Temperature-dependent nonlinear Hall effect in macroscopic Si-MOS antidot array

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By measuring magnetoresistance and the Hall effect in a classically moderate perpendicular magnetic field in a Si-MOSFET-type macroscopic antidot array, we found a nonlinear with field, temperature- and density-dependent Hall resistivity. We argue that this nonlinearity originates from low mobility shells of the antidots with a strong temperature dependence of the resistivity and suggest a qualitative explanation of the phenomenon.

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Low-field Hall resistance (R_{xy}) is broadly used to determine electron density. In two dimensions, one has $R_{xy} = B/ne$ [where n is the electron density, B is the magnetic field, perpendicular to the two-dimensional (2D) plane, and e is the electron charge]. Remarkably, within the classical treatment, Hall resistance coincides with Hall resistivity and does not depend on geometry, e.g., if a hole is drilled in the 2D gas, the current flow will be redistributed while the R_{xy} value will stay the same. On the other hand, such geometrical constriction effectively admixtures Corbino geometry to the ordinary Hall bar, thus leading to huge positive magnetoresistance [1].

One of the ways to experimentally tune the parameters of the drilled holes and 2D gas independently is by a double-gated antidot array (AA), where the electron density at the dots and residual 2D gas is controlled by two independent gate electrodes. Antidot arrays give a vast parametrical space (mean free path, densities of the 2D gas and antidots, sizes of the antidots, and distances between them). Their transport properties might be studied in various magnetic fields at various temperatures.

As a rule, the bare 2D gas that hosts the AA has a rather high carrier mobility that allows one to study oscillatory effects: either quantum interference effects (Altshuler-Aharonov-Spivak oscillations [2]) or quantization effects (commensurability oscillations [3]). Quenching of the low-field Hall effect in AA was also addressed within classical billiard models [4] and was even detected experimentally [3].

In order to suppress the above-mentioned ballistical effects, we study classically large micrometer-sized antidots in low-mobility 2D gas, i.e., both the size and distance between the antidots are much larger than the mean free path and magnetic length for reasonable fields (a few T). The boundaries of our antidots are also classically smooth (compared to those needed for clear oscillatory effects—see, e.g., Ref. [5] and references therein).

Theoretically, the problem of the Hall effect in a two-dimensional inhomogeneous system is a very longstanding one (see, e.g., Ref. [6] for a review). The most recent theoretical calculations were performed by Bulgadaev and Kusmartsev within the effective media approach [7,8] and by Parish and Littlewood within the random lattice method [9]. Both approaches open the possibility for the nonlinear field dependence of the Hall resistance.

Samples used. We used lithographically defined Hall-bar-shaped Si-MOSFETs containing an AA with two independent gates: one for the antidot array V_a and one for the remaining 2D gas (R2DEG) V_g. Independently on the same chip we defined separate Hall bars without antidots obtained within the same processes. We therefore could independently measure the transport properties of both pristine 2D gas and R2DEG. The samples were produced similarly to those used in Ref. [10]. The substrate was doped and preserved a certain conductivity ($R \lesssim 1$ GΩ) down to 20 K, therefore all measurements were performed at lower temperatures.

The dimensions of the Hall bars with AA were 0.4 mm × 0.4 mm; the dimensions of the Hall bars with pristine 2DEG were 0.1 mm × 1 mm. All distances and the schematic geometry of the antidots are shown in Fig. 1. About 400 chips were defined lithographically on one 4 in. wafer. There was a variation in sample properties (mobility, gate oxide thickness) from chip to chip, probably caused by temperature gradients during the sample fabrication (growth of the thermal oxide, distribution of impurities). The effects discussed below, however, were reproducibly observed on several samples with various parameters.

Measurements. The measurements were carried out in a temperature range 2–20 K using a PPMS-9 cryomagnetic system. The measurement current was in the range 50–200 nA to ensure the absence of overheating. All measurements were performed in the frequency range 13–18 Hz using lock-in amplifiers. The resistivity (Hall resistivity) data were collected during field sweeps (typically from −7 to 7 T) and then symmetrized (antisymmetrized).

