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Acoustic wave absorption as a probe of dynamical geometrical response of fractional quantum Hall liquids

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We show that an acoustic crystalline wave gives rise to an effect similar to that of a gravitational wave to an electron gas. Applying this idea to a two-dimensional electron gas in the fractional quantum Hall regime, this allows for experimental study of its intra-Landau level dynamical response in the long-wavelength limit. To study such response we generalize Haldane's geometrical description of fractional quantum Hall states to situations where the external metric is time dependent. We show that such time-dependent metric (generated by acoustic wave) couples to collective modes of the system, including a quadrapolar mode at long wavelength, and magnetoroton at finite wavelength. Energies of these modes can be revealed in spectroscopic measurements, controlled by strain-induced Fermi velocity anisotropy. We argue that such geometrical probe provides a potentially highly useful alternative probe of quantum Hall liquids, in addition to the usual electromagnetic response.

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Introduction. Fractional quantum Hall (FQH) liquid is the prototype topological state of matter. Haldane [1] pointed out recently that the description of FQH liquids in terms of topological quantum field theories, while capturing the universal and topological aspect of the physics, is incomplete in the sense that an internal geometrical degree of freedom responsible for the intra-Landau level dynamics of the system is not included. This geometrical degree of freedom, or internal metric, couples to anisotropy in the interaction between electrons [2–4] or the electron band structure [5], and its expectation value is determined by energetics of the system. In a recent work [6] we showed that this internal metric parameter manifests itself as the anisotropy of a composite fermion Fermi surface, which is measurable. Our quantitative result compares favorably with recent experiments, in which electron mass anisotropy is induced and controlled by an in-plane magnetic field [7–9]. This demonstrates the observability of this internal geometry. It has also been argued [1,10–12] that this internal metric may be viewed as a *dynamical* degree of freedom, whose long-wavelength dynamics corresponds to the collective excitations of the system that can be viewed as "gravitons" [13]. In a parallel stream of works, much effort has been devoted to studying FQH liquids in a curved background space [14–23], following earlier seminal work by Wen and Zee [24].

In the existing theoretical studies [2–6,25,26], the background geometry (or metric) provided by electron-electron interaction and/or band structure is static. The main purpose of the present work is to generalize this to the case where the background metric is time dependent, and show that the dynamics of the metric couples to the intra-Landau level collective modes of the FQH liquid. In particular, we show that such time-dependent metric can be generated by acoustic waves, which play a role very similar to the gravitational wave in this context. Such gravitational wave naturally couples to graviton and other collective modes of the system. This allows for spectroscopic measurements of collective mode energies, in particular graviton energy, using acoustic wave absorption.

Existing work on this subject [2–6,25,26] has thus far focused on nonrelativistic electrons. On the other hand, graphene has emerged as a new arena to study quantum Hall physics [27]. A second purpose of the present work is to show that much of the considerations in the present and earlier works carry over to Dirac electrons and thus graphene straightforwardly, once we identify the anisotropy of Fermi velocity with the external metric. We thus start our discussion below with a description of how the Fermi velocity anisotropy of Dirac electrons translate into a background metric for the FQH states they form.

External metric of Dirac and Schrodinger electrons. Consider the Hamiltonian

$$H = T + V, \tag{1}$$

with the kinetic energy for massless Dirac electrons in a magnetic field taking the form

$$T = \sum_j v^{\mu\nu}\sigma_\mu \Pi_\nu^j, \tag{2}$$

where j is electron index, $\sigma_{\mu=1,2}$ are the Pauli matrices, and $v^{\mu\nu}$ is the (real) Fermi velocity matrix. Repeated Greek indices are summed over.

$$\Pi = \mathbf{p} + \frac{e}{c}\mathbf{A}(\mathbf{r}) \tag{3}$$

is the mechanical momentum, $\nabla \times \mathbf{A}(\mathbf{r}) = -B\hat{z}$, thus the electrons move in a uniform perpendicular magnetic field. The two components of Π satisfy the commutation relation

$$[\Pi_x, \Pi_y] = -\frac{i\hbar e}{c}(\partial_x A_y - \partial_y A_x) = \frac{i\hbar e B}{c} = \frac{i\hbar^2}{\ell^2}, \tag{4}$$

where $\ell = \sqrt{\hbar c/(eB)}$ is the magnetic length.

