Spin dynamics in SiGe quantum dot lines and ring molecules

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Semiconductor quantum dot (QD) structures can be used as a model for understanding the effect of the microscopic structure, symmetry of crystals, and molecules on their macroscopic properties. In this work, the results of two theoretical approaches demonstrate that the spin dynamics in ordered QD structures depends on the size, spatial configuration, and topology of the object built of QDs. It was shown that the spin dynamics in QD structures with the hopping regime of conductivity significantly differs from the spin dynamics in two-dimensional (2D) and three-dimensional (3D) structures being at the other side of the metal-insulator transition. The special character of the effective magnetic field $\delta\mathbf{H}$ fluctuations appearing only during tunneling between quantum dots is responsible for the insensitivity of spin relaxation times to the magnitude of the external magnetic field in infinite QD structures (2D square lattice and 1D linear QD chain). In finite QD structures (QD rings and linear chains), an external magnetic field $\mathbf{H}_0$ is directly involved in the spin relaxation process and spin is lost due to interaction with a special combination of fields $\Delta\mathbf{H} \sim [\mathbf{H_0} \times \delta\mathbf{H}]/\delta H$ that leads to an unusual orientation dependence of ESR linewidth, recently observed for QD chains. It was shown that the ordering of QD structures can be used for the conservation of spin orientation. For 1D finite quantum dot chains, the ordering can provide the stabilization of all spin components S_x, S_y, and S_z, while for ringlike molecules only S_z polarization can be stabilized. The results obtained in this work can be useful for development of novel semiconductor devices and in quantum information processing.

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I. INTRODUCTION

Quantum dots (QDs) created during heteroepitaxy have both properties making them similar to real atoms, and the properties that are very different from those of ordinary atoms. One of the nonordinary properties is related to the asymmetry of the quantum dot shape (similar to pyramids), which is a consequence of the growth process of quantum dots on a semiconductor substrate. This asymmetry affects primarily the spin degree of freedom of charge carriers localized in quantum dots. The crystals built of such asymmetric atoms have no inversion center, which immediately leads to spin-orbit-induced modification of the band structure of these crystals, such as spin splitting of the energy spectrum. The Ge/Si heterosystem with Ge quantum dots is very suitable for the study of the effects of asymmetry of "atoms," since Ge and Si are centrally symmetric materials and the emerging spin splitting is related to the asymmetry of QDs, but not to the asymmetry of the lattice of the constituent elements, like in A_3B_5-based heterosystems. The present paper is devoted to the effects in spin dynamics due to asymmetry of QDs in the Ge/Si heterosystem with Ge quantum dots.

For an electron in a crystal with some velocity v the spin splitting of the energy spectrum is associated with an effective magnetic field in which the electron spin will precess with a frequency Ω_k, depending on the wave vector $\mathbf{k}$. In conventional natural crystals the existence of this magnetic field leads to the spin relaxation of free carriers through the Dyakonov-Perel (DP) mechanism [1]. Spin relaxation can be described by the dephasing of spin precession in this effective magnetic field at random collisions of charge carriers. Because of this mechanism, the spin orientation is rather quickly lost in structures without an inversion center. The same mechanism can work in the structures built of artificial atoms, QDs, where electron transport is going through the hopping between QDs. Shklovskii [2] has shown the principal possibility of Dyakonov-Perel spin relaxation in the systems with hopping transport. This is confirmed by ESR experiments on Ge/Si structures with tunnel-coupled Ge QDs, where the stochastic spin precession at hopping between QDs was found to be the main mechanism of the spin relaxation. Each hop is accompanied by spin turning through the small angle and a sequence of these turnings can be considered as spin precession in an effective magnetic field. Such precession can not lead to spin relaxation if the direction of electron motion does not change. Randomness of tunneling directions during carrier movement across a nonordered QD array leads to the dephasing of the spin precession and finally results in the loss of spin orientation. Then the randomness of hopping caused by the inhomogeneity of the nonordered QD array is a key factor of efficiency of DP spin relaxation. It is natural to assume that in ordered QD structures, the suppression of DP spin relaxation should be observed and new interesting effects in spin dynamics will appear.

The first sign of principal difference in the spin relaxation in ordered QD structures (QD molecules) is an unusual narrowing of the ESR line obtained recently for QD linear chains [3]. Usually, the ESR study of 2D electron gas structures with structure-induced asymmetry demonstrates a broadening of ESR line in the external magnetic field deviated from the growth direction of the structure [4]. Such behavior of the ESR linewidth ΔH is a direct consequence of the DP mechanism domination and occurs due to a special in-plane arrangement of effective magnetic fields (spin-orbit fields), leading to the anisotropy of spin relaxation processes in the system. It is

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easy to show that the transverse spin relaxation time T_2 should decrease in such type of 2D systems in tilted magnetic fields [5]. Since $\Delta H \sim 1/T_2$ for homogeneously broadened ESR lines, a special orientation dependence of ESR linewidth, namely ESR line broadening in tilted magnetic fields, should be observed. A similar orientation dependence was observed for a two-dimensional array of tunnel-coupled QDs [6], suggesting that the mechanism of spin relaxation is the same in both types of structure, with hopping and diffusive regime of transport. However, the results obtained in the Ref. [3] questioned this hypothesis. The authors demonstrated the unusual orientation behavior of the transverse spin relaxation time for finite QD linear chains experimentally, by means of ESR measurements, and theoretically, by means of Monte-Carlo simulations: in the external magnetic field applied along the growth direction T_2 is smaller than the one for the in-plane magnetic field. At the same time for the infinite linear QD chain, the behavior of T_2 was qualitatively the same as for 2D arrays of tunnel-coupled QDs and 2D electron gas structures. To explain this result the authors introduce a new magnetic field $\Delta\mathbf{H}$, being a special combination of external magnetic field $\mathbf{H}_0$ and effective magnetic field $\delta\mathbf{H}$, $\Delta\mathbf{H} \sim [\mathbf{H_0} \times \delta\mathbf{H}]/\delta H$. The orientation dependence of the ESR linewidth obtained for finite QD lines confirmed that namely the field $\Delta\mathbf{H}$ defines the spin dynamics in this system. This result demonstrates that ordering of QDs provides a significant change in the spin dynamics in a QD system.

In the present work, we try to reveal the cause of this change by the example of linear chains and circular molecules of quantum dots. The hopping spin relaxation in ordered QD structures is not studied theoretically until today, though the importance of spatially close configurations of localized centers, similar to molecules, was emphasized in the studies of spin dynamics in the regime of hopping conductivity in n-type semiconductors with spin-split spectrum [7–9]. The spin behavior in closely spaced donor pairs was noted to be unusual: the longitudinal magnetic field accelerates the spin relaxation, while for conduction electrons in A_3B_5 semiconductors, the opposite effect of suppression of spin relaxation should be observed [10]. Unfortunately, the closely spaced donor groups were studied only as a part of a large inhomogeneous system, like n-type impurity-doped GaAs, and many interesting effects were lost in light of the peculiarities of the hopping transport in disordered systems.

To make the present work more comprehensive, we study the angular dependencies of spin relaxation times, because these dependencies are most strongly related to the mechanism of spin relaxation in the system and can assist in experiments to determine the dominating mechanism in the system [4,6]. The second essential feature being very helpful in the understanding of spin dynamics is the relation of the spin relaxation time to the external magnetic field magnitude. For example, the spin relaxation in the systems with domination of the DP mechanism is very sensitive to the magnitude of the external magnetic field [10]. So, the study of these characteristics can help us understand how the ordering can change the mechanism of spin relaxation in this system and to find the distinctive features of the spin dynamics in QD structures. The study is carried out on the example of the Ge/Si heterosystem. To describe the spin dynamics, we use two methods: (i) solving the eigenmodes problem and (ii) finding the spin relaxation times by means of Fourier transforms of perturbing fields.