Results. Bare 2D gas manifested a mobility of about 1 m^2 V/s at helium temperatures. The magnetoresistance $\rho_{xx}(B)$ was weak and negative: In the lowest fields (< 0.4 T) this negative magnetoresistance originates from the suppression of weak localization, in larger fields negative magnetoresistance is always observed in high-mobility Si-MOSFETs [11–13]—it is probably related to electron-electron interactions and is still not understood. Starting from ∼ 2 T, Shubnikov–de Haas oscillations develop.

Hall resistance is a linear function of the magnetic field, as it should be (see Fig. 2). In the lowest fields (as indicated in the inset to Fig. 2) there is a small feature close to $B = 0$. This feature was already thoroughly analyzed by us (see Appendix

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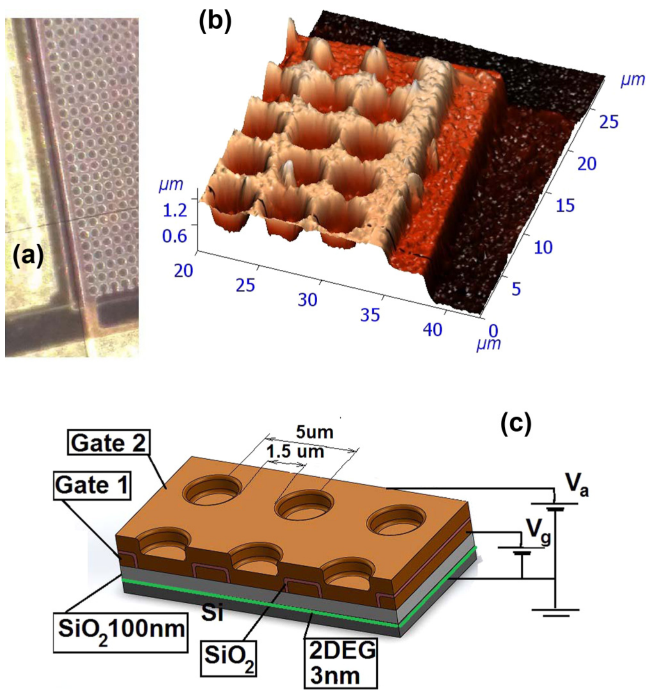

FIG. 1. (a) Optical image, (b) atomic force microscopy (AFM) image, and (c) schematics of the gating of the studied antidot array.

B in Ref. [14]): We cannot explain it with any existing theory, although we can state that it is probably related to weak localization, similarly to the feature observed by Minkov *et al.* [15]. The density of the electrons extracted from the Hall slope is roughly proportional to the gate voltage $n \propto V_g$. Resistivity of the bare 2DEG does not show any hysteresis with the gate.

The resistivity of R2DEG from the same chip with the same oxide thickness (sample AA1, Fig. 3) is larger than that for bare 2DEG (this is evident because of the constricted geometry and effectively larger l/w ratio). The deviations of the *magnetoresistivity* tensor for R2DEG from that for the bare 2D gas become more and more pronounced as AA gets more and more depleted. The V_a value, however, should not be negative in order to avoid irreversible changes, accompanied by long equilibration due to recharging of the impurity states. R2DEG demonstrated relaxation processes after changing the gate voltage. The equilibration time increased with decreasing temperature. In order to ensure an equilibrium state we always swept the gate voltages at elevated temperatures. We present below the most representative figures for $V_a = 0$ in a wide range of densities (V_g) [see Figs. 3(a), 3(b), 3(d), and 3(e)].

With antidots the behavior of the Hall effect and magnetoresistance changed considerably: Magnetoresistance became essentially positive, and the Hall effect started to deviate from a linear-in-B behavior. Positive magnetoresistance in a nonuniform system is not surprising (see, e.g., Ref. [16]) and follows from simple geometrical constriction [9], whereas the correction to the Hall effect is much less trivial.

