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Analysis in temporal regime of dispersive invisible structures designed from transformation optics

B. Gralak,* G. Arismendi, B. Avril, A. Diatta, and S. Guenneau

CNRS, Aix-Marseille Université, Centrale Marseille, Institut Fresnel, 13397 Marseille, France

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A simple invisible structure made of two anisotropic homogeneous layers is analyzed theoretically in temporal regime. The frequency dispersion is introduced and analytic expression of the transient part of the field is derived for large times when the structure is illuminated by a causal excitation. This expression shows that the limiting amplitude principle applies with transient fields decaying as the power $-3/4$ of the time. The quality of the cloak is then reduced at short times and remains preserved at large times. The one-dimensional theoretical analysis is supplemented with full-wave numerical simulations in two-dimensional situations which confirm the effect of dispersion.

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In 2006, Pendry *et al.* [1] and Leonhardt [2] designed an invisibility cloak for electromagnetic radiation by blowing up a hole in optical space and hiding an object inside it. These proposals have been validated by microwave experiments [3]. However, these metamaterials are subject to an inherent frequency dispersion which may affect the quality of the optical function designed in time harmonic regime. Hence, there is a renewed interest in the propagation in dispersive media, originally investigated by Brillouin [4]. The effect of dispersion has been addressed in the cases of the flat lens [5–9] and cylindrical invisibility cloaks [10–13].

In this Rapid Communication, a regularized version of Pendry's transform [14] is implemented for the design of the simplest possible system of invisible layers. With this transform, infinities are avoided in the material parameters of the cloak which consists of two homogeneous anisotropic slabs. Frequency dispersion is introduced, which is a required model for metamaterials whenever the permittivity (or permeability) is lower than that of vacuum (i.e., when the phase velocity is greater than c or negative). The effect of dispersion is analyzed with electromagnetic sources with sinusoidal time dependence that are switched on at an initial time. Such an illumination has been originally used by Brillouin [4] in homogeneous dispersive media, and more recently in the case of the negative index flat lens [5–7].

The originality of our approach is to consider a simple invisibility system made of two layers allowing analytic calculations. Indeed, the invisible nature of the system leads to a simple expression of the transmitted field, since there is no reflection at the interfaces. Also, the absence of branch cut in the integral expression of the time dependent field in multilayered structures is exploited. The method is presented in detail and the derivation of the transient regime shows that the electromagnetic field includes contributions generated by the singular values of the permittivity and permeability (zeros and infinities). An explicit expression of the transient fields is obtained for long times, which is similar to the one obtained by Brillouin [4] for wave fronts (forerunners). Next, the limiting amplitude principle is considered to show that cloaking can be addressed in temporal regime after the transient regime. These results are supplemented with numerical simulations in the case of a two-dimensional cylindrical layered cloak, where the presence of additional modes is confirmed in the transient regime.

We start with the definition of a system of invisible layers. Let $\boldsymbol{x} = (x_1,x_2,x_3)$ be a Cartesian coordinate system in the space $\mathbb{R}^3$. At the oscillating frequency ω, the electric field amplitude $\boldsymbol{E}(\boldsymbol{x})$ is governed in free space by the Helmholtz equation

$$-\nabla \times \nabla \times \boldsymbol{E}(\boldsymbol{x}) + \omega^2 \mu_0 \varepsilon_0 \, \boldsymbol{E}(\boldsymbol{x}) = \boldsymbol{0}, \tag{1}$$

where ε_0 and μ_0 are the vacuum permittivity and permeability. The invisible layered structure is then deduced using the coordinate transform $\boldsymbol{x} \to \boldsymbol{x}'$ (see Fig. 1):

$$\begin{aligned} x_1' &= \frac{a}{\alpha} x_1, & 0 \leqslant x_1 \leqslant \alpha, \\ x_1' &= a + \frac{b-a}{b-\alpha}(x_1 - \alpha), & \alpha \leqslant x_1 \leqslant b, \\ x_1' &= x_1, & x_1 \leqslant 0, \quad b \leqslant x_1, \end{aligned} \tag{2}$$

