Universal spin Hall conductance fluctuations in chaotic Dirac quantum dots

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We present complete analytical and numerical results that demonstrate the anomalous universal fluctuations of the spin Hall conductance in chiral materials such as graphene and topological insulators. We investigate both the corresponding fluctuations, the universal fractionated and the universal quantized, and also the open channel orbital number crossover between the two regimes. In particular, we show that the Wigner-Dyson symmetries do not properly describe such conductances and the preponderant role of the chiral classes on the Dirac quantum dots. The results are analytical and solve outstanding issues.

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I. INTRODUCTION

The electronic transport through diffusive and ballistic mesoscopic devices has long been the subject of many theoretical [1,2] and experimental investigations [3]. The mesoscopic transport engenders some peculiar physical features [1,2], with special focus on universal conductance fluctuations (UCFs) [4–7]. The UCFs occur owing to quantum interferences and chaotic scattering processes, which give rise to the sample-to-sample fluctuations of charge conductances. It was found that in metals and semiconductors the electronic transport description is accommodated in Wigner-Dyson universal classes in the framework of random matrix theory (RMT) [1,8]. The universal classes are characterized by the presence or absence of two fundamental symmetries of nature: time-reversal (TRS) and spin-rotation symmetries (SRS) [8].

However, the control [9] of novel Dirac materials (such as graphene and topological insulators) introduces new fundamental symmetries, such as chiral/subÂ-lattice/mirror (CHS) and particle-hole symmetries, which became protagonists of a myriad of interesting quantum effects [10]. Accordingly, novel RMT ensembles emerge, which are known as chiral [11] and Atland-Zimbauer [12] universal classes. The chiral universal classes can be applied to bipartite systems, such as hexagonal and square lattices, whose main examples are graphene structures and topological insulators, respectively. There are three chiral classes: a chiral circular orthogonal ensemble (chCOE), characterized by the presence of CHS, TRS, and SRS ($\beta = 1$), a chiral circular unitary ensemble (CUE), which preserves CHS and has the TRS broken by an external magnetic field ($\beta = 2$), and a chiral circular symplectic ensemble (chCSE), which is characterized by the presence of CHS, TRS, and has the SRS broken by a strong spin-orbit interaction (SOI) ($\beta = 4$). Moreover, Atland-Zimbauer universal classes can be applied to electronic devices in contact with a superconductor.

With the generation and control of pure spin current through mesoscopic devices, investigations on its universal fluctuations became very compelling [13]. Motivated by Ref. [14], which found the survival of the spin Hall effect (SHE) in disordered two-dimensional (2D) mesoscopic devices, Ren *et al.* [15] show numerically, using tight-binding Hamiltonian models, that diffusive mesoscopic samples with strong SOI exhibit universal fractionated spin Hall conductance fluctuations (UFSCFs). The authors find a universal amplitude given by $\mathrm{rms}[G^f_{\mathrm{sH}}] \approx 0.18e/4\pi$. In order to give an explanation in the framework of RMT, Bardarson *et al.* [16] used a Landauer-Büttiker approach and a Wigner-Dyson universal class with strong SOI to obtain a analytical expression for UFSCFs of a chaotic quantum dot, confirming the result of Ref. [15].

Furthermore, the 2D Dirac material exhibits the famous *quantized* spin Hall effect, which mean that spin Hall conduction G^q_{sH} takes only integer multiples of $e/4\pi$ [17,18]. Motivated by this novel feature, Qiao *et al.* [19] show numerically, using three tight-binding Hamiltonian models, that diffusive 2D Dirac samples with and without strong SOI exhibit universal quantized spin Hall conductance fluctuations (UQSCFs) with an amplitude given by $\mathrm{rms}[G^q_{\mathrm{sH}}] = 0.285 \pm 0.005e/4\pi$, which does not follow the conventional value obtained in Refs. [15,16]. In recent work, Choe and Chang [20] study electronic transport numerically in a 2D Dirac device with strong SOI and CHS. Nevertheless, their studies were not enough to reach a definitive understanding of the behavior in terms of a UQSCF obtained in Ref. [19]. Thus, an understanding of UFSC remains open to date.