The Hall slope versus magnetic field is shown in Figs. 3(c) and 3(f). It differs dramatically from those in the inset to Fig. 2. The features of the Hall slope are as follows: (i) Similarly to the bare 2DEG, the Hall effect has a low-field feature and traces of Landau quantization; (ii) the zero-field Hall slope is temperature dependent, and it appears as carriers are frozen out as T decreases; (iii) the coefficient c in the low-field

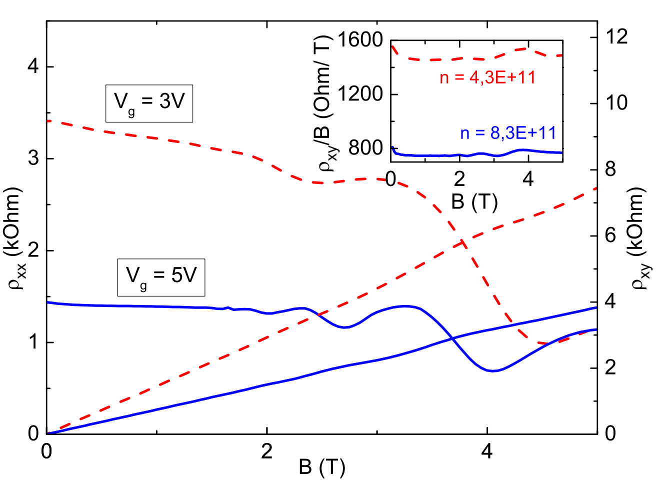

FIG. 2. Magnetoresistance and Hall resistance Si-MOSFET of Hall bar geometry for two representative gate voltages at $T = 2$ K. The inset shows the Hall slope (R_{xy}/B) vs magnetic field.

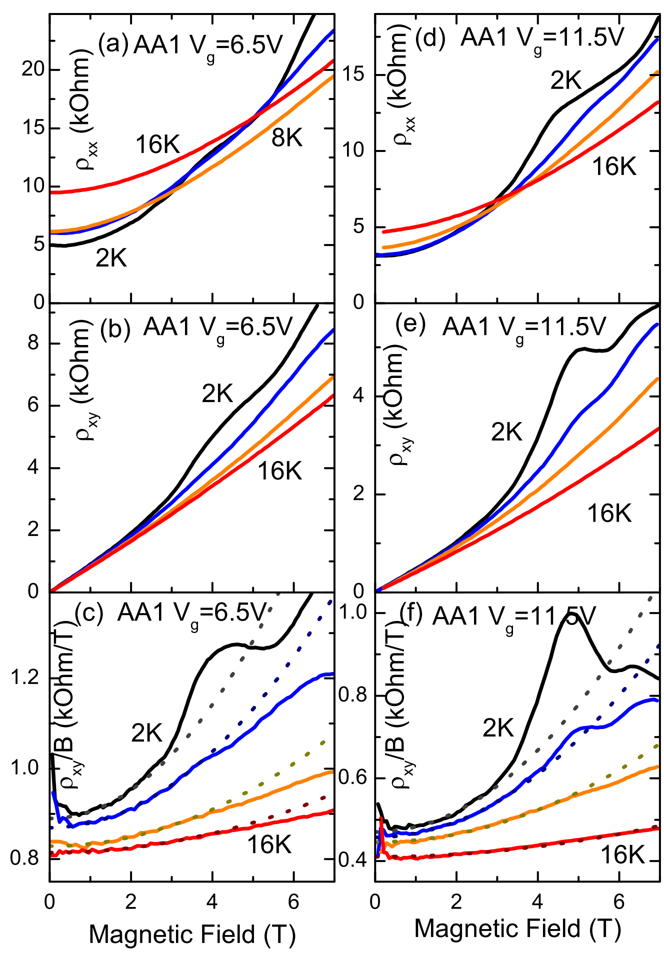

FIG. 3. (a), (d) Magnetoresistance, (b), (e) Hall resistance, and (c), (f) Hall slope for R2DEG (sample AA1) at (a)–(c) $V_g = 6.3$ V, $V_a = 0$ V and (d)–(f) $V_g = 11.5$ V, $V_a = 0$ V for various temperatures (red, 16 K; orange, 8 K; blue, 4 K; black, 2 K). In (c) and (f) the dotted lines indicate the parabolic-in-field, low-field behavior of the Hall coefficient (see text).