where $0 < a < \alpha < b$, $x_2' = x_2$, and $x_3' = x_3$ being invariant. The effect of this geometric transform is to map the layer $0 \leqslant x_1 \leqslant \alpha$ onto the layer $0 \leqslant x_1' \leqslant a$ (denominated as layer A), and the layer $\alpha \leqslant x_1 \leqslant b$ onto $a \leqslant x_1' \leqslant b$ (denominated as layer B). Note that such a geometric transform, adapted from [15], regularizes the original transform for an invisibility cloak proposed in [1]. The corresponding transform is applied to the Helmholtz equation (1):

$$-\nabla' \times \frac{1}{\mu(x_1')} \nabla' \times \boldsymbol{E}'(\boldsymbol{x}') + \varepsilon(x_1')\, \omega^2 \mu_0 \varepsilon_0 \, \boldsymbol{E}'(\boldsymbol{x}') = \boldsymbol{0}, \tag{3}$$

where the relative permittivity and permeability are both equal to the tensor $\nu \equiv \varepsilon = \mu$ (as in [8]) taking constant values in each layer:

$$\begin{aligned} \varepsilon(x_1') = \mu(x_1') = \nu(x_1') &= \nu_a \quad \text{if} \quad 0 \leqslant x_1' \leqslant a, \\ \varepsilon(x_1') = \mu(x_1') = \nu(x_1') &= \nu_b \quad \text{if} \quad a \leqslant x_1' \leqslant b, \\ \varepsilon(x_1') = \mu(x_1') = \nu(x_1') &= 1 \quad \text{if} \quad x_1' \leqslant 0, \quad b \leqslant x_1'. \end{aligned} \tag{4}$$

The constant values in layers A and B are given by

$$\nu_{a,b} = \begin{bmatrix} \nu_{a,b}^{\perp} & 0 & 0 \\ 0 & \nu_{a,b}^{\parallel} & 0 \\ 0 & 0 & \nu_{a,b}^{\parallel} \end{bmatrix}, \tag{5}$$

*boris.gralak@fresnel.fr

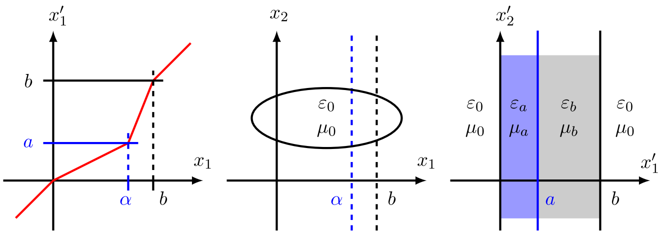

FIG. 1. Coordinate transform for invisible layers. Left: Change of coordinate $x_1 \to x_1'$. Center: Free space before coordinate transform. Right: invisible set of homogeneous anisotropic layers after coordinate transform.

where the components parallel and perpendicular to the plane interfaces, respectively, denoted by the superscripts $\|$ and $\perp$, are

$$\nu_a^\perp = 1/\nu_a^\| = a/\alpha, \quad \nu_b^\perp = 1/\nu_b^\| = (b-a)/(b-\alpha). \tag{6}$$

The transformed Helmholtz equation (3) can be reduced to a set of two independent scalar equations using the symmetries of the geometry, namely, the invariances under the translations and rotations in the plane (x_2',x_3'). After a Fourier decomposition from (x_2',x_3') to (k_2',k_3'), Eq. (3) becomes

$$\frac{\partial}{\partial x}\frac{1}{\nu^\|(x)}\frac{\partial U}{\partial x}(x) - \frac{k^2}{\nu^\perp(x)}U(x) + \frac{\omega^2}{c^2}\nu^\|(x)U(x) = 0, \tag{7}$$

