Magnetic behavior of dirty multiband superconductors near the upper critical field

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Magnetic properties of dirty multiband superconductors near the upper critical field are studied. The parameter κ_2 characterizing magnetization slope is shown to have a significant temperature variation which is quite sensitive to the pairing interactions and relative strengths of intraband impurity scattering. In contrast to single-band superconductors the increase of κ_2 at low temperatures can be arbitrarily large determined by the ratio of maximal and minimal diffusion coefficients in different bands. Temperature dependencies of $\kappa_2(T)$ in two-band MgB_2 and iron-based superconductors are shown to be much more sensitive to the multiband effects than the upper critical field $H_{c2}(T)$.

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I. INTRODUCTION

Recently a number of multiband superconductors have been discovered where the pairing of electrons is supposed to take place simultaneously in several bands overlapping at the Fermi level [1–7]. One of the first such superconductors found was MgB_2 [1] which has two distinct superconducting gaps residing on different sheets of the Fermi surface [8–11]. Up to date MgB_2 has the highest critical temperature $T_c = 40$ K among simple binary compounds [12]. Later on multiband superconductivity has been established in iron-pnictides [4–6] with the highest T_c above 100 K detected in atomically thin films of FeSe [13]. There strong interband interactions mediated by antiferromagnetic excitations have been suggested to play the dominant role in pairing resulting in the peculiar $s_\pm$ symmetry of the order parameter.

Besides their high T_c both the two-gap MgB_2 and iron-based superconductors have remarkable magnetic properties. The interplay of several pairing channels in multiband superconductors was predicted to produce convex-shaped temperature dependencies of the upper critical field [14–17]. Due to their anomalous shapes the $H_{c2}(T)$ curves can reach much larger values at $T \to 0$ than was expected from a one-gap theory [18,19]. This explains an enormous enhancement of the upper critical field in MgB_2 by nonmagnetic impurities [20–23]. Experiments measuring upper critical fields in disordered MgB_2 [20] and certain iron-based superconductors [24] are consistent with theoretical calculations using a two-gap model [14,15,17]. Therefore a convex shape of $H_{c2}(T)$ dependencies is considered as one of the hallmarks of multiband pairing [24–28]. However it is not a universal feature of multiband superconductors since concave $H_{c2}(T)$ curves were observed in MgB_2 with lower impurity concentration [12] as well as in many iron-pnictide compounds [29–32].

In order to find a robust test for mutiband pairing it is natural to look for an unusual behavior of magnetization at $H < H_{c2}$. In the vicinity of H_{c2} magnetization M_z can be characterized by the parameter $\kappa_2(T)$ introduced by Maki [33]

$$M_z = \frac{H - H_{c2}}{4\pi\beta_L\left(2\kappa_2^2 - 1\right)}, \tag{1}$$

where the z axis is directed along the magnetic field, and β_L is an Abrikosov parameter equal to 1.16 for a triangular lattice [34]. In single-band superconductors the parameter κ_2 has been studied extensively in the clean [35] and dirty limits [36,37], for the arbitrary strength of impurity scattering [38,39], and taking into account strong electron-phonon coupling effects [40]. Dirty single-band superconductors were shown to have a universal behavior characterized by a slow monotonic increase of κ_2 [33,36,37] with cooling from $\kappa_2(T = T_c) = \kappa_{\mathrm{GL}}$ to $\kappa_2(T = 0) \approx 1.2\kappa_{\mathrm{GL}}$, where κ_{GL} is the Ginzburg-Landau parameter at $T = T_c$. The theoretical calculations were found to be in good agreement with experimentally measured $\kappa_2(T)$ dependencies in several superconducting alloys [41–43].

The parameter κ_2 is a basic quantity of type-II superconductors determining their thermodynamic [44] and transport properties [45,46] near H_{c2}. In the present paper we use the Usadel theory [14] to calculate κ_2 in dirty multiband superconductors with high concentration of nonmagnetic impurities. Such a description is not universal but appropriate for a certain class of multiband materials including MgB_2 [14,16] and iron-pnictides [24]. This theory does not fit for heavy-fermion superconductors like $CePt_3Si$ [7] whose crystal lattice lacks a center of inversion. In noncentrosymmetric compounds multiple bands originate from strong spin-orbital coupling which lifts spin degeneracy of electron states. In order to describe gap structure, magnetic properties, and the influence of impurity scattering in such superconductors one needs to use a different formalism [47].

The structure of this paper is as follows. In Sec. II basic equations of the multiband Usadel theory are introduced. General formulas describing the high-field magnetic response of dirty multiband superconductors are derived in Sec. III including equations for the H_{c2} in Sec. III A and the magnetization in Sec. III B. Several examples of two-band superconductors are considered in Sec. IV. Results are discussed in Sec. V and conclusions are given in Sec. VI.

