Chirality and current-current correlation in fractional quantum Hall systems

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We study current-current correlation in an electronic analog of a beam splitter realized with edge channels of a fractional quantum Hall liquid at Laughlin filling fractions. In analogy with the known result for chiral electrons [M. Büttiker, Phys. Rev. B **46**, 12485 (1992)], if the currents are measured at points located after the beam splitter, we find that the zero frequency equilibrium correlation vanishes due to the chiral propagation along the edge channels. Furthermore, we show that the current-current correlation, normalized to the tunneling current, exhibits clear signatures of the Laughlin quasiparticles' fractional statistics.

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I. INTRODUCTION

With the theoretical explanation of the Fractional Quantum Hall (FQH) effect at filling $\nu = 1/m$, with m odd [1], Laughlin made the remarkable prediction that the elementary charged excitation of a FQH system is a quasiparticle carrying fractional charge $q = \nu e$ [2]. Moreover, Laughlin's quasiparticles (LQP) were predicted to carry fractional statistics, as well, that is, on exchanging two of them with each other, the relative wave function must acquire a statistical phase $\theta \neq \{\pi, 2\pi\}$, with $\theta = \pi$ corresponding to fermions, $\theta = 2\pi$ corresponding to bosons. As a result, they behave as Abelian anyons with fractional charge q [3].

Even richer structures are possible in the case of non-Abelian anyons—the braiding of one quasiparticle by another one will result in the system to be sent into a different quantum state and not only in the relative wave function to acquire a statistical phase [4].

A FQH system is fully gapped in the bulk, with gapless branches of chiral excitations at its edges, supporting current flow across the sample. The elementary charge carrier is the boundary analog of an LQP and, therefore, it carries a fractional charge q, as well [5]. Because of such a correspondence, it was possible to experimentally estabilish the fractional charge of LQPs by means of shot-noise measurements on a FQH-bar [6,7]. Nevertheless, a direct observation of their fractional statistics is still the subject of ongoing experimental efforts [8,9].

Correlation measurements of light intensities in optics [10,11] and electrical currents in solid-state physics [12] have provided an important tool to investigate the difference between the two "classical" statistics of quantum elementary particles: bosonic and fermionic. In the pursue of evidence for fractional statistics in FQH systems, a number of works have been putting forward the use of solid-state analogs of Fabry-Perot [13–19], Mach-Zehnder [20–31] and more elaborated Hanbury Brown and Twiss [32–37] interferometers to address the statistical properties of fractional quantum Hall anyons.

In this paper, we confine our attention to Abelian anyons, emerging as elementary charged excitations of an FQH state at a Laughlin filling ν. We focus on a simple measurement with LQPs colliding at a beam splitter-like device, such an experiment is not subject to some of the intricacies found in interferometric setups. In order to illustrate our approach, we start by considering a simple, but instructive, example. With reference to Fig. 1, we consider a beam splitter where particles are injected from sources S_1 and S_2 and measured at detectors D_1 and D_2. An incoming particle from S_1 can be either transmitted to D_2 with scattering amplitude t, or reflected to D_1 with scattering amplitude r. Similarly, an incoming particle from S_2 can be either transmitted to D_1 with scattering amplitude t', or reflected to D_2 with scattering amplitude r', so that the scattering matrix describing these processes is given by

$$S = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix}. \tag{1}$$

Because of the particle number conservation, S must be unitary, which enables us to use the following parameterization,

$$S = \begin{pmatrix} \sqrt{\mathcal{R}} & \sqrt{\mathcal{T}} \\ \sqrt{\mathcal{T}} & -\sqrt{\mathcal{R}} \end{pmatrix}, \tag{2}$$

with $\mathcal{T}$ and $\mathcal{R}$ respectively being the transmission and reflection coefficients. Let n_{D_1} and n_{D_2}, respectively, be the particle number operators at D_1 and at D_2. Let us assume that the particles considered are either fermions or bosons.

Following Ref. [38], and considering a toy model with just one quantum mode per arm, one can calculate the correlation between the number of particles measured at D_1 and at D_2, i.e., $\langle\langle n_{D_1} n_{D_2}\rangle\rangle = \langle n_{D_1} n_{D_2}\rangle - \langle n_{D_1}\rangle\langle n_{D_2}\rangle$. Notwithstanding that in optics very special incoming states can be realized, in typical experiments, such as in transport measurements in solid-state physics—the appropriate tool to investigate Abelian anyons at a FQH edge—particles colliding at a beam splitter emerge from thermal reservoirs.

Assuming that the particles are emitted from two independent reservoirs S_1 and S_2, respectively characterized by (thermal) distribution functions n_1 and n_2, one obtains

$$\langle\langle n_{D_1} n_{D_2}\rangle\rangle = \pm \mathcal{R}\mathcal{T}(n_1 - n_2)^2, \tag{3}$$

where the plus and minus sign refer to bosons and fermions, respectively. It is worth stressing that, as it is apparent from Eq. (3), when $n_1 = n_2$, the correlations vanish, irrespectively of the underlying quantum statistics [39]. In this paper, we

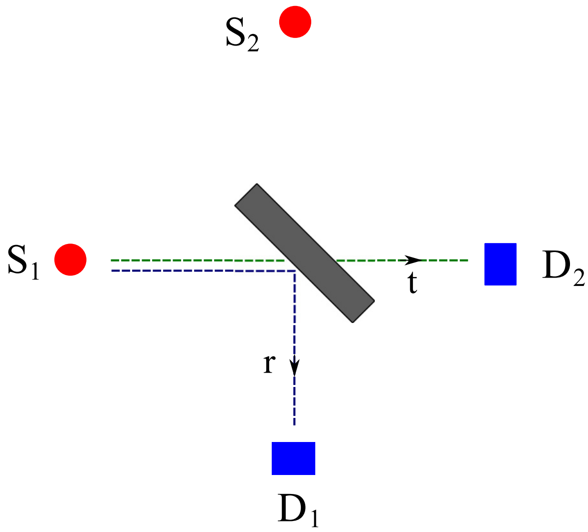

FIG. 1. Beam splitter. Particles emitted from source S_1 can be transmitted to detector D_2 with transmission amplitude t or reflected to detector D_2 with amplitude r. Similarly, for particles emitted from source S_2 (these processes are not illustrated in this figure). For photons, which obey Bose-Einstein statistics, the physical realization of a beam splitter is a partially silvered mirror. In the case of electrons, which obey Fermi-Dirac statistics, a beam splitter can be realized using edges of an integer quantum Hall liquid impinging on a quantum point contact.

derive the analog of Eq. (3) for LQPs originating from sources (FQH edges