Massless surface waves between two different superfluid phases of ³He

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An interface between two media is a topologically stable two-dimensional object where 3D-symmetry breaks which allows for the existence of many exotic excitations. A direct way to explore surface excitations is to investigate their interaction with the surface waves, such as very well known capillary-gravity waves and crystallization waves. Helium remains liquid down to absolute zero where bulk excitations are frozen out and do not mask the interaction of the waves with the surface states. Here we show the possibility of the massless wave which can propagate along the surface between two different superfluids phases of ³He. The displacement of the surface in this wave occurs due to the transition of helium atoms from one phase to another, so that there is no flow of particles as densities of phases are equal. We calculate the dispersion of the wave in which the inertia is provided by spin supercurrents, and the restoring force is magnetic field gradient. We calculate the dissipation of the wave and show the preferable conditions to observe it.

Watching the waves on the surface of the ocean is probably one of the oldest physical observations. Similar waves can propagate along the surface between any two immiscible fluids, and particularly between liquid and its vapor, see Fig. 1(a). The inertia of these waves is due to the motion of the liquid while the restoring force is due to gravity (long waves) and surface tension (short waves). Unlike most of known waves, dispersion relation of a surface wave is strongly nonlinear,

$$\rho \omega^2 = \rho g q + \alpha q^3, \tag{1}$$

so that there is no phononlike spectra even at $q \to 0$. Here ρ is the density of the liquid, ω is the angular frequency, g is the gravitational acceleration, α is the surface tension, and q is the wave vector. Capillary-gravity waves have been observed on the surface of ⁴He and ³He and have been utilized for high-precision measurements on the surface tension. In this way critical exponents near the critical liquid-gas point have been measured [1].

For the helium liquid-solid interface there opens a possibility for quite an exotic wave, a crystallization wave. At low enough temperatures, where liquid phase is superfluid and thus provides extremely fast mass and heat transport, the interface between solid and liquid becomes mobile enough to support a wave of crystallization and melting. The dispersion relation is similar to that of waves on a free liquid surface, and the only difference is that for the same amplitude of the wave, liquid phase carries smaller mass flux [see Fig. 1(b)] which is proportional to density difference $\Delta \rho$ between the solid and the liquid,

$$\frac{\Delta \rho^2}{\rho} \omega^2 = \Delta \rho g q + \alpha q^3. \tag{2}$$

Crystallization waves have been predicted by Andreev and Parshin in 1978 [2] and discovered by Keshishev *et al.* two years later in ⁴He below 0.5 K [3]. By measuring crystallization

waves at surfaces of different orientations the singularity of the surface tension at the basal *c*-facet orientation has been observed [4].

In this paper we show the possibility of the kind of surface wave which does not have any associated mass flux. Imagine an interface between two immiscible liquids which both consist of the same atoms or molecules. Such an interface will support, in addition to the usual capillary-gravity wave described above, another type of wave which is associated with the transition of particles near the surface from one phase to another one. Basically, such a wave is an analog of the crystallization wave as one of the phases locally "grows" into another phase. The hydrodynamical motion in both liquids is thus only due to the difference in mass densities of the phases.

Generally, it is possible that both phases have the same or nearly the same mass densities. Then such a wave will not have any hydrodynamical inertia and its frequency will be infinitely high at any finite wave vector q unless some other kind of inertia is involved. The simplest example of the discussed system is liquid in contact with its vapor close to the critical liquid-vapor point where densities of both phases equalize. Moreover, at the critical point surface tension also vanishes, and thus there is no restoring force unless some other force stabilizing the surface is introduced.

Although it seems impossible to satisfy all the conditions for the discussed massless wave, nevertheless, there is an example of such a system, a surface between two superfluid phases of helium-3, stabilized with the magnetic field [5,6]. These two phases, being referred to as "A" and "B" phases, have the same mass densities, and the described phase wave on the A-B interface does not require mass transport in the bulk phases, see Fig. 1(c).

However, the A phase has larger magnetic susceptibility and thus prefers large magnetic field. By applying vertical magnetic field gradient ∇H one can stabilize the A phase in the high field region on top of the B phase. The field gradient is therefore an analog of gravity and plays a role of the restoring force of the wave.