In this paper, we investigate analytically the UQSCF in a chaotic quantum dot with CHS, also known as a chaotic Dirac quantum dot [21,22], at low temperature. We assume a preserved TRS, SRS broken by a strong SOI (chCSE), and a mean dwell time of electrons in the quantum dot that is larger than the SOI time, $\tau_{\mathrm{dwell}} \gg \tau_{\mathrm{so}}$. We identified two regimes: The first one, when the CHS is broken, gives rise to the UFSCF and exhibits an amplitude $\mathrm{rms}[G^f_{\mathrm{sH}}] \approx \sqrt{2} \times 0.18e/4\pi$; The second one, when CHS is preserved, gives rise to the UQSCF, which assumes an amplitude $\mathrm{rms}[G^q_{\mathrm{sH}}] \approx 0.283e/4\pi$. The last is in agreement with Ref. [19].

II. THEORETICAL FRAMEWORK

We consider a multiterminal 2D device with CHS symmetry where the electrons flow under the influence of a strong SOI at low temperature. The 2D device is connected by ideal point contacts to four independent electronic reservoirs, as depicted in Fig. 1. We use the Landauer-Büttiker approach to write the α-direction spin-resolved current through the terminals as [23]

$$I^\alpha_i = \frac{e^2}{h} \sum_{j,\alpha'} \tau^{\alpha\alpha'}_{i,j} (V_i - V_j). \qquad (1)$$

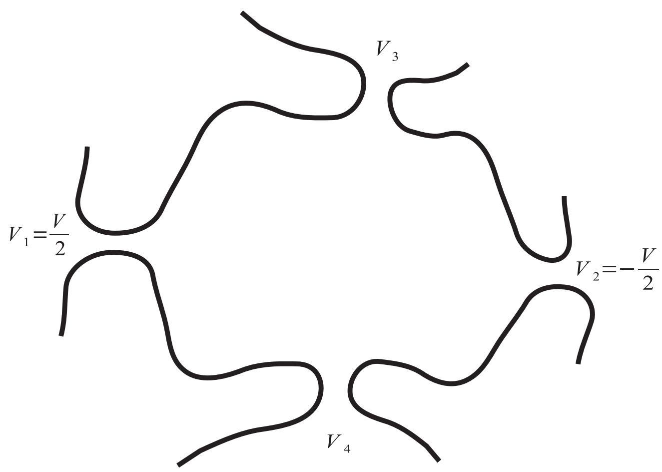

FIG. 1. A chaotic Dirac quantum dot connected through four leads to electron reservoirs with spin resolution. The 1 and 2 leads have specific potentials, $V/2$ and $-V/2$, while the 3 and 4 leads have potentials adjusted in a way that prevents the charge current from flowing. Therefore, a charge current between the 1 and 2 leads induces a spin current between the 3 and 4 leads.

The spin-dependent transmission coefficients can be obtained through $\tau_{i,j}^{\alpha\alpha'} = \sum_{m\in i,n\in j} |\mathcal{S}_{m,\alpha;n,\alpha'}|^2$, where α and α' are the spin projections in the $\alpha = x$, y, or z direction and $\mathcal{S}$ is the scattering matrix of order $2\bar{N}_T \times 2\bar{N}_T$. The $\mathcal{S}$ matrix describes the transport of electrons through the chaotic Dirac quantum dot. The total number of open orbital scattering channels is $\bar{N}_T = 2N_T = \sum_{i=1}^4 2N_i$, where $2N_i$ is the number of open channels in the ith lead point contact and the factor 2 came from two sublattices of the Dirac materials [24].