for $U(x)$, the (Fourier transformed) electric field component along direction $(-k_3,k_2)$. Here, x denotes x_1', k^2 equals $k_2^2 + k_3^2$ (with $k_2 = k_2'$ and $k_3 = k_3'$), $c = 1/\sqrt{\varepsilon_0\mu_0}$ is the light velocity in vacuum, and functions $\nu^\|(x)$ and $\nu^\perp(x)$ are the components of $\nu(x)$ parallel and perpendicular, respectively, to the plane interfaces. Notice that, since $\varepsilon = \mu$, the second scalar equation derived from the Helmholtz equation is fully identical to (7), except that $U(x)$ should be the (Fourier transformed) magnetic field component along direction $(-k_3',k_2')$ [or $(-k_3,k_2)$].

In this Rapid Communication, the system is analyzed using a transfer matrix formalism [16]. Equation (7) is formulated as

$$\frac{\partial}{\partial x}F(x) = -iM(x)F(x), \tag{8}$$

where

$$F = \begin{bmatrix} U \\ \dfrac{i}{\nu^\|}\dfrac{\partial U}{\partial x}\end{bmatrix}, \quad M = \begin{bmatrix} 0 & \nu^\| \\ \dfrac{\omega^2}{c^2}\nu^\| - \dfrac{k^2}{\nu^\perp} & 0\end{bmatrix}. \tag{9}$$

The transfer matrices T_a and T_b, associated with layers A and B, defined by $F(a) = T_aF(0)$ and $F(b) = T_bF(a)$, are given by

$$T_a = \exp[-iM_0\alpha], \quad T_b = \exp[-iM_0(b-\alpha)], \tag{10}$$

the matrix M_0 being the value taken by the matrix $M(x)$ in vacuum, i.e., when $\nu^\|(x) = \nu^\perp(x) = 1$. This implies that the transfer matrix $T_bT_a = \exp[-iM_0b]$, associated with layers A and B, is exactly the same as that of a vacuum layer of thickness b. Hence the system of layers A and B is invisible to any incident field.

Nevertheless, as pointed out by Veselago when he introduced negative index materials [17], causality principle and passivity require that permittivity and permeability be frequency dispersive when they take relative value below unity [18,19]. According to this requirement, frequency dispersion is introduced in the components of ν_a and ν_b with value below unity, assuming the simple Drude-Lorentz model [19]:

$$\begin{aligned} \nu_a^\perp(\omega) &= 1 - \frac{\Omega_a^2}{\omega^2-\omega_a^2}, & \Omega_a^2 &= \frac{\alpha-a}{\alpha}(\omega_0^2-\omega_a^2), \\ \nu_b^\|(\omega) &= 1 - \frac{\Omega_b^2}{\omega^2-\omega_b^2}, & \Omega_b^2 &= \frac{\alpha-a}{b-a}(\omega_0^2-\omega_b^2). \end{aligned} \tag{11}$$

Under this assumption, the functions $\nu_a^\perp(\omega)$ and $\nu_b^\|(\omega)$ take the appropriate values for the invisibility at $\omega = \omega_0$. Notice that the resonance frequencies ω_a and ω_b must be smaller than the operating frequency ω_0 in order to ensure that the oscillator strengths Ω_a^2 and Ω_b^2 are positive. For frequencies different from ω_0, the system has no reason to be invisible.

The effect of dispersion is analyzed using illumination with sinusoidal time dependence oscillating at ω_0 and switched on at an initial time. Such a "causal" incident field, originally used by Brillouin [4] and more recently in [5–7], is assumed to be in normal incidence for simplicity. Hence the following current source is considered:

$$S(x,t) = S_0\delta(x-x_0)\theta(t)\sin[\omega_0 t], \tag{12}$$

where δ is the Dirac "function," $\theta(t)$ the step function (equal to 0 if $t<0$ and 1 otherwise), and S_0 the constant component of the source parallel to the field component $U(x)$.