II. THEORETICAL RESULTS AND DISCUSSION

A. Eigenmodes problem: model

The first part of our theoretical research consists in finding the eigenmodes of the spin polarization decay in a system of QD chains (linear or circular). The model and its parameters were chosen to be close to the experimental system with GeSi quantum dots. This system principally differs from the previously studied system with hopping between donor states in n-type GaAs [11]. In the last case, it was shown that the stochastic spin precession during random hopping does not dominate in this system, and other mechanisms of spin relaxation, due to anisotropic exchange interaction [12] and hyperfine interaction [13], define the spin lifetime. In the case of the Ge/Si system with quantum dots, the hyperfine interaction can be neglected due to the low concentration of ^{29}Si isotopes (<5%). As concerning the anisotropic exchange interaction, its contribution depends on the filling factor of QDs ν. If $\nu \lesssim 1$, this interaction also can not be considered as a main source of spin relaxation. We will concentrate namely on this case, which corresponds to the real experimental structures, where not all QDs are filled by electrons and the spin relaxation occurs mainly at the hopping between QDs [6]. Thus we neglect the electron-electron interaction and treat our model for one electron.

The Hamiltonian of the system is

$$\hat{H} = \sum_a \varepsilon_a (n_{a\uparrow} + n_{a\downarrow}) + \sum_{a,b,\sigma,\sigma'} t_{a\sigma,b\sigma'} \hat{a}^\dagger_{a\sigma} \hat{a}_{b\sigma'} + \sum_{a,\sigma,\sigma'} \hat{a}^\dagger_{a\sigma} (\Omega_L \hat{\boldsymbol{\sigma}})_{\sigma\sigma'} \hat{a}_{a\sigma'} + \hat{H}_{ph} + \hat{V}_{e\text{-}ph},$$

where ε_a is the energy of electron localized in QD, $t_{a\sigma b\sigma'} = t_{ab} \exp(i\alpha \mathbf{e_h} \hat{\boldsymbol{\sigma}}/2)$, t_{ab} is the tunneling coupling of electrons in neighboring QDs, $\mathbf{e}_h$ is the unit vector along the axis of the effective magnetic field fluctuations, $\hat{\boldsymbol{\sigma}} = (\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z)$ denotes the Pauli matrices, α is the rotation angle, $\hat{H}_{ph}$ is the phonon Hamiltonian, and $\hat{V}_{e\text{-}ph}$ is the Hamiltonian of electron-phonon interaction. It is clearly seen that the spin-orbit interaction is hidden in the second term of the Hamiltonian that describes the spin rotation at the tunneling between QDs.

We shall not concretize the terms related to phonons, because this is not the principal question of our work, and these terms were well described in many previous works related to hopping [14–16]. The main point is that the phonons assist the hopping between QDs with some average time of hopping τ_h. It should be noted that the time τ_h plays the same part in the spin dynamics in a QD system as the time of momentum scattering τ_k in the systems with conduction electrons, because after these times, the spin-orbit field randomly changes its direction. The time τ_h can be taken from the experimental results of the ESR study of two-dimensional GeSi QD arrays. The authors of Ref. [6] estimated the characteristic time of hopping between QDs using the data of angular dependence of ESR linewidth. For a typical QD array with density $\simeq 10^{11}$ cm^{-2}, the time of hopping is about $\tau_h \simeq 3 \times 10^{-11}$ s.

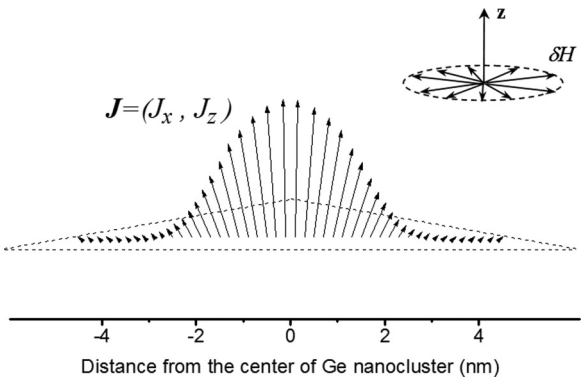

FIG. 1. The distribution of the angular momentum for the ground state of the hole localized in a Ge QD (obtained from tight-binding calculations in our previous work, Ref. [17]). The arrows schematically show the direction and magnitude of the vector $\langle\psi|\hat{\mathbf{J}}|\psi\rangle$ along the line passing through the center of the QD base. For a better representation, we multiply the deviation angle by 5. The inset shows schematically the distribution of the effective magnetic field arising due to the absence of inversion symmetry. The shape of the QD is shown by dashed lines.

Hopping between any neighboring QDs is permitted with an equal probability for the back and forth motion. Each tunneling transition is accompanied by a spin rotation by a small fixed angle α. Before the determination of its magnitude, the nature of spin rotation in a Ge/Si QD system should be discussed. The first question is how the spin rotation occurs in this system. Hopping transport can be considered as a sequence of fast tunnel hops from one localized state to another alternating with exponentially long waiting periods in each localized state. While waiting in the localized state, an electron has velocity $v = 0$ and, therefore, the average effective magnetic field is zero. However, this is an incomplete picture of the effective magnetic field for a localized state. The quantum confinement leading to uncertainty of the wave vector $\mathbf{k}$ provides the uncertainty of the effective magnetic field. For a better understanding, one can look at Fig. 1, where some results of our previous study in the tight-binding (TB) approach of spin relaxation during the tunneling between Ge/Si QDs are presented [17]. Here the distribution of the spin polarization vector across the Ge nanocluster for a localized hole state is shown. Here, the x profile of this distribution is shown, but this distribution is similar for all profiles, passing perpendicularly to $\mathbf{Z}$ axis (the growth direction of QDs). One can see that the asymmetry of the QD shape leads to the deviation of the hole angular momentum from the Z direction. The special distribution of the angular momentum can be explained by the distribution in the xy plane of the effective magnetic fields, as shown in the inset of Fig. 1. Obviously, the average value of the effective magnetic field is $\langle\delta\mathbf{H}\rangle = 0$ for the localized QD state. During tunneling of carriers between two QDs, the preferential direction of tunneling $\mathbf{n}$ defines the direction of the effective magnetic field as $\delta\mathbf{H} \sim [\mathbf{n} \times \mathbf{e}_z]$. The vector $\mathbf{n}$, like the vector $\mathbf{k}$ in a two-dimensional (2D) electron gas, controls the direction of $\delta\mathbf{H}$. However, the consideration of the spin behavior during tunneling between dots as the spin rotation around this field is not quite correct. The spin deviation at the exponential tails of the wave function is transferred during tunneling to the neighboring quantum dot. A similar situation arises for hopping between donor states in GaAs. The nature of spin rotation is clearly reflected in the expression for the donor electron wave function proposed by Kavokin [8]. This expression contains a multiplier that has the form of an operator of finite spin rotation in an effective magnetic field that arises due to the absence of inversion symmetry of GaAs. However, for the Ge/Si QD system, this expression can not be applied because the effective magnetic field in a QD system is local, it arises only in the vicinity of QDs, which are sources of symmetry breaking in a SiGe heterostructure with Ge QDs. Then the correct estimation of spin deviation in a Ge/Si QD system is possible only on the basis of atomistic calculations. According to TB calculations, the angle of spin turning (deviation) during one tunneling event α is defined by the aspect ratio h/l, where h and l is the height and lateral size of QD [17]. Usually, all QDs in an array formed by self-assembling have the same aspect ratio [18], hence, there is one characteristic turning angle α for all tunneling events in the QD array. If an electron continues in the direction of tunneling (for example, along a one-dimensional QD chain), the sequence of small turns that follow can be considered as a precession in the effective magnetic field. It should be noted that the resulting angle of the spin rotation depends on the number of QDs which were visited by the electron during hopping. The larger the density of QDs, the larger the resulting rotation angle. This is the principal difference of spin relaxation in a QD system from the case of hopping spin relaxation in an impurity band of n-type GaAs. In the last case, the angle is defined by the length of the path that the electron has gone [8], because the effective magnetic field arises due to the absence of inversion symmetry in GaAs and is global within the studied sample. Thus the value of α is taken based on the results of tight-binding calculations performed for the holes localized in Ge/Si QDs [17], giving $\alpha = 0.1$ for *hut*-cluster shaped QDs ($h/l = 0.1$). Since the experimental study of spin relaxation was performed in this system only for electrons, which are localized in the Si vicinity of Ge QDs, we have scaled the value of α for electrons taking into account the weakness of spin-orbit interaction in Si. Then we take $\alpha = 0.01$.