The easiest way to obtain the Landau level energies and corresponding wave functions is to square the kinetic energy of a single electron:

$$[v^{\mu\nu}\sigma_\mu \Pi_\nu]^2 = (vv^T)^{\alpha\beta}\Pi_\alpha\Pi_\beta - \frac{\hbar^2}{\ell^2}(v^{11}v^{22} - v^{12}v^{21})\sigma_z, \tag{5}$$

from which it is clear that the zero energy Landau level (0LL, which will be the focus of the rest of this Rapid Communication) wave functions only have weight in one of the two components, and the symmetric matrix vv^T plays a

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role identical to the inverse effective mass matrix for quadratic bands:

$$T = \frac{1}{2}(m^{-1})^{\mu\nu}\Pi_\mu \Pi_\nu = \frac{g^{\mu\nu}\Pi_\mu \Pi_\nu}{2m_0}, \tag{6}$$

where m^{-1} is the inverse effective mass tensor, $1/m_0$ is the geometric mean of the eigenvalues of m^{-1}, and the (space-only) metric tensor g is defined by the second equality above, which is symmetric and unimodular.

For massless Dirac electrons, we may therefore diagonalize this matrix vv^T to obtain its eigenvalues av_F^2 and v_F^2/a, with $\sqrt{a}v_F$ and $v_F^2/\sqrt{a}$ the Fermi velocities along the two principle directions, defined to be the x and y directions hereafter, and v_F is their geometric mean. $|a-1|$ is a measure of the anisotropy. It is known [28] that strain and ripple modifies the v matrix in graphene, and thus v^F and in particular the anisotropy a, which plays a role very similar to the effective mass anisotropy parameter in a quadratic band (here the notation is the same as that of Ref. [6]). In ordinary semiconductors we expect strain of lattice also induces or modifies effective mass anisotropy. We thus discuss the massless Dirac and (massive) Schrodinger electrons on equal footing in the remainder of the Rapid Communication. In the notation of Eq. (6), a strain induces a change of metric and thus geometry (of spaces), and a time-dependent strain plays a role similar to a gravitational wave, which can excite the "gravitons" of the FQH systems as we will see below.

Geometrical coupling of intra-Landau level dynamics and external metric. The intra-Landau level degrees of freedom are described by guiding center coordinates

$$\mathbf{R} = \mathbf{r} - (\ell^2/\hbar)\hat{z} \times \Pi, \tag{7}$$

which commute with Π. The interaction term

$$V = \sum_{i<j} V(\mathbf{r}_i - \mathbf{r}_j) = \frac{1}{2}\sum_{\mathbf{q}} V_{\mathbf{q}}\rho_{\mathbf{q}}\rho_{-\mathbf{q}}, \tag{8}$$

where $V_{\mathbf{q}}$ is the Fourier transform of electron-electron interaction potential $V(r)$ (assumed to be isotropic) and

$$\rho_{\mathbf{q}} = \sum_i e^{i\mathbf{q}\cdot\mathbf{r}_i} \tag{9}$$

is the density operator. In the large B limit, Landau level spacing overwhelms V, and the electron motion is confined to a given Landau level. In this case it is appropriate to project V onto 0LL that results in a reduced Hamiltonian involving the $\mathbf{R}$'s only [6]:

$$\tilde{V} = \frac{1}{2}\sum_{\mathbf{q}} V_{\mathbf{q}} e^{-(1/2)(aq_x^2+q_y^2/a)\ell^2}\bar{\rho}_{\mathbf{q}}\bar{\rho}_{-\mathbf{q}}, \tag{10}$$

