Scaling of macroscopic superpositions close to a quantum phase transition

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It is well known that in a quantum phase transition (QPT), entanglement remains short ranged [Osterloh *et al.*, Nature (London) **416**, 608 (2005)]. We ask if there is a quantum property entailing the whole system which diverges near this point. Using the recently proposed measures of quantum macroscopicity, we show that near a quantum critical point, it is the effective size of macroscopic superposition between the two symmetry breaking states which grows to the scale of system size, and its derivative with respect to the coupling shows both singular behavior and scaling properties.

Introduction. Superposition of states is the most important and distinctive feature of the microscopic world: Atoms, electrons, and photons, and even molecules [1,2], can exist in a superposition of two or more physical states. This property lies at the heart of all intriguing quantum phenomena that we know, from wave-particle duality to entanglement and nonlocality. Nevertheless, everyday objects of macroscopic size do not exist in superposition of their different states. We do not see states which represent a cat in a superposition of dead and alive states [3]. This may be due to the fact that quantum mechanics is modified at a certain scale [4,5], or it may be due to the extremely rapid decoherence of such states as the result of their macroscopic number of constituents with the outside world [6]. There are now intensive experimental [1,2] and theoretical investigations [5] to reveal if there is a distinct border between the micro and macro world.

Despite the absence of quantum superpositions on macroscopic scales, we see many macroscopic phenomena which are the result of collective quantum mechanical behavior of their microscopic constituents. An important example is the phenomenon of superconductivity which arises when pairs of electrons interact with phonons and tend to entangle and form Cooper pairs [7], which subsequently undergo Bose condensation. Without this microscopic superposition of atomic entities, we do not have the phenomenon of superconductivity at the macroscopic level. Nevertheless this cannot be called a witness of quantum mechanics at macroscopic scale. It was Leggett [8] who first emphasized the difference between this type of macroscopic quantum effect, which is the result of collective superposition of microscopic entities, and the one which was discussed above, i.e., when a macroscopic system, a large macromolecule [1], a virus or a cat [3], is in a superposition of two macroscopically distinct states.

To put all this in very concrete terms, one can take a number $N \sim$ Avogadro number of spin 1/2 particles. Then the simplest conceivable example of the former is a state like $[\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)]^{\otimes N}$ and of the latter a Greenberger-Horne-Zeilinger state [9] $|GHZ\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle^{\otimes N} + |\downarrow\rangle^{\otimes N})$. It is only the latter kind that is called a quantum superposition on a genuinely macroscopic scale. We should stress the difference between three concepts: macroscopic quantum superposition, entanglement, and quantum correlation. By macroscopic superposition we mean a state like $|\psi_1\rangle + |\psi_2\rangle$, where $|\psi_1\rangle$ and $|\psi_2\rangle$ are macroscopically distinct [10], in which case a

measurement of any single particle can reveal the difference between $|\psi_1\rangle \equiv |\uparrow\rangle^{\otimes N}$ and $|\psi_2\rangle \equiv |\downarrow\rangle^{\otimes N}$. States with this property are *a priori* extremely rare in many ensembles of pure states, especially there are highly entangled states that feature vanishing macroscopic superposition as pointed out in Ref. [11]. Finally this property is distinct from quantum correlation, which can exist even in separable mixed states and is measured by quantum discord [12].

It is quite conceivable that there is a spectrum of states between these two extremes. Therefore, like many other quantities, once the qualitative difference and the typical examples of these two classes of states are understood, the next step is to define reasonable measures to quantify exactly how macroscopic a quantum superposition is. These measures are called measures of quantum macroscopicity. To this end, various proposals have been suggested [10,13–20]. The basic idea used in all these proposals is that macroscopic superposition entails a large amount of uncertainty when a suitably chosen macroscopic observable is measured. The way this large variance scales with N defines a measure of macroscopicity.

We now come to the main questions asked in this paper: Is there macroscopic superposition of symmetry breaking states in a quantum phase transition (QPT), and if yes, how this macroscopicity behaves near the critical point? How the relevant critical exponents are related with this scaling? What happens exactly near a point of quantum phase transition? The intuition behind these questions is the observation that in a classical phase transition, say in the Ising model, large areas of up and down spins coexist. So one expects that in a quantum phase transition, this coexistence appears in the form of large scale superpositions. Therefore an exponent like ν which sets the scale of divergence of correlation length in classical phase transition, may appear here in some form of divergence of macroscopic superposition.