We include in the model the Larmor precession caused by the external magnetic field. The main feature of a QD system is that the Larmor precession in an external magnetic field during the short time of tunneling results in a very small rotation angle and can be neglected. Then the precession in the effective magnetic field and the precession in the external magnetic field with a frequency Ω_L can be considered as separated in time and acting independently from each other, the spin-orbit field being nonzero during a tunneling event, the external magnetic field acts during the time between tunneling events. To study the angular dependence of spin relaxation times, the direction of the external magnetic field is changed between $\mathbf{e}_z$ and $\mathbf{e}_x$. We consider the following configurations of QD structures: (i) a 2D square lattice of QDs in xy plane; (ii) an infinite linear QD chain oriented along the y direction; (iii) linear QD chains with different number of QDs oriented along the y direction; (iv) a circular QD chain (a QD ring) in the xy plane; and (v) a QD ring with one broken bond, crossing the x axis, the center of the ring coincides with the origin of the coordinates.

Let N be the number of QDs in the structure. For one QD ($N = 1$), the spin dynamics is described by the simple equation

$$\frac{d\mathbf{P}}{dt} = \Omega_L \times \mathbf{P},$$

where $\mathbf{P}$ is a spin-polarization vector $\mathbf{P} = (p_x, p_y, p_z)$. In a system of two coupled QDs ($N = 2$), this equation is modified to

$$\frac{d}{dt}\begin{pmatrix}\mathbf{P}_1\\ \mathbf{P}_2\end{pmatrix} = \begin{pmatrix}\Omega_L \times \mathbf{P}_1 & 0\\ 0 & \Omega_L \times \mathbf{P}_2\end{pmatrix} + \begin{pmatrix}-\tau_{12}^{-1}\hat{I} & \tau_{21}^{-1}\hat{R}_{21}\\ \tau_{12}^{-1}\hat{R}_{12} & -\tau_{21}^{-1}\hat{I}\end{pmatrix}\begin{pmatrix}\mathbf{P}_1\\ \mathbf{P}_2\end{pmatrix}.$$

Here, $\mathbf{P}_i$ is the polarization vector on the ith QD; τ_{12} and τ_{21} are the characteristic times of the tunneling from the first to the second quantum dot and backwards; $\hat{R}_{12}$ and $\hat{R}_{21}$ are the matrices of rotation during tunneling from the first to the second quantum dot and backwards. These equations for spin dynamics can be easily generalized to an arbitrary number of QDs.

The spin relaxation times T_1 and T_2 were found by solving the eigenvalue problem

$$d\mathbf{P}/dt = -\lambda\mathbf{P}$$

and selecting the eigenmodes $P = P_0 \exp(-\lambda t)$ with the slowest decay. All eigenmodes can be divided into two groups, corresponding to longitudinal and transverse spin polarization. The longitudinal spin does not rotate in an external magnetic field, hence, in the first group $\mathrm{Im}\,\lambda \ll \Omega_L$. The eigenmodes with $\mathrm{Im}\,\lambda = \pm\Omega_L$ correspond to transverse spin components rotating in an external magnetic field. The longitudinal spin relaxation time T_1 is defined by the smallest value of $\mathrm{Re}\,\lambda$ in the first group, $(T_1)^{-1} = \min(\mathrm{Re}\,\lambda)$. The transverse spin relaxation time is defined similarly $(T_2)^{-1} = \min(\mathrm{Re}\,\lambda)$, where the minimum is the smallest value of $\mathrm{Re}\,\lambda$ in the second group. To compare the spin dynamics in QD structures and in 3D and 2D structures with absence of inversion symmetry, we study the spin behavior depending on the magnitude and orientation of the external magnetic field.

B. Eigenmodes problem: results

The results of solving the eigenmodes problem demonstrate that the spin relaxation in QD structures essentially differs from the spin relaxation of delocalized charge carriers in 3D and 2D structures with absence of inversion symmetry. In the last case, the Dyakonov-Perel mechanism controls the spin relaxation and the increase of external magnetic field provides the longer spin relaxation time T_1. For infinite QD structures (2D square lattice and 1D infinite chain), both spin relaxation times, T_1 and T_2, turned out to be independent from the external magnetic field magnitude (Fig. 2, the calculations were done for $\mathbf{H}_0 \| \mathbf{Z}$). The times are of the order of τ_h/α^2 for all magnitudes of the magnetic field. For a 2D lattice, the times are two times smaller (not shown in Fig. 2) than for a 1D infinite chain. Thus our results show twofold increase of spin relaxation times at transition from 2D QD lattice to 1D infinite chain, that is, in a good agreement with existing results of nanowire investigations demonstrating an increase of spin lifetimes in 1D structures [19].

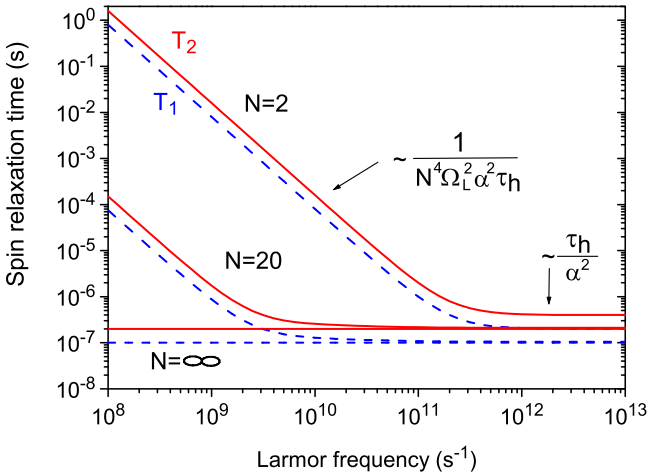

FIG. 2. Longitudinal (T_1) and transverse (T_2) spin relaxation time dependencies on the Larmor frequency of the external magnetic field for linear chains with a different number of quantum dots. The external magnetic field is applied along the Z direction (the growth direction). For finite QD structures, a quadratic dependence on the Larmor frequency Ω_L is observed in the limit $\Omega_L\tau_h \ll 1$.

In the case of finite linear QD chains for small enough magnetic fields, the spin relaxation times decrease when the external magnetic field magnitude increases following the quadratic dependence $T_{1,2} \sim 1/\Omega_L^2$. After some critical point ($\Omega_L\tau_h \simeq 1$), the times become independent from the magnitude of the external field. They reach the characteristic values $\simeq\tau_h/\alpha^2$, and remain constant at larger magnetic fields. The critical point position depends on the number of dots in the linear chain. The larger the number of QDs in the chain, the earlier (at smaller Larmor frequencies) the transition to the constant rate of spin relaxation occurs.

In the case of circular QD chains, the dependence on the magnitude of the external magnetic field is changed depending on the topology of an object (Fig. 3). For the unclosed ring, the frequency dependence of both spin components is similar to the one for finite line chains. For the closed ring, the behavior of T_1 remains the same, while the T_2 frequency dependence is changed. At small frequencies, $\Omega_L \ll 1/\tau_h$, the transverse spin relaxation rate becomes independent from Ω_L and tends to a constant: $1/\tau_s \simeq \alpha^4 N^2/\tau_h$.

The dependence on the external magnetic field orientation is presented in Fig. 4. For the infinite 2D QD array with square lattice packing, the angular behavior of T_2 and T_1 is very similar to the spin behavior observed for 2D electron gas structures with inversion asymmetry of the confining potential. For an external magnetic field applied along the growth direction of the structure, the time T_2 is two times larger than T_1, while for the in-plane magnetic field, the relation between T_1 and T_2 changes, T_2 becomes smaller than T_1.

For the infinite QD linear chain, the T_2 angular dependence remains the same as for the 2D QD array, while the time T_1 increases drastically for the in-plane magnetic field.

