

## Superconductivity in dense carbon-based materials

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Guided by a simple strategy in search of new superconducting materials, we predict that high-temperature superconductivity can be realized in classes of high-density materials having strong  $sp^3$  chemical bonding and high lattice symmetry. We examine in detail sodalite carbon frameworks doped with simple metals such as Li, Na, and Al. Though such materials share some common features with doped diamond, their doping level is not limited, and the density of states at the Fermi level in them can be as high as that in the renowned  $MgB_2$ . Together with other factors, this boosts the superconducting temperature ( $T_c$ ) in the materials investigated to higher levels compared to doped diamond. For example, the  $T_c$  of sodalitelike  $NaC_6$  is predicted to be above 100 K. This phase and a series of other sodalite-based superconductors are predicted to be metastable phases but are dynamically stable. Owing to the rigid carbon framework of these and related dense carbon materials, these doped sodalite-based structures could be recoverable as potentially useful superconductors.

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### I. INTRODUCTION

The discovery of superconductivity at 39 K in  $MgB_2$  [1] opened up a new perspective in searching for new materials with high critical temperatures  $T_c$ . Being well described in terms of the conventional Bardeen-Cooper-Schrieffer (BCS)-Eliashberg theory [2–4],  $MgB_2$  indicates the clear ways how to make similar or even better superconductors. One of the key factors is to have strongly covalent-bonding/antibonding states crossing the Fermi energy  $E_F$ . Such states that are similar to  $\sigma$  electrons in  $MgB_2$  could effectively interact with phonons and therefore lead to high electron-phonon coupling, denoted by the constant  $\lambda$  and thus high  $T_c$ . Another important factor is the existence of specific phonons that couple strongly to the covalent states; usually such phonons are bond-stretching vibration modes.

It thus came as no surprise that superconductivity in  $YbC_6$  at 6.5 K [5] and in  $CaC_6$  at 11.5 K [5,6] was discovered a few years after that in  $MgB_2$ —electronically and structurally these materials are similar to  $MgB_2$ . They belong to the class of materials called graphite intercalation compounds (GICs) where intercalant (metallic) atoms form layers between the graphene sheets [7]. In GICs, the metal donates some electrons to the graphite antibonding  $\pi^*$  band to create superconducting transport [7]. The strength of the electron-phonon coupling on the  $\pi^*$  band alone, however, is insufficient to explain the observable  $T_c$ 's [8–10]. In  $CaC_6$ , for example, this contribution is only  $\sim 50\%$  to the total value of  $\lambda$ ; the other half comes from an intercalant-derived interlayer band as well as from the  $\pi^*$ -interlayer interband interactions via out-of-plane carbon

vibrations [8]. In GICs, the critical temperature  $T_c$  increases with pressure due to the fact that the electron-phonon coupling is sensitive to the distance between the graphene layers [7]. For this reason,  $MgC_6$  is expected to have a higher  $T_c$  than  $CaC_6$ ; however, attempts to fabricate pure  $MgC_6$  have thus far apparently failed.

Whereas the GIC  $LiC_6$  is not superconducting, it has recently been shown that two-dimensional Li-decorated graphene is a superconductor with a  $T_c$  of 5.9 K [11]. In the latter, Li adatoms transfer their electrons to the honeycomb carbon network, thereby shifting the Fermi energy above the Dirac points in the direction of graphene-derived conduction  $\pi^*$  band. Adatoms also strongly modify the phonon spectrum of the system, leading to the relatively high  $\lambda$  of  $\sim 0.58$ . The pairing energy gap opens up on the piece of the  $\pi^*$  Fermi surface around the K-point. The authors propose that superconductivity in Li-decorated graphene is of the same nature as of GIC  $CaC_6$  [11].

At ambient pressure, among all the carbon-based compounds, the highest  $T_c$ 's exhibit trivalent alkali-metal doped fullerides  $A_3C_{60}$  [12,13], with  $RbCs_2C_{60}$  being an absolute record holder (33 K). All such materials have a high [face-centered-cubic (fcc)] symmetry and three electrons introduced in the triply degenerate band having  $t_{1u}$  symmetry. Noticeably,  $Cs_3C_{60}$  has a different (noncubic, base-centered tetragonal) structure than all the other  $A_3C_{60}$  systems and is not superconducting at ambient conditions. However, it becomes superconducting under pressure, and its  $T_c$  reaches the highest value among all  $A_3C_{60}$  compounds (40 K) [12,13]. Importantly, the application of pressure pushes the  $Cs_3C_{60}$  system in the direction of cubic symmetry; this suggests that the cubic symmetry (or closeness to it) is an essential factor for reaching these relatively high  $T_c$ 's in trivalent fullerides.