In fact, it was anticipated that near a quantum critical point, we will see long range entanglement (a quantum parallel of diverging correlation length in classical phase transition). However it came as a big surprise [21] that near a critical point, entanglement always remained short ranged. Instead what becomes singular and shows scaling behavior is the derivative of this short-range entanglement. Later elaborations revealed a more detailed picture. First it was shown that while entanglement remains short range in a QPT, other kinds of quantum correlations, measured by discord, can be long

ranged [22]. Then it was shown that depending on whether the QPT is first order or continuous, the entanglement itself or its derivative is the quantity which is relevant in detecting it [23,24].

Therefore this result gives some weight to the question we are asking. In fact by using the measures of macroscopic superposition which have been recently developed [14,19], we show that near the point of phase transition, the system, which is undecided between the two symmetry breaking phases, actually goes into a macroscopic superposition of them and shows singular behavior and scaling properties in the derivative of the effective size of this superposition. Our findings on macroscopic superposition, in conjunction with other results on short-range entanglement in quantum phase transitions [21,25–30], confirm the recent results of Ref. [11] according to which macroscopicity is rather rare in ensembles of random pure states despite having large geometric entanglement.

The Ising model in transverse field. We now consider a paradigmatic example of quantum phase transitions [31], namely the Ising model in transverse field, described by the Hamiltonian

$$H = -\lambda \sum_{i=1}^{N} \sigma_i^x \sigma_{i+1}^x - \sum_{i=1}^{N} \sigma_i^z, \qquad (1)$$

where σ_i^μ is the μth Pauli matrix ($\mu = x,y,z$) at site i and λ is the inverse strength of the external field. Besides the obvious translational symmetry, the Hamiltonian is real and has a global phase flip symmetry $[U,H] = 0$, where $U = \prod_{i=1}^{N} \sigma_i^z$. The operator U flips σ_i^x (and also σ_i^y) and leaves σ_i^z unchanged. Let the state vectors $\{|0\rangle,|1\rangle\}$ and $\{|+\rangle,|-\rangle\}$ be the eigenstates of the σ^z and σ^x, respectively. In the limit $\lambda = 0$ the unique ground state $|00\cdots0\rangle$ is unchanged under the U symmetry, while in the limit $\lambda \longrightarrow \infty$ the symmetry is broken (in the thermodynamic limit) and the two degenerate ground states $|++\cdots+\rangle$ and $|--\cdots-\rangle$ are mapped to each other by U. The model is solved exactly by using the Jordan-Wigner transformation which turns it into a free fermion model [32,33]. At zero temperature and in the limit $\lambda = 0$, the system goes to the unique ground state $|00\cdots0\rangle$ where all the spins are aligned in the z direction. In this phase $\langle\sigma^x\rangle$ as a local order parameter vanishes. As we gradually increase the value of λ, at $\lambda = 1$, the system undergoes a quantum phase transition, the symmetry break downs, and the system chooses one of the two degenerate ground states, finally in the limit $\lambda \longrightarrow \infty$ system goes to $|++\cdots+\rangle$ or $|--\cdots-\rangle$, where each of these two states shows superpositions (in the $\{|0\rangle,|1\rangle\}$ basis), but on a microscopic scale of each individual spin.

Measures of macroscopic superposition. We now remind the reader of a few basic facts about two measures of macroscopicity which we mainly use in our analysis. A simple calculation shows that the variance of any additive operator like $\mathcal{A} = \sum_{i=1}^{N} A_i$ on any product state $|\Psi\rangle = |\psi\rangle^{\otimes N}$ is proportional to N, that is $\mathcal{V}_\Psi(\mathcal{A}) = N\mathcal{V}_\psi(A)$, where $\mathcal{V}_\phi(X) := \langle\phi|X^2|\phi\rangle - \langle\phi|X|\phi\rangle^2$ is the variance of the observable X on the state $|\phi\rangle$. However quantum states which are in macroscopic superpositions show quadratic behavior when the variance of suitable additive operators are measured on them. An example is the GHZ state. For this state the observable $M_z := \sum_{i=1}^{N} \sigma_i^z$ (e.g., the magnetization in the z direction)