II. MULTIBAND USADEL THEORY

We consider multiband superconductors in a dirty limit using the Usadel theory [14]. Each kth band is described in terms of the quasiclassical Green's function matrix $\hat{g}_k =$

$\hat{g}_k(\varepsilon,\boldsymbol{r})$ which is defined as follows:

$$\hat{g}_k = \begin{pmatrix} g_k & f_k \\ -f_k^+ & -g_k \end{pmatrix} \quad (2)$$

and subject to the normalization constraint $\hat{g}_k^2 = 1$. The matrix Usadel equation reads [48]

$$D_k \hat{\partial}_{\boldsymbol{r}}(\hat{g}_k \hat{\partial}_{\boldsymbol{r}} \hat{g}_k) - [\omega\tau_3 + \hat{\Delta}_k, \hat{g}_k] = 0, \quad (3)$$

where D_k is the diffusion constant and the covariant differential superoperator is defined by $\hat{\partial}_{\mathbf{r}}\hat{g} = \nabla\hat{g} - ie\boldsymbol{A}[\tau_3,\hat{g}]$. We assume that the pairing takes place only in the spin-singlet channel so that the gap operator in the kth band has a spin-independent form $\hat{\Delta}_k(\boldsymbol{r}) = |\Delta_k|\tau_2 e^{-i\theta_k\tau_3}$.

The model described by Eq. (3) is based on several simplifying assumptions. First, we neglected paramagnetic depairing. This approximation is not sufficient for certain iron-pnictide compounds with a large critical field [28,31,49–51] that can reach a paramagnetic limit leading to the possibility of a Fulde-Ferrel-Larkin-Ovchinnikov transition [17]. At the same time these materials have short coherence lengths $\xi \sim$ 1–3 nm [17] which are not consistent with the dirty limit approximation considered in the present paper. Furthermore, we have neglected interband impurity scattering which couples the Green function in various bands [14]. By neglecting these effects we assume that the interband scattering rate is much smaller compared to the orbital depairing energy eD_kH_{c2}. This condition is satisfied not too close to T_c in MgB_2 where the interband scattering was predicted to be small [52]. Due to the same reason we neglect the influence of spin-flip scattering at paramagnetic impurities and inelastic electron-phonon scattering [53,54]. These depairing mechanisms are known to be important near T_c but can be neglected at lower temperatures where the $\kappa_2(T)$ anomalies are expected to appear.

The 12 component of the matrix Eq. (3) is

$$\frac{D_k}{2i}(g_k\hat{\boldsymbol{\Pi}}^2 f_k - f_k\nabla^2 g_k) = \Delta_k g_k - i\omega f_k, \quad (4)$$

where $\hat{\boldsymbol{\Pi}} = \nabla - 2ie\boldsymbol{A}$. A similar equation given by the 21 component of (3) yields $f^+(\boldsymbol{r},\omega) = -f^*(\boldsymbol{r},\omega)$. The gap in each band is determined by self-consistency equations

$$\Delta_k(\boldsymbol{r}) = 2i\pi T\sum_{j=1}^{N}\sum_{n=0}^{N_D}\lambda_{kj} f_j(\omega_n), \quad (5)$$

where $\hat{\lambda}$ is the $N\times N$ coupling matrix satisfying general symmetry relations $\nu_k\lambda_{kj} = \nu_j\lambda_{jk}$ and the sum by Matsubara frequencies $\omega_n = (2n+1)\pi T$ is taken in the limits $N_D(T) = \Omega_D/(2\pi T)$ set by the Debye frequency Ω_D. The electric current density is given by

$$\boldsymbol{j} = i\pi T\sum_{k=1}^{N}\sum_{n=0}^{\infty}\frac{\sigma_k}{e}\mathrm{Tr}[\tau_3\hat{g}_k(\omega_n)\hat{\partial}_{\boldsymbol{r}}\hat{g}_k(\omega_n)], \quad (6)$$

where the partial conductivities are $\sigma_k = 2e^2\nu_k D_k$ and ν_k are the densities of states per one spin projection. The sum over frequencies in Eq. (6) converges therefore no cutoff is needed. The magnetization of a superconducting sample $\boldsymbol{M}$ is determined by the current (6) according to the usual relation $\nabla\times\boldsymbol{M} = \boldsymbol{j}$.

III. MULTIBAND SUPERCONDUCTORS IN LARGE MAGNETIC FIELDS

A. The upper critical field H_{c2}

At large magnetic fields $H_{c2} - H \ll H_{c2}$ we can apply approximations related to the smallness of the order parameter $|\Delta_k| \propto \sqrt{1 - H/H_{c2}}$. To calculate the structure of a vortex lattice in a two-band superconductor let us consider the linear integral-differential system consisting of Usadel Eqs. (4) linearized with respect to the normal state solution

$$\hat{L}_\omega f_k = i\Delta_k, \quad \hat{L}_\omega = \frac{D_k}{2}\hat{\boldsymbol{\Pi}}_0^2 - |\omega|, \quad (7)$$

supplemented by the self-consistency relation (5). In a linearized theory the magnetic field is not perturbed by the vortex currents, therefore we put $\boldsymbol{B}_0 = H_{c2}\boldsymbol{z}$ and choose a Landau gauge in Eq. (7), $\boldsymbol{A}_0 = H_{c2}x\boldsymbol{y}$. Then the gradient term in Eq. (7) is $\hat{\boldsymbol{\Pi}}_0 = \nabla - 2ie\boldsymbol{A}_0$. A periodic vortex lattice is described by the Abrikosov solution of Eqs. (5) and (7) which in general has the following form:

