Tunable and direction-dependent group velocities in topologically protected edge states

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(Received 7 October 2015; revised manuscript received 12 May 2016; published 26 May 2016)

Topological effects continue to fascinate physicists since more than three decades. One of their main applications are high-precision measurements of the resistivity. We propose to make also use of the spatially separated edge states. It is possible to realize strongly direction-dependent group velocities. They can also be tuned over orders of magnitude so that robust and tunable delay lines and interference devices are within reach.

I. INTRODUCTION

The edge states of topological insulators represent a fascinating field of research, which has been founded over thirty years ago with the discovery of the integer and the fractional quantum Hall effects [1,2]. In particular, the link to topological invariants has been an important step forward [3–6] relying in the end on the notion of gauge-invariant quantum mechanical phases [7].

Recently, the field has significantly gained interest due to the discovery of topological insulators driven by spin-orbit interaction, i.e., without necessity of an external magnetic field [8–11]. This experimental progress has been anticipated in a model put forward by Haldane [12], which is governed by nontrivial phases, but not induced by a magnetic field. This model is still the simplest example of a tight-binding model with nontrivial bands, i.e., with bands of nonzero Chern number. Since then, there is an abundance of studies of these effects, for instance the inclusion of spin to restore time-reversal symmetry [13–15]. A very recent promising progress is that films of solid state systems realize ferromagnetic Chern insulators representing essentially two-dimensional lattice systems, which display the quantum anomalous Hall effect (QAHE) [9–11] *without* external magnetic field. Thus experiment has by now come very close to the original idea of Haldane.

Our aim is to suggest two ways to put the topological robustness of edge states to use beyond the high-precision measurement of resistitivity in the Hall effect. Our focus are Chern insulators, which do not require an external magnetic field. The idea is based on the previous finding [16] that the Fermi velocity of the edge states along the edge is not a universal quantity, but depends crucially on the details of the edge. We aim here at a proof-of-principle illustration of the potential of this idea. Thus we perform calculations for the archetypical model of the field, the Haldane model on the honeycomb lattice [12]. For the same reason, we do not consider spin though the obtained results will carry over to spin currents in Kane-Mele models [13–15].

Of course, in standard quantum Hall setups the different transport behavior of the spatially separated edge states has been discussed, for instance, in strongly differing localization [17] or in switchable quantum dots coupled to single edge states [18]. Various quantum interferometer devices and quantum gates based on edges states have been realized [19,20].

We put forward two effects which to our knowledge have not yet been studied: (i) the Fermi velocity can be made extremely direction-sensitive, i.e., it can differ from one edge to the other by orders of magnitude. (ii) The Fermi velocity can be tuned by suitable voltages to vary by orders of magnitude such that one can control transport properties realizing tunable delay lines and interference devices for precise measurements of delays. Finally, we discuss various routes to realize the proposed effects.

II. MODEL

The tight-binding model considered reads

$$H = H_{\text{strip}} + H_{\text{decor}}, \tag{1a}$$

$$H_{\text{strip}} = t \sum_{\langle l,j \rangle} c_l^\dagger c_j + t_2 \sum_{\langle\langle l,j \rangle\rangle} e^{i\phi_{lj}} c_l^\dagger c_j \tag{1b}$$

$$H_{\text{decor}} = \sum_j \left[\lambda t \left(c_{d(j)}^\dagger c_j + c_j^\dagger c_{d(j)} \right) + \delta c_{d(j)}^\dagger c_{d(j)} \right], \tag{1c}$$

where the underlying lattice is shown in Fig. 1. The hopping on the strip of the honeycomb lattice is given by H_{strip} where $\langle,\rangle$ stands for nearest-neighbor (NN) hopping while $\langle\langle,\rangle\rangle$ stands for next-nearest-neigbor (NNN) hopping. The elements t and t_2 are real; the former serves as energy unit. The nontrivial topology is induced by breaking the time-reversal symmetry by the phases $\phi_{lj} = \pm\phi$. The minus sign applies to the red (light gray) arrows while the plus sign applies to the blue (dark gray) arrows in Fig. 1. Starting from the other sublattice, the colors of the arrows are interchanged.