shows an anomalously large variance. In fact straightforward calculation shows that $\mathcal{V}_{GHZ}(M_z) := \langle M_z^2\rangle - \langle M_z\rangle^2 = N^2$. These anomalous large fluctuations are the signature of a macroscopic quantum superposition. In view of the fact that superposition is the characteristic feature of quantum mechanics against classical mechanics, it is usually said that the scaling of an additive operator with system size N is taken to depict its classical or quantum behavior. However it should be noted that this quantum versus classical division is to be interpreted as superposition versus product state. There are other divisions with respect to entanglement or more generally the type of correlations [34] which, although they have their root in the superposition property, are defined and characterized in a different way. Here we confine ourselves to this specific meaning mentioned above. Based on this concept, the $p-$ index of a pure state $|\psi\rangle$ is defined as [14]:

$$\max_{A \in \mathcal{A}} \mathcal{V}_\psi(A) = \mathcal{O}(N^p), \quad N \text{ large}, \qquad (2)$$

where $\mathcal{A}$ is the set of all additive operators $A = \sum_{i=1}^{N} A_i$ such that every operator A_i acts nontrivially on the ith particle and $||A_i|| = 1$. A fully product state has $p = 1$. This means that $p > 1$ is an entanglement witness for pure states. The state with $p = 2$ contains superposition of macroscopically distinct states, because in this case a Hermitian additive operator has a "macroscopically large" fluctuation in the sense that the relative fluctuation does not vanish in the thermodynamic limit.

For such pure states, the fluctuation of an observable means the existence of a superposition of eigenvectors of that observable corresponding to different eigenvalues with the largest difference. An example is the GHZ state which is the superposition of two states which are eigenvectors of the additive operator $\sum_{i=1}^{N} \sigma_z$ corresponding to N and $-N$.

As another measure of macroscopicity, Fröwis and Dür defined the quantum fisher information [19] which detects a certain kind of correlation. It is well known that a separable state has a Fisher information which scales at most linearly with the system size N for every local operator like A [35], $\mathcal{F}(\rho_{sep},A) \leqslant 4N$. (For the definition of Fisher information and its properties see Ref. [36].) On the other hand, for GHZ, $\mathcal{F}(GHZ,A) = 4N^2$. The authors of Ref. [19] introduce the concept of an "effective size" N_{eff}. For a general state ρ of N particles, this measure is defined as

$$N_{\text{eff}}(\rho) := \max_{A \in \mathcal{A}} \mathcal{F}(\rho,A)/(4N). \qquad (3)$$

In other words, N_{eff} defines the scale over which macroscopic superposition and hence quantum behavior prevails. If $N_{\text{eff}}(\rho) = \mathcal{O}(N)$, we have macroscopic quantum behavior while if $N_{\text{eff}}(\rho) = \mathcal{O}(1)$, then quantum behavior, if existing at all, exists at the microscopic level. For pure states $\rho = |\psi\rangle\langle\psi|$, the Fisher information reduces to four times the variance $\mathcal{V}_\psi(A) = \langle\psi|A^2|\psi\rangle - \langle\psi|A|\psi\rangle^2$, and the effective size takes the form

$$N_{\text{eff}}(\psi) = \max_{A \in \mathcal{A}} \mathcal{V}_\psi(A)/N. \qquad (4)$$

In the sequel we will use these two measures to quantify the macroscopic superposition of the ground state of the Ising model in transverse field, when it undergoes a quantum phase transition.

Macroscopic superposition and its scaling behavior. Let us denote the ground state of the transverse Ising model (1), by $|\psi\rangle$, which simplifies the more detailed notation $|\psi_\lambda(N)\rangle$. Let A be of the form $A_\mathbf{n} = \sum_{i=1}^N \boldsymbol{\sigma}_i \cdot \mathbf{n}$, where $\mathbf{n} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$. In order to use measure of macroscopicity (4), we have to first determine the direction $\mathbf{n}$ for which maximum variance is obtained. Using the definition of variance we find

$$
\begin{aligned}
\mathcal{V}_\psi(A_\mathbf{n}) = {} & \sin^2\theta[\cos^2\phi\,\langle X^2\rangle + \sin^2\phi\,\langle Y^2\rangle] + \cos^2\theta\langle Z^2\rangle \\
& + \tfrac{1}{2}\sin 2\theta[\cos\phi\langle ZX + XZ\rangle + \sin\phi\langle ZY + YZ\rangle] \\
& + \tfrac{1}{2}\sin^2\theta\sin 2\phi\,\langle XY + YX\rangle \\
& - \langle X\sin\theta\cos\phi + Y\sin\theta\sin\phi + Z\cos\theta\rangle^2, \quad (5)
\end{aligned}
$$