$$\Delta_k(\boldsymbol{r}) = \Delta b_k\Psi(\boldsymbol{r}), \quad (8)$$

$$\Psi(\boldsymbol{r}) = \sum_n C_n e^{inpy}\Psi_0(x - nx_0), \quad (9)$$

where $|C_n| = 1$, $x_0 = p/(2eH_{c2})$, and parameter p is determined by the lattice geometry. The lowest Landau level wave function $\Psi_0(x) = 2L_H\sqrt{\pi}\exp(-x^2/2L_H^2)$ satisfies $(L_H^2\partial_x^2 - x^2/L_H^2 + 1)\Psi_0 = 0$, where the magnetic length is $L_H = 1/\sqrt{2eH_{c2}}$. The gaps Δ_k are determined by the common amplitude Δ and a normalized set of components $\sum_k b_k^2 = 1$.

The solution of Eq. (7) yields

$$f_k(\boldsymbol{r},\omega_n) = \frac{\Delta_k(\boldsymbol{r})}{i(q_k + |\omega_n|)}, \quad (10)$$

where $q_k = eH_{c2}D_k$. Substituting the ansatzes (8), (9), and (10) to the self-consistency equation (5) we get the homogeneous linear system $\hat{A}(b_1,\ldots,b_N)^T = 0$ for the order parameter amplitudes where

$$\hat{A} = \hat{\Lambda}^{-1} - \hat{I}[G_0 + \ln(T_c/T) + \psi(1/2) - \psi(1/2 + \hat{\rho})]. \quad (11)$$

Here $\psi(x)$ is a di-gamma function and the diagonal matrix $\hat{\rho}$ is given by $(\hat{\rho})_{ij} = \delta_{ij}q_i/(2\pi T)$. The solvability condition $\det\hat{A} = 0$ determines the upper critical field H_{c2} of a dirty multiband superconductor.

It is instructive to consider in more detail Eq. (11) in two-band superconductors. In this case $G_0 = (\mathrm{Tr}\,\Lambda - \lambda_0)/2w$, where $w = \lambda_{11}\lambda_{22} - \lambda_{12}\lambda_{21}$, $\lambda_0 = \sqrt{\lambda_-^2 + 4\lambda_{12}\lambda_{21}}$, and $\lambda_- = \lambda_{11} - \lambda_{22}$. The equation $\det\hat{A} = 0$ can be resolved in terms of the $\ln(T/T_c)$ yielding in general two different solutions:

$$\ln(T/T_c) = -(U_1 + U_2 + \lambda_0/w)/2 + [(U_1 - U_2 - \lambda_-/w)^2/4 + \lambda_{12}\lambda_{21}/w^2]^{1/2}, \quad (12)$$

$$\ln(T/T_c) = -(U_1 + U_2 + \lambda_0/w)/2 - [(U_1 - U_2 - \lambda_-/w)^2/4 + \lambda_{12}\lambda_{21}/w^2]^{1/2}, \quad (13)$$

where $U_k = \psi(1/2+\rho_k) - \psi(1/2)$. Taking the limit $T \to T_c$ one can see that the physical solutions are (i) (12) in case when $w \equiv \det\hat{\Lambda} > 0$ and (ii) (13) in case when $w \equiv \det\hat{\Lambda} < 0$. While case (i) corresponds to the coupling parameters of MgB_2 [14,15], case (ii) describes multiband superconductors with interband-dominated pairing when $\lambda_{12}\lambda_{21} > \lambda_{11}\lambda_{22}$ such as iron-pnictide compounds [4,6,24].

B. The magnetization slope dM_z/dH

Magnetic field created by vortex currents (6) can be found using the solution (8). Taking into account that Green's functions $f_k(\boldsymbol{r})$ given by (10) satisfy the relation

$$i\partial_x f_k = (\partial_y - 2ieA_y) f_k, \tag{14}$$

we obtain the multiband expression for magnetization

$$4\pi M_z(\boldsymbol{r}) = -\sum_k \frac{\sigma_k}{eT}\psi_k' |\Delta_k|^2. \tag{15}$$

The order parameter amplitudes Δ_k can be found according to the following straightforward algorithm. First, nonlinear corrections $\tilde{f}_k$ are obtained from Eq. (4) taking into account higher-order terms in Δ_k:

$$\begin{aligned}\hat{L}_\omega \tilde{f}_k = &-\frac{i\Delta_k|\Delta_k|^2}{2(q_k+|\omega|)^2} \\ &+\frac{eD_k\{\hat{\boldsymbol{\Pi}}_0, \boldsymbol{A}_1\}\Delta_k}{(q_k+|\omega|)} \\ &+\frac{i(2q_k\Delta_k|\Delta_k|^2 + D_k\Delta_k\nabla^2|\Delta_k|^2)}{4(q_k+|\omega|)^3}.\end{aligned} \tag{16}$$