The results obtained for finite QD lines are rather different. The angular dependencies of T_1 and T_2 are not typical of systems with dominant Dyakonov-Perel mechanism. The behavior of transverse spin relaxation time T_2 is in a good

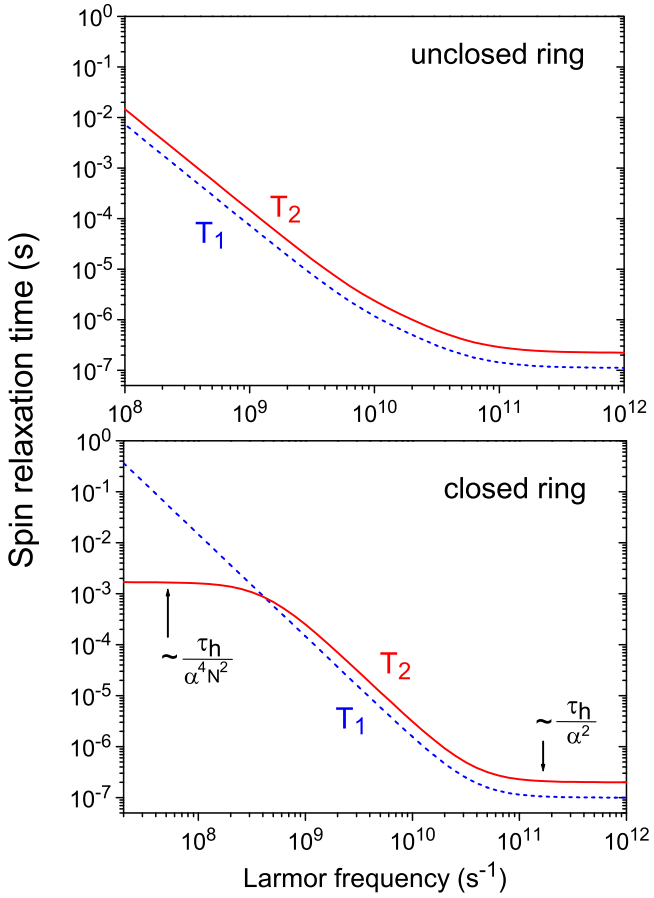

FIG. 3. Longitudinal (T_1) and transverse (T_2) spin relaxation time dependencies on the Larmor frequency of the external magnetic field for the closed (top panel) and the unclosed QD ring (bottom panel). The external magnetic field is applied along the Z direction (growth direction). The number of QDs $N = 10$.

agreement with the one recently reported in Ref. [3], where an ESR study of the samples with QD line chains was performed. T_2 increases for an in-plane orientation of the external magnetic field. The present simulations show that the behavior of T_1 is similar to T_2 one, T_1 increases following the same functional dependence by keeping the ratio $T_1/T_2 = 0.5$.

The results of the angular dependence study for circular QD chains directly demonstrates the connection between the topology of the object and the spin behavior (see Fig. 5). The orientation dependence of T_2 and T_1 for the unclosed ring demonstrates an increase of these times with the deviation of the magnetic field from Z that is similar to the case of finite QD linear chains. However, this increase is not so large as for QD lines. For the closed ring, the behavior of T_2 remains the same as in the case of the unclosed ring, while the T_1 behavior is changed. For an external magnetic field applied along the growth direction, the time T_1 increases, i.e., the S_z component of the spin tends to stabilize. However, such difference between spin behavior in the closed and unclosed ring is observed only in the frequency range of $\Omega_L < 10^9$ s^{-1}. Figure 5 demonstrates the spin behavior at frequency $\Omega_L = 10^8$ s^{-1}. At larger frequencies, the spin behavior in both cases of closed and unclosed rings is qualitatively the same.

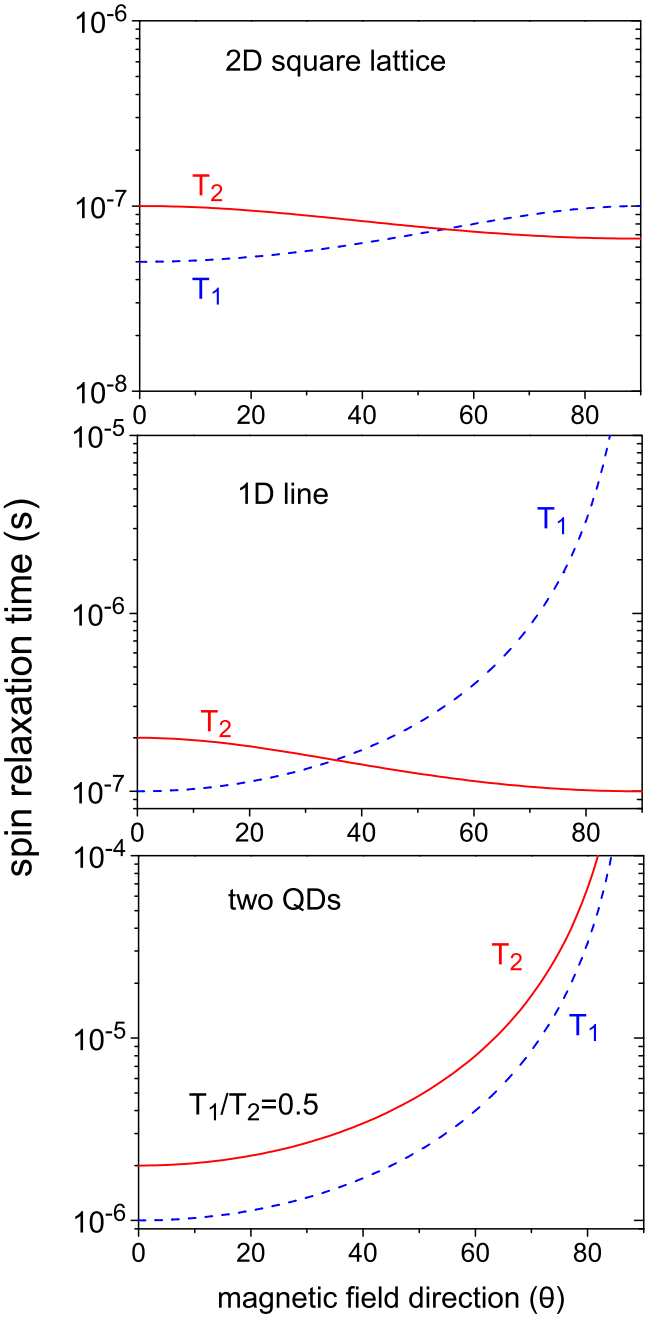

FIG. 4. Longitudinal (T_1) and transverse (T_2) spin relaxation times dependencies on the magnetic field direction for different configurations of quantum dot structures. θ is the angle between the z axis and magnetic field direction, which is varied in the xz-plane. The Larmor frequency is $\Omega_L = 10^{11}$ s^{-1}.

C. Eigenmodes problem: discussions

Let us discuss the results of T_1 and T_2 angular dependence study (Figs. 4 and 5). To understand the obtained results, it is necessary to describe the nature of perturbing field responsible for spin relaxation. In the 2D QD array case, the angular dependence of T_1 and T_2 can be explained by an in-plane distribution of fluctuating spin-orbit fields arising during tunneling between dots. When an electron randomly hops between QDs, only in-plane components δH_x and δH_y

