_1 + n_2 e_2 + n_3 e_3) \cdot (e_1 + e_2 + e_3) = \sqrt{3}n \cdot e_{[111]}$ with $e_{[111]}$ being the unit vector along the [111] direction of the original diamond lattice in the Fu-Kane-Mele-Hubbard model.

Electric-field-induced antiferromagnetic resonance. In order to realize and detect the electric-field-induced AF resonance, we consider the AF insulator/HM bilayer system in the presence of an ac electric field $E_{ac}(t) = E_{ac}e^{i\omega_0 t}e_z$ and a static magnetic field $H = g\mu_B Be_z$ ($B = Be_z$) with B being much weaker than both the AF exchange coupling and easy-axial anisotropy. Here e_z is the unit vector parallel to the easy axis of the AF order. A schematic figure of our setup is shown in Fig. 1(a). The essential point is the coupling of the Néel field and an electric field through Eq. (5). Now we study the dynamics of m and n phenomenologically, i.e., based on the Landau-Lifshitz-Gilbert equation [49]. From the total free energy of the system $F_{AF} = F_0 + F_{\theta}$, the effective fields for nand m are given by

$$f_{n} = -\frac{\delta F_{AF}}{\delta n} = An \times (\nabla^{2}n \times n) + Kn_{z}e_{z} - (n \cdot H)m$$
$$+ \eta_{\theta}(E_{ac} \cdot B)e_{[111]}, \qquad (6)$$
$$f_{m} = -\frac{\delta F_{AF}}{\delta m} = -am + n \times (H \times n),$$

where $\eta_{\theta} = (e^2/2\pi h)(\sqrt{3}Un_0/M_0)$. Since the AF insulator has the HM contact, in the resonance state a pure spin current is injected into the HM layer through the interface, which enhances the Gilbert damping constant [32]. The Landau-Lifshitz-Gilbert equation is given by

$$\dot{\boldsymbol{n}} = (\gamma \boldsymbol{f}_m - \boldsymbol{G}_1 \dot{\boldsymbol{m}}) \times \boldsymbol{n},$$

$$\dot{\boldsymbol{m}} = (\gamma \boldsymbol{f}_n - \boldsymbol{G}_2 \dot{\boldsymbol{n}}) \times \boldsymbol{n} + (\gamma \boldsymbol{f}_m - \boldsymbol{G}_1 \dot{\boldsymbol{m}}) \times \boldsymbol{m} + \tau_{\rm SP},$$
(7)

where $\gamma = 1/\hbar$, G_1 and G_2 are dimensionless Gilbertdamping parameters, and $\tau_{SP} = -G_{SP}(\dot{n} \times n + \dot{m} \times m)$ is the additional damping torque with a spin pumping parameter G_{SP} [32,50].

To obtain the resonance state, where all the spins are precessing uniformly, we assume the dynamics of the Néel vector and the total magnetization around the easy axis as $\mathbf{n}(t) = \mathbf{e}_z + \delta \mathbf{n}(t)$ and $\mathbf{m}(t) = \delta \mathbf{m}(t)$, denoting that $\delta \mathbf{n}(t)$ and $\delta \mathbf{m}(t)$ are the small precession components ($|\delta \mathbf{n}|, |\delta \mathbf{m}| \ll 1$). After the linearization and the Fourier transform $\delta \mathbf{n}(t) =$

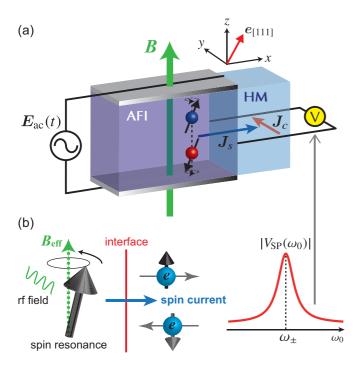


FIG. 1. (a) Schematic figure of a possible experimental setup to observe the electric-field-induced AF resonance in our system. The direction of the AF order (i.e., the easy axis) is defined as the z direction. $e_{[111]}$ is the unit vector along the [111] direction of the diamond lattice. An ac electric field $E_{ac}(t) = E_{ac}e^{i\omega_0 t}e_z$ is applied to the parallel-plate capacitor, and a static magnetic field $B = Be_z$ is also applied to the AF insulator. The pumped spin current J_s into the attached HM such as Pt can be detected through the inverse spin Hall effect as a direct current J_c (i.e., the voltage V_{SP}) across the transverse direction. (b) Schematic illustration of the spin pumping mechanism. Spin resonance injects a pure spin current into the attached layer via the exchange interaction at the interface.