Like layered  $sp^2$ -bonded systems, corresponding  $sp^3$  systems can also be turned into superconductors by bringing

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strong covalent states to the Fermi energy  $E_F$  [14]. Thus, when diamond is heavily doped with boron, which acts as a charge acceptor, the diamond becomes a superconductor with a  $T_c$  of 4 K [15]. Due to higher symmetry, diamond has more bond stretching phonon branches than does MgB<sub>2</sub>: three out of six versus two out of nine in MgB<sub>2</sub> [16]. This advantage, however, is more than compensated by two negative factors. First, in going from two to three dimensions, the critical phonons become harder, reducing the strength of  $\lambda$  [17]. Second, B-doped diamond has a rather small density of states (DOS) at the Fermi level ( $\sim 0.06$  states (eV atom spin)<sup>-1</sup> for 2% doping [16]), and as a result, relatively few carriers are available for the superconducting state—this further lowers  $T_c$ . Unfortunately, the doping level cannot be increased arbitrarily in the covalent systems, and the introduction of too many dopant atoms can lead to severe mechanical stresses accompanied by misfit dislocations. This problem, however, can be overcome by going from diamond to  $sp^3$  silicon or carbon clathrate structures which allow intercalation (e.g., Refs. [18,19]). In doped clathrates, the ratio of intercalated-to-host network atoms can be as large as 8/46 [20], and the initial Fermi level can be considerably displaced in the valence or conduction bands. For example, in the  $n$ -doped clathrate Ba<sub>8</sub>Si<sub>46</sub>,  $E_F$  is located 1 eV above the conduction band minimum, which is associated solely with Si [18]. Direct calculations of  $T_c$  and  $\lambda$  in this system show that its superconductivity is indeed an intrinsic property of the  $sp^3$  network and not directly related to the Ba 5d states [18]. In contrast to Ba<sub>8</sub>Si<sub>46</sub>, in  $p$ -doped FC<sub>34</sub>,  $E_F$  is located 0.25 eV below the valence band maximum because an electron is transferred from the carbon network to the guest ion [19]. Here, FC<sub>34</sub> is predicted to have the highest  $T_c$  (77 K) of the silicon and carbon clathrate compounds that have been considered.

Another interesting example is graphane, which is a two-dimensional polymer of carbon and hydrogen, and an  $sp^3$ -bonded material like diamond. Due to its reduced dimensionality and in contrast to diamond, the DOS in graphane exhibits a step function with respect to energy  $E$  (as measured from the band edge) [21]. As a result, the number of available carriers is sufficient to induce superconductivity even at relatively low doping. Apart from the strong chemical bonds between the carbon atoms and large DOS at  $E_F$ , graphane also exhibits a giant Kohn anomaly in optical phonon dispersions [21]. These three factors combined together push the superconducting  $T_c$  in graphane above 90 K [21].

The pursuit of higher temperature superconductors has also led to considering hydrogen-rich materials [22], including hydrides that may exhibit high  $T_c$  behavior analogous to that predicted for metallic hydrogen [23]. Most notably, compressed H<sub>2</sub>S was predicted to exhibit very high  $T_c$  superconductivity at pressures of 100 GPa [24]. Remarkably, subsequent electrical conductivity and Meissner effect measurements of the H-S system indicate a high  $T_c$  reaching 203 K near 200 GPa, the highest critical temperature yet reported for any material [25]. Of particular relevance here is the prediction of very high  $T_c$  of 220–235 K for CaH<sub>6</sub> at 150 GPa [26]. This system is in fact an intercalated structure, where the unusual hydrogen sodalite cages contain enclathrated Ca. Similar to doped Si and C clathrates, in CaH<sub>6</sub>, the host-derived covalent bands dominate at  $E_F$ . It is important that these bands are degenerate

at the zone center: the resultant dynamic Jahn-Teller effect enhances  $\lambda$ , leading to high  $T_c$ .

The above discussions suggest the following possible simple strategy in the search for new potential high- $T_c$  superconductors. In all of the materials mentioned above, (i) the bonding or antibonding states of covalent nature are expected to intersect the Fermi level, and (ii) such states are maximally degenerate at the points of symmetry in the Brillouin zone. Here, we show by first-principles calculations that both the conditions can be met in high symmetry classes of  $sp^3$  bonded carbon framework structures. The properties and relative stabilities of such carbon frameworks are reviewed in Refs. [27,28]. This includes so-called sodalite carbon frameworks which have a body-centered-cubic (bcc) structure [29] and doped with Li, Na, Al metal atoms. Electron-phonon calculations based on BCS theory predict that these compounds may possess very high superconducting temperatures, i.e. above 100 K. Our calculations also suggest that such materials could be synthesized under modest pressures achievable with large volume presses and recoverable as superconductors at pressure.