where $X = \sum_{i=1}^N \sigma_i^x$ with similar definitions for Y and Z. The U symmetry [by which $(X, Y, Z) \longrightarrow (-X, -Y, Z)$], implies that $\langle Y\rangle = \langle X\rangle = \langle XZ + ZX\rangle = \langle YZ + ZY\rangle = 0$. As for $\langle XY + YX\rangle$, it vanishes due to the reality of the Hamiltonian and Hermiticity of the operator $XY + YX$ and the fact that $Y^* = -Y$. Therefore the variance (5) is reduced to

$$
\begin{aligned}
\mathcal{V}_\psi(A_\mathbf{n}) = {} & \sin^2\theta[\langle X^2\rangle\,\cos^2\phi + \langle Y^2\rangle\sin^2\phi] \quad (6) \\
& + \cos^2\theta[\langle Z^2\rangle - \langle Z\rangle^2].
\end{aligned}
$$

To find the direction $\mathbf{n}$ for which this variance is maximized, we note that since $\mathbf{n}$ is defined on the compact surface of a 2-sphere, the maximum will be a local one which is determined by examining the first and second derivatives of $\mathcal{V}_\psi(A_\mathbf{n})$ as a function of θ and ϕ. For simplicity let us denote this quantity simply by $\mathcal{V}$. It is then found that

$$
\begin{aligned}
\frac{\partial\mathcal{V}}{\partial\theta} &= \sin 2\theta[\cos^2\phi\langle X^2\rangle + \sin^2\phi\langle Y^2\rangle - \langle Z^2\rangle + \langle Z\rangle^2], \\
\frac{\partial\mathcal{V}}{\partial\phi} &= \sin^2\theta\,\sin 2\phi[\langle Y^2\rangle - \langle X^2\rangle]. \quad (7)
\end{aligned}
$$

The extrema are obtained by setting these two derivatives equal to zero which yield three solutions ($\theta = 0$, ϕ = irrelevant) or $\mathbf{n} = \mathbf{z}$, ($\theta = \frac{\pi}{2}, \phi = 0$) or $\mathbf{n} = \mathbf{x}$ and ($\theta = \frac{\pi}{2}, \phi = \frac{\pi}{2}$) or $\mathbf{n} = \mathbf{y}$. Examining the Hessian matrix (of second derivatives) shows that $\mathbf{n} = \mathbf{x}$ is indeed the point of maximum. Note that while the model has a U symmetry, it lacks rotational symmetry and hence in general $\langle X^2\rangle \neq \langle Y^2\rangle$. In other words, the U operator cannot change an operator's X into Y or vice versa. Therefore the observable which detects macroscopic superposition is $A_\mathbf{x} = \sum_{i=1}^N \boldsymbol{\sigma}_i \cdot \mathbf{x}$, which from (4) gives the effective size of superpositions as

$$
N_{\text{eff}} \equiv \frac{\mathcal{V}_\psi(A_\mathbf{x})}{N} = \frac{\langle X^2\rangle}{N} = \sum_{n=1}^N \langle\sigma_1^x\sigma_n^x\rangle, \quad (8)
$$

where in the last equality we have used translational invariance of the system. Therefore determination of the macroscopic measure of superposition is reduced to calculation of the two point functions $G^{xx}(n) := \langle\sigma_1^x\sigma_{1+n}^x\rangle$. Note that while the model is a free fermion, determination of the two-point spin functions is quite nontrivial due to the nonlocal nature of the Jordan-Wigner transformation. These two-point functions have been determined in Refs. [32] and [33]. They are

given by

$$
G^{xx}(n) := \begin{vmatrix}
G_{-1} & G_{-2} & \ldots & G_{-n} \\
G_0 & G_{-1} & \ldots & G_{-n+1} \\
. & . & & . \\
. & . & & . \\
. & . & & . \\
G_{n-2} & G_{n-1} & \ldots & G_{-1}
\end{vmatrix},
$$

with $G_n = L_n + \lambda L_{n+1}$, and $L_n = \frac{2}{N}\sum_{k>0}\lambda_k^{-1}\cos(kn)$, where $\Lambda_k = \sqrt{1 + \lambda^2 + 2\lambda\cos(k)}$, $m = 0, 1, \ldots, \frac{1}{2}(N-1)$.