The Hamiltonian of the additional sites at the upper edge is given by H_{decor}. If j is a protruding site at the upper edge of the honeycomb we denote its adjacent additional site by $d(j)$. The hopping between them is given by λt where $0 \leqslant \lambda \leqslant 1$ reduces this hopping relative to the standard NN hopping. The local energy of the additional sites is tuned by a gate voltage denoted by δ. For $\lambda = 0$, one retrieves the standard Haldane model on a strip of honeycomb lattice. Since this model has been studied before [12,14], we refrain from computing its nontrivial Chern numbers explicitly. We choose $t_2 = 0.2t$ and $\phi = \pi/2$ because this implies a sizable gap between the Chern bands and the bands are relatively flat.
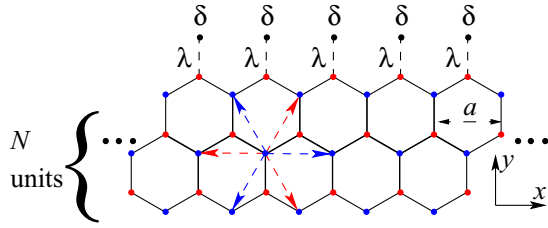
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FIG. 1. Sketch of a strip of the honeycomb lattice with NN hopping (black bonds) and NNN hopping (colored arrows). Note that the color of the arrows is swapped starting from the other sublattice. The strip height is N units. The upper boundary is decorated by weakly coupled ($\lambda \leqslant 1$) additional sites with local potential δ. The lattice constant a is set to unity.

III. DIRECTION-SENSITIVE FERMI VELOCITY

We show the nontrivial properties by directly computing the dispersion of the upper and lower edge modes. This allows us to address their Fermi velocities. To this end, we consider a strip of finite height, typically of $N = 80$ units, see Fig. 1. The translational invariance in x direction is preserved such that k_x continues to be a good quantum number. Then, we diagonalize the resulting $(4N + 3) \times (4N + 3)$ matrix numerically for given k_x. This procedure yields Fig. 2 where we focus on the edge states. The continuum of states above the lower band edge $\Delta_{\text{low}}(k_x) := \min_{k_y} \omega_{\text{up}}(k_x, k_y)$ are indicated by the light hatched/red colored region. The continuum of states below the upper band edge $\Delta_{\text{up}}(k_x) := \max_{k_y} \omega_{\text{low}}(k_x, k_y)$ are indicated by the dark hatched/blue colored region.

The left and right moving modes in Fig. 2 show a distinctly different behavior. This stems from the different structure of the upper and the lower edges of the honeycomb strip as shown in Fig. 1. The additional decorating sites have a pronounced effect. Without them, the dispersion of the edge mode crosses the Fermi level at the Brillouin zone boundary,
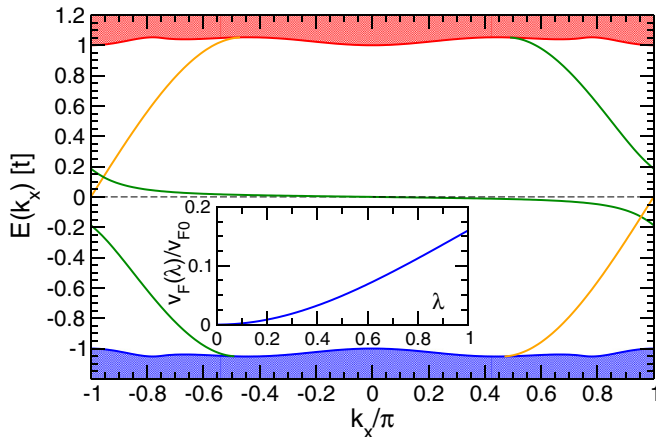

FIG. 2. Dispersion of the edge modes: in green (dark gray), the left mover at the upper edge and in orange (light gray) the right mover at the lower edge for $\lambda = 0.2$. The shaded areas show the continua from the Chern bands. The inset shows the Fermi velocity $v_F = \partial \omega / \partial k_x |_{\epsilon_F}$ relative to the one at the undecorated edge vs the relative coupling λ at $t_2 = 0.2t$ and $\phi = \pi/2$.

see right moving dispersion. If they are present, however, the Fermi level is hit at the zone center, see left moving dispersion. The remarkably flat dispersion results from the weak coupling $\lambda < 1$ of the otherwise isolated decorating sites. If $\lambda = 0$, the decorating site would host a completely local, i.e., momentum independent mode.