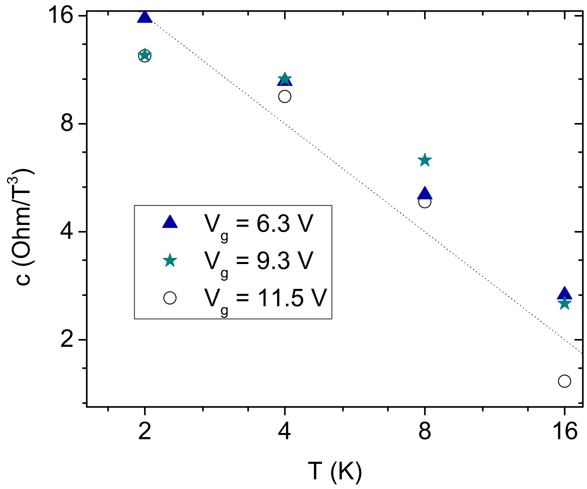

FIG. 4. Temperature dependencies of the fitting parameter c (see text and Fig. 3) for sample AA1 and various gate voltages, shown in the panel. The dotted line indicates $1/T$ dependence.

expansion of the Hall slope ($\rho_{xy}/B \approx a + cB^2$) is temperature dependent and behaves approximately as $1/T$ (see Fig. 4); (iv) the coefficient c weakly depends on carrier density; and (iv) the value of the Hall slope for the RDEG itself is essentially higher than one would expect from bare 2D gas [e.g., in Fig. 3(c) the Hall slope is more than 1 kΩ per T, whereas the expected value of the density $n = 10^{12}$ cm^{-2}, which corresponds to $V_g = 6.3$ V for bare 2DEG, should give a Hall slope about 630 Ω per T].

Discussion. The above observations imply that the AA gives rise to corrections to the Hall resistance, similarly to additional scatterers adding up to longitudinal resistance. This correction seems to be density independent in a certain density range and sensitive to temperature and magnetic field.

An explanation of the phenomenon is required. The essential ingredients of such an explanation are as follows:

1. The antidots themselves are nonconductive ($V_a = 0$), however, they are surrounded with a weakly conductive shell with decreased electron density. The typical width of the shell might be about 50–200 nm. The width is determined by the gate oxide thickness.

2. The diameter of the antidot is about 3.5 μm, and the distance between antidots is effectively about 1 μm (see the AFM image in Fig. 1). The typical density of the R2DEG is 5–10 $\times$ 10^{11} cm^{-2}, the mean free path is about 80 nm, and in the shells the mean free path is smaller.

3. For Si-MOSFETs of similar mobility ($\sim$ 1 m^2/V s), the resistivity has a strong "metallic" temperature dependence in the range of densities $n \sim 2$–5×10^{11} cm^{-2} and temperatures $T \sim 2$–10 K.

We note that qualitatively similar behavior (i.e., the Hall coefficient growing with field) was obtained theoretically in all cases for a nonuniform system in Ref. [9]. This behavior is opposite to that for the frequently used two-liquid model: In a mixture of two types of carriers of the same sign but with different mobilities, the low-field Hall coefficient c_{LF} is determined by higher-mobility carriers n_{HM}^{-1}, which is always larger than the high-mobility Hall coefficient c_{HF} that is