As the magnetization of the A phase is larger than the magnetization of the B phase, the motion of the A-B interface
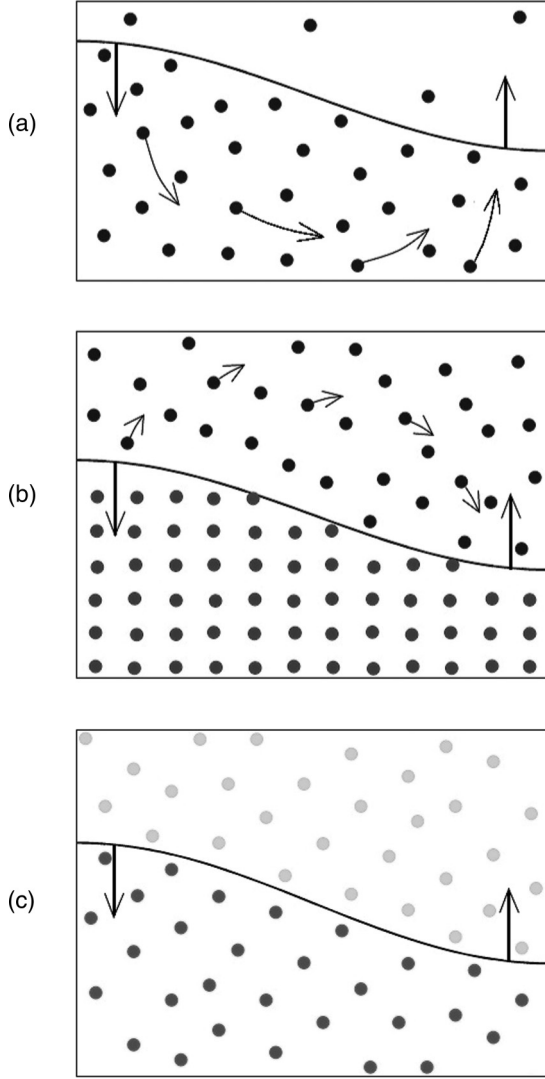
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FIG. 1. Three types of surface waves. (a) Usual capillary-gravity wave: Oscillations of the surface are accompanied with the motion of the liquid in the bulk. (b) Crystallization wave: Oscillations of the surface are accompanied with the much slower motion of the liquid because density of liquid and solid are similar. (c) Phase wave on the surface between two superfluid phases of ^{3}He: No mass transport is needed because densities of both superfluid phases are equal.

is accompanied with the spin currents in both phases. As it was noted by Andreev, spin currents have a kinetic energy [7], and thus play the role of inertia. We therefore have a wave on the surface between two liquids in which there is no mechanical motion at all: ^{3}He particles transit from one phase to another staying at rest, and both kinetic and potential energy of the wave are provided by the spin degree of freedom.

As it was shown by Leggett [8], the spin dynamics in the longitudinal direction can be described in the terms of the angle θ of rotation of the order parameter vector $\vec{d}$ around the field direction. The corresponding equations for the A phase ($z > 0$) and for the B phase ($z < 0$) are [7]:

$$\ddot{\theta}_a - c_a^2 \frac{\partial^2 \theta_a}{\partial z^2} + \Omega_a^2 \theta_a = 0, \quad \ddot{\theta}_b - c_b^2 \frac{\partial^2 \theta_b}{\partial z^2} + \Omega_b^2 \theta_b = 0. \quad (3)$$

Here Ω_a, Ω_b, c_a, and c_b are longitudinal NMR frequencies and spin waves velocities of A and B phases, correspondingly. We consider a slow wave, $\omega \ll \Omega_a, \Omega_b$, for which the solution of (3) decaying at $z = \pm\infty$ is

$$\theta_{a,b}(z,t) = \theta_{0a,0b}(t) \exp\left(\mp \frac{\Omega_{a,b}}{c_{a,b}} z\right). \quad (4)$$

Following Andreev [7], we write the energy density of the spin waves as

$$\varepsilon_{a,b} = \frac{\chi_{a,b}}{2\gamma^2}\left[c_{a,b}^2\left(\frac{\partial \theta_{a,b}}{\partial z}\right)^2 + \Omega_{a,b}^2 \theta_{a,b}^2\right], \quad (5)$$

where χ_a and χ_b are magnetic susceptibilities of the phases, and $\gamma = 2\mu/\hbar$ is the gyromagnetic ratio. After integration over z we find the total kinetic energy of the wave,

$$E_{\text{kin}} = \frac{1}{2}\frac{\chi_a c_a \Omega_a}{\gamma^2}\theta_{0a}^2 + \frac{1}{2}\frac{\chi_b c_b \Omega_b}{\gamma^2}\theta_{0b}^2. \quad (6)$$

The fluxes of the z component of the spin at the boundary are given by

$$j_{a,b} = -\frac{\chi_{a,b}c_{a,b}^2}{\gamma^2}\frac{\partial \theta}{\partial z} = \pm\frac{\chi_{a,b}c_{a,b}\Omega_{a,b}}{\gamma^2}\theta_{0a,0b}. \quad (7)$$