where

$$\bar{\rho}_{\mathbf{q}} = \sum_i e^{i\mathbf{q}\cdot\mathbf{R}_i} \tag{11}$$

is the guiding center density operator, and we choose $\hat{x}$ and $\hat{y}$ directions to be the diagonal directions of m^{-1} or vv^T, with anisotropy possibly induced by lattice distortion. We note the *only* place that the background geometric parameter a enters $\tilde{V}$ is in the Gaussian form factor of 0LL. Once confined to the 0LL the difference between Dirac and Schrodinger electrons disappears, and our discussions below apply to both.

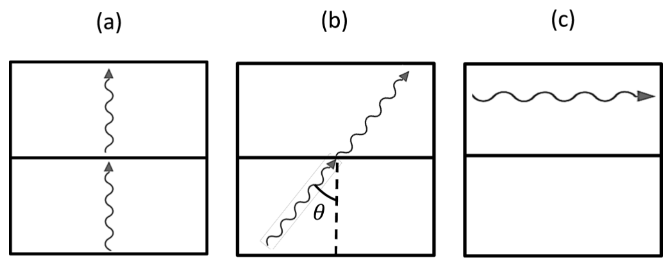

FIG. 1. Illustration of experimental setup. Square boxes represent 3D crystals, and thick solid lines represent 2D electron gas. Wavy lines represent acoustic waves whose effects are very similar to gravitational waves. (a) Bulk acoustic wave propagating perpendicular to the 2D layer, inducing a uniform strain seen by the 2D electron. (b) Bulk acoustic wave propagating at an angle θ to the normal direction of the 2D layer, inducing a nonuniform strain seen by the 2D electron. (c) Surface acoustic wave propagating parallel to the 2D layer.

Electron dispersion in GaAs and graphene is isotropic under ambient condition and thus $a = 1$. Now let us start by considering a particularly simple case, namely, a small *uniform* anisotropy induced by either strain (in either GaAs or graphene) or ripple (in graphene), which is possibly time dependent:

$$a = 1 + \xi(t). \tag{12}$$

This corresponds to space distortion induced by a long-wavelength "gravitational wave," in the gravity analogy. Physically it can be induced by a lattice wave that is either of long wavelength, or with wave vector perpendicular to the two-dimensional electron gas (2DEG) plane so that the electrons see a uniform lattice distortion (see Fig. 1, and more on this later).

We assume the frequency of $\xi(t)$ is low compared to Landau level spacing, thus no inter-Landau level transition is induced. Then the main physical effect of this time-dependent geometry comes from its coupling to intra-Landau level dynamics of the electrons. This results in a time-dependent perturbation in the intra-Landau level Hamiltonian:

$$\delta\tilde{V}(t) = \frac{\xi(t)}{4}\sum_{\mathbf{q}}\left(q_y^2 - q_x^2\right)V_{\mathbf{q}}e^{-(1/2)q^2\ell^2}\bar{\rho}_{\mathbf{q}}\bar{\rho}_{-\mathbf{q}}. \tag{13}$$

The 2DEG (assumed to be in its ground state) will absorb energy from the "gravitational wave," with a rate determined by the spectral function

$$I(\omega) = \sum_n |\langle n|\hat{O}|0\rangle|^2\delta(\omega - \omega_n), \tag{14}$$

where $|0\rangle$ is the ground state, $|n\rangle$ is an excited state with excitation energy $\hbar\omega_n$, and

$$\hat{O} = \sum_{\mathbf{q}}\left(q_y^2 - q_x^2\right)V_{\mathbf{q}}e^{-(1/2)q^2\ell^2}\bar{\rho}_{\mathbf{q}}\bar{\rho}_{-\mathbf{q}} \tag{15}$$

describes the coupling between 2DEG in a FQH state to the lattice distortion/geometry. It is interesting to note that the $\mathbf{q}$ dependence of the term being summed over above takes a d-wave form, indicating $\hat{O}$ carries angular momentum $L = 2$.