We now want to use (8) and see how this effective size changes as we change the coupling λ. In the two limiting case, the behavior is simple and expected. When $\lambda = 0$, the ground state is $|\psi\rangle_{\lambda=0} = |00\cdots0\rangle$, for which $\langle\sigma_1^x\sigma_n^x\rangle_{n\neq1} = 0$. Therefore we obviously have $N_{\text{eff}} = \langle\sigma_1^x\sigma_1^x\rangle = 1$ and hence $N_{\text{eff}} = 1$, implying no superposition. On the other hand in the limit of very large λ, the first term of the Hamiltonian (1) dominates and the ground state goes to $|\psi\rangle_{\lambda\to\infty} \approx \frac{1}{\sqrt{2}}(|+ + \cdots +\rangle + |- - \cdots -\rangle)$, where it is expected that $N_{\text{eff}} \longrightarrow N$. This is indeed the case as a simple calculation from (8) shows.

Remark. Note that in the limit $\lambda \longrightarrow \infty$, as long as N is finite, no symmetry breaking happens and so the ground state in the limit does not break the U symmetry. So starting at $\lambda = 0$ with the state $|00\cdots0\rangle$ which is an eigenstate of U with eigenvalue 1, when we continuously change λ, we always remain in the same eigenspace and hence in the limit $\lambda \longrightarrow \infty$, we have the superposition $|\psi\rangle_{\lambda\to\infty} \approx \frac{1}{\sqrt{2}}(|+ + \cdots +\rangle + |- - \cdots -\rangle)$ and not one of its individual components. It is only in the thermodynamic limit that the symmetry breaks down and only one of the two states is chosen and hence the macroscopic measure again gives a zero value as $\lambda = 0$.

In the absence of symmetry breaking, which only happens for an infinite system, we have to use (8) and do a finite size scaling. The results are shown in Fig. 1. It is clear from this figure that $\frac{N_{\text{eff}}}{N}$ sharply goes from 0 to 1 at a point $\lambda_m(N)$. As N approaches ∞, this transition becomes discontinuous. The inset of Fig. 1 expresses this in an alternative way by showing
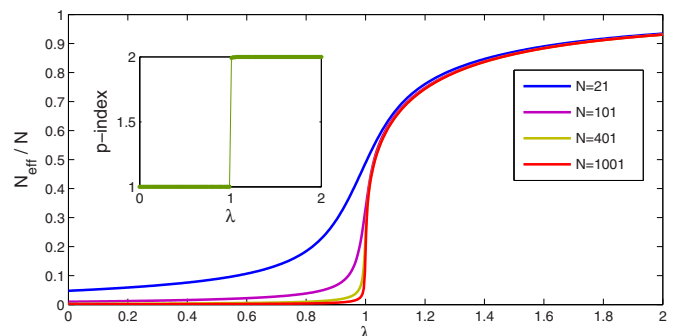

FIG. 1. The macroscopic superposition as measured by effective size $N_{\text{eff}} := \frac{\langle X^2\rangle}{N}$ in the transverse Ising model as a function of the coupling λ. The curves from top to bottom, correspond to $N = 21, 101, 401$ and $N = 1001$. The inset shows the p index as a function of λ. We considered different system sizes, $1000 < N < 2000$, for which a sharp transition in the p index could be detected. It is seen that both N_{eff} and p change discontinuously near the point $\lambda_c = 1$.
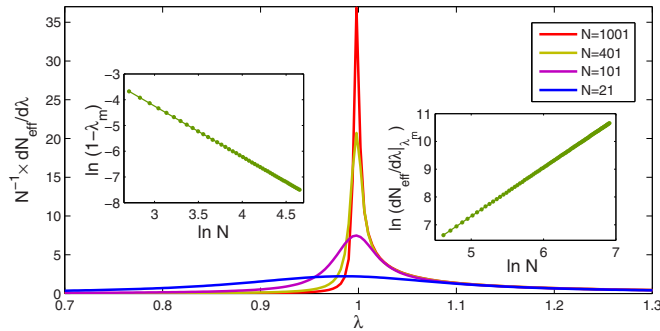
FIG. 2. The derivative of macroscopicity as measured by $dN_{\text{eff}}/d\lambda$, obtains its maximum value at point $\lambda_m(N)$. The left inset shows that this point approaches the actual critical point $\lambda_c := \lambda_m(\infty) = 1$ as in (9) and the right inset shows that the maximum value itself diverges with N as in (10).