The dependence of the Fermi velocity v_F of the flat dispersion on λ is studied quantitatively in the inset of Fig. 2. As discussed above, it vanishes for $\lambda = 0$ and grows quadratically if the coupling is switched on. In this way, the coupling λ provides an excellent control parameter to tune the Fermi velocity at one edge of the sample. Since the position of the edge determines the direction of motion the Fermi velocities become strongly direction-sensitive. In particular, for small values of λ, the value for $v_{F,\text{right}}$ can differ by orders of magnitude from $v_{F,\text{left}}$. This opens interesting avenues to applications which have not yet been realized so far. We stress that the robustness of the topological edge mode protects them from being destroyed or blocked completely by disorder effects. Of course, they will be influenced by them on a quantitative level. But the qualitative features will persist, for instance, no localization occurs because back-scattering cannot occur. Only if the disorder becomes very large so that scattering from one edge to the other sets in, see, for instance, Ref. [17], and the advocated effect vanishes.

The direction-sensitivity of the Fermi velocity appears to be a static property once the system is given. Next, we illustrate that it can be tuned as well.

IV. TUNABLE FERMI VELOCITY

The guiding idea is to continuously tune the system *with* decorating sites towards the system without them. To this end, the decorating sites shall be switched off. This can be achieved by pushing them up in energy so that the electrons do not visit them anymore. The knob to do so is the local potential δ in (1c), which can be thought to be realized by a gate voltage.

In the upper panel of Fig. 3, we depict the effect of a finite gate voltage $\delta = 0.1t$. Clearly, the intended effect takes place and the dispersion of the left moving mode is shifted upwards by about δ. In addition, the precise shape of the dispersion is modified. As expected, the right moving mode at the other edge is almost not influenced. The strip analyzed is $N = 80$ units high and for this value no effect of the gate voltage on the right moving mode occurred, even in the ninth digit.

Due to the shift, the part of the left moving dispersion crossing the Fermi level at zero energy is changed. Its slope is increased as we argued before on physical grounds. Note that due to the particle-hole symmetry of the model a negative shift would have the same effect, i.e., it produces the same increase of the Fermi velocity.

In the lower panel of Fig. 3, we study the increase of the Fermi velocity quantitatively. Indeed, it is possible to drive the Fermi velocity to the value of the undecorated edge. For large enough gate voltage δ, the Fermi velocity relative to the undecorated one saturates at unity, i.e., the system behaves as if the decorating sites were not present at all. The gate voltages at which the saturation sets in is of the order of λt because δ has to counteract the hybridization between the protruding edge sites j and the attached decorating sites $d(j)$.
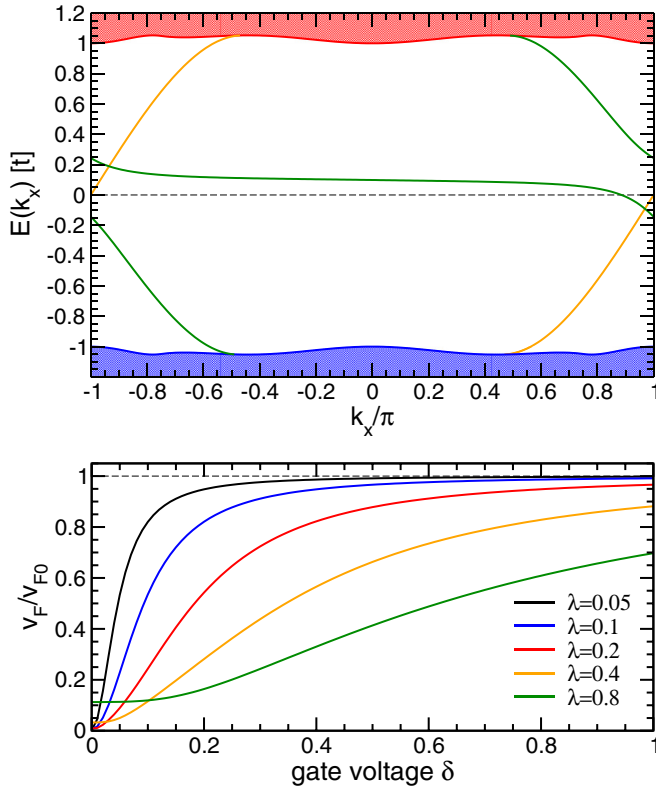
FIG. 3. (Top) Dispersion of the edge modes; in green (dark gray) the left mover at the upper edge and in orange (light gray) the right mover at the lower edge for $\lambda = 0.2$ and $\delta = 0.1t$. The shaded areas show the continua from Chern bands. (Bottom) Fermi velocity $v_F = \partial\omega/\partial k_x|_{\epsilon_F}$ relative to the one at the undecorated edge vs the gate voltage δ for various values of the relative coupling λ at $t_2 = 0.2t$ and $\phi = \pi/2$.
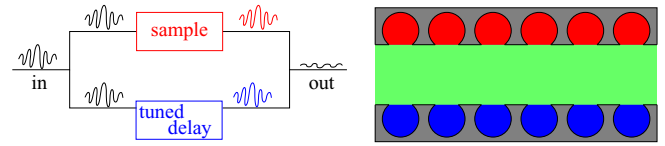