## II. COMPUTATIONAL DETAILS

Our calculations have been mainly performed for  $XC_6$  compounds in cubic  $Im-3m$  structure (2 f.u. unit cell<sup>-1</sup>), where  $X$  atoms organize a Bravais bcc lattice and carbon atoms form squared C<sub>4</sub> and hexagonal C<sub>6</sub> rings that share some common sides [Fig. 1(a)]. These C<sub>4</sub> and C<sub>6</sub> rings are interlinked in such a way that they form a sodalite framework with an  $X$  atom enclathrated at the center of each cage consisting of 24 atoms. Each carbon atom is  $sp^3$  bonded to four carbon atoms with C-C-C angles of 90° and 120°. The optimized lattice parameters were found using density functional theory within the Perdew-Burke-Ernzerhof (PBE) [39] generalized gradient approximation as implemented in the Quantum-ESPRESSO package [40]. Phonon dispersion and electron-phonon coupling calculations were performed with density functional perturbation theory using the Quantum-ESPRESSO package. Ultrasoft pseudopotentials for Na and C were employed with a kinetic energy cutoff of 80 Ry.

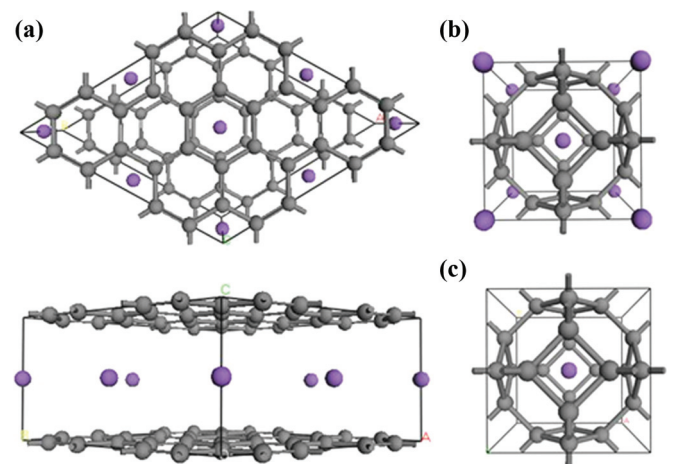


FIG. 1. Crystal structures of (a) GIC- $XC_6$ , (b) sodalite-type  $XC_6$ , and (c) sodalite-type  $XC_{12}$ .

A  $q$ -mesh of  $4 \times 4 \times 4$  in the first Brillouin zone was used in the electron-phonon coupling calculations. A  $k$ -mesh of  $16 \times 16 \times 16$  were adopted to ensure  $k$ -points sampling convergence with a Gaussian width of 0.04 Ry in order to approximate the zero-width limit in the calculations of  $\lambda$ . The real part susceptibility of noninteracting electrons  $\text{Re } \chi_0(q)$  was calculated by using the highly precise analytic tetrahedron method. In order to reach high precision in the  $\text{Re } \chi_0(q)$ , the irreducible part of the Brillouin zone was divided into 16000 tetrahedra. Additional computational details can be found in the Supplemental Material.

### III. RESULTS

The total energy and electronic structure of a series of high-symmetry  $sp^3$  bonded carbon framework structures containing different metal atoms were examined. These calculations revealed dynamical stability of sodalite-structured carbides (Fig. 1). We focus on structures with composition  $XC_6$ , an important stoichiometry also found in the GICs. We begin with a discussion of  $\text{NaC}_6$  as a model system. At zero pressure, its optimized lattice constant is 4.57 Å, with C-C bond of 1.61 Å, which is slightly longer than the 1.54 Å found in diamond due to the strained square faces (Fig. 1). From the electron localization function, one can see that there is strong covalent bonding between the C atoms and only a weak interaction between Na and C atoms. To further understand its electronic properties, we compare the band structures of the empty  $C_6$  cage ( $\text{Na}_0C_6$ ) and  $\text{NaC}_6$  (Fig. 2). Without Na, the empty  $C_6$  bcc sodalite framework is a semiconductor with a calculated indirect band gap of 2.48 eV. The top of the valence band is located at the H symmetry point in the first Brillouin zone. This result contrasts with diamond where the valence band

maximum is located at  $\Gamma$ . The electronic dispersion curves of  $\text{NaC}_6$  are almost identical to that of  $C_6$  with the Fermi level in  $\text{NaC}_6$  shifted up by 3.57 eV due to the additional electrons donated from the Na atoms. Therefore, the doped Na donates electrons to  $C_6$ , and the resulting cations only act as spectators and do not significantly alter the band topology of the framework.