An applied bias voltage V between longitudinal electrodes 1 and 2 gives rise to a longitudinal electronic current I and, due to the *quantized* spin Hall effect, to spin currents $I_i^\alpha = I_i^1 - I_i^1 \neq 0$ at the transversal contacts 3 and 4 with $\alpha = x,y,z$ [25], as depicted in Fig. 1. Moreover, we consider the absence of net electric charge current at the transverse 3 and 4 leads, i.e., $I_i^0 = I_i^1 + I_i^1 = 0$. Therefore, the charge conservation implies $I = I_1^0 = -I_2^0$. Using those constraints in Eq. (1), it was shown in Ref. [16] that the transversal spin currents can be written as

$$J_i^\alpha = \frac{1}{2}\left(\tau_{i2}^\alpha - \tau_{i1}^\alpha\right) - \sum_{j=3,4} \tau_{ij}^\alpha \bar{V}_j, \qquad (2)$$

for which we introduce the dimensionless currents $J = h/e^2(I/V)$ and the effective transverse voltages $\bar{V}_i = V_i/V$, given by

$$\bar{V}_i = \frac{1}{2}\frac{\tau_{ij}^0\left(\tau_{j2}^0 - \tau_{j1}^0\right) + \left(\tau_{i2}^0 - \tau_{i1}^0\right)\left(4N_j - \tau_{jj}^0\right)}{\tau_{43}^0\tau_{34}^0 - \left(4N_3 - \tau_{33}^0\right)\left(4N_4 - \tau_{44}^0\right)}, \qquad (3)$$

where here $i,j = 3,4$ with $i \neq j$.

III. RANDOM MATRIX THEORY

Our calculation consists in the obtention of the average and the fluctuation amplitude of transversal spin currents, Eq. (2), for a chaotic Dirac quantum dot, within the framework of RMT. Therefore, the spin-dependent transmission coefficient can be written in an appropriate way through the following relation,

$$\tau_{ij}^\alpha = \mathrm{Tr}\left[\mathcal{P}_i^\alpha \mathcal{S} \mathcal{P}_j^0 \mathcal{S}^\dagger\right], \quad \alpha = 0,x,y,z \qquad (4)$$

where the scattering matrix $\mathcal{S}$ is a member of the chCSE ensemble, which means the system preserves the TRS (absence of magnetic field) and the SRS is broken by strong SOI [11]. The matrix $\mathcal{P}_i^\alpha = \mathcal{P}_i \otimes \sigma^\alpha$ represents a projector operator over the ith terminal. Its dimension is $2\bar{N}_T \times 2\bar{N}_T$ and its entries are $(\mathcal{P}_i^\alpha)_{m\mu,n\gamma} = \delta_{mn}\sigma_{\mu\gamma}^\alpha$, while $\sum_{j=1}^{i-1} 2N_j < m < \sum_{j=1}^{i} 2N_j$ and 0 otherwise [16]. The σ^0 and σ^α are the identity matrix and Pauli matrices, respectively.

The scattering matrix of Eq. (4) has an additional CHS symmetry, implying it satisfies the following commutation relation [10],

$$\mathcal{S} = \Sigma_z \mathcal{S}^\dagger \Sigma_z, \quad \Sigma_z \equiv \begin{bmatrix} \mathbf{1}_{\bar{N}_T} & 0 \\ 0 & -\mathbf{1}_{\bar{N}_T} \end{bmatrix}, \qquad (5)$$

at the Dirac point (null Fermi energy). To obtain the average of Eq. (4), it is convenient to decompose $\mathcal{S}$ as a function of the $\mathcal{U}$ matrix, which is a symplectic matrix of order $2\bar{N}_T \times 2\bar{N}_T$, as $\mathcal{S} = \Sigma_z \mathcal{U}^\dagger \Sigma_z \mathcal{U}$, as can be seen in Ref. [26]. Hence, Eq. (4) can be rewritten as

$$\tau_{ij}^\alpha = \mathrm{Tr}\left[\mathcal{P}_i^\alpha \mathcal{U}^\dagger \Sigma_z \mathcal{U} \mathcal{P}_j^0 \mathcal{U}^\dagger \Sigma_z \mathcal{U}\right], \quad \alpha = 0,x,y,z. \qquad (6)$$

Using the method of Ref. [22], developed for diagrammatic integration over chaotic Dirac quantum dots, we calculate the average and covariance of the spin-dependent transmission coefficient of Eq. (6). First, for the average of Eq. (6), we obtain

$$\left\langle \tau_{ij}^\alpha \right\rangle = 4\delta_{\alpha 0}\frac{N_T(4N_iN_j - \delta_{ij}N_i) + \delta_{ij}N_i}{(2N_T - 1)(N_T + 1)}. \qquad (7)$$