In the domain of complex frequencies $z = \omega + i\eta$, the electric field radiated in vacuum by this source is

$$U_0(x,z) = \frac{S_0\mu_0 c}{2}\frac{\omega_0}{z^2-\omega_0^2}\exp[iz|x-x_0|/c]. \tag{13}$$

The positive imaginary part η has been added to the frequency ω to ensure a correct definition of the Fourier transform with respect to time of the source (12). The time dependent incident field radiated in vacuum is, with $z = \omega + i\eta$,

$$\begin{aligned} E_0(x,t) &= \frac{1}{2\pi}\int_{\mathbb{R}} d\omega\, \exp[-izt]U_0(x,z) \\ &= -\frac{S_0\mu_0 c}{2}\theta(t-|x-x_0|/c) \\ &\quad \times \sin[\omega_0(t-|x-x_0|/c)]. \end{aligned} \tag{14}$$

The next steps are to compute the time dependent field transmitted through the system, and to analyze the behavior of the field when the time t tends to infinity. According to the limiting amplitude principle, the solution should have an asymptotic behavior corresponding to the time harmonic frame oscillating at the frequency ω_0. Let $T(\omega)$ be the transmission coefficient of the system made of layers A and B. Then, the time dependent electric field is, for $x > b$,

$$E_T(x,t) = \frac{1}{2\pi}\int_{\mathbb{R}} d\omega\, \exp[-iz(t-\{x-b\}/c)]\,U_0(0,z)T(z). \tag{15}$$

At this stage, it is stressed that, for a fixed incident angle, the transmission coefficient $T(z)$ does not contain any square root of the permittivities and permeabilities of the layered system and of the complex frequency z. This remarkable property, which remains true for any multilayered structure, underpins the present technique since it removes all branch cuts in the evaluation of the integral of the transmitted field. This is an advantage in comparison with the method used by Brillouin for the analysis of wave propagation in dispersive media [4]. The expression of the transmitted field is thus given by the sum of the contributions from all the poles in the function $f(z)$ under the integral in (15).

The poles of the factor $U_0(0,z)$ at $z = \pm\omega_0$ [see Eq. (13)] provide the contribution at the operating frequency ω_0,

$$E_T^{(0)}(x,t) = -\frac{S_0\mu_0 c}{2}\theta(t - \{x - x_0 + \alpha - a\}/c) \times \sin[\omega_0(t - \{x - x_0\}/c)], \tag{16}$$

corresponding to the time harmonic solution for which the system is invisible. This contribution vanishes for times such that ct is smaller than $x - x_0 + \alpha - a = x + |x_0| + \alpha - a > x + |x_0|$, instead of $x - x_0 = x + |x_0|$. This is not surprising since the dispersion has not been taken into account in both parallel permittivity and permeability $\varepsilon_a^\parallel = \mu_a^\parallel = \nu_a^\parallel > 1$ of layer A: hence the corresponding delay $(\alpha - a)/c$ is retrieved in the above expression.

The poles of the transmission coefficient are determined from the expression

$$T(z) = \exp[iz\{\alpha + (b - a)\nu_b^\parallel(z)\}/c]. \tag{17}$$

Next, replacing $\nu_b^\parallel(z)$ by the dispersive model (11) yields

$$T(z) = \exp[iz(\alpha + b - a)/c] \exp\left[-i\frac{z(b - a)}{c}\frac{\Omega_b^2}{z^2 - \omega_b^2}\right]. \tag{18}$$

Thus the transmission coefficient has two isolated singularities at $z = \pm\omega_b$. It is shown in the Supplemental Material [20] that the residues associated with these singularities exist, and can be estimated for large values of the relative time

$$\tau = t - \frac{x - x_0 + \alpha - a}{c} \gg \beta = \frac{(b - a)\Omega_b^2}{2\omega_b^2 c}. \tag{19}$$