the p index as a function of λ where again a sharp transition is found at $\lambda_m(N)$. Therefore the quantum phase transition is concomitant with a divergence of the derivative of N_{eff} which sets the scale of macroscopic superpositions in the system, that is, near a critical point, quantum superposition of the two degenerate states entails the whole system. Obviously this divergence is not seen directly for finite N. Instead what we see is that $\frac{dN_{\text{eff}}}{d\lambda}$ is a rapidly increasing function of N. Let us see the position of the maximum $\lambda_m(N)$ and the maximum value of the derivative behave with N. Figure 2 shows that by increasing N toward the thermodynamic limit, the position of this transition point approaches the actual critical value $\lambda_c := \lambda_m(\infty) = 1$. This approach toward the actual critical point is governed by a power law in the form

$$1 - \lambda_m(N) \sim N^{-1.96}. \quad (9)$$

We note in passing that the same scaling behavior, albeit with different powers, has been reported in other works. In particular in Ref. [21] where the nearest neighbor concurrence or entanglement is considered the relation is $1 - \lambda_m(N) \sim N^{-1.87}$ and in Ref. [27] where negativity of three consecutive particles is considered the relation is like $1 - \lambda_m(N) \sim N^{-2.19}$. Finally in Ref. [28] where the maximum of geometric phase of the ground state is considered the approach toward the actual critical point is like $1 - \lambda_m(N) \sim N^{-1.803}$.

Furthermore, the maximum value of the derivative at $\lambda_m(N)$ diverges with N as follows

$$\frac{dN_{\text{eff}}}{d\lambda}(\lambda_m, N) \sim N^{1.75}, \quad (10)$$

leading to a divergent behavior in the thermodynamic limit ($N \longrightarrow \infty$). Finally the most important exponent, namely $\nu = 1$ which puts the model in the universality class of the Ising model is obtained when the data for the function $\frac{dN_{\text{eff}}}{d\lambda}(\lambda, N)$ collapse to a single curve by choosing a suitable scaling function. Figure (3) shows such a scaling function. It is clearly seen that

$$\frac{dN_{\text{eff}}}{d\lambda}(\lambda, N) - \frac{dN_{\text{eff}}}{d\lambda}(\lambda_m, N) = N^{1.89} Q(N(\lambda - \lambda_m)), \quad (11)$$

where Q is a universal function derived numerically and shown in Fig. 3. When this is compared with the scaling
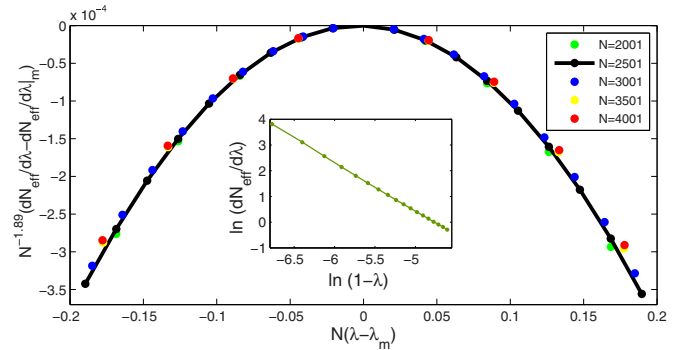
FIG. 3. The collapse of the derivative of macroscopicity on a universal function of $N(\lambda - \lambda_m)$, equation (11), which shows that $\hat{\mathbf{n}} = 1$. The inset shows the asymptotic behavior of $dN_{\text{eff}}/d\lambda$ in the vicinity of the quantum criticality as in equation (12), the result is for $N = 4001$.

function with variable $N^{1/\nu}(\lambda - \lambda_m)$ [37], it leads to $\nu = 1$, in agreement with the well known results for the Ising model in transverse field [32,33,38]. On the other hand, we can find the asymptotic behavior of $\frac{dN_{\text{eff}}^*}{d\lambda}(\lambda) := \frac{dN_{\text{eff}}}{d\lambda}(\lambda, N \to \infty)$ in terms of the coupling $\lambda_c - \lambda = 1 - \lambda$. For this we have to take very large values of N to mimic the thermodynamic limit $N \longrightarrow \infty$. For $N = 4001$, the result is

$$\frac{dN_{\text{eff}}^*}{d\lambda}(\lambda) \sim (1 - \lambda)^{-1.89}, \quad (12)$$

as we see the exponent 1.89 is fully consistent with the equation (11).