Equation (7) is quite distinct from the result of Ref. [16], which studied the UFSCF of a Schrödinger (Wigner-Dyson) chaotic quantum dot. Second, following the same method of Ref. [22], the covariance of Eq. (6) reads

$$\mathrm{covar}[\tau_{ij}^\alpha, \tau_{kl}^\beta] = \left\langle \left(\tau_{ij}^\alpha - \left\langle\tau_{ij}^\alpha\right\rangle\right)\left(\tau_{kl}^\beta - \left\langle\tau_{kl}^\beta\right\rangle\right)\right\rangle$$

and prompted us to find 11 025 diagrams, of which 6300 are non-null. Therefore, the overall result for the covariance of the spin-dependent transmission coefficient involves 6300 terms and the algebraic final expression is cumbersome. Nevertheless, for the relevant configuration wherein $\alpha = \beta \neq 0$ and $i = k$, the general expression simplifies to

$$\frac{\mathrm{covar}\left[\tau_{ij}^\alpha, \tau_{il}^\alpha\right]}{D} = \begin{cases} (N_T - N_i)(4N_iN_T - 2N_T - 3), & i = j = l, \\ -4N_jN_lN_T, & i \neq j \neq l, \\ N_j\left(4N_T^2 - 4N_jN_T - 2N_T - 3\right), & i \neq j = l, \\ -N_l(4N_iN_T - 2N_T - 3), & i = j \neq l, \\ -N_j(4N_iN_T - 2N_T - 3), & i = l \neq j, \end{cases} \qquad (8)$$

where $D = \frac{128N_iN_T}{(4N_T+3)(16N_T^2-1)(2N_T-3)(2N_T-1)}$.

IV. UNIVERSAL SPIN HALL CONDUCTANCE FLUCTUATIONS

We begin by using Eq. (7) to obtain the ensemble average of the transverse spin currents of Eq. (2). The result is

$$\langle J_i^\alpha \rangle = \frac{1}{2}(\langle \tau_{i2}^\alpha \rangle - \langle \tau_{i1}^\alpha \rangle) - \sum_{j=3,4} \langle \tau_{ij}^\alpha \rangle \langle \bar{V}_j \rangle = 0, \tag{9}$$

as expected even for $i = 3,4$ and $\alpha = x,y,z$. The ensemble average of the effective transverse voltages, Eq. (3), is obtained with the previous spin-dependent transmission coefficient general result for the covariance. We obtain

$$\langle \bar{V}_{3,4} \rangle = \frac{1}{2} \frac{N_1 - N_2}{N_1 + N_2}. \tag{10}$$

The same expression was obtained in Ref. [16], indicating that the effective transverse voltage is CHS independent.

Although the transversal spin current ensemble average vanishes, Eq. (10) establishes an effective non-null transverse voltage. Accordingly, the transversal spin current J_i^α assumes finite values. The amplitude of the fluctuations of J_i^α is given by [16]

$$\begin{aligned} \mathrm{var}[J_i^\alpha] = {} & \frac{1}{4} \sum_{j=1,2} \mathrm{var}[\tau_{ij}^\alpha] - \frac{1}{2} \sum_{j=1,2} \mathrm{covar}[\tau_{i1}^\alpha, \tau_{i2}^\alpha] \\ & + \sum_{j=3,4} \left(\mathrm{var}[\tau_{ij}^\alpha] \langle \bar{V}_j^2 \rangle + \mathrm{covar}[(\tau_{i1}^\alpha - \tau_{i2}^\alpha), \tau_{ij}^\alpha] \langle \bar{V}_j \rangle \right) \\ & + 2\, \mathrm{covar}[\tau_{i3}^\alpha, \tau_{i4}^\alpha] \langle \bar{V}_3 \rangle \langle \bar{V}_4 \rangle. \end{aligned} \tag{11}$$