FIG. 4. (Left) Sketch of a circuit to use the tunable delay line in an interference measurement for the determination of the delay of signal transmission in a sample; almost destructive interference is shown. (Right) Sketch of a decorated Hall sample to realize tunable and direction-dependent Fermi velocities. The black hatched area is impenetrable for the electrons, while the potential of the colored areas (blue (dark gray) and red (light gray)) can be tuned independently by gate voltages δ_{low} and δ_{up}, while the bulk of the sample (green) remains unchanged. A magnetic field is applied perpendicular to the plane shown to generate the edge states. It should be approximately so large that circular Landau orbits fit into the bays.

This observation shows that for small values of λ the gate voltage δ provides an excellent control parameter to tune the Fermi velocity. Rather small values of the gate voltage are sufficient to modify v_F by orders of magnitude. There is a small offset at $\delta = 0$ given by $v_F(\lambda, \delta = 0)$. But for increasing δ a linear regime is entered in which $v_F \propto \delta$ holds until the saturation regime is reached for $\delta > \lambda t$. This linear regime is proposed to be put to use in applications. Here the Fermi velocity, which is the group velocity of an electric signal, can be tuned by a third gate so that a tunable delay line can be realized. The gate voltage varies δ so that a setup can be realized where the group velocity can be tuned on the fly. In this way, the time a electric signal requires to pass through the device can be fine-tuned to obtain destructive or constructive interference with a signal which has passed along another path, for instance, through a sample of which the transmission properties shall be measured. Due to the topological protection of the edge states, disorder will not destroy the effects so that they are qualitatively robust against imperfections.

In Fig. 4(a), a circuit is sketched which permits to measure the delay occurring in the unknown sample in parallel to the tunable delay line based on a Chern insulator with tailored edges. The idea is to look for destructive and constructive interference of an oscillating electric signal at the output. In this way, very precise detection of small delays should be

possible. Note that we do not advocate quantum interference here in contrast to many studies in the literature, e.g., Ref. [19].

Furthermore, this effect is direction-dependent. One can push the device one step further by decorating also the lower edge of the sample (not shown). Then two independent gate voltages can be used to control the Fermi velocities in both directions left and right independently.

In addition, one can feed the signal itself not only to the input of the Chern insulator, but also to the gate voltage itself. This yields a tunable nonlinear feedback: depending on the bias on the control gate, larger amplitudes travel faster or slower than smaller amplitudes. By suitable tuning of this effect one can realize a wave equation similar to the Korteweg-de Vries equation, which allows for soliton solutions. Thus particularly stable signals can be transmitted.

V. POSSIBLE REALIZATIONS

About three years ago the chances of realizing a Chern insulator in a microscopic lattice were still considered slim [21]. But the observation of the QAHE in thin films of ferromagnetic Chern insulators has changed the game [9,10]. The temperatures at which the QAHE is observable has increased from several milliKelvin to a few degrees of Kelvin [11] and theoretical calculations indicate that even room temperature should be within reach [22,23]. Thus it appears perfectly reasonable that the proposed lattices or similar analogues can be designed and realized. Detailed calculations show that the tailored design of super lattices of gold atoms on single-vacancy graphene provide a promising candidate to realize high-temperature Chern insulators [23,24]. Thus the tailored design of differing edges will also possibly open a route to realize the proposed strips of lattices and to measure the advocated effects.

An alternative route is open in tailored optical lattices where it has been recently possible to realize the Haldane model (1a) itself and to measure some of its basic properties [25]. Thus a proof-of-principle realization of decorated strips of Haldane models such as shown in Fig. 1 appears to be well within reach. While this is an attractive prospect such a realization is probably not so close to the application as a measuring device.