Note that in the light of (11), equation (10) is consistent with a recent result of Ref. [39] on measuring multipartite entanglement by dynamical susceptibilities, where it is reported that $f_Q := \frac{F_Q}{N} \sim N^{0.75}$. Here F_Q is the Fisher information which for pure states reduces to four times the variance. The consistency comes about by noting that $f_Q = 4N_{\text{eff}}$ and a relation like $f_Q \sim N^{0.75} h((\lambda - \lambda_m)N)$, leads to the relation $\frac{df_Q}{d\lambda} \equiv \frac{dN_{\text{eff}}}{d\lambda} \sim N^{1.75} h'((\lambda - \lambda_m)N)$, where h' is the derivative of h.

Finally it is instructive to study a hypothetical transition between the ground state at $\lambda = 0$ ($|0\rangle^{\otimes N}$) and the ground state at $\lambda \longrightarrow \infty$ ($\frac{1}{\sqrt{2}}[|+\rangle^{\otimes N} + |-\rangle^{\otimes N}]$), in the form of

$$|\psi_n\rangle = \frac{1}{\sqrt{2}}[|+\rangle^{\otimes n} + |-\rangle^{\otimes n}] \otimes |0\rangle^{N-n}, \quad (13)$$

where a domain wall has been created at site n. The state is unzipped at point n and as n grows, the hypothetical state changes from the ground state at $\lambda = 0$ to the ground state at $\lambda \longrightarrow \infty$. It is a simple calculation to calculate N_{eff} for this state. It turns out to be

$$N_{\text{eff}} = \frac{n(n-1)}{N} + 1, \quad (14)$$

which shows that the effective size raises from 1 to N in a smooth way.

In summary, by studying a paradigmatic example of quantum phase transitions and using the recently defined measures of macroscopic superpositions we have shown that the degree of macroscopic superposition diverges near the

point of quantum phase transitions. Physically this means that quantum phase transitions and the ensuing symmetry breaking does not happen on a microscopic scale which then would propagate through the whole system, but the entire system goes into a macroscopic superposition of symmetry breaking states. This change happens in a very sharp way which becomes discontinuous in the thermodynamic limit. These findings verify, by different tools, the scenario suggested in Ref. [40] according to which, 'a topological defect can be put in a nonlocal superposition, where the order parameter of the system is "undecided" by being in a quantum super-

position of conflicting choices of the broken symmetry.' The final stage of quantum phase transition is achieved by the rapid environment-induced decoherence ($\tau_{dec} \sim \frac{1}{N}$) of this macroscopic superposition to a statistical mixture of the two symmetry breaking phases. Such statistical mixtures also show a nonanalytical behavior, measured by the quantum Fisher information, but they do not show diverging behavior [41]. These findings on macroscopic superposition, in conjunction with the result of Ref. [21] on short-ranged entanglement, may be an example of the interplay between the two properties, recently mentioned in Ref. [11].

[1] M. Arndt, O. Nairz, J. Vos-Andreae, C. Keller, G. Van der Zouw, A. Zeilinger, Nature (London) **401**, 680 (1991).

[2] B. Brezger, L. Hackermüller, S. Uttenthaler, J. Petschinka, M. Arndt, and A. Zeilinger, Phys. Rev. Lett. **88**, 100404 (2002).

[3] E. Schrödinger, Die Naturwissenschaften **23**, 807 (1935).

[4] G. C. Ghirardi, A. Rimini, and T. Weber, Phys. Rev. D **34**, 470 (1986).

[5] A. Bassi, K. Lochan, S. Satin, T. P. Singh, and H. Ulbricht, Rev. Mod. Phys. **85**, 471 (2013).

[6] J. P. Paz and W. H. Zurek, Proceedings Les Houches **111A**, 409 (2000).