Here $\boldsymbol{A}_1 = \boldsymbol{A} - \boldsymbol{A}_0$. Then the self-consistency equation (5) yields a nonhomogeneous linear system for the corrections:

$$\hat{C}_{kj}\tilde{\Delta}_j = 2\pi i T\sum_{n=0}^{\infty}\nu_k \tilde{f}_k(\omega_n), \tag{17}$$

$$\hat{C} = \hat{\nu}\hat{\Lambda}^{-1} + 2\pi T\sum_{n=0}^{N_D}\hat{\nu}\hat{L}_{\omega_n}^{-1}, \tag{18}$$

where $\hat{\nu}_{kj} = \nu_k\delta_{kj}$. Since the matrix $\hat{\nu}\Lambda^{-1}$ is symmetric, the operator $\hat{C}$ is Hermitian. Moreover, the solution (8) of linearized gap equation belongs to the kernel $\hat{C}_{kj}\Delta_j = 0$. Hence multiplying the left-hand side of a nonhomogeneous Eq. (17) by Δ_k^* we get $\sum_{k,j}\langle\Delta_k^*\hat{C}_{kj}\tilde{\Delta}_j\rangle = 0$. Thus Eq. (17) is solvable if its right-hand side is orthogonal to the linear solution

$$\sum_k\sum_{n\geqslant 0}\nu_k\langle\Delta_k^*\tilde{f}_k(\omega_n)\rangle = 0. \tag{19}$$

To calculate each term in the sum (19) we multiply Eq. (16) by Δ_j^* and average over space coordinates taking into account the relations

$$\langle\Delta_k^*\hat{L}_\omega\tilde{f}_k\rangle = -(|\omega|+q_k)\langle\Delta_k^*\tilde{f}_k\rangle, \tag{20}$$

$$\langle\Delta_k^*\{\hat{\boldsymbol{\Pi}}_0, \boldsymbol{A}_1\}\Delta_k\rangle = -i\langle B_1|\Delta_k|^2\rangle, \tag{21}$$

$$2q_k\langle|\Delta_k|^4\rangle + D_k\langle|\Delta_k|^2\nabla^2|\Delta_k|^2\rangle = 0, \tag{22}$$

where $B_1 = -\delta H + 4\pi M_z$ and $\delta H = H_{c2} - H$. The relations (20) and (21) can be obtained by a straightforward calculation, while (22) is less trivial although it has been used in the theory of single-band superconductors [54]. The detailed derivation of Eq. (22) is shown in the Appendix.

To simplify the further derivation let us consider from the beginning a high-κ limit when $\sigma_k D_k \ll 1$. In this case we can neglect the magnetization in Eq. (21) to get finally

$$\begin{aligned}4i\pi^2T^3\sum_{n\geqslant 0}\langle\Delta_k^*\tilde{f}_k(\omega_n)\rangle = &\, eTD_k\psi_k'\delta H\langle|\Delta_k|^2\rangle \\ &-2\sigma_k D_k\psi_k'^2\tilde{\kappa}_k^2\langle|\Delta_k|^4\rangle,\end{aligned} \tag{23}$$

where $\tilde{\kappa}_k$ are single-band parameters given by [33,36]

$$\tilde{\kappa}_k = \left[\frac{-\psi_k''}{16\pi\sigma_k D_k\psi_k'^2}\right]^{1/2}. \tag{24}$$

Combining Eqs. (19) and (23) we obtain the order parameter amplitude in Eq. (8) given by

$$\Delta = \left[\frac{eT\delta H}{2\beta_L}\frac{\sum_k \nu_k b_k^2 D_k\psi_k'}{\sum_k \nu_k b_k^4\sigma_k D_k\psi_k'^2\tilde{\kappa}_k^2}\right]^{1/2}, \tag{25}$$

where the Abrikosov parameter is $\beta_L = \langle|\Psi|^4\rangle/\langle|\Psi|^2\rangle^2$.

The derived amplitude Δ is a basic parameter for calculations of thermodynamic and transport properties of superconductors near H_{c2}. In particular, using Eq. (15) we obtain an expression for the space-averaged magnetization $M_z = -\delta H(dM_z/dH)$ where the slope is given by

$$\frac{dM_z}{dH} = \frac{1}{8\pi\beta_L}\frac{\left(\sum_k \nu_k D_k\psi_k' b_k^2\right)^2}{\sum_k \nu_k^2 D_k^2\psi_k'^2 b_k^4\tilde{\kappa}_k^2}. \tag{26}$$

Comparing Eq. (26) with the conventional parametrization (1) in the limit $\kappa_2 \gg 1$ we find an effective parameter

$$\kappa_2 = \frac{\sqrt{\sum_k \nu_k^2 D_k^2\psi_k'^2 b_k^4\tilde{\kappa}_k^2}}{\sum_k \nu_k D_k\psi_k' b_k^2}. \tag{27}$$

Close to the critical temperature κ_2 reduces to the Ginzburg-Landau parameter $\kappa_2(T_c) = \kappa_{\rm GL} \equiv \lambda_L/\xi$, where λ_L is the London penetration length and $\xi = 1/\sqrt{2eH_{c2}}$ is the coherence length in dense vortex lattices near H_{c2}.