The resulting contribution $E_T^{(b)}$ in the transmitted field is

$$E_T^{(b)}(x,t) \underset{\tau/\beta\to\infty}{\approx} -2S_0\mu_0\pi c\frac{\omega_0\omega_b}{\omega_b^2 - \omega_0^2}\theta(\tau)\frac{1}{\sqrt{\tau/\beta}} \times J_1(2\omega_b\beta\sqrt{\tau/\beta})\cos[\omega_b(\tau + \beta/2)], \tag{20}$$

where J_1 is the Bessel function (see the Supplemental Material). It is stressed that a similar behavior, given by the Bessel function J_1 with argument proportional to $\sqrt{\tau}$, has been highlighted by Brillouin [4] but for short relative time τ (forerunners). In both cases, J_1 is a consequence of the dispersion given by the Drude-Lorentz model (11), but for different frequency ranges: near the resonance frequencies $\pm\omega_b$ in the present case, and for the high frequencies in the case considered by Brillouin (forerunners). Forerunners at $\tau \to 0$ can be also characterized here.

The asymptotic form $J_1(u) \approx \sqrt{2/(\pi u)}\cos[u - 3\pi/4]$ provides an explicit expression for long time $\tau \gg \beta$. The contribution in the transmitted field becomes

$$E_T^{(b)}(x,t) \underset{\tau/\beta\to\infty}{\approx} -2S_0\mu_0 c\frac{\omega_0\omega_b}{\omega_b^2 - \omega_0^2}\frac{\sqrt{\pi}}{\sqrt{\omega_b\beta}}\theta(\tau) \times (\tau/\beta)^{-3/4}\cos[2\omega_b\beta\sqrt{\tau/\beta} - 3\pi/4] \times \cos[\omega_b\beta(\tau/\beta + 1/2)]. \tag{21}$$

This expression shows that this second contribution has a first factor oscillating at the frequency ω_b and a second factor with more complex oscillating behavior with argument $\Omega_b\sqrt{2(b - a)\tau/c}$. The amplitude of this contribution decreases like $(\omega_b\tau)^{-3/4}$, and thus the total transmitted electric field

$$E_T(x,t) \underset{\tau/\beta\to\infty}{\approx} -\frac{S_0\mu_0 c}{2}\theta(\tau)\sin[\omega_0(\tau + \{\alpha - a\}/c)] \tag{22}$$

tends to the field radiated in vacuum (14) for long enough time τ, and cloaking is addressed. Hence the limiting amplitude principle applies here, unlike for the perfect lens [5,7].

The situation where small absorption is included can be considered: the resonance frequencies $\pm\omega_b$ are replaced by $\pm\omega_b - i\gamma$ with $\gamma > 0$ in (11) while Ω_b remains positive. The main change in the second contribution (21) is the presence of the additional factor $\exp[-\gamma\tau]$, which makes the permanent regime (purely oscillating at the operating frequency ω_0) easier to handle. Notice that the argument of the Bessel function, $2\omega_b\beta\sqrt{\tau/\beta} = \Omega_b\sqrt{2(b - a)\tau/c}$, is independent of ω_b and thus absorption has no influence on the behavior governed by this function. Finally, it is stressed that the introduction of small absorption affects the transmission coefficient at the operating frequency ω_0 by an attenuation of $\exp[-\gamma(b - a)/c]$, which results in a signature of the invisible structure.

TABLE I. Relative permittivity values of the layered cloak from inside (layer 1) to outside (layer 20).

Layer	1	2	3	4	5	6	7	8	9	10
$\varepsilon/\varepsilon_0$	0.0012	8.0	0.02	8.0	0.07	8.0	0.12	8.0	0.18	8.0
Layer	11	12	13	14	15	16	17	18	19	20
$\varepsilon/\varepsilon_0$	0.24	8.0	0.3	8.0	0.38	8.0	0.44	8.0	0.5	8.0