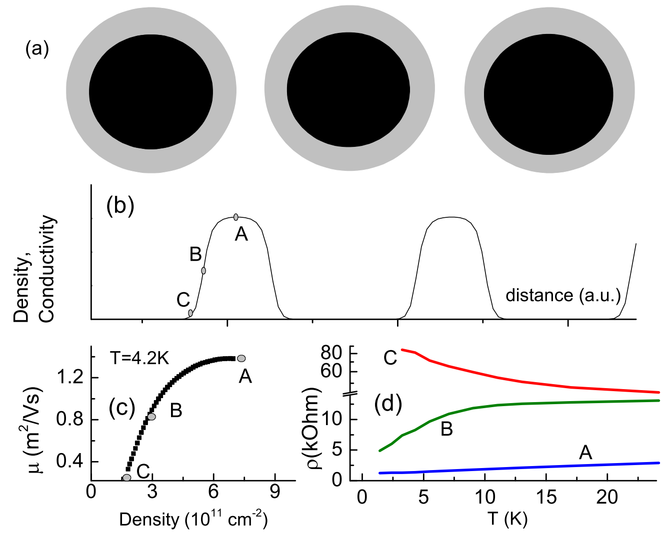

FIG. 5. (a) Schematics of the antidots with shell. (b) Carrier density (or local conductivity) distribution in a lateral direction with three typical points, indicated A, B, and C. (c) Density dependence of the mobility for the bare 2DEG of the sample AA1. (d) Temperature dependence of the resistivity for points A, B, and C, indicated in (b) and (c).

determined by the total carrier density $(n_{\mathrm{HM}} + n_{\mathrm{LM}})^{-1}$; see, e.g., Ref. [17].

On the basis of the above facts, we suggest the following possible explanation for the phenomenon: In 2DEG with an AA [shown schematically in Fig. 5(a)], the zero-field density or local $B = 0$ conductivity distribution is shown in Fig. 5(b). In a zero magnetic field, electrons in the shells [point B in Fig. 5(c)] have both a lower density and mobility than R2DEG [point A in Fig. 5(c)], and weakly participate in the charge transport, because the conductivity $\sigma = ne\mu$ is small. A nonmonotonic $\mu(n)$ dependence, and particularly a drop in mobility at low densities, is well known for Si-MOSFETs [18] and crucial for further explanations.

Current redistribution occurs in the magnetic field . Although modeling of magnetotransport through a nonuniform system is a complicated task, an idea why the lower-mobility shells contribute more in higher fields may follow from the simple Drude theory. Indeed, in uniform 2D systems, in a perpendicular magnetic field, the conductivity tensor components are expressed as $\sigma_{xx} = ne\mu(1 + \mu^2B^2)^{-1}$ and $\sigma_{xy} = \mu B\sigma_{xx}$. It means that the conductivity of lower-mobility shells drops with field not as rapidly as the conductivity of the higher-mobility R2DEG, i.e., as the magnetic field increases, the contribution of the shells to transport grows and the Hall resistance becomes larger because the density in the shells is lower than in R2DEG.

A clue to the explanation of the strong temperature dependence of the effect might be the strong temperature dependence of the resistivity ($1/\sigma \propto R \propto A + BT$; see Ref. [19]): Figure 5(d) shows the temperature dependencies of the bare 2DEG resistivity in points A, B, and C from Fig. 5(b), corresponding to the R2DEG, the active part of the shell, and the insulating part of the shell, respectively. The temperature dependence of the resistivity in the shell is much stronger than that in R2DEG. At lower temperatures the mobility of the active part of the

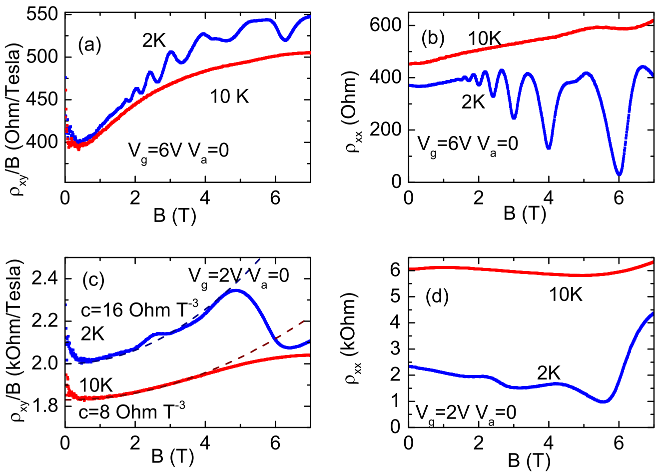

FIG. 6. (a), (c) Hall slope and (b), (d) magnetoresistance for sample AA2 at 2 K (blue curves) and 10 K (red curves). $V_a = 0$ everywhere. (a), (b) $V_g = 6$ V. (c), (d) $V_g = 2$ V. In (c) the dashed lines indicate a parabolic-in-field correction to the Hall slope. Constant c of this parabola is indicated in the panel.

shells increases, and their contribution even at zero field also increases, which explains why the low-field Hall coefficient grows as T decreases.