[7] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. **106**, 162 (1957).

[8] A. J. Leggett, Prog. Theor. Phys. Suppl. **69**, 80 (1980).

[9] Jian-Wei Pan, D. Bouwmeester, M. Daniell, H. Weinfurter, and A. Zeilinger, Nature (London) **403**, 515 (2000).

[10] J. I. Korsbakken, K. B. Whaley, J. Dubois, and J. I. Cirac, Phys. Rev. A **75**, 042106 (2007).

[11] M. C. Tichy, Chae-Yeun Park, M. Kang, H. Jeong, and K. Mølmer, Phys. Rev. A **93**, 042314 (2007).

[12] H. Ollivier and W. H. Zurek, Phys. Rev. Lett. **88**, 017901 (2001).

[13] W. Dür, C. Simon, and J. I. Cirac, Phys. Rev. Lett. **89**, 210402 (2002).

[14] A. Shimizu and T. Miyadera, Phys. Rev. Lett. **89**, 270403 (2002).

[15] G. Björk and P. G. L. Mana, J. Opt. B **6**, 429 (2004).

[16] A. Shimizu and T. Morimae, Phys. Rev. Lett. **95**, 090401 (2005).

[17] F. Marquardt, B. Abel, and J. von Delft, Phys. Rev. A **78**, 012109 (2008).

[18] C. W. Lee and H. Jeong, Phys. Rev. Lett. **106**, 220401 (2011).

[19] F. Fröwis and W. Dür, New J. Phys. **14**, 093039 (2012).

[20] B. Yadin and V. Vedral, Phys. Rev. A **92**, 022356 (2015).

[21] A. Osterloh, L. Amico, G. Falci, and R. Fazio, Nature (London) **416**, 608 (2005).

[22] J. Mazieroa, L. C. Céleria, R. M. Serraa, and M. S. Sarandy, Phys. Lett. A **376**, 1540 (2012).

[23] L.-A. Wu, M. S. Sarandy, and D. A. Lidar, Phys. Rev. Lett. **93**, 250404 (2004).

[24] L.-A. Wu, M. S. Sarandy, D. A. Lidar, and L. J. Sham, Phys. Rev. A **74**, 052335 (2006).

[25] T. J. Osborne and M. A. Nielsen, Phys. Rev. A **66**, 032110 (2002).

[26] G. Vidal, J. I. Latorre, E. Rico, and A. Kitaev, Phys. Rev. Lett. **90**, 227902 (2003).

[27] M. Hofmann, A. Osterloh, and O. Gühne, Phys. Rev. B **89**, 134101 (2014).

[28] Shi-Liang Zhu, Phys. Rev. Lett. **96**, 077206 (2006).

[29] M. Cozzini, R. Ionicioiu, and P. Zanardi, Phys. Rev. B **76**, 104420 (2007).

[30] S. Alipour, V. Karimipour, and L. Memarzadeh, Phys. Rev. A **75**, 052322 (2007).

[31] S. Sachdev, in *Quantum Phase Transitions* (Cambridge University Press, Cambridge, 1999).

[32] E. Lieb, T. Schultz, and D. Mattis, Ann. Phys. (N.Y.) **16**, 407 (1961).

[33] P. Pfeuty, Ann. Phys. (N.Y.) **57**, 79 (1970).

[34] N. Li and S. Luo, Phys. Rev. A **78**, 024303 (2008).

[35] L. Pezzé and A. Smerzi, Phys. Rev. Lett. **102**, 100401 (2009).

[36] M. G. A. Paris, Int. J. Quantum Inf. **7**, 125 (2009).

[37] M. N. Barber, in *Phase Transitions and Critical Phenomena* Vol. 8, edited by C. Domb and J. L. Leibovitz (Academic, London, 1983), pp. 146–259.

[38] E. Barouch and B. McCoy, Phys. Rev. A **3**, 786 (1971).

[39] Ph. Hauke, M. Heyl, L. Tagliacozzo, and P. Zoller, Nature Physics (2016).

[40] J. Dziarmaga, W. H. Zurek, and M. Zwolak, Nature (London) **8**, 49 (2012).

[41] T. L. Wang, L. N. Wu, W. Yang, G. R. Jin, N. Lambert, and F. Nori, New J. Phys. **16**, 063039 (2014).