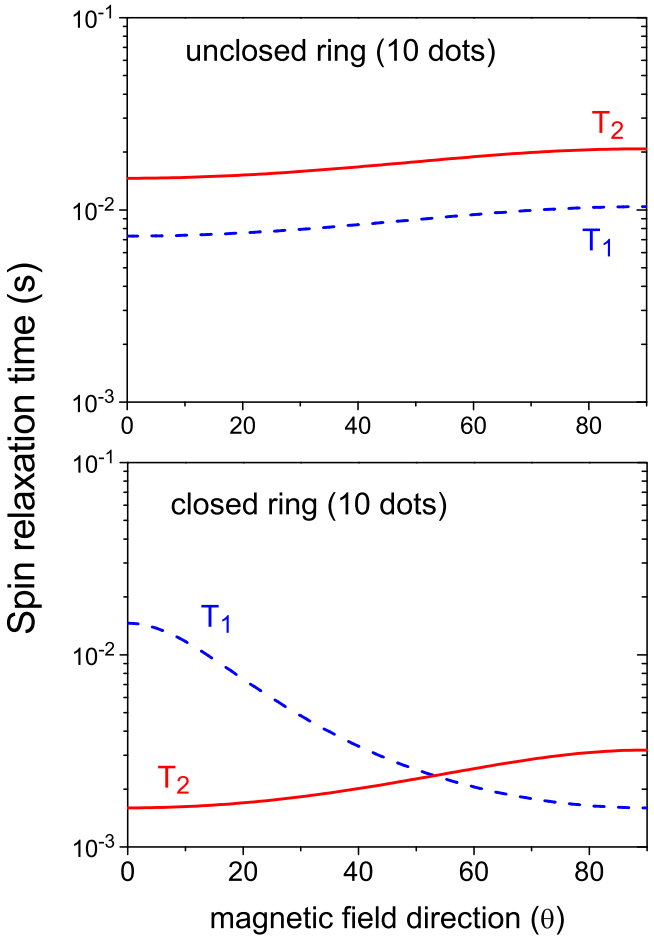

FIG. 5. Longitudinal (T_1) and transverse (T_2) spin relaxation times dependencies on the magnetic field direction for circular configurations of quantum dot structures, for unclosed and closed QD rings. θ is the angle between the z axis and magnetic field direction, which is varied in the xz-plane. The Larmor frequency is chosen to be $\Omega_L = 10^8\ \mathrm{s}^{-1}$, because, for $\Omega_L > 10^8\ \mathrm{s}^{-1}$ the spin behavior is the same for all circular configurations.

appear, $\delta H_z = 0$. For the external magnetic field applied along the z axis, the spin relaxation times can be written as [20]

$$1/T_1 \propto \delta H_x^2 + \delta H_y^2,$$

$$1/T_2 \propto \frac{1}{2}\left(\delta H_x^2 + \delta H_y^2\right) + \delta H_z^2,$$

where $\delta H_z = 0$. For the tilted magnetic field, part of the in-plane spin-orbit field contributes to δH_z (with the z direction defined by H_0). Thus T_2 should decrease, while T_1 should increase for $\theta \neq 0$. Thus the spin behavior is the same as for conduction electrons in 2D structures with spin-split spectrum [4].

If we consider an infinite 1D QD chain, an electron motion occurs only in one dimension. Then one of the components of fluctuating field disappears, in our geometry this is the δH_y component (QD line is directed along the y axis). If the magnetic field is applied along the x direction ($\theta = 90^\circ$ in our notation), the directions of spin-orbit field and external magnetic field coincide and the perturbing field can not provoke the relaxation of the longitudinal spin component. Then T_1 increases for the in-plane external magnetic field. Generally, such spin behavior should be observed also in the case of 1D structures, like nanowires lacking inversion symmetry, with domination of the DP spin relaxation mechanism. Indeed, a similar orientation dependence of the spin relaxation rate was obtained in the work of Pramanik *et al.* [21] where spin transport in nanowires was studied using a semiclassical approach. The depolarization rate is found to depend strongly on the initial spin polarization. If the initial polarization is along the axis of the wire, the spin depolarizes ~100 times slower compared to the case when the initial polarization is transverse to the wire axis. If one takes into account that the spin orientation along the nanowire axis (y axis) is physically the same as the orientation along z axis (in both cases the spin is perpendicular to the spin-orbit field), one can see the good agreement between the results of the work of Pramanik *et al.* and the T_1 orientation behavior obtained by us for the 1D case (see central panel in Fig. 4).

Our results can be helpful in understanding the two works, where qualitatively similar results were obtained, but the quantitative difference between them was very large. In the work of Kiselev and Kim [22], an increase of the spin relaxation time by orders of magnitude was obtained for a narrow two-dimensional strip with decreasing channel width. The authors investigated the relaxation of the spin component directed along the channel width, i.e., perpendicular to the strip. This corresponds to the T_1 behavior at $\theta = 90^\circ$ in our notation. If one compares the 2D and 1D cases at this angle in Fig. 2, one will see that T_1 increases by orders of magnitude at the transition from the 2D case to the 1D case, as in the work of Kiselev and Kim. As concerning the point $\theta = 0^\circ$, it corresponds to the experimental conditions in the work of Holleitner *et al.* [23], where the S_z component relaxation in submicrometer InGaAs wires was studied by pump and probe spectroscopy. For structures with nanowires, Holleitner obtained that the spin relaxation time increased approximately two times as compared to 2D quantum well structures. The same result can be obtained from comparison of T_1 values in the 2D and 1D cases at $\theta = 0^\circ$ in Fig. 2.

In the case of finite QD lines for an in-plane magnetic field, there is not only an increase of T_1 but also that of T_2. To understand this, one should take into account two factors. The first is coincidence of the in-plane magnetic field direction with the axis along which the spin-orbit field fluctuates. The second is the independence of spin rotations around the external magnetic field and around the spin-orbit field, they are separated in time and go consecutively. It must also be remembered that the angle of rotation in the spin-orbit field is always the same, i.e., it is the same for all tunneling events (see Ref. [14]). In these conditions in the reference frame rotating with the Larmor frequency around the external magnetic field, the angle of transverse spin rotation during the electron motion along the QD chain in the forward direction ($+\alpha$) compensates the rotation angle during the motion in the backward direction ($-\alpha$). In other words, due to finiteness of the object, the number of turns during the forward motion is equal to the number of turns during the backward motion. As result, the spin polarization at each dot in the chain does not change. An alternative explanation can be applied if one

uses the concept proposed in the recent work devoted to ESR in QD chains [3]. The authors proposed to describe the spin behavior in finite QD structures in terms of an interaction with a transverse effective magnetic field. This field is the vector product $\Delta\mathbf{H} \sim [\mathbf{H_0} \times \delta\mathbf{H}]/\delta H$ and it decreases with a deviation of the external magnetic field from the growth direction according to the law $\sim \sin\varphi$, where φ is the angle between $\delta\mathbf{H}$ and $\mathbf{H_0}$. Namely, this dependence defines the T_1 and T_2 orientation behavior, obtained by us independently.

For the unclosed ring, the orientation dependence can be explained in the same way. The increase of T_1 and T_2 for the in-plane magnetic field is provided by the finiteness of the object and the coincidence of the directions of the external and spin-orbit fields. The diminished rise, as compared with linear QD chains, can be explained by the curvature of the object, which leads to a nonzero $\Delta\mathbf{H}$ even in the case of the in-plane external magnetic field.

The difference in the angular dependencies of the spin relaxation times for unclosed and closed rings, namely the increasing T_1 with decreasing θ in the case of the closed ring, can be explained in the following way. The sequence of small rotations of spin at the circular electron movement leads to an effective rotation of spin around the Z axis. One can interpret this as an appearance of a secondary effective magnetic field along the z-direction $\tilde{H}_z$, which leads to an additional stabilization of S_z (see Fig. 6).

The appearance of $\tilde{H}_z$ also changes the frequency dependence of T_2 for closed rings, as compared with the case of unclosed rings and linear chains (see Fig. 3). For small frequencies, the rate of transverse spin relaxation becomes independent from Ω_L and tends to the constant $1/T_2 \simeq \alpha^4 N^2/\tau_h$. To understand this limitation, let us consider the simpler case of the infinite 1D linear chain or a 2D QD array.

In the case of infinite 1D or 2D structures, the spin relaxation rate is independent from the Larmor frequency of an external magnetic field applied along the growth direction. The simulations give that T_1 and T_2 are defined only by the magnitude of spin-orbit field, $T_{1,2} \simeq \tau_h/\alpha^2$ with $\alpha = \gamma\delta H\tau_t$, where $\gamma = g\mu_B/\hbar$ is the gyromagnetic ratio, τ_t is the time of tunneling between QDs. This limitation can be obtained from a simple consideration. In the reference frame rotating with the Larmor frequency, the interaction with $\mathbf{H}_0$ can be omitted; the direction of spin-orbit field is randomized in-plane, because it is determined by the random direction of hopping along the QD chain (forward or backward) and by the random angle accumulated during time τ_h due to the rotation of the QD chain with the Larmor frequency. So, the spin behavior can be described in terms of an interaction with a randomly distributed in-plane spin-orbit field δH. Let the spin orientation be lost during N hopping events, $\tau_s \simeq N\tau_h$. The turning angle accumulated over N random hops is $\varphi \simeq \alpha\sqrt{N} \simeq 1$, where $\alpha = \gamma\delta H_q\tau_t$ with τ_t being the time of tunneling between dots. Then N can be expressed through α, $N \simeq 1/\alpha^2$. Substituting this N in the expression for τ_s, one can obtain $\tau_s \simeq \tau_h/\alpha^2$.