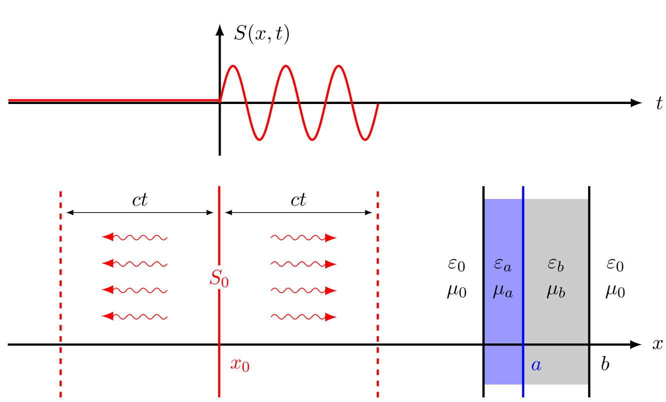

FIG. 2. Excitation of the system. Top: Causal current source with sinusoidal time dependence. Bottom: Field radiated by the causal source and illuminating the invisible layers.

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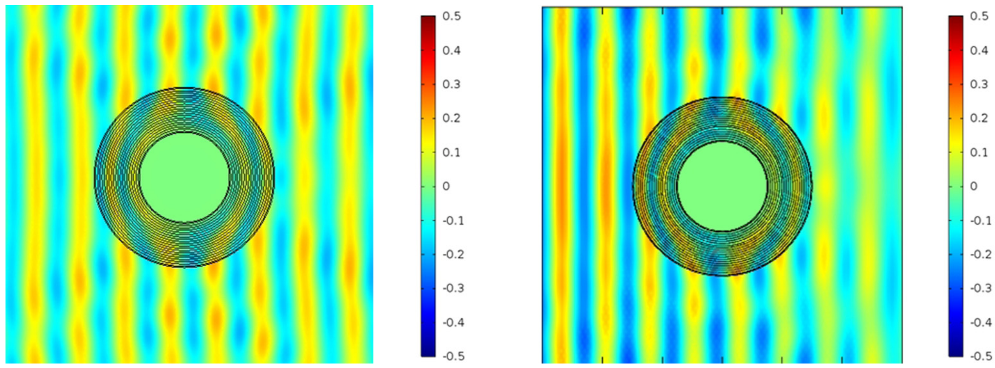

FIG. 3. Magnetic field in the presence of the cylindrical cloak when illuminated by a time harmonic plane wave (left) and by the causal incident field given by Eq. (12) and Fig. 2 (right).

In oblique incidence, expressions are more complicated since reflections occur at the different interfaces. However, the term $-k^2/\nu_a^\perp$ in (9) leads to a singularity at the frequency ω_p for which $\nu_a^\perp$ vanishes:

$$\nu_a^\perp(\omega_p) = 0, \quad \omega_p = \pm\sqrt{\omega_a^2 + \Omega_a^2}. \tag{23}$$

This singularity generates an additional contribution at the frequency ω_p, as well as the singularity at ω_b. It is found that both singularities $\nu \to 0$ and $\nu \to \infty$ lead to additional contributions of the field in temporal regime. This result confirms the well-known difficulties associated with cloak's singularities [15].

The analytical results are numerically tested in the case of a cylindrical cloak designed using homogenization techniques [21,22]. This cloak is a concentric multilayered structure of inner radius R_1 and outer radius $R_2 = 2R_1$, consisting of 20 homogeneous layers of equal thickness $R_1/20$ and made of nondispersive dielectrics (see Table I for the values of relative permittivities, the relative permeability being unity).

The left panel of Fig. 3 shows that the cylindrical cloak works almost perfectly in time harmonic regime oscillating at the frequency $\omega_0 = 2\pi c/\lambda_0$, where $\lambda_0 = R_2/2$. Note that a purely dielectric structure is used for this two-dimensional (2D) cloak, and thus interfaces between different concentric layers are subject to reflections producing effective dispersion. Hence, it is expected to observe an effect of dispersion even if all the dielectric layers are nondispersive [16]. The right panel of Fig. 3 shows the longitudinal magnetic field amplitude when the cloak is illuminated by the causal incident field given by Eq. (12) and Fig. 2.