The fluxes are connected by the boundary condition

$$j_a - j_b = \frac{H}{\gamma}(\chi_a - \chi_b)\dot{\zeta}, \quad (8)$$

where $\zeta(x,t) = \zeta_0(t)\exp(iqx)$ is the displacement of the boundary from its equilibrium $z = 0$ position. Using (8) and minimizing kinetic energy (6) we obtain amplitudes of the spin currents,

$$\theta_{0a} = \theta_{0b} = \frac{\gamma H(\chi_a - \chi_b)}{\chi_a c_a \Omega_a + \chi_b c_b \Omega_b}\dot{\zeta}. \quad (9)$$

Substituting amplitudes (9) to (6) we find the kinetic energy of the wave, $E_{\text{kin}} = (1/4)M\dot{\zeta}^2$ with the effective "mass" of spin currents

$$M = \frac{(\chi_a - \chi_b)^2 H^2}{\chi_a c_a \Omega_a + \chi_b c_b \Omega_b}. \quad (10)$$

The potential energy of the wave is due to the field gradient and due to the small surface tension α, $E_{pot} = (1/4)[(\chi_a - \chi_b)H\nabla H + \alpha q^2]\zeta^2$, and the dispersion relation of the wave is

$$\omega^2 = \frac{(\chi_a - \chi_b)H\nabla H + \alpha q^2}{M}. \quad (11)$$

Note that in the limit of long waves frequency does not depend on the wave vector and is determined by only magnetic parameters of superfluid phases and by the geometry of the magnetic field. At low temperatures and low pressure the A-B interface is stabilized in the field of $H_{AB} \approx 0.34$ T [9], and the surface tension is $\alpha \approx 5$ nJ/m^2 [10]. If we assume the field gradient of $\nabla H \sim 10$ T/m, then the characteristic wavelength at which the two terms in Eq. (11) equalize is

$$\lambda_c \sim 2\pi\sqrt{\frac{\alpha}{(\chi_a - \chi_b)H\nabla H}} \sim 1 \text{ mm}. \quad (12)$$

Longer waves are inherently magnetic, and their frequency $\omega_0 \sim \sqrt{c\Omega\nabla H/H}$ does not depend on wave vector but can be tuned by the magnetic field gradient.

The wave can be exited by applying an oscillating magnetic field $h = h_0 \exp(i\omega t)$ parallel to the constant stabilizing field H. This kind of experiment has been done in Lancaster by Bartkowiak *et al.*, who have measured the heat produced by the oscillating A-B interface in the ultra low temperature limit at zero pressure [11]. The oscillating field h causes the equilibrium vertical position of the interface to oscillate with the amplitude $\delta\zeta = h/\nabla H$. The equation of motion of the interface can be written as

$$M\ddot{\zeta} + \Gamma\dot{\zeta} = -H\nabla H(\chi_a - \chi_b)(\zeta - \delta\zeta \exp(i\omega t))$$
$$= -M\omega_0^2(\zeta - h/\nabla H \exp(i\omega t)) \quad (13)$$

where Γ is the friction coefficient. The frequency dependence of the power dissipated in the wave per unit area is given by

$$P = \Gamma\overline{|\dot{\zeta}|^2} = \Gamma\omega^2\overline{|\zeta^2|}$$
$$= \frac{\Gamma}{2}\frac{\omega_0^4\omega^2}{\left(\omega_0^2 - \omega^2\right)^2 + (\omega\Gamma/M)^2}\frac{h^2}{\nabla H^2}. \quad (14)$$

The quality factor of the wave $Q = \omega_0 M/\Gamma$ is proportional to the square root of the effective mass of the wave M which is very small because of the smallness of nuclear susceptibilities, $\chi_a = 3.8 \times 10^{-8}$, $\chi_b = 1.2 \times 10^{-8}$ at zero bar [12]. The longitudinal NMR frequencies in the low temperature limit at high pressure have been measured to be $\Omega_a|_{hp} = 2\pi \times 100$ kHz, $\Omega_b|_{hp} = 2\pi \times 250$ kHz [13–16].