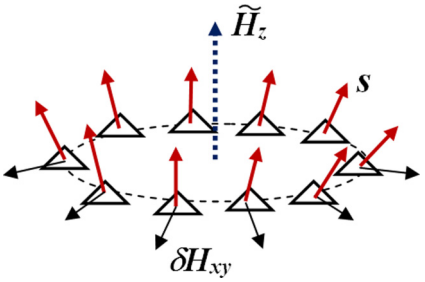

FIG. 6. The spin behavior at electron movement along a circular chain of QDs. The appearance of secondary effective magnetic field $\tilde{H}_z$ is shown.

For finite linear QD chains, the rate of spin relaxation becomes sensitive to the external magnetic field magnitude (see Fig. 2) and the field dependence is described as $1/T_{1,2} \propto \Omega_L^2$. A qualitatively similar result was obtained by Lyubinskii for donor pairs in GaAs [9]. There, the spin relaxation during hopping in pairs was accelerated by external magnetic field and saturated at large Larmor frequencies Ω_L. However, Lyubinskii obtained not the quadratic dependence of relaxation rate but the linear dependence $1/\tau_s \propto \Omega_L$, because he considered the donor pairs as part of a large inhomogeneous system and took only the optimal pairs with $\tau_h \approx 1/\Omega_L$ that make the main contribution to the spin relaxation in the system.

For finite QD structures, the external magnetic field is directly involved in the spin relaxation process. The external field disturbs the balance of spin deviations when the rotation angle $+\alpha$, caused by hopping from one end of QD chain to another, is compensated by the rotation angle $-\alpha$, caused by hopping in the opposite direction. In the presence of an external magnetic field, the Larmor precession is added to the precession in spin-orbit fields appearing only during tunneling between dots. Since these precessions go independently around different axes and are separated in time, the consideration becomes rather complicated (except the case of in-plane magnetic field, discussed above). To simplify the situation, one can remove the spin-orbit field by a simple trick. In the reference frame rotating with spin around the spin-orbit field, the latter can be considered as absent, because the spin does not deviate. But the external magnetic field swings in this reference frame (see Fig. 7), and this swinging can be described as an appearance of some additional transverse field, changing its direction at the opposite sides of a molecule. Now the spin relaxation will be induced by interaction with this field $\Delta\mathbf{H}$. The magnitude of this field depends on the external magnetic field and on the length of the chain, $\Delta\mathbf{H} \approx \alpha(N-1)[\mathbf{H_0} \times \delta\mathbf{H}]/(2\delta H)$. The longer the chain, the larger the spin deviation at the ends of the molecule and correspondingly the larger the effective field introduced to describe this deviation. For finite QD chains, namely this field is the main source of spin relaxation at $\Omega_L\tau_h < 1$. At larger frequencies, the spin deviation caused by the tunneling between dots is averaged by the Larmor precession in the external magnetic field, and, consequently, the additional field $\Delta H = 0$, which means switching off the main mechanism of spin relaxation described in Ref. [3]. The position of switching off point depends on the number of dots in the linear chain. The larger the number of QDs in the chain, the earlier (at smaller frequencies) the transition to the constant rate occurs. After the transition point, there is no correlations of spin-orbit field at the time scale τ_h and the spin dynamics is defined by stochastic spin precession in random spin-orbit fields δH. Therefore there is no dependence on the magnitude of the external field like in the case of infinite

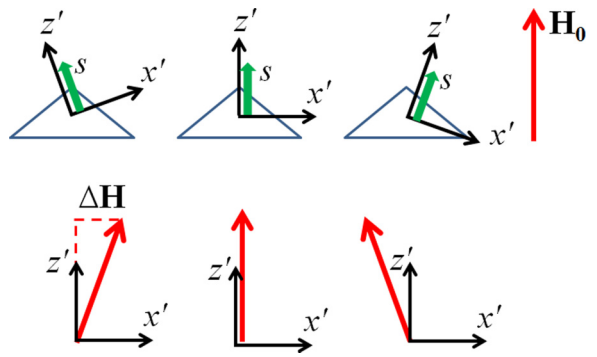

FIG. 7. Demonstration of the spin rotation around the effective magnetic field during tunneling between dots in the case of three QDs chain (top). The change of the external magnetic field orientation (big arrows) in the reference frame rotating together with the spin around the effective magnetic field at each hopping event (bottom). In this reference frame, the external magnetic field deviates at a small angle from the original direction at each hopping event, and this provides the spin relaxation. The magnetic field $\mathbf{H}'$ in the new reference frame is $\mathbf{H}' = \mathbf{H_0} + \Delta\mathbf{H}$, where $\mathbf{H_0}$ is the magnetic field in the fixed reference frame, $\Delta\mathbf{H} = 0$ at the central QD and $\Delta\mathbf{H} \approx \pm\alpha[\mathbf{H_0} \times \mathbf{e_h}]$ at the ends of the QD chain. Here, $\mathbf{e_h}$ is a unit vector along the axis of the effective magnetic field fluctuations.

QD chain. The change of mechanism occurs when $\Omega_L \tau_c \simeq 1$, where τ_c is the characteristic time of traversing the QD chain. Due to randomness of electron motion, this time is defined as $\tau_c = N^2\tau_h$. Then the tenfold increase of the length of QD chain leads to a two orders smaller critical frequency of the mechanism change. The frequency dependence for unclosed rings is qualitatively the same as for finite QD linear chains.

Now, having in mind the results for infinite chains and infinite QD structures, let us return to the case of the closed ring. The fluctuations of $\tilde{H}_z$ in the case of the closed ring will be the same as for the infinite chain: $+\tilde{H}_z$ for clockwise motion, and $-\tilde{H}_z$ for counterclockwise motion. For small Ω_L, only $\tilde{H}_z$ will provoke the transverse spin relaxation, since the effect of δH_x and δH_y will be the same as in the case of finite size QD structures (their action can be replaced by the action of the field $\Delta\mathbf{H} \sim [\mathbf{H_0} \times \delta\mathbf{H}]/\delta H$, which is decreasing when $\Omega_L \to 0$). The magnitude of $\tilde{H}_z$ can be estimated in the following manner. At circular movement, the spin rotates by the angle $\varphi \simeq N^2\alpha^2/4\pi$ around the Z axis at a full electron turn along the ring. On the other hand, the same angle can be written as $\varphi = \tilde{\Omega}\tau_t N = \gamma\tilde{H}_z\tau_t N$. Equating these magnitudes one can obtain

$$\tilde{H}_z = N\alpha^2/(4\pi\gamma\tau_t). \tag{1}$$

The turning angle produced by precession in this field is $\alpha_z \sim \tilde{H}_z\tau_t$. Using the expression $1/\tau_s \simeq \alpha_z^2/\tau_h$ for the spin relaxation time, one can obtain the required limitation $1/\tau_s \simeq \alpha^4 N^2/\tau_h$ in the small Ω range.

In general, the frequency dependence of T_1 and T_2 for the closed ring reflects the change of three spin relaxation mechanisms. The first prevails in the small frequency range $\Omega_L > 10^9$ s^{-1} and consists of the interaction with field $\tilde{H}_z$. The second is dominant in the frequency range $10^9 < \Omega_L < 10^{11}$ s^{-1} and induces the spin relaxation by means of the interaction with field $\Delta\mathbf{H}$. The third mechanism works in the range $\Omega_L > 10^{11}$ s^{-1} and defines the spin dynamics by the stochastic spin precession in a random magnetic field. It should be noted that in all cases of QD structures considered here, the main reason of the peculiarities of T_1 and T_2 frequency dependencies is the independence of spin rotations in the spin-orbit and external magnetic fields.