The cloaking effect appears to be of similar quality in both panels of Fig. 3. We now analyze the magnetic field at short times. In Fig. 4, cylindrical modes are excited in the multilayers when the incident front wave reaches the cloak (left), what produces a superluminal concentric wave (see [23] for a design without supraluminal component). These modes can propagate in the cloak faster than the front wave in vacuum since the frequency dispersion is not introduced in the dielectrics, especially those with index values below unity. The cylindrical modes excited in the multilayers then radiate cylindrical waves outside the cloak, as evidenced by the right panel in Fig. 4, which explains the tiny perturbation of the field observed in right panel of Fig. 3 (the field perturbation is smoothed down at long times, in agreement with the analytical part). In addition, Fig. 4 shows a picture of the transient part of the field produced by the causal source. Here, we take benefit of the supra-luminal propagation of the modes in the cloak to observe that the radiated transient part is almost isotropic. We deduce that the radial dependence of this transient part does not correspond to the function J_1 found by Sommerfeld and Brillouin [4], and exhibited in the present Eq. (20). There is no contradiction since the J_1 dependence is clearly related to the Drude-Lorentz model of the dispersion, while the transient field around the 2D cloak is related to the effective dispersion produced by the cylindrical multilayered geometry. Nonetheless, one

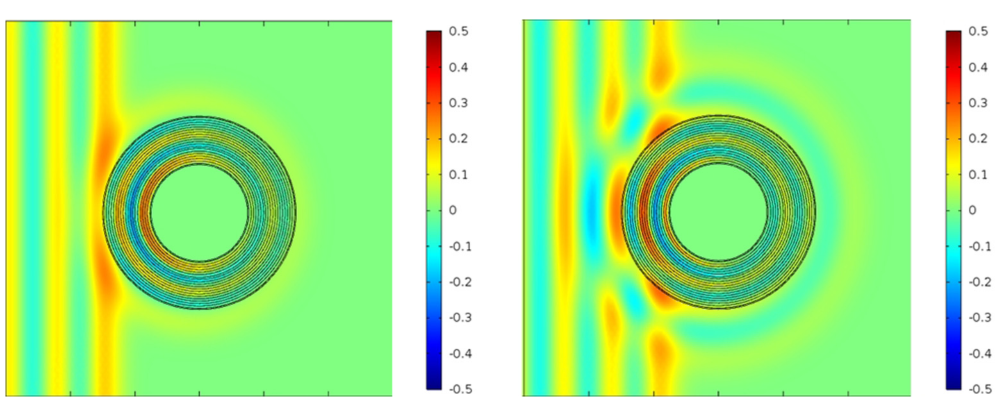

FIG. 4. Magnetic field in the presence of the cylindrical cloak when illuminated by the causal incident field at two time steps in the transient regime. Cylindrical modes inside the cloak generate a supra-luminal concentric wave.

can conclude that both situations considered in this Rapid Communication attest that the quality of cloaking deteriorates at short times under illumination by a causal incident field.

In summary, a new method to analyze propagation of electromagnetic waves in dispersive media has been proposed. The major ideas are to consider a layered structure to eliminate branch cuts, and an invisible structure (with $\varepsilon = \mu$) to eliminate reflections in normal incidence. In this situation, the transient regime can be highlighted and, especially, an explicit expression is obtained in the long time limit. As a result the amplitude of the transient part decreases like $(t - x/c)^{-3/4}$. Hence the technique proposed in this Rapid Communication brings new elements to the method used by Brillouin [4], where wave fronts (forerunners) can be simply exhibited. The analysis of the transient regime in the situation of the invisible structure has shown that the singularities of the permittivity and permeability generate additional contributions to the electric field. However, in normal incidence, the contributions vanish in the long time limit, thus cloaking is achieved after the transient regime. Finally, numerical simulations for a two-dimensional cylindrical layered cloak confirm the effect of dispersion, which affects the quality of cloaking at short times when it is illuminated by a causal incident field.

The proposed method opens new possibilities for investigating transient regime of dispersive systems, notably structures designed from transformation optics like cloaks, carpets, concentrators, and rotators. This method can be also applied to optical systems moving at constant relativistic velocity [24] and to other wave equations.

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