As it was shown by Leggett, the longitudinal resonance frequency is scaled as $\Omega \propto \sqrt{N(0)a}T_c$ where $N(0)$ is the density of states at the Fermi surface, and a is the ratio of the relative specific heat jump to its BCS value (1.43) [17]. Using $N(0)|_{hp} = 1.26 \times 10^{51}$ J^{-1}m^{-3}, $N(0)|_{0bar} = 0.54 \times 10^{51}$ J^{-1}m^{-3} [12], $a|_{hp} = 1.4$, $a|_{0bar} = 1$, $T_c|_{hp} = 2.49$ mK, and $T_c|_{0bar} = 0.93$ mK [18–20] we find $\Omega_a|_{0bar} \simeq 2\pi \times 25$ kHz, and $\Omega_b|_{0bar} \simeq 2\pi \times 65$ kHz. For the estimation of the velocity c of spin waves one can use the relation $c = l(\Omega/2\pi)$ where $l \approx 10\ \mu$m is the dipole healing length. Finally, we find the mass of the wave at high pressures and at zero bar, $M|_{0bar} \approx 8 \times 10^{-9}$ kg/m^2 and $M|_{hp} \approx 2 \times 10^{-9}$ kg/m^2.

Another kind of the inertial mass M^* of the interface originates from the time variation of the order parameter near the moving interface. This mass has been first considered by Yip and Leggett who have given a rough estimate $M^* \sim 10^{-11}$kg/m^2 [21,22]. Roughly, this mass is smaller than the magnetic mass by the factor ξ_0/l, where $\xi_0 = 18\ldots 88$ nm is the coherence length.

There also exists usual material mass of the wave because of the density difference $\Delta\rho_{AB} \sim 10^{-6}$ kg/m^3 between phases [22]. The applied magnetic field increases the density difference because the A phase has large susceptibility. The magnetostriction effect can be estimated as the additional effective pressure $\delta P = (1/2)\chi H^2$ which causes compression of the liquid by an amount $\delta\rho/\rho = \beta\delta P$; where $\beta = 5.7 \times 10^{-8}$ 1/Pa is the compressibility [23]. The additional density difference due to magnetostriction is thus $\Delta\rho_{AB}(H) = (1/2)(\chi_A - \chi_B)H^2\beta\rho \sim 10^{-8}$ kg/m^3 which is negligible compared to zero field difference $\Delta\rho_{AB}$. The contribution to the mass of the wave from density difference of the phases is given by $m = \Delta\rho_{AB}^2/(\rho q) \sim 10^{-17}$ kg/m^2 which is nine orders of magnitude smaller than the spin current mass M.

With the estimated above mass $M \sim 10^{-8}$ kg/m^2 and with the field gradient of few Tesla per meter we can estimate the resonant frequency of the wave ω to be of the order of $2\pi \times 1$ kHz. This value is indeed much smaller than the longitudinal NMR frequency, and thus the condition of slow wave used for calculations of the mass is valid. In the opposite case of fast wave (very short wavelength and/or very strong field gradient), the magnetizations of the bulk phases will not have time to reach their equilibrium values, and both the solution (4) and the boundary condition (8) fail. For this case more complicated analysis is needed.

The friction coefficient Γ is determined by scattering of quasiparticles in the A phase and decreases at ultra low temperatures as T^4 [21,24]. It has been measured at temperatures close to the A-B transition at high pressures in Los Alamos [25], $\Gamma|_{0.75T_c,hp} = 0.07$ kg/(m^{2}s). This value is in very good agreement with theoretical estimation of Kopnin, $\Gamma \approx 7\pi^4 N(0)T^4/(30v_F\Delta_A^2)$ (v_F is Fermi velocity, Δ_A is the BSC energy gap) [24]. The low temperature value of the friction coefficient can be found from the Lancaster experiment [11]. According to Eq. (14), in the case of strong damping $\Gamma/M \gg \omega_0$, the dissipation has a plateau at high frequencies,

$$P_{pl} = \frac{1}{2\Gamma}\frac{\omega_0^4 M^2 h^2}{\nabla H^2} = h^2 H^2(\chi_a - \chi_b)^2\frac{1}{2\Gamma}, \quad (15)$$

which depends only on the amplitude h of the oscillating field and on the friction coefficient Γ. Indeed, such a plateau has been observed in Lancaster at frequencies larger than 10 Hz with the level $P_{pl} = 2 \times 10^{-7}$ W/m^2 independent on the field gradient [11]. The amplitude of the oscillating field used in the Lancaster experiment was $h = 0.64$ mT which gives $\Gamma|_{0.17T_c,0bar} = 3 \times 10^{-5}$kg/(m^{2}s). This is about three times larger than the theoretical estimate.