A third alternative is to construct lattice models artificially by design of tailored semiconductor structure. For instance, antidot structures can already be synthesized with high precision [26]. So it is conceivable to realize dot or antidot lattices which have the discrete translational invariance we are considering here. Applying a perpendicular field to such planar modulated electron gases yields the desired topologically nontrivial Chern bands with the corresponding gapless edge modes. We recall that lattices in a finite magnetic field of certain strengths correspond to Haldane type of models as illustrated before for the kagome lattice [16].

A fourth alternative realization can be based on more standard quantum Hall setups. A quantum Hall sample in a magnetic field provides a Chern insulator with spatially separated edge states. Thus, if one is able to pattern the edges differently and to control them independently by gate voltages, the effect of tunable Fermi velocities will be observable.

To the author's knowledge, no such experiment has been performed so far. However, the degree of control of gate voltages in time and space on the nanoscale is remarkable so that it appears that the proposed experiment is well possible. For instance, it is possible to switch the coupling of quantum dots to one edge mode on and off yielding electrons on demand [18]. If this kind of setup is extended to a periodic chain of quantum dots, see Fig. 4 b, all gated by the same voltage, a setup is realized, which is a continuum version of the lattice model in Fig. 1. The patterning on the nanoscale is also possible, see for instance the Mach-Zehnder interferometer realized by Karmakar *et al.* [19]. Note that the interometer discussed in that work uses the nanoscale structures to make the two edge states interfere. Thus the setup is not the one advocated here.

VI. SUMMARY

The first idea of the present work is the finding that the relevant group velocity of signals transferred in topologically protected and thus robust edge modes may differ strongly depending on whether they propagate at the upper or lower edge. The reason is that the structure of the edges may differ and that this influences the nonuniversal transport properties. For disorder or a (de)coupled quantum dot such phenomena were discussed and observed before [17,18], but not for a periodic structure with well-defined Fermi velocity, which will allow for faithful signal transmission.

The second idea was that this difference can be enhanced and manipulated by special design of the edges. For instance, weakly linked local energy levels, decorating the edges, render the mode at the Fermi velocity arbitrarily slow. This leads to direction-dependent group velocities because the mode at one edge moves into one direction while the mode at the other edge moves into the other direction and the link strength can be chosen very differently at the edges.

The third idea is to tune the group velocities by gate voltages which can effectively switch the weakly linked decorating

levels on and off. By varying the gate voltages, one may easily tune the group velocities. This can even be done for left and right movers independently. By this mechanism the group velocity, relevant for the delay time in signal transmission, can be varied by a gate voltage. Signals can be delayed and/or modulated.

The above ideas have been demonstrated on the proof-of-principle level by explicit calculations for the archetypical model for nontrival Chern bands [12]. This could be decorated by weakly coupled additional sites at the edges with local energy given by the gate voltage. The relative coupling λ and the gate voltage δ are the tuning parameters.

Furthermore, we discussed and suggested ways to realize the above proposal. Such realizations appear possible in many fields ranging from quantum Hall systems with external magnetic field over ultracold atoms in optical lattices with artificial gauges to thin films of ferromagnetic topological insulators [9–11,23]. The latter represent lattice models very close to the ones studied here theoretically.

VII. OUTLOOK

The findings presented have established the fundamental idea on the proof-of-principle level. Depending on the route favored for experimental realization further calculations are called for. For instance, on the one hand in optical lattices the Chern insulator is realized so far on square lattice, not on honeycomb lattice. On the other hand, proposals in solid state physics favor honeycomb lattice systems which break time-reversal symmetry by spontaneous ferromagnetism. Thus differing specific calculations will be useful.

The topological protection of the edges limits the influence of weak disorder and weak interactions to a quantitative level. The qualitative effect to have tunable velocities remains robust. Nevertheless, it would be necessary to assess quantitatively by theoretical considerations how robust the tunable velocities are. Such studies must be adapted to the envisaged experimental realization; thus they cannot be made on general level.

The fundamental idea advocated here can be extended also towards spintronics. The inclusion of spin, for instance as in the models put forward by Kane and Mele [13–15] will enable to pass from currents and signals expressed in charges to currents and signals of spin, i.e., to pass from electronics to spintronics [27]. So there appear ample ways to explore the applicability of topologically protected edge states propagating along tailored and tuned edges.

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