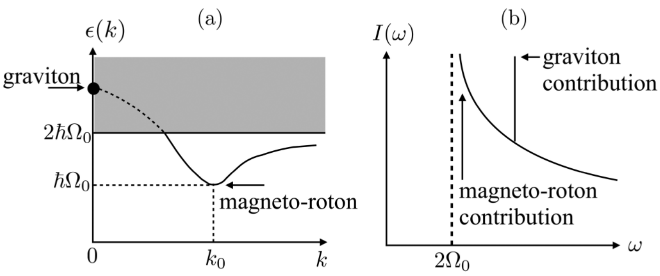

FIG. 2. (a) Illustration of excitation spectrum of Laughlin-type fractional quantum Hall states. The solid line represents the magnetoroton mode, and the shaded region represents the two-roton continuum. The magnetoroton mode continues into the two-roton continuum with decreasing wave vector k (now represented as a dashed line, all the way to $k = 0$, ending at the black dot that is the graviton mode which is the main focus of this Rapid Communication. (b) Spectral function of $\hat{O}$ defined in Eq. (14), revealing the presence of both the graviton and magnetoroton modes.

It will thus couple to excitations with $L = 2$, which is the case for gravitons.

Excitation spectrum of Laughlin-type FQH states is illustrated schematically in Fig. 2(a). The lowest-energy elementary excitations are magnetorotons (referred to as roton from now on) whose dispersion takes the form

$$\Omega(k) = \Omega_0 + A(k - k_0)^2, \tag{16}$$

where Ω_0 is the (minimum) roton frequency (or roton gap), k_0 is the momentum of roton minimum, and A is a constant. To a very good approximation [29] a roton with momentum $\mathbf{k}$ is created by $\bar{\rho}_{\mathbf{k}}$ for $k = |\mathbf{k}| \approx k_0$, and exhausts its spectral weight. We thus find among other excitations, $\hat{O}$ creates a *pair* of rotons with total momentum zero, and the roton pair contribution to the spectral function $I(\omega)$ takes the form

$$I_{\text{roton}}(\omega) \propto \sum_{|\mathbf{q}|\approx k_0} \left(q_y^2 - q_x^2\right)^2 |V_{\mathbf{q}}|^2 e^{-q^2\ell^2} \langle 0|\bar{\rho}_{\mathbf{q}}\bar{\rho}_{-\mathbf{q}}|0\rangle \times \delta[\omega - 2\Omega(q)]. \tag{17}$$

It is easily seen that $I_{\text{roton}}(\omega)$ has a threshold frequency at $2\Omega_0$, and diverges for $\omega \to 2\Omega_0 + 0^+$:

$$I_{\text{roton}}(\omega) \propto \int dq\, \delta[\omega - 2\Omega_0 - 2A(q - q_0)^2] \propto \frac{1}{\sqrt{\omega - 2\Omega_0}}. \tag{18}$$

Thus the roton gap Ω_0 is clearly visible in $I(\omega)$ [see Fig. 2(b)]. This provides an alternative method of measuring the magnetoroton gap, in addition to earlier attempts using optical methods [30,31].

What is more interesting, however, is the long-wavelength mode with $k \to 0$, which is the graviton mode that is of primary interest in this Rapid Communication [32]. It is known [12,29,33] that $\langle 0|\rho_{\mathbf{k}}\rho_{-\mathbf{k}}|0\rangle \propto (k\ell)^4$ as $k \to 0$, thus it is very difficult to probe the collective mode in this regime using electromagnetic/optical probes that couple to electron density. In particular, the mode with $\mathbf{k} = 0$ simply has no coupling to the ground state through the density operator, as the cyclotron mode exhausts the spectral weight of the latter (Kohn's theorem). Another way to understand this is that the graviton has spin-2, and cannot be excited by spin-1 photons. On the other hand, the perturbation induced by an acoustic/gravitational wave in Eq. (13) is indeed an angular momentum-two operator, and can excite the graviton mode (it is only natural that gravitons are excited by gravitational waves). We thus expect the graviton shows up as a *sharp* resonance in the spectral function (14), allowing its energy to be measured spectroscopically [see Fig. 2(b)]. Ongoing numerical calculation of $I(\omega)$ defined in Eq. (14) indeed finds a pronounced peak corresponding to the graviton, which will be presented elsewhere [34].