IV. EXAMPLES OF TWO-BAND SUPERCONDUCTORS

The general formalism developed in previous sections can be applied to study magnetic properties of particular multiband compounds. To begin with we consider a two-band model of MgB_2 characterized by the coupling parameters [10] $\lambda_{11} = 0.81$, $\lambda_{22} = 0.285$, $\lambda_{12} = 0.119$, $\lambda_{21} = 0.09$. Temperature dependencies of $H_{c2}(T)$ and $\kappa_2(T)$ are shown in Fig. 1 for different values of (a) and (b) $D_1/D_2 = 1$; 0.5; 0.25, (c) and (d) $D_1/D_2 = 0.05$, and (e) and (f) $D_1/D_2 = 20$. For equal diffusion coefficients $D_1/D_2 = 1$ the single-band behaviors [18,19,36] of H_{c2} and κ_2 are recovered, see Figs. 1(a) and 1(b). As will be shown below this result is valid for any number of bands and arbitrary pairing matrix.

The disparity of diffusion coefficients $D_1/D_2 \neq 1$ results in significant variations of κ_2. Comparing Figs. 1(a) and 1(b) one can see that $\kappa_2(T)$ is much more sensitive to the ratio

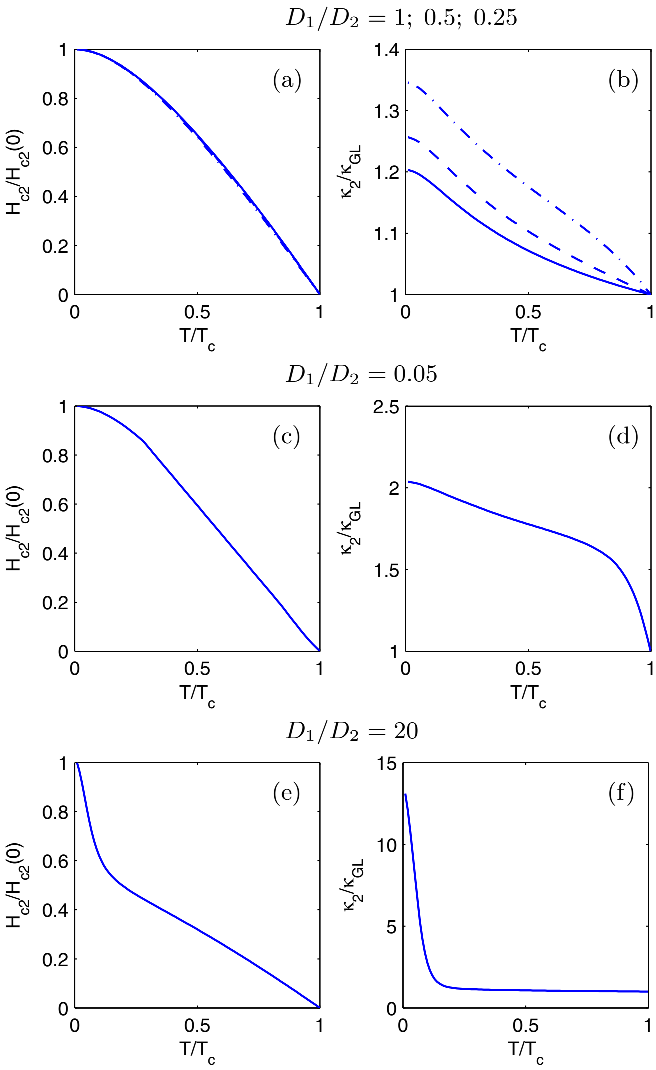

FIG. 1. Magnetic properties of the two-band superconductor MgB_2 with coupling parameters mentioned in the text. The panels show (a), (c), and (e) $H_{c2}(T)$ and (b), (d), and (f) $\kappa_2(T)$ as given by Eqs. (12) and (27) for different values of the ratio D_1/D_2. In (a) and (b) solid, dashed, and dash-dotted lines correspond to $D_1/D_2 = 1;\ 0.5;\ 0.25$, respectively. In (a) these curves are almost undistinguishable. (c) and (d) $D_1/D_2 = 0.05$ and (e) and (f) $D_1/D_2 = 20$.

of diffusivities than the second critical field. The curvature variations of $H_{c2}(T)$ are noticeable only in the limit $D_1 \gg D_2$ as shown Fig. 1(e). As demonstrated in Fig. 1(c) the opposite limit $D_1 \ll D_2$ yields ordinary concave curves $H_{c2}(T)$ almost within the entire temperature domain except for the small vicinity of T_c. On the contrary, temperature dependencies of $\kappa_2(T)$ shown for the same parameters in Figs. 1(b) and 1(d) are drastically different from the single-band case. Of particular interest is a sharp increase of $\kappa_2(T \to 0)$ which is most pronounced under the condition $D_1 \gg D_2$ relevant to MgB_2 [14] [see Fig. 1(f)]. Physically this means that the slope of the magnetization curve becomes much less steep as shown in Fig. 3. The low-temperature increase of κ_2 provides a feasible probe to study magnetic signatures of multiband pairing.