Substituting Eqs. (8) and (10) in Eq. (11), we obtain the following expression,

$$\mathrm{var}[J_i^\alpha] = \frac{128 N_i N_1 N_2 N_T (4N_T^2 - 2N_T - 3)}{(4N_T + 3)(16N_T^2 - 1)(2N_T - 3)(2N_T - 1)(N_1 + N_2)}, \tag{12}$$

which is the main result of our work. Equation (12) is quite distinct from the main result of Ref. [16], and reveals the full difference between the spintronics of a chaotic Dirac (CHS) quantum dot and a chaotic Schrödinger (Wigner-Dyson) quantum dot. In Fig. 2, we plot the average (9) and variance (12) of transverse spin current J_i^α for $i = 3,4$ and $\alpha = x,y,z$ as a function of both symmetric open leads, $N_1 = N_2 = N_3 = N_4 = N$, and asymmetric open leads, $N_1 = N_3 = 2N_2 = 2N_4 = N$.

Using Eq. (12), we can analyze two relevant regimes of the chaotic Dirac quantum dots: the first one for the broken CHS, Eq. (5), and the second one when it is preserved. In accordance with Refs. [27,28], the CHS is relevant for the systems at zero Fermi energy and/or if there are few open channels in the leads. However, if the Fermi energies are away from zero and/or if there are many open channels in the terminals, the Wigner-Dyson universality classes and the chiral universality classes lead presumably to the same results.

We first investigate the setup with broken CHS employed whenever the system has a large number of open channels or high Fermi energy. For this system, we fix symmetric terminals, $N_1 = N_3 = N_2 = N_4 = N$, from the general result with a large number of open channels $N \gg 1$ in Eq. (12), and we obtain $\mathrm{var}[J_i^\alpha] = 2 \times 1/32$. Therefore, the spin current fluctuates universally with amplitude $\mathrm{rms}[I_3^z] = 0.25 e^2 V/h$, which can be used to write the universal fluctuations of transversal spin conductance as

$$\mathrm{rms}[G_{\mathrm{sH}}^f] \approx \sqrt{2} \times 0.18 \frac{e}{4\pi}. \tag{13}$$

In this regime, the universal conductance fluctuations has an amplitude $\sqrt{2}$ times higher than the result obtained in Refs. [15,16] for the Wigner-Dyson universality classes. Furthermore, the result is in agreement with Ref. [19] when the Fermi energies are $|E| > 1$ and with the studies concerning the universal conductance fluctuations in the two-dimensional topological insulators with strong SOI of Ref. [20].

However, without the CHS symmetry (Wigner-Dyson ensembles), the universal fluctuations of the transversal spin conductance of a chaotic Dirac quantum dot does not describe the result obtained in Ref. [19], i.e., $\mathrm{rms}[G_{\mathrm{sH}}^q] = (0.285 \pm 0.005) e/4\pi$, complying only if the number of channels is very small (quantized). For this reason, we fix from our general analytical result a small number of channels in order to preserve the CHS. Using a symmetric configuration with $N_1 = N_3 = N_2 = N_4 = 1$, known as a high quantum regime, in Eq. (12), we obtain $\mathrm{rms}[I_3^z] \approx 0.283 e^2 V/h$. The result can be rewritten in terms of the universal fluctuations of transversal spin conductance as

$$\mathrm{rms}[G_{\mathrm{sH}}^q] \approx 0.283 \frac{e}{4\pi}, \tag{14}$$

which is agreement with Ref. [19] if the Fermi energies are $|E| < 1$. In the top panel of Fig. 3, we plot the universal fluctuations of transversal spin conductance as a function of