D. Method of Fourier transforms of the perturbing fields

The dependence of spin relaxation times on the magnitude of the external magnetic field can be obtained using the traditional approach that allows to find the spin relaxation rate through Fourier transforms (FT's) of the time-dependent components of the perturbing fields or, strictly speaking, through FTs $f_{qq}(\omega)$ of the correlation function of these fields [5]:

$$\begin{aligned} \frac{1}{T_1} &= \gamma^2[f_{xx}(\Omega_L) + f_{yy}(\Omega_L)], \\ \frac{1}{T_2} &= \gamma^2[f_{zz}(0) + f_{yy}(\Omega_L)], \end{aligned} \tag{2}$$

where Ω_L is the Larmor frequency of the external magnetic field.

In a 2D system with dominant Dyakonov-Perel mechanism, the autocorrelation function of perturbing fields represents a simple exponential decay, $\overline{\delta H_q(t)\delta H_q(t+\tau)} \sim \exp(-|\tau|/\tau_k)$, resulting from momentum scattering (see Fig. 8). Then the

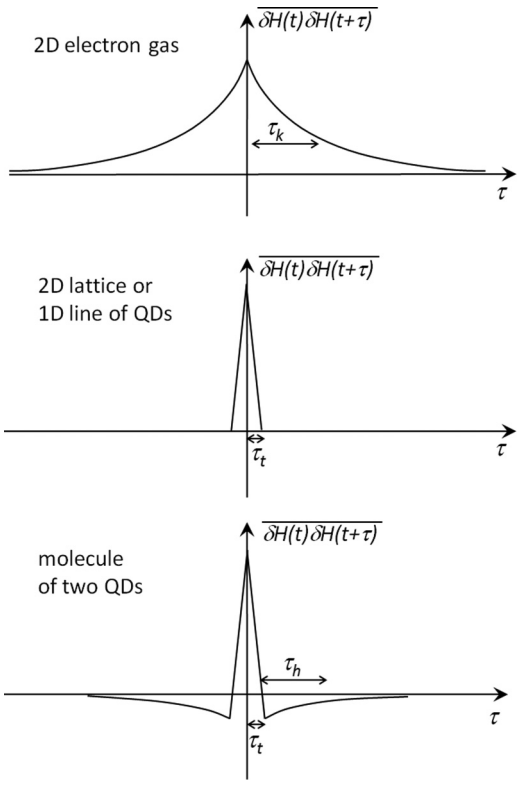

FIG. 8. Sketch of the autocorrelation function $\overline{\delta H(t)\delta H(t+\tau)}$ for different types of structures lacking inversion symmetry: 2D electron gas (top), 2D lattice of QDs (central), molecule consisting of two QDs (bottom).

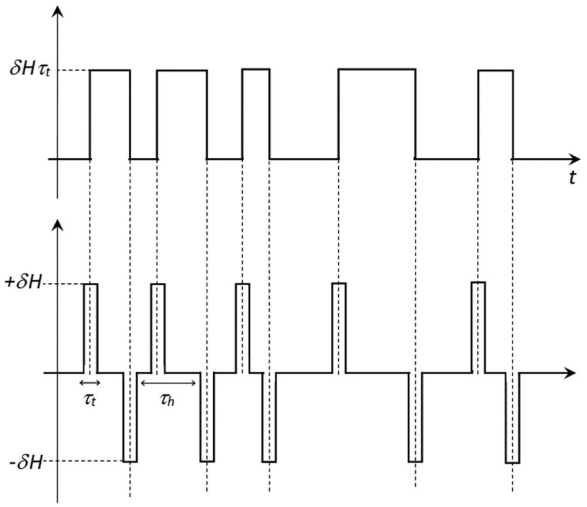

FIG. 9. The top panel demonstrates the random telegraph noise used for obtaining the fluctuations of the effective magnetic field for a QD molecule consisting of two QDs (bottom) by derivation and following convolution with a rectangle. The amplitude of this noise is chosen so that the amplitude of the effective magnetic field for a QD molecule would be δH.

spin relaxation rates correspond to expressions

$$\frac{1}{T_1} = \gamma^2\left(\overline{\delta H_x^2} + \overline{\delta H_y^2}\,\cos^2\theta_H\right)\frac{\tau_k}{1+\Omega_L^2\tau_k^2},$$
$$\frac{1}{T_2} = \gamma^2\overline{\delta H_y^2}\,\sin^2\theta_H\,\tau_k + \frac{1}{2T_1}, \tag{3}$$

where τ_k is the characteristic correlation time, θ_H is the angle between the external magnetic field and the growth axis Z, δH_x, δH_y are the components of the fluctuating effective magnetic field.

In the QD case, the effective magnetic field appears only during tunneling events and it is not zero only during the short time of tunneling between QDs τ_t. And such bursts are randomly repeated with characteristic time τ_h, an average time between tunneling events (for example, see Fig. 9, where the fluctuations of the effective magnetic field for a QD molecule, consisting of two QDs, are presented in the bottom panel). Despite the fact that in both cases (2D electron gas structures and QD structures) the random processes (momentum scattering or random tunneling between QDs) cause a change of the effective magnetic field direction, providing spin dephasing, the spin dynamics in QD structures is rather different.

To clarify this, let us consider the correlation function of a perturbing field δH for the infinite QD structures (2D lattice or 1D line of QDs, see Fig. 8):

$$\overline{\delta H_q(t)\delta H_q(t+\tau)} = \begin{cases}(\tau_t - |\tau|)\cdot\overline{\delta H_q^2}/\tau_h, & \text{if } |\tau| \leqslant \tau_t,\\ 0, & \text{if } |\tau| \geqslant \tau_t,\end{cases}$$

where τ_t is the time of tunneling between quantum dots, τ_h is the characteristic time between tunneling events, q takes on values corresponding to the components x, y in 2D case and x in 1D case (for QD line along y direction).

The FT of this function is

$$f_{qq}(\omega) = \frac{\delta H_q^2\tau_t^2}{\tau_h}\frac{\sin^2(\omega\tau_t/2)}{(\omega\tau_t/2)^2}. \tag{4}$$

During tunneling, the spin rotation angle due to the Larmor precession, $\Omega_L\tau_t$, is negligibly small, then the dependence on the Ω_L disappears [$\sin(x)/x \approx 1$ at small x]. This result was obtained independently by finding the eigenmodes in the framework of the model described above.

So, the independence from the external magnetic field magnitude results from the smallness of time interval τ_t. When $\tau_t \to \tau_h$, Eq. (4) has to transform to $f_{qq}(\omega)$ similar to the one for the DP mechanism (FT of correlation function of random telegraph noise):

$$f_{qq}(\omega) = \delta H_q^2\frac{\tau_h}{1+\omega^2\tau_h^2}. \tag{5}$$

To verify this, it is enough to show that at $\tau_t = \tau_h$, Eqs. (4) and (5) in the limit cases $\omega \to 0$ and $\omega \to \infty$ give the same results (the same functional dependence on Ω):

$$\frac{\sin^2(\omega\tau_t/2)}{(\omega\tau_t/2)^2} \xrightarrow[\tau_t=\tau_h]{} \begin{cases}1, & \text{if } \omega \to 0,\\ \sim 1/(\omega^2\tau_h), & \text{if } \omega \to \infty.\end{cases}$$

Let us consider the finite QD structure, for example, the simple case of QD molecule consisting of two QDs. The main difference from the previous case is the change of $\delta\mathbf{H}$ direction to the opposite one at each tunneling event. In other words, the anticorrelations of the effective magnetic field appear on the time scale compared with τ_h (see Fig. 8, bottom panel). The time dependence $\delta H_x(t)$ represents the sequence of rectangular peaks with amplitudes $\pm\delta H$ and width τ_t (see Fig. 9, bottom panel). This functional dependence can be obtained from a random telegraph noise by deriving and following convolution with a rectangular peak. To satisfy the condition that the amplitude of the effective magnetic field is $\pm\delta H$, one has to take a telegraph noise with amplitude $\delta H\tau_t$ (see Fig. 9, top panel). Then the FT of autocorrelation function in the case of a two QD molecule can be easily obtained from the FT of the well-known correlation function of the random telegraph noise [Eq. (5)] by differentiation and following convolution with a rectangle.