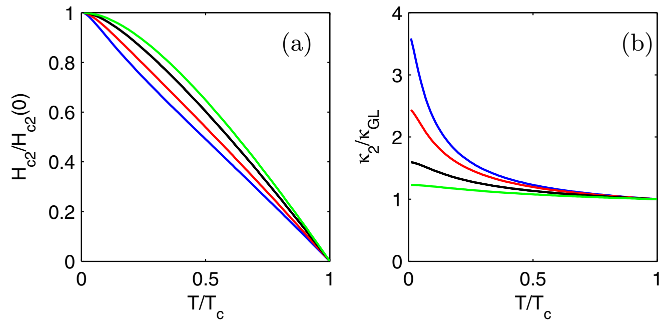

FIG. 2. Magnetic properties of a two-band superconductor with interband-dominated pairing $\lambda_{11} = \lambda_{22} = 0$, $\lambda_{12} = \lambda_{21} = -0.5$ corresponding to iron-pnictide superconductors. (a) $H_{c2}(T)$ curves from *top to bottom* correspond to $D_1/D_2 = 1;\ 0.25;\ 0.1;\ 0.05$. (b) The magnetization parameter $\kappa_2(T)$ as given by Eq. (27). The curves from *bottom to top* correspond to the same sequence of D_1/D_2 as in (a).

Next we consider two-band superconductors with pairing from interband repulsion $\lambda_{ii} = 0$ and $\lambda_{12} = \lambda_{21} = -0.5$ resulting in the $s_{\pm}$ superconducting state [4,6]. Such a model has been used to describe an unconventional convex behavior of $H_{c2}(T)$ observed experimentally in iron-based compounds [24].

Here we suggest that an independent and more sensitive test for the multiband physics in iron-pnictides can be implemented by measuring temperature dependencies of $\kappa_2(T)$. As can be seen in Fig. 2(b) κ_2 demonstrates a sharp increase at low temperatures even for not too small values of the ratio D_1/D_2 and deviates strongly from the single-band behavior shown by the green down-most curve. For the same parameters $H_{c2}(T)$ dependencies have only tiny deviations from the single-band one shown by the green up-most line in Fig. 2(a).

Summarizing the above examples one can see that even a moderate disparity of diffusion constants when $D_1/D_2 \sim 1$ in two-band superconductors results in a significant increase of $\kappa_2(T)$ at low temperatures as compared to its value at the critical temperature $\kappa_2(T_c) = \kappa_{GL}$. As a result the magnetization slope dM_z/dH becomes much less steep as compared to single-band superconductors. This behavior is illustrated in Fig. 3 which shows the slopes dM_z/dH normalized to their

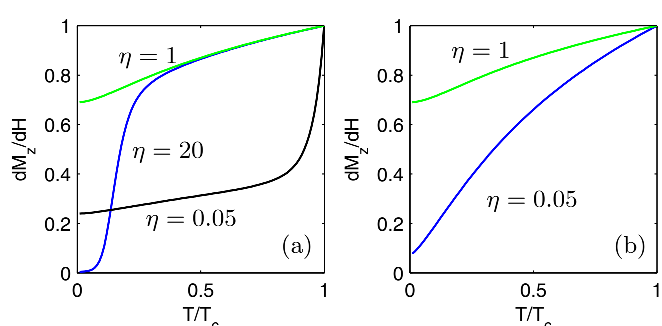

FIG. 3. Slopes of the magnetization curves dM_z/dH in two-band superconductors with coupling parameters corresponding to (a) MgB_2, $\lambda_{11} = 0.81$, $\lambda_{22} = 0.285$, $\lambda_{12} = 0.119$, $\lambda_{21} = 0.09$ and (b) iron-pnictides, $\lambda_{11} = \lambda_{22} = 0$, $\lambda_{12} = \lambda_{21} = -0.5$. The value of $\eta = D_1/D_2$ is marked near each curve. The slopes are normalized to their values at T_c.

values at T_c as functions of temperature for different values of D_1/D_2. In both Figs. 3(a) and 3(b) the up-most green curves show a single-band behavior which is reproduced universally for $D_1 = D_2$ irrespective of the coupling parameters. The changes in magnetization slopes can be directly measured and yield important information about multiband pairing and diffusion constants in different bands.

V. DISCUSSION

The convex shape of $H_{c2}(T)$ curves is often considered as a signature of multiband pairing [24,25,27]. However, as demonstrated by the above two-band examples, the conditions for having pronounced convexity, such as shown in Fig. 1(c), are quite restrictive. If the disparity of diffusivities is not extreme $D_1/D_2 \sim 1$ then deviations of $H_{c2}(T)$ from the conventional single-band theory are not significant. However, even in this case it is possible to detect signatures of multiband pairing in the magnetic response measuring the magnetization slopes at high fields. Shown in the right columns of Figs. 1 and 2 $\kappa_2(T)$ dependencies are quite sensitive to variations of diffusivities even in the range of parameters when $H_{c2}(T)$ curves look almost the same as the single-band one.