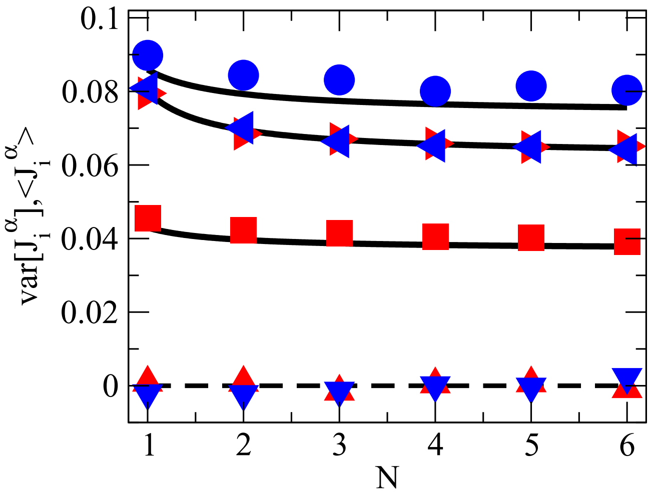

FIG. 2. Average (9) and variance (12) of transverse spin current J_i^α for $i = 3,4$ and $\alpha = x,y,z$ as a function of open channels N. The analytical results are represented by solid lines, while the numeric simulations, obtained by RMT, are represented by the symbols. Symmetric terminals ($N_1 = N_2 = N_3 = N_4 = N$): $\mathrm{var}[J_3^\alpha]$ (triangle left), and $\mathrm{var}[J_4^\alpha]$ (triangle right). Asymmetric terminals ($N_1 = N_3 = 2N_2 = 2N_4 = N$): $\langle J_3^\alpha \rangle$ (triangle up), $\langle J_4^\alpha \rangle$ (triangle down), $\mathrm{var}[J_3^\alpha]$ (square), and $\mathrm{var}[J_4^\alpha]$ (circle).

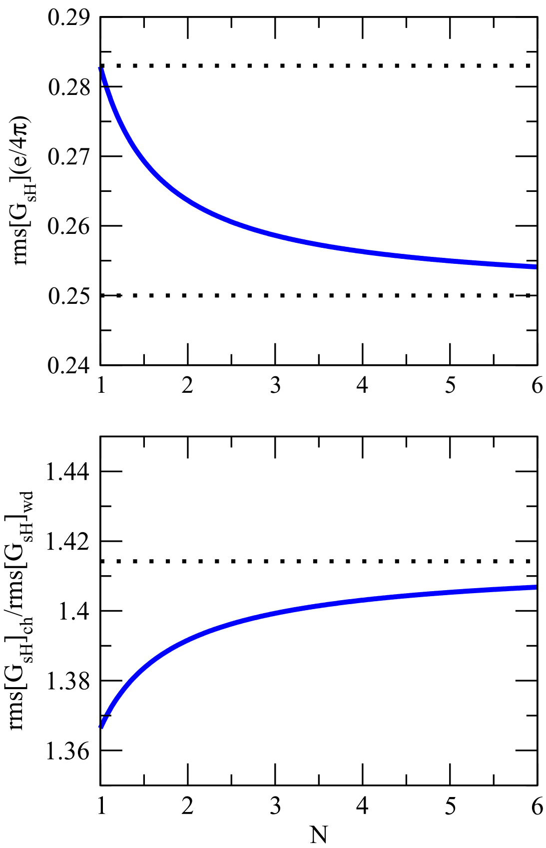

FIG. 3. Top: The universal fluctuations of transversal spin conductance plotted as an open channel crossover from Eq. (14) to Eq. (13). Bottom: The ratio between universal fluctuations of transversal spin conductances of the chiral universal classes and the one of the Wigner-Dyson universal classes ($\mathrm{rms}[G_{\mathrm{sH}}]_{\mathrm{ch}}/\mathrm{rms}[G_{\mathrm{sH}}]_{\mathrm{wd}}$); the $\sqrt{2}$ limit as a function of N.

N, which shows the break of CHS as an open channel crossover from Eq. (14) to Eq. (13).

Importantly, the result of Eq. (14) for the the variance of transversal spin current is incompatible with the one of Ref. [16], obtained for the Wigner-Dyson universal classes. In a high quantum regime, the result of Ref. [16] multiplied for $\sqrt{2}$ yields $\sqrt{2} \times \mathrm{rms}[G_{\mathrm{sH}}]_{\mathrm{wd}} \approx 0.292e/4\pi$, which is therefore outside the error bar of the tight-binding simulation of Ref. [19] that is valid for graphene and other chiral systems. In the bottom panel of Fig. 3, we depict the ratio between universal fluctuations of transversal spin conductances of the chiral universal classes and the Wigner-Dyson ones. Notice the ratio ($\mathrm{rms}[G_{\mathrm{sH}}]_{\mathrm{ch}}/\mathrm{rms}[G_{\mathrm{sH}}]_{\mathrm{wd}}$) tends to $\sqrt{2}$ as a function of N from the general analytical result.