As result, we have a FT for the case of a two QD molecule:

$$f_{qq}(\omega) = \delta H_q^2\tau_t^2\frac{\omega^2\tau_h}{\left(1+\omega^2\tau_h^2\right)}\frac{\sin^2(\omega\tau_t/2)}{(\omega\tau_t/2)^2}. \tag{6}$$

In the limit $\Omega_L \gg 1/\tau_h$, the frequency dependence disappears and $f_{qq}(\Omega_L) \to H_q^2\tau_t^2/\tau_h \sim \alpha^2/\tau_h$. In the opposite case, $\Omega_L \ll 1/\tau_h$, $f_{qq}(\Omega_L) \to \delta H_q^2\tau_t^2\Omega_L^2\tau_h$. This is in good correspondence with the frequency dependence of T_1 and T_2 obtained by derivation of the eigenmodes problem for a two QD molecule (see Fig. 3).

For structures with N quantum dots, correlations appear as an electron traverses the full length of the QD chain $N\cdot l$, where l is the distance between the neighboring dots. The characteristic time of fluctuations is increased N^2 times, since, for traversing the QD chain, N^2 random jumps between dots are needed, $\tau_c \to N^2\tau_h$. Also the rotation angle increased effectively, because when an electron passes from the beginning to the end of a QD chain, spin rotations accumulate:

$\alpha \to N\alpha$. In general, this case can be reduced to the case of two QDs with the modification of τ_c and α :

$$f_{qq}(\omega) = N^2 \delta H_q^2 \tau_t^2 \frac{\omega^2 N^2 \tau_h}{(1+\omega^2 (N^2 \tau_h)^2)}. \quad (7)$$

Here, for simplicity, we omit the part of Eq. (6) related to $\sin(x)/x$. In the limit $\Omega_L \gg 1/\tau_h$, the frequency dependence disappears and $f_{qq}(\Omega_L) \to N^2 H_q^2 \tau_t^2 / N^2 \tau_h = \alpha^2/\tau_h$. In the opposite case, $\Omega_L \ll 1/\tau_h$, $f_{qq}(\Omega_L) \to N^2 \delta H_q^2 \tau_t^2 \Omega_L^2 N^2 \tau_h = N^4 \alpha^2 \Omega_L^2 \tau_h$.

The QD rings are a more complicated case of finite QD structures due to the appearance of the secondary effective magnetic field originating from the circular movement of carriers. As we mentioned above, the sequence of small turns of spin at the circular electron movement leads to an effective rotation of the spin around the z axis that is equivalent to an appearance of the z component of an effective magnetic field. The estimations of the magnitude of this field made in the previous section [see Eq. (1)]. The fluctuations of $\tilde{H}_z$ will be the same as for the infinite chain, for clockwise motion $+\tilde{H}_z$, for counterclockwise motion $-\tilde{H}_z$. As concerning H_x, H_y components, their correlation function will be the same as for the finite chain up to a factor defining a transformation from the linear chain to the circular one. Using $f_{zz}(0)$ obtained for the infinite chain [Eq. (4)] and $f_{yy}(\Omega_L)$ obtained for the finite chain [Eq. (7)], one can write the following expression for T_2 at $\theta = 0$:

$$1/T_2 = \gamma^2 \left[\frac{\delta \tilde{H}_z^2 \tau_t^2}{\tau_h} + N^2 \overline{\delta H_y^2} \tau_t^2 \frac{\Omega_L^2 N^2 \tau_h}{\left(1+\Omega_L^2 (N^2 \tau_h)^2\right)} \right].$$

Here, for simplicity, we omit the part of Eq. (4) related to $\sin(x)/x$. One can see that in the limit $\Omega_L \ll 1/\tau_h$, $T_2 \to \tau_h/(N^2\alpha^4)$, in good accordance with the results obtained by derivation of the eigenmodes problem. So, the examination of our results by a well known method of Fourier transforms of the perturbing fields proves the correctness of the results obtained by derivation of the eigenmodes problem.

III. COMPARISON WITH EXPERIMENTAL DATA

The theoretical results are in a good agreement with the experimental results of ESR studies of ordered QD structures. For homogeneously broadened ESR lines, the ESR line width can be used as a parameter that characterizes the behavior of transverse spin relaxation time T_2. Our previous results obtained for two-dimensional nonordered QD arrays with a high density of QDs demonstrate that the ESR line is broadened when an external magnetic field deviates from the growth direction of the structures [6]. The widest ESR lines were observed for the in-plane magnetic field, which corresponds well to the calculated orientation dependence of T_2 (see Fig. 4). Recently, for one-dimensional QD chains with finite sizes, the inverse orientation dependence of ESR linewidth has been observed [3], which confirms the modeling performed in the present work. Concerning the ringlike QD molecules, the fourfold increase of T_1, as compared with dense 2D QD arrays, was obtained experimentally in the recent work by spin echo measurements [24]. The orientation dependence of the ESR linewidth obtained in the same work does not allow to obtain a confirmation of the increasing T_2 at the tilted magnetic field because of the weakness of the electron localization potential induced in Si by mixed GeSi QDs. Quantum dots were grown at relatively high temperatures and have a large lateral size and a low Ge content. Then, the localization radius of electron is about 50 nm and it becomes sensitive to the external magnetic field magnitude. The external magnetic field applied perpendicularly to the plane of a QD array makes the electron localization on QDs stronger due to the wave function shrinking effect [25]. At the deviation of the magnetic field from the growth direction, the localization becomes weaker. The probability of hopping between dots increases, and, up to some critical angle ($\theta = 30^\circ$), the hopping between QDs is going inside QD ringlike groups. In this interval of θ, the narrowing of ESR line is observed. At larger θ, the localization radius overcomes some critical value and the random hopping between QD ringlike groups becomes very intensive. This leads to the broadening of a ESR line like in 2D QD arrays. So, when electrons move inside QD finite groups, the narrowing of ESR line is observed, that can be considered as a confirmation of the modeling results. However, the same narrowing can occur due to averaging of local magnetic fields (for example, induced by ^{29}Si isotopes) at an increase of the electron localization radius. To separate these mechanisms of narrowing, it needs to create QD rings with a stronger localization of electrons, where the localization radius has no sensitivity to the external magnetic field. An ESR experiment on QD structures with a strong localization of electrons within QD rings can give an additional confirmation of the theoretical results obtained in the present work.

IV. CONCLUSIONS

In summary, our results demonstrate that there is a principal difference between the spin relaxation of delocalized electrons in semiconductor structures with the absence of inversion symmetry and localized electrons providing the hopping conductivity in QD structures. This difference leads to an insensitivity of spin relaxation rate to the external magnetic field magnitude in infinite QD structures and an unusual field dependence obtained for finite QD structures. These effects originate from the nature of the effective magnetic field (spin-orbit field) δH. In QD system, this field represents short bursts appearing only during tunneling of carriers between QDs, $\delta H \neq 0$ in the time interval τ_t. For delocalized electrons with $k \neq 0$, the effective magnetic field is nonzero all the time, and in presence of the external magnetic field the spin dynamics is controlled by precession with sum frequency $\Omega_L + \Omega_k$, that leads to a well-known suppression of DP spin relaxation in a longitudinal magnetic field [10]. In QD structures, these fields do not act in a simple additive way, because they are essentially separated in time, and this results in an unusual field dependence of the spin relaxation rates.

The external magnetic field itself can not affect the spin relaxation, but in presence of autocorrelations of the effective magnetic field, it is involved in spin relaxation process. For delocalized electrons, the autocorrelations of spin-orbit field are present on a time scale compared with the momentum scattering time τ_k. In infinite QD structures, there are no autocorrelations of spin-orbit field on the time scale compared with

the characteristic time between hopping events τ_h, which plays an equivalent part in the spin dynamics as τ_k. Therefore there is no dependence on the external magnetic field H_0 for infinite QD structures if the dependence of τ_h on H_0 is neglected.

In QD structures with finite sizes, the autocorrelation function of the spin-orbit field significantly changes, the autocorrelations of this field appear on a time scale comparable with the time interval between hopping events, thus causing a dependence of spin relaxation rate on the external magnetic field. The spin relaxation is provided by the vector product of the spin-orbit field and the external magnetic field and this leads to unusual angular dependencies of spin relaxation times. The reported results show that the ordering of QDs and an appropriate choice of the external magnetic field direction enables switching the main spin relaxation mechanism off and a sufficient increase of the spin relaxation times in QD structures.

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