To understand qualitative features of $\kappa_2(T)$ dependencies it is instructive to consider several characteristic cases. First let us recover single-band results for H_{c2} and κ_2 assuming all diffusivities to be equal $D_k = D$ for $k = 1,\dots,N$. In this case $\rho_k = \rho$ so that Eq. (11) reduces to $\hat{A} = \hat{\Lambda}^{-1} - \hat{I}[G_0 - U(\rho) - \ln(T/T_c)]$. The solvability condition $\det\hat{A} = 0$ yields a single-band equation for the upper critical field [18,19] $U(\rho) + \ln(T/T_c) = 0$. The corresponding eigenvector is temperature independent and determined by the equation $(\hat{\Lambda}^{-1} - \hat{I}G_0)\boldsymbol{b} = 0$. Then taking into account that $\kappa_2(T_c) = \kappa_{\rm GL}$ from Eq. (27) we obtain the analytical expression $\kappa_2^2/\kappa_{\rm GL}^2 = -\pi^4\psi''/[56\zeta(3)\psi'^2]$ coinciding with the single-band result [36].

To explain significant variations of κ_2 in case of different diffusivities let us compare a low- and high-temperature asymptotic of Eq. (27):

$$\kappa_2(T_c) = \sqrt{\frac{7\zeta(3)}{4\pi^5 e^2}}\frac{\sqrt{\sum_k \nu_k b_k^4}}{\sum_k \nu_k D_k b_k^2}, \tag{28}$$

$$\kappa_2(0) = \frac{1}{\sqrt{32\pi}e}\frac{\sqrt{\sum_k \nu_k b_k^4 D_k^{-2}}}{\sum_k \nu_k b_k^2}, \tag{29}$$

where we have used that $\psi_k' \approx \rho_k^{-1}$, $\psi_k'' \approx -\rho_k^{-2}$ at $T \to 0$ and $\psi_k' = \pi^2/2$, $\psi_k'' = -14\zeta(3)$ at $T = T_c$.

Equations (28) and (29) demonstrate that in the limit of a strong disparity between diffusivities the value of $\kappa_2(T_c)$ is determined by the maximal diffusivity while $\kappa_2(0)$ is determined by the minimal one. Therefore $\kappa_{\rm GL} = \kappa_2(T_c) \sim 1/D_1$ and $\kappa_2(0) \sim 1/D_2$ so that the low temperature increase of $\kappa_2(0)/\kappa_{\rm GL} \sim D_1/D_2 \gg 1$ is determined by the ratio of maximal and minimal diffusivities $D_1 = \max(D_k)$ and $D_2 = \min(D_k)$, respectively.

Such a behavior can be qualitatively understood as follows. Near the critical temperature Eq. (11) reduces to $\hat{A} = \hat{\Lambda}^{-1} - \hat{I}G_0$ so that gap amplitudes b_k are determined only by the coupling matrix. The magnetic field is small so that $\rho_k \ll 1$ and its influence on the ratio of gap amplitudes is negligible. However, the contributions to superconducting current and magnetization (6) and (15) from each band are proportional to the corresponding diffusion coefficients. Hence in the limit of strong disparity $D_1/D_2 \ll (\gg) 1$ the band with the largest diffusivity provides a dominant contribution to the magnetization near T_c. On the other hand, at low temperatures the magnetic field is large so that $\rho_k \gg 1$ and from Eq. (10) one can see that the anomalous function amplitude is smaller in bands with larger diffusivities. Hence at $T \to 0$ the most significant contribution to the magnetic response and κ_2 is determined by the band with the smallest diffusivity.

Finally, iron-pnictide compounds with large critical field [28,31,49–51] have short coherence lengths $\xi \sim 1$–3 nm [17] which are not consistent with the dirty limit approximation considered here. It is possible however to develop a theory for κ_2 in superconductors with arbitrary impurity concentration [38,39]. In single-band superconductors $\kappa_2(T \to 0)$ diverges in the clean limit but for experimentally relevant impurity concentrations the changes are not dramatic as compared to the dirty limit [41]. On the other hand, in the multiband case an interplay between different Fermi velocities and impurity scattering rates should result in nontrivial modifications of $\kappa_2(T)$ temperature dependencies.

VI. CONCLUSION

To conclude we have calculated the parameter κ_2 characterizing magnetization slopes dM_z/dH in dirty multiband superconductors at high fields $H_{c2} - H \ll H_{c2}$. The developed theory describes any number of superconducting bands and arbitrary set of pairing constants. We have shown quite generally that in contrast to the dirty single-band superconductors the temperature dependencies of $\kappa_2(T)$ have remarkable features which are highly sensitive to the multiband effects. The low-temperature increase of κ_2 as compared to its value at T_c is found to be strongly pronounced even for the moderate disparity of diffusion coefficients in different bands. This effect should be particularity appealing for experimental identification since it could unambiguously confirm unconventional magnetic behavior of multiband superconductors. We have considered several examples of two-band materials like MgB_2 and iron-pnictides and demonstrated that κ_2 is much more sensitive than H_{c2} to the ratio of diffusion coefficients in different bands. The derived expressions for κ_2 provide a basis to study thermodynamic and transport properties of multiband superconductors in high magnetic fields.