When depicted in an extended Brillouin zone, the Fermi surface associated with the lowest conduction band appears as six identical distorted balls “welded” together at  $H$  points. In the conventional Brillouin zone, however, this six-piece surface looks like a set of six separate “vases” rather than balls (Fig. 2). One may expect that the nesting features of the Fermi surface leads to strong Kohn anomalies in the phonon spectra. A standard approach to check this is to calculate the real part of the noninteracting susceptibility  $\text{Re } \chi_0(q)$  [30,31]. In such a calculation, the peaks or maxima in  $\text{Re } \chi_0(q)$  indicate the nesting vectors  $\mathbf{q}_n$  or positions of possible Kohn anomalies. The calculations show that the highest points of the  $\text{Re } \chi_0(q)$  curve along the  $\Gamma$ -H,  $\Gamma$ -N, and  $\Gamma$ -P directions correspond to  $\mathbf{q} = 0.63 \mathbf{q}_{\text{max}}$ ,  $\mathbf{q} = 0.66 \mathbf{q}_{\text{max}}$ , and  $\mathbf{q} = 0.60 \mathbf{q}_{\text{max}}$ , respectively;  $\mathbf{q}_{\text{max}}$  is the maximal vector along the particular direction [Fig. 3(b)]. These positions correlate well with the positions of Kohn anomalies in the acoustic phonon branches calculated along the corresponding directions [Fig. 3(a)]. As for other directions  $H$ -N and  $P$ -N, the predicted positions of Kohn anomalies along them are at  $\mathbf{q} = 0.66 \mathbf{q}_{\text{max}}$  and  $\mathbf{q} = 0.74 \mathbf{q}_{\text{max}}$ , respectively, again in agreement with the calculated phonon spectrum.

The band structure of  $\text{NaC}_6$  shows the existence of sharp and flat bands crossing the Fermi energy level. The phonon density of states (PHDOS), Eliashberg spectral function  $\alpha^2 F(\omega)$ , and electron-phonon coupling strength  $\lambda(\omega)$  for  $\text{NaC}_6$  are shown in Fig. 3. The absence of any imaginary phonon modes indicates dynamical stability of the structure, which we find at both ambient and high pressures. A gap at  $300 \text{ cm}^{-1}$  [Fig. 3(a)] separates the phonon spectrum into two regions. The lower frequency branches are associated with the motions of Na and C, whereas the higher frequency branches are mainly associated with C atoms. The combined contribution (43 and 57%, respectively) gives a  $\lambda$  of 2.92.

To illustrate the contributions associated with different phonon modes, circles with radii proportional to the corresponding electron phonon coupling  $\lambda_{qj}$  are plotted in Fig. 3(a). It is found that nearly all phonon modes contribute to  $\lambda$ , except acoustic modes at relatively small vectors  $\mathbf{q}$ . Though the latter strongly couple with the electrons at long wavelengths ( $\mathbf{q} \rightarrow 0$ ), they contribute negligibly to the electron-phonon spectral function  $\alpha^2 F(\omega)$  for small  $\omega$  because their PHDOS is proportional to  $\omega^2$ . The superconductivity is facilitated by the Kohn anomalies realized in the phonon dispersion along different directions in the Brillouin zone (in systems where such anomalies are strong). A large  $\lambda$  also benefits from the high DOS at the Fermi level, which is simply  $\text{Re } \chi_0(0)$  provided that all the bands that intersect the Fermi level are taken into account.

The superconducting  $T_c$  was estimated from the spectral function  $\alpha^2 F(\omega)$  determined from the numerical solution of the Eliashberg equation [32] using typical choices of the Coulomb pseudopotential  $\mu^*$  (0.1 to 0.13). The  $T_c$  calculated for  $\text{NaC}_6$

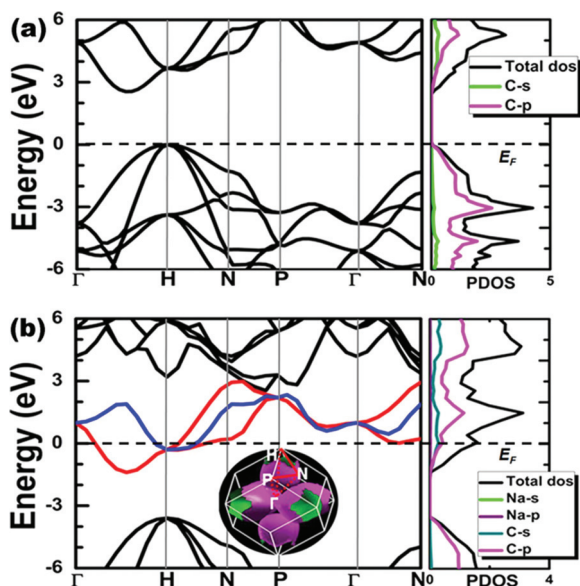


FIG. 2. Electronic band structures. (a) Band structure of  $C_6$ . (b) Band structure of  $\text{NaC}_6$ . The horizontal dotted lines indicate the Fermi level. The red and blue bands are crossing the Fermi level. Insert shows the Fermi surface corresponding to the red and blue bands along  $H$ -N.