Acoustic wave as gravitational wave, and its other effects. The experimental setup is schematically illustrated in Fig. 1. The acoustic waves propagate either inside the three-dimensional (3D) bulk crystal or along its surface, and interact with the mobile electrons that live in a 2D layer through the lattice distortion or strain they induce. There are several mechanisms for this (electron-phonon) interaction. As discussed above, the strain-induced change of electron effective mass tensor corresponds to a geometric or gravitational interaction, which is the main focus of the present work. The specific geometric perturbation considered above corresponds to the setup of Fig. 1(a), that induces a *uniform* (but oscillating) strain in the 2D plane [35]. There are, however, two other more familiar sources of coupling between strain and electrons which are likely to be more important under generic situations [36]: (i) The strain induces a deformation potential that couples directly to the density of electrons. (ii) For noncentrosymmetric crystals, strain induces an electric polarization and corresponding electric field due to the piezoelectric effect. We argue that the effects (i) and (ii) may be eliminated by using the setup of Fig. 1(a). In this case since the strain is *uniform* in the 2D plane, the deformation potential is also uniform and thus has no effect. Similarly the piezoelectric effect induces a *uniform* electric field, which couples to the *center of mass* of the electron gas. Kohn's theorem guarantees that the dipole coupling of the electron gas to such uniform electric field can only cause inter-Landau level transition, and no absorption will occur through such coupling as long as $\omega < \omega_c$ (cyclotron frequency).

We now briefly consider the more generic case in which the lattice distortion is a function of both time and space, with the latter dictated by a 2D wave vector $\mathbf{k}_0$. Such a distortion is induced by a 3D lattice wave propagating with wave vector $\mathbf{K}_0$, whose 2D projection is $\mathbf{k}_0$ [Fig. 1(b)], or by a surface acoustic wave [Fig. 1(c)]. In the gravity analogy we then have a (2D) gravitational wave with wave vector $\mathbf{k}_0$. However, in these cases the other effects of (i) and (ii) mentioned in the previous paragraph are generically present [although their effects are expected to vanish as $(k_0\ell)^4$], and in the case of graphene nonuniform strain can also induce a pseudogauge field. Thus, additional effort is needed to isolate the gravitational response. If this is possible, we are able to measure not only the graviton energy at zero wave vector, but also its dispersion.

The acoustic wave absorption experiment proposed here has some similarity to earlier phonon absorption experiments [37]. After all, an acoustic wave is made of coherent phonons or a phonon version of laser. But there are a couple of fundamental differences here. (i) In earlier experiments phonons are generated by a heat pulse, and thus come with energies following a thermal distribution. This makes it impossible for spectroscopy measurement. (ii) More importantly, since the thermal phonons come with uncontrolled momenta and polarizations, their effects are dominated by the deformation potential and piezoelectric polarization induced by the strain [36]. Here the wave vector and polarization of the acoustic wave are carefully controlled so that the dominant effect of the strain is on the electron effective mass tensor or metric.

Summary. In this Rapid Communication we propose acoustic wave as an alternative probe of fractional quantum Hall liquids, and demonstrate that it contains effects similar to those of gravitational wave. It allows for a direct measurement of graviton energy, which is not possible using electromagnetic probes. While we focused on Laughlin-type states for their simplicity, the graviton as well as magnetoroton modes are expected to exist in all fractional quantum Hall liquids, and can be probed using the methods described in this Rapid Communication.

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