$$\int \delta \tilde{\boldsymbol{n}}(\omega) e^{-i\omega t} d\omega/(2\pi), \text{ Eq. (7) reduces to}$$

$$2i\omega_H \omega \delta \tilde{\boldsymbol{n}}/\omega_a + \left[\left(\omega^2 + \omega_H^2 \right) / \omega_a - \omega_K + i\omega \tilde{G}_2 \right] \boldsymbol{e}_z \times \delta \tilde{\boldsymbol{n}}$$

$$= \boldsymbol{W} \delta(\omega_0 - \omega), \qquad (8)$$

where $\omega_H = \gamma g \mu_B B$, $\omega_a = \gamma a$, $\omega_K = \gamma K$, $W = \omega_\theta e_{[111]} \times e_z$ with $\omega_\theta = \gamma \eta_\theta E_{ac} B$, ω_0 is the frequency of the applied ac electric field $[E_{ac}(t) = E_{ac}e^{i\omega_0 t}e_z]$, and $\tilde{G}_2 = G_2 + G_{SP}$. It can be shown that for $\tau \gg \hbar/J$ (τ is the typical time scale of AF dynamics and $J \propto A$ is the AF exchange coupling constant), the G_1 term in Eq. (7) becomes unimportant, enabling the disregard of the damping term $-G_1 \dot{m} \times n$ [51]. The resonance frequencies are obtained as [52]

$$\omega_p = \sqrt{\omega_a \omega_K} + p \omega_H, \tag{9}$$

where p = + (-) corresponds to the excitation of the righthanded (left-handed) mode. Note that these frequencies do not depend on the parameters of the θ term. This is because the θ term acts only as the driving force to cause the resonance, as is seen from Eq. (8).

Detection of the antiferromagnetic resonance. So far we have shown that the AF resonance can be realized by ac electric fields, which is in sharp contrast to conventional methods using ac magnetic fields. How can we detect this

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electrically driven AF resonance? Regarding its detection, we can employ a standard method. Namely, we can observe the spin-pumping-induced voltage signal in the HM layer. One of the advantages of employing this method in our system is that we can identify its detection easily. Since the system we consider is insulating, we are free from additional dc voltages from the anisotropic magnetoresistance and the anomalous Hall effect [55]. As shown in Fig. 1(b), the spin pumping generates a pure dc spin current J_s flowing across the AF insulator/HM interface as $J_s = (\hbar/e)\Gamma_{\text{eff}}^r \langle \mathbf{n} \times \dot{\mathbf{n}} \rangle_t$ with $\langle \cdots \rangle_t$ indicating time average [56]. Here, Γ_{eff}^r is the real part of the effective mixing conductance (reflecting the influence of a back flow spin current) per unit area [26,56]. Its magnitude $J_s = |J_s|$ is given by

$$J_s(\omega_0) = 2\frac{\hbar}{e} \Gamma_{\text{eff}}^r \omega_0 \operatorname{Im}[\delta \tilde{n}_x(\omega_0) \delta \tilde{n}_y^*(\omega_0)].$$
(10)

The spin polarization in the HM decays due to spin relaxation with the length scale characterized by the spin diffusion length [57]. Here, we assume that the spin relaxation is included in Γ_{eff}^r of the HM. The spin current is converted into an electric voltage across the transverse direction via the inverse spin Hall effect [27]: $V_{\text{SP}}(\omega_0) = \rho d_H \alpha_{\text{SH}} J_s(\omega_0)$, where d_H is the thickness of the HM and α_{SH} is the spin Hall angle. Using Eq. (10), $V_{\text{SP}}(\omega_0)$ is written explicitly as [58]

$$V_{\rm SP}(\omega_0) = -\frac{1}{8}\rho d_H \alpha_{\rm SH} \frac{\hbar}{e} \Gamma^r_{\rm eff} \omega_0 \sum_{p=\pm} p \frac{\omega_H}{\omega_K} \times \sqrt{\frac{\omega_a}{\omega_K}} \frac{\omega_\theta^2 \sin^2 \theta_{[111]}}{\left(g_2^p \omega_0\right)^2} \operatorname{Lor}(\omega_0, \omega_p), \qquad (11)$$

where $\text{Lor}(\omega_0, \omega_p) = (g_2^p \omega_0)^2 / [(\omega_0 - \omega_p)^2 + (g_2^p \omega_0)^2]$ is a symmetric spectrum function (Lorentzian), g_2^p is a constant, and $\theta_{[111]}$ being the angle between $e_{[111]}$ and e_z . For example, in the case of B = 0.1 T and $E_{ac} = 1$ V/m with possible (typical) values of the parameters, we find the magnitude of V_{SP} in the resonance state as $V_{\text{SP}}(\omega_{\pm}) \sim 10 \ \mu\text{V}$ [58]. This value is experimentally observable. Furthermore, it should be noted that the above value of the ac electric field, $E_{ac} = 1$ V/m, is small. Namely, from the viewpoint of lower energy consumption, our proposal has an advantage compared to conventional "current-induced" methods that require such high-density currents as $\sim 10^{10} \text{ A/m}^2$ [38].