V. NUMERIC SIMULATION

In order to confirm the analytical results, Eqs. (9) and (12), we use numerical simulations from the RMT [11]. The massless Dirac Hamiltonian satisfies the following anticommutation relation [10,11]

$$\mathcal{H} = -\lambda_z \mathcal{H} \lambda_z, \quad \lambda_z = \begin{bmatrix} \mathbf{1}_{2M} & 0 \\ 0 & -\mathbf{1}_{2M} \end{bmatrix}. \tag{15}$$

The $\mathcal{H}$ matrix has dimension $4M \times 4M$. The anticommutation relation, Eq. (15), implies a Hamiltonian member that can be written as

$$\mathcal{H} = \begin{pmatrix} \mathbf{0} & \mathcal{T} \\ \mathcal{T}^\dagger & \mathbf{0} \end{pmatrix}. \tag{16}$$

The quaternionic $\mathcal{T}$-matrix block of the $\mathcal{H}$ matrix has dimension $2M \times 2M$. The RMT establishes that the entries of the $\mathcal{T}$ matrix have a Gaussian distribution

$$P(\mathcal{T}) \propto \exp\left\{-\frac{2M}{\lambda^2}\mathrm{Tr}(\mathcal{T}^\dagger \mathcal{T})\right\}, \tag{17}$$

where $\lambda = 4M\Delta/\pi$ is the variance related to the electronic single-particle level spacing Δ. The Hamiltonian model for the scattering matrix can be written as [29]

$$\mathcal{S} = \mathbf{1} - 2\pi i \mathcal{W}^\dagger(\epsilon - \mathcal{H} + i\pi \mathcal{W}\mathcal{W}^\dagger)^{-1}\mathcal{W}, \tag{18}$$

which satisfies Eq. (5). The $\mathcal{W} = (\mathcal{W}_1, \ldots, \mathcal{W}_4)$ matrix is a $4M \times 2\bar{N}_T$ deterministic matrix, describing the coupling of the resonances states of the chaotic Dirac quantum dot with the propagating modes in the four terminals. This deterministic matrix satisfies the nondirect process, i.e., the orthogonality condition $\mathcal{W}_p^\dagger \mathcal{W}_q = \frac{1}{\pi}\delta_{p,q}$ holds. Accordingly, we consider the relation $\lambda_z \mathcal{W} \Sigma_z = \mathcal{W}$, indicating the scattering matrix is symmetric (5). We consider the system on the Dirac point ($\epsilon = 0$), and, to ensure the chaotic regime and consequently the universality of the observable, the number of resonances inside the quantum dot is large ($M \gg N_T$).

The numerical simulations produce Fig. 2, which shows symbols obtained through 25×10^3 realizations compared with the analytical results, Eqs. (9) and (12). We use the $\mathcal{T}$ matrices, with dimension 800×800 ($M = 400$), and the corresponding $\mathcal{H}$ matrices with dimension 1600×1600 (1600 resonances).

VI. CONCLUSIONS

To summarize, we present a complete analytical study of UQSCFs of a chaotic Dirac quantum dot in the framework of RMT for the chiral universal symmetries in the absence of magnetic field. We show that the effective transverse voltage is CHS independent, Eq. (10). Moreover, in the regime of broken CHS, the UFSCFs exhibit a value $\mathrm{rms}[G_{\mathrm{sH}}^f] \approx \sqrt{2} \times 0.18e/4\pi$. However, the system with preserved CHS exhibits a UQSCF with amplitude $\mathrm{rms}[G_{\mathrm{sH}}^q] \approx 0.283e/4\pi$, which is in agreement with the numerical simulation of Ref. [19]. We can conclude that the quantized spin Hall effect in the absence of magnetic field is described by chiral universal classes in the framework of RMT. Perspectives of our work include the study of the universal classes of the UQSCFs in the presence of magnetic field, preserving the particle-hole symmetry.

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