The lateral width of the shell is determined by the geometry of the system (gate thickness). If the density, and hence conductivity, of the 2D gas becomes very high, the size and contribution of the high-mobility, low-density area should become negligible. Figures 6(a) and 6(b) demonstrate the effect of increased density for sample AA2, which has a thinner oxide layer compared to AA1 and hence thinner shells of the antidots. It is easy to see that although Hall effect nonlinearity exists, its temperature dependence weakens, as expected. Because of large carrier density, a whole palette of magneto-oscillations from the bare 2D gas becomes clearly seen. We note that in sample AA2, in spite of such high electron densities ($\sim 1.5 \times 10^{12}$ cm^{-2}), another fingerprint of a nonuniform system, i.e., positive magnetoresistance, is observed, similarly to sample AA1.

When we decrease the density [see Figs. 6(c) and 6(d)], the behavior of magnetoresistance and Hall slope in sample AA2 changes: Magnetoresistance at low T becomes negative due to 2DEG, as it used to be close to the metal-insulator transition, and the correction to the Hall coefficient becomes temperature dependent. The dependence is weaker than $1/T$ ($c = 8\ \Omega\,\mathrm{T}^{-3}$ at 10 K and $c = 16\ \Omega\,\mathrm{T}^{-3}$ at 2 K). We believe this is because 2DEG itself has its own strong $R(T)$ dependence, so the contribution of the shells to the Hall resistance on top of the R2DEG is not as distinguishable as it is in sample AA1. The strength of "metallicity" is also well seen from Fig. 6(d): As T increases by a factor of 5, the resistivity triples.

It worth noting that since the mean free path and coherence length are much less than the size of the antidots and their period, spatial ordering of the AA is not essential for nonlinearity of the Hall effect. In order to maximize the effect, one should merely fabricate the sample with a maximal fraction of the shells, i.e., the gate insulator thickness d_G should be increased, whereas the diameter of the antidot should be decreased down to a minimal possible value ($\sim 2d_G$, otherwise depletion in the antidots will not be achieved). The density of the antidots should also be maximized, e.g., by arranging the antidots in a triangular lattice.

In order to observe the nonlinear Hall effect we expect that the local conductivity approximation should be valid, i.e., the mean free path has to be smaller than all other length scales, which means that the effect should be preferably observed in low-mobility systems. It also means that the coexistence of our phenomenon and well-known commensurability oscillations is impossible, because the latter are essentially ballistic phenomena and require that the cyclotron radius $r_c \sim l_B = 26\ \mathrm{nm} \times \sqrt{B}$ (B is measured in T) should be comparable with the period of the antidot lattice, and no scattering should occur during the cyclotron rotation, i.e., $\omega_c\tau \equiv \mu B > 1$.

To summarize, we have observed an interesting effect in magnetotransport of the macroscopic gate-defined antidot array: a positive and temperature-dependent correction to the Hall resistance. We believe that this correction originates from the shells of the antidots, which have a lower carrier mobility than the bare 2D gas. We suggest a qualitative explanation of the phenomenon: In a perpendicular magnetic field, the conductivity of the 2D gas drops—the larger the mobility, the larger is the drop. Correspondingly, with increasing magnetic field, a current redistribution occurs in favor of the lower-mobility, lower-density shell regions, and those in turn increase the total Hall resistance. The temperature dependence of the effect originates from a strong temperature dependence of the resistivity in Si-MOSFETs. This qualitative model requires further justification by microscopic theory.

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