Due to the smallness of the mass of the wave, the characteristic attenuation rate of the wave is very fast even at ultra low temperatures, $\Gamma|_{0.17T_c,0bar}/M \approx 4 \times 10^3$ s^{-1} which should be compared to $\omega_0 \approx 1.4 \times 10^3$ s^{-1} for the strongest field gradient $\nabla H = 2$ T/m used in Lancaster experiments. To observe resonance of the A-B wave one should increase the field gradient by at least an order of magnitude to shift the resonant frequency ω_0 above the dissipation rate.

The excess by factor of three of the dissipation measured in Lancaster over the theoretical value might be the indication of the contribution of the surface states which should dominate at zero temperature limit. The A-B interface between two fermionic superfluids is probably the richest surface in nature and promises to support a variety of surface states such as Majorana fermions and anyons (particles which are neither bosons nor fermions) [26]. The proposed magnetic massless surface wave could open access to these exotic surface states which may cause additional dissipation and contribute to the mass of the wave.

[1] M. Iino, M. Suzuki, and A. J. Ikushima, J. Low Temp. Phys. **63**, 495 (1986).

[2] A. F. Andreev and A. Y. Parshin, JETP **48**, 763 (1978).

[3] K. O. Keshishev, A. Y. Parshin, and A. V. Babkin, JETP Lett. **30**, 56 (1980).

[4] E. Rolley, C. Guthmann, E. Shevalier, and S. Balibar, J. Low Temp. Phys. **99**, 851 (1995).

[5] D. I. Bradley, Y. M. Bunkov, D. J. Cousins, M. P. Enrico, S. N. Fisher, M. R. Follows, A. M. Guénault, W. M. Hayes, G. R. Pickett, and T. Sloan, Phys. Rev. Lett. **75**, 1887 (1995).

[6] R. Blaauwgeers, V. B. Eltsov, G. Eska, A. P. Finne, R. P. Haley, M. Krusius, J. J. Ruohio, L. Skrbek, and G. E. Volovik, Phys. Rev. Lett. **89**, 155301 (2002).

[7] A. F. Andreev, JETP Lett. **58**, 715 (1993).

[8] A. J. Leggett, Rev. Mod. Phys. **47**, 331 (1975).

[9] M. Bartkowiak, S. W. J. Daley, S. N. Fisher, A. M. Guénault, G. N. Plenderleith, R. P. Haley, G. R. Pickett, and P. Skyba, Phys. Rev. Lett. **83**, 3462 (1999).

[10] M. Bartkowiak, S. N. Fisher, A. M. Guénault, G. R. Pickett, M. C. Rogge, and P. Skyba, J. Low Temp. Phys. **126**, 533 (2002).

[11] M. Bartkowiak, S. N. Fisher, A. M. Guénault, R. P. Haley, G. N. Plenderleith, G. R. Pickett, and P. Skyba, Physica B **284–288**, 240 (2000).

[12] J. C. Wheatley, Rev. Mod. Phys. **47**, 415 (1975).

[13] D. D. Osheroff and W. F. Brinkmann, Phys. Rev. Lett. **32**, 584 (1974).

[14] R. A. Webb, R. L. Kleinberg, and J. C. Wheatley, Phys. Rev. Lett. **33**, 145 (1974).

[15] H. M. Bozler, M. E. R. Bernier, W. J. Gully, R. C. Richardson, and D. M. Lee, Phys. Rev. Lett. **32**, 875 (1974).

[16] D. D. Osheroff, Phys. Rev. Lett. **33**, 1009 (1974).

[17] A. J. Leggett, Phys. Rev. Lett. **29**, 1227 (1972).

[18] W. P. Halperin, C. N. Archie, F. B. Rasmussen, T. A. Alvesalo, and R. C. Richardson, Phys. Rev. B **13**, 2124 (1976).

[19] T. A. Alvesalo, T. Haavasoja, M. T. Manninen, and A. T. Soinne, Phys. Rev. Lett. **44**, 1076 (1980).

[20] D. S. Greywall, Phys. Rev. B **33**, 7520 (1986).

[21] S. Yip and A. J. Leggett, Phys. Rev. Lett. **57**, 345 (1986).

[22] A. J. Leggett and S. K. Yip, *in Helium Three*, edited by W.P. Halperin and L.P. Pitaevskii (North Holland, Amsterdam, 1990).

[23] E. R. Grilly, J. Low Temp. Phys. **4**, 615 (1971).

[24] N. B. Kopnin, Sov. Phys. JETP **65**, 1187 (1987).

[25] D. S. Buchanan, G. W. Swift, and J. C. Wheatley, Phys. Rev. Lett. **57**, 341 (1986).

[26] A. Stern, Nature (London) **464**, 187 (2010).