One can confirm the electric-field-induced AF resonance in this study as follows. The experimental setup we propose is for measuring the magnetic-field angle dependence of the induced dc voltage in the case where the applied static magnetic field **B** is much weaker than both the AF exchange coupling and uniaxial anisotropy. In this case, the induced voltage $V_{SP}(\omega_{\pm}) \propto -(\gamma \eta_{\theta} E_{ac} B)^2 \cos^2 \psi$ oscillates as a function of the relative angle ψ between E_{ac} and **B**. For magnetic field rotations in the x-y plane, where $E_{ac} \perp B$, one will not find any electrical signals since $\psi = \pi/2$. Such a ψ dependence is a direct consequence of the existence of the topological term (5) which is essential for the realization of the electric-field-induced AF resonance in this study.

Discussions. First, we comment on a possible realization of our prediction in real materials. It is suggested theoretically that the AF insulator phases, in which there exists the θ term

and the value of θ is proportional to the Néel field as in our case, can be realized in the magnetically doped Bi₂Se₃ family [43] and transition-metal oxides with the corundum structure [44]. Recently, AF insulator phases have been observed experimentally in Ga_xBi_{2-x}Se₃ [59] and Ce_xBi_{2-x}Se₃ [60]. These could be candidate materials to observe the electric-field-induced AF resonance.

Next, we discuss the mechanism of the AF resonance in this study. Recall that the presence of the θ term (3) results in the topological magnetoelectric effect (in ground states), i.e., an electric polarization density in the bulk is obtained as P = $(e^2/2\pi h)\theta B$ [2–4,46]. In the case of static magnetic fields, the time derivative of both sides reads $\dot{P} = (e^2/2\pi h)\dot{\theta}B$. There is no electric-field screening since the system we consider is insulating, which means that the electric polarization in the bulk can be flipped by external ac electric fields. Namely, we have demonstrated that nonzero \dot{P} realized by external ac electric fields induces nonzero $\dot{\theta}$, i.e., a time dependence of the Néel field *n*. On the other hand, a recent study has proposed a novel phenomenon, the "dynamical chiral magnetic effect" in AF insulators with SOC that possess the θ term [25]. The dynamical chiral magnetic effect indicates an alternating electric current generation by magnetic fields such that j = $(e^2/2\pi h)\dot{\theta}B$, and emerges as a consequence of the realization of the dynamical axion field $\theta(t)$ in condensed matter. Here, nonzero $\hat{\theta}$, i.e., a time dependence of the Néel field *n* realized by external ac magnetic fields, induces a polarization current $j(=\dot{P})$ in the bulk. Therefore, the electric-field-induced AF resonance in this study can be understood as the inverse process of the dynamical chiral magnetic effect. In other words, we have proposed a way to electrically induce the dynamical axion field in condensed matter.

We can also confirm this mechanism from another viewpoint. As shown in Fig. 2, it is known that the θ term represents the chiral anomaly described by a triangle Feynman diagram. In this triangle diagram, one of the three vertices is the Néel field $i\gamma^5 M_{5f}$ and the other two are the electromagnetic fields $e\gamma^{\mu}A_{\mu}$ [25]. Both the AF resonance in this study and the dynamical chiral magnetic effect are described by the θ term. Figure 2 shows a diagrammatic comparison of these two phenomena. In the dynamical chiral magnetic effect, the Néel field and the magnetic field are the inputs, and the electric field is the observable. On the other hand, in the AF resonance state, the magnetic and electric fields are the inputs, and the Néel

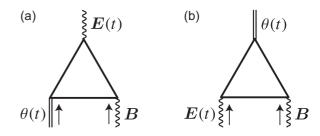


FIG. 2. (a) Diagrammatic representation of the dynamical chiral magnetic effect. $E(t)[\propto j(t)]$ represents an alternating current induced by external $\theta(t)$ and B. (b) That of the inverse process of the dynamical chiral magnetic effect. $\theta(t)$ represents an AF resonance state induced by external E(t) and B. These two phenomena are described by the θ term $S_{\theta} = (e^2/2\pi h) \int dt d^3r \,\theta(t)E \cdot B$ with $\theta(t) \propto \sum_f n_f(t)$. Arrows indicate the inputs induced by external forces. Solid lines indicate the noninteracting Green's function of the electrons.

field is the observable. Hence, we see that the AF resonance state can be understood as the inverse process of the dynamical chiral magnetic effect.

Summary. In summary, we have demonstrated that the AF resonance can be realized by ac electric fields in 3D AF insulators with SOC. This is in sharp contrast to conventional methods using ac magnetic fields. It is found that weak ac electric fields $\sim 1 \text{ V/m}$ are enough to cause the resonance. The essential point is the existence of the θ term which arises as a consequence of strong SOC. The mechanism of the AF resonance in this study can be understood as the inverse process of the dynamical chiral magnetic effect. In other words, we have proposed a way to electrically induce the dynamical axion field in condensed matter. Also, the observation of the electric-field-induced AF resonance indicates the existence of the chiral magnetic effect. The spin-pumping-induced voltage signal via the inverse spin Hall effect, which is an observable quantity to verify our prediction, is characterized by the angle dependence between the applied ac electric field and static magnetic field. Our study opens up a new direction in possible applications of topological materials in spintronics.

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