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APPENDIX: PROOF OF THE RELATION EQ. (22)

We use the relations $\nabla = (\boldsymbol{\partial}_+ + \boldsymbol{\partial}_-)/2$ introducing the operators $\partial_\pm = \boldsymbol{x}\partial_x + \boldsymbol{y}(\partial_y \pm 2ieA_y)$ so that $\partial_\pm^2 = \partial_x^2 + (\partial_y \pm 2ieA_y)^2$. The gap functions satisfy

$$\partial_+^2 \Delta^* = -2eH_{c2}\Delta^*, \tag{A1}$$

$$\partial_-^2 \Delta = -2eH_{c2}\Delta. \tag{A2}$$

Due to the relations

$$\partial_x \Delta = i(\partial_y - 2ieA_y)\Delta, \tag{A3}$$

$$\partial_x \Delta^* = -i(\partial_y + 2ieA_y)\Delta^*, \tag{A4}$$

we get

$$(\partial_-\Delta)^2 = 0, \quad (\partial_+\Delta^*)^2 = 0. \tag{A5}$$

Then the average is given by

$$4\langle|\Delta|^2\nabla^2|\Delta|^2\rangle = \langle|\Delta|^2(\partial_+^2 + \partial_-^2 + 2\partial_+\partial_-)|\Delta|^2\rangle.$$

Let us consider the three terms separately:

(i) $\langle|\Delta|^2\partial_+^2|\Delta|^2\rangle$

$$= \langle|\Delta|^2(\Delta^*\partial_+^2\Delta + \Delta\partial_+^2\Delta^* + 2\partial_+\Delta\partial_+\Delta^*)\rangle$$
$$= -2eH_{c2}\langle|\Delta|^4\rangle + \langle|\Delta|^2\Delta^*\partial_+^2\Delta\rangle + 2\langle|\Delta|^2\partial_+\Delta\partial_+\Delta^*\rangle, \tag{A6}$$

(ii) $\langle|\Delta|^2\partial_-^2|\Delta|^2\rangle$

$$=\langle|\Delta|^2(\Delta^*\partial_-^2\Delta + \Delta\partial_-^2\Delta^* + 2\partial_-\Delta\partial_-\Delta^*)\rangle$$
$$= -2eH_{c2}\langle|\Delta|^4\rangle + \langle|\Delta|^2\Delta\partial_-^2\Delta^*\rangle + 2\langle|\Delta|^2\partial_-\Delta\partial_-\Delta^*\rangle, \tag{A7}$$

(iii) $-2\langle|\Delta|^2\partial_+\partial_-|\Delta|^2\rangle$

$$\begin{aligned}
&=\langle(\partial_+|\Delta|^2)^2\rangle\rangle + \langle(\partial_-|\Delta|^2)^2\rangle\\
&=\langle(\partial_+\Delta)^2\Delta^{*2}\rangle + \langle(\partial_+\Delta^*)^2\Delta^2\rangle + 2\langle|\Delta|^2(\partial_+\Delta)(\partial_+\Delta^*)\rangle\\
&\quad + \langle(\partial_-\Delta)^2\Delta^{*2}\rangle + \langle(\partial_-\Delta^*)^2\Delta^2\rangle + 2\langle|\Delta|^2(\partial_-\Delta)(\partial_-\Delta^*)\rangle\\
&=\langle(\partial_+\Delta)^2\Delta^{*2}\rangle + 2\langle|\Delta|^2(\partial_+\Delta)(\partial_+\Delta^*)\rangle\\
&\quad + \langle(\partial_-\Delta^*)^2\Delta^2\rangle + 2\langle|\Delta|^2(\partial_-\Delta)(\partial_-\Delta^*)\rangle,
\end{aligned} \tag{A8}$$

where we took into account Eqs. (A5). Collecting all terms we get

$$\begin{aligned}4\langle|\Delta|^2\nabla^2|\Delta|^2\rangle = &-4eH_{c2}\langle|\Delta|^4\rangle + \langle\Delta^{*2}[\Delta\partial_+^2\Delta - (\partial_+\Delta)^2]\rangle\\ &+ \langle\Delta^2[\Delta^*\partial_-^2\Delta^* - (\partial_-\Delta^*)^2]\rangle.\end{aligned} \tag{A9}$$

The last two terms here can be transformed in a similar way as follows:

$$\Delta\partial_+^2\Delta - (\partial_+\Delta)^2 = \Delta\partial_-^2\Delta - (\partial_-\Delta)^2 = -4eH_{c2}\Delta^2, \tag{A10}$$

$$\Delta^*\partial_-^2\Delta^* - (\partial_-\Delta^*)^2 = \Delta^*\partial_+^2\Delta^* - (\partial_+\Delta^*)^2 = -4eH_{c2}\Delta^{*2}, \tag{A11}$$

so that finally we get

$$\langle|\Delta|^2\nabla^2|\Delta|^2\rangle = -2eH_{c2}\langle|\Delta|^4\rangle,$$

which proves Eq. (22).

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