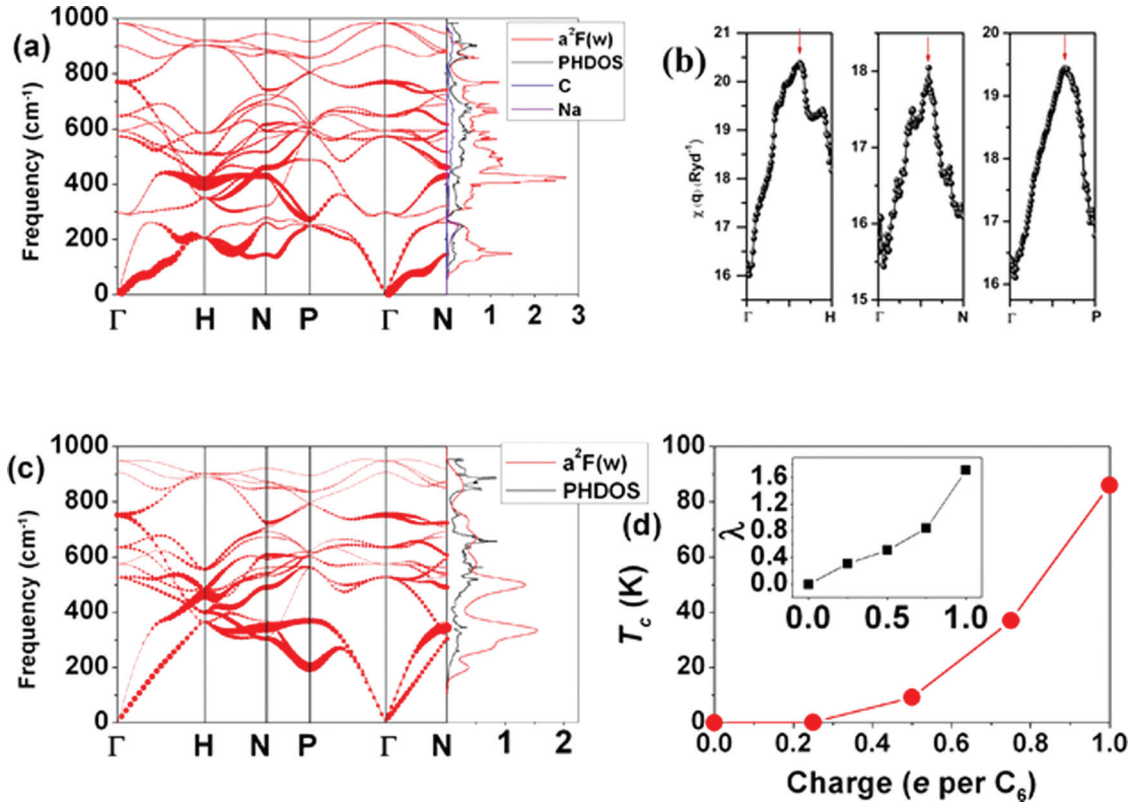


FIG. 3. Phonon properties and Eliashberg spectral function. (a) Phonon dispersions, phonon density of states PHDOS, Eliashberg spectral function  $\alpha^2 F(\omega)/\omega$  and electron-phonon coupling integration of  $\lambda(\omega)$  of NaC<sub>6</sub>. (b)  $\text{Re } \chi_0(q)$  along the  $\Gamma$ -H,  $\Gamma$ -N, and  $\Gamma$ -P lines of the Brillouin zone in NaC<sub>6</sub>. Only contributions from the lowest conduction band are taken into account because they define the shape of  $\text{Re } \chi_0(q)$ . The red arrows indicate the positions of the peaks that correlate with the positions of the Kohn anomalies in the phonon dispersions [panel (a)]. (c) Same as for (a) but for C<sub>6</sub> with one additional negative charge. (d) The calculated superconducting temperature  $T_c$  and the electron phonon coupling parameter as a function of pressure. The size of red circles in panels (a) and (c) is proportional to  $\lambda_{qj}$  which is given by formula (4) in the Supplemental Material.

is 116–127 K at ambient pressure [Fig. 4(a)], which is an exceptionally high value. We have also examined potential effects of pressure on the superconducting  $T_c$ . The calculated  $\lambda$  decreases with increasing pressure, while the average phonon frequency increases [Fig. 4(b)]. Thus, the predicted  $T_c$  drops from 116 K at zero pressure to 101 K at 75 GPa, with a pressure coefficient ( $dT/dP$ ) of 0.2 K GPa<sup>-1</sup>. This change is mainly due to the fact that  $\lambda$  sharply decreases from 2.92 at zero pressure to 1.94 at 25 GPa, and to 1.49 at 75 GPa.

#### IV. DISCUSSION

According to the Allen-Dynes modified McMillan equation [33], the critical temperature  $T_c$  is determined by two main contributions. Here,  $T_c$  is linearly dependent on a characteristic vibrational frequency, the logarithmic average of the phonon spectrum, and exponentially dependent on the electron-phonon coupling parameter  $\lambda$ . Since hydrogen is the lightest element, it is expected to possess the highest vibrational frequencies in the compounds. Therefore, the search of high  $T_c$  superconductivity has been focused on solid hydrogen and hydrogen-rich materials. However, even a small increase in  $\lambda$  can raise the  $T_c$  significantly; this is the case of the sodalite-structured carbide materials investigated here. Remarkably, to the best of our knowledge, NaC<sub>6</sub> has the largest predicted  $\lambda$  (2.92) among

all the carbon-based materials. For instance, the  $\lambda$  of FC<sub>34</sub> is predicted to be 0.85 [19].

We now examine the common properties of LiC<sub>6</sub>, NaC<sub>6</sub>, AlC<sub>6</sub>, and SiC<sub>6</sub> that make them all potential high  $T_c$  superconductors. As already noted, in the case of NaC<sub>6</sub>, the phonons that suffer Kohn anomaly contribute significantly to the Eliashberg function and therefore to  $\lambda$  [Fig. 3(a)]. Though such or even more pronounced anomalies are also seen in the phonon spectra of LiC<sub>6</sub>, they are not so evident in the case of AlC<sub>6</sub> and SiC<sub>6</sub> (Fig. S4 in the Supplemental Material [34]). So the presence of strong Kohn anomalies is desirable but is not a necessary factor to achieve a relatively high  $T_c$ . On the other hand, all of these systems have a relatively high DOS at the Fermi level. In the case of NaC<sub>6</sub> and AlC<sub>6</sub>, for example,  $N(E_F) = 0.10$  states (eV atom spin)<sup>-1</sup>; this quantity is close to the 0.12 states (eV atom spin)<sup>-1</sup> found in MgB<sub>2</sub> [3,4]. The fact that the high DOS at  $E_F$  does not depend significantly on the doping level or charge transfer from metal elements is related to the ability of flat  $\sigma^*$  carbon bands to accommodate up to 6 electrons before the dip in  $N(E_F)$  is reached [Fig. 2(b)].

To confirm this observation, we have performed additional electron phonon coupling calculations on pure sodalite carbon (C<sub>6</sub>) structure with additional negative charge [Figs. 3(c) and 3(d)]. In C<sub>6</sub> with one additional electron per C<sub>6</sub> unit,  $\lambda$  is lower than that found for NaC<sub>6</sub>. However, it is still rather large

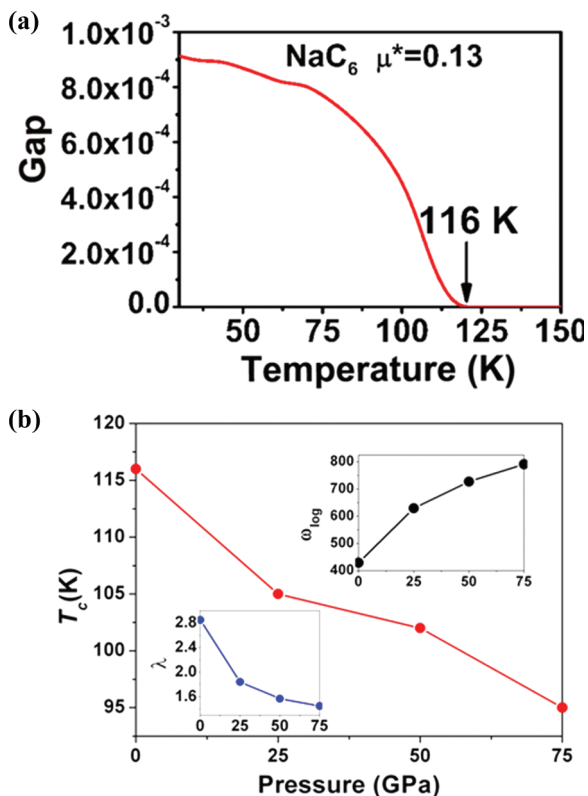


FIG. 4. Predicted superconductivity in  $\text{NaC}_6$ . (a) Superconductivity gap of  $\text{NaC}_6$  at ambient pressure. (b)  $T_c$  as a function of pressure. The coupling parameters, the average phonon frequencies  $\omega_{\log}$  and superconducting temperatures are also shown as a function of pressure.

(1.71 versus 2.92), indicating the electron phonon coupling is mainly derived from the metallic carbon-carbon bonds. Moreover,  $\lambda$  is enhanced with increasing the negative charges (up to a critical point  $>1$ ) as shown in Fig. 3(d).

Another common feature of the  $\text{LiC}_6$ ,  $\text{NaC}_6$ ,  $\text{AlC}_6$ , and  $\text{SiC}_6$  systems is that they all have degenerate states close to  $E_F$  at some particular points of symmetry. These points are  $H$  ( $\text{LiC}_6$  and  $\text{NaC}_6$ ),  $\Gamma$  ( $\text{AlC}_6$ ), and  $P$  ( $\text{SiC}_6$ ) (Fig. S5 in the Supplemental Material [34]). The degenerate states are likely to be subject to Jahn-Teller distortion. The Jahn-Teller effect is the coupling between the electronic and nuclear degrees of freedom leading to distortion in the geometric structure and the consequent lifting of orbital degeneracy. If the Jahn-Teller distortion is dynamic, the related vibration as a strong electron-phonon coupling process will lead to superconductivity. The same mechanism has been invoked to explain superconductivity in boron-doped diamond [15,35]. According to our electronic band structure calculations, the latter is also metallic with four bands crossing the Fermi energy, consistent with robust superconductivity. Our results show that, for the sodalite structures, charge transfer between incorporated elements and carbon cage is a key factor in boosting  $T_c$ , but only when the transferred charge does not exceed roughly 1 electron. Thus,  $\text{PC}_6$ ,  $\text{SC}_6$ ,  $\text{ClC}_6$ ,  $\text{SiC}_6$ , and  $\text{FC}_6$  have lower critical temperatures than  $\text{NaC}_6$ ,  $\text{LiC}_6$ , and  $\text{AlC}_6$  due to weaker charge transfer (Fig. S2 in the Supplemental Material [34]). On the other hand,  $\text{AlC}_6$  has larger charge transfer than  $\text{NaC}_6$ , yet a

lower  $T_c$ . To further clarify this point, in addition to  $\text{NaC}_6$ , we examined  $\text{NaC}_{12}$  (Fig. S6 in the Supplemental Material [34]), which is obtained from the former by removing the Na atoms at corner positions in the unit cell. Here,  $\text{NaC}_{12}$  in this structure is also found to be superconducting with a  $T_c$  of 3.8 K ( $\lambda = 0.45$ ). The lower  $T_c$  can be explained by the fact that the charge transfer in  $\text{NaC}_{12}$  is significantly lower than in  $\text{NaC}_6$  (0.41 electron/ $\text{C}_6$  versus 0.82 electrons/ $\text{C}_6$ , respectively).

The potentially good superconductors  $\text{LiC}_6$ ,  $\text{NaC}_6$ ,  $\text{AlC}_6$ , and  $\text{SiC}_6$  bear some analogy to the fullerides, GICs,  $n$ -doped silicon clathrates, and  $\text{MgB}_2$ , which in fact can also be viewed as an intercalated material with parallels to graphite. In the case of the aforementioned sodalite systems, metallic elements act as donors with respect to carbon, similar to intercalants in fullerides, graphite, or Mg relative to B. The degree of doping, however, is different. Whereas Li and Na donate almost all valence ( $s$ ) electrons to carbon (similar to alkali metals in fullerides or Mg in  $\text{MgB}_2$  [4]), Al and Si only partially transfer their valence electrons, similar to GICs [7,9,10,33], Ba-doped silicon clathrates [20] and Ca- and Yb-doped graphene [36,37]. As a result, no dopant-derived bands form near the Fermi energy in  $\text{LiC}_6$  and  $\text{NaC}_6$ , but such bands do form in  $\text{AlC}_6$  and  $\text{SiC}_6$ . They cross the Fermi level and strongly hybridize with the host-atom (C) states like the analogous bands in GICs, silicon clathrates, or doped graphene. As pointed out in previous studies, such hybridization leads to a strong enhancement of the electron-phonon interaction and contributes to superconductivity [10,20,33,35,36]. In the case of GICs, for example, electrons associated with the intercalant-derived bands interact strongly with carbon atom vibrations perpendicular and metal vibrations parallel to graphite layers [9,10,33]. A similar effect is possible in  $\text{AlC}_6$  and  $\text{SiC}_6$ , where the sodalite-carbon network has some parallels to the graphite structure, in the sense of having ideal hexagons and squares sharing common sides. In contrast to GICs and other similar systems just discussed above, there are no occupied metal-derived bands in  $\text{MgB}_2$ ; instead the B-derived bonding  $\sigma$  band is only partly filled, and the holelike  $\sigma$  states strongly interact with the bond-stretching phonons, providing the necessary conditions to realize high-temperature superconductivity [3,4]. Similar to fullerides and  $\text{MgB}_2$ , in  $\text{LiC}_6$  and  $\text{NaC}_6$ , the antibonding C-derived  $\sigma^*$  states are partly occupied and strongly interact with a number of different phonons, especially with those exhibiting a Kohn anomaly.

Our results indicate that the existence of degenerate states (especially those triply degenerate) at high symmetry points in the Brillouin zone should be considered as a promoting factor in achieving high  $T_c$ . To strengthen this point, we also investigated the effect of distortions on the electronic properties of  $\text{NaC}_6$  (Fig. S7 in the Supplemental Material [34]). We found that, in the tetragonal structure with  $a = c = 4.57 \text{ \AA}$  and  $c = 4.67 \text{ \AA}$ , the electron-phonon coupling constant  $\lambda$  drops to 2.07 compared to 2.92 in initial cubic structure. The corresponding calculated  $T_c$  is 82 K versus 116 K in the cubic phase. In the context of the role of cubic symmetry, our  $\text{XC}_6$  systems are parallel to fcc alkali-doped fullerides, where the  $t_{1u}$  band is very similar to the C-derived  $\sigma^*$  band in the systems studied. As discussed above, the cubic symmetry is crucial for realizing superconductivity with enhanced  $T_c$ 's in  $\text{A}_3\text{C}_{60}$  systems [12,13].

The triple degeneracy in the sodalite framework structure is easily realized, even if the degenerate bands are  $1s$ -derived, as is the case for  $\text{CaH}_6$  [26]. Indeed, any atom occupying Wyckoff positions  $12d$  in the  $Im\bar{3}m$  space group (e.g., C or H for  $\text{NaC}_6$  and  $\text{CaH}_6$ ) will inevitably lead to at least to one of the four possible triply degenerate Bloch states at the  $H$  point ( $H_4^+$ ,  $H_4^-$ ,  $H_5^+$ , and  $H_5^-$ ). At the same time, all atoms whose orbitals transform according to the irreducible representations  $A_1$ ,  $B_1$ , and  $E$  can induce the Bloch states  $\Gamma_4^-$ ,  $\Gamma_5^-$ , which are triply degenerate at  $\Gamma$  [38]. This common symmetry, in addition to the low- $Z$  character of the atomic framework, is an important factor underlying the high  $T_c$  calculated at the same level of theory for the two materials.

We now comment on the stability of these structures. The  $\text{XC}_6$  phases examined have higher enthalpy than the elemental assemblage of diamond plus and solid alkali metal (Fig. S3 in the Supplemental Material [34]). The higher energy arises from strained  $sp^3$  bonds of the sodalite cage. The dynamical stability of  $\text{NaC}_6$  and other sodalite-based carbon structures ( $\text{XC}_6$ ,  $X = \text{H, Li, N, O, F, Al, Si, P, S, and Cl}$ ) under ambient pressures and the strong covalently bonded carbon structures indicate they should possess large kinetic energy barriers with respect to transformations to lower density forms. Thus, by analogy to diamond and other  $sp^3$  bonded carbon structures that are formed under pressure, these materials may be recoverable to ambient conditions (Figs. S3 and S4 in the Supplemental Material [34]).

## V. CONCLUSIONS

By following some simple criteria for the existence of phonon-mediated high- $T_c$  superconductors, we discovered a new class of materials that potentially can have  $T_c$  of the order of 100 K and higher—doped sodalite-structured carbide

compounds. These materials are characterized by strong  $sp^3$  bonding, high crystal symmetry, and large number of electrons transferred from a dopant to the host lattice. All the mentioned features contribute to their strong electron-phonon couplings in ( $\lambda \sim 1$ ). Among all the studied  $\text{XC}_6$  systems,  $\text{NaC}_6$  has the highest predicted  $T_c$  at ambient pressure (above 100 K). This is followed by  $\text{AlC}_6$  and  $\text{LiC}_6$ , where  $T_c$  can reach 85–95 K. As rigid  $sp^3$  bonded structures, carbon sodalite networks may be kinetically stable at ambient pressure. Dense superconductive states, such as those reported here, may be favored in other mixtures of elemental metals plus carbon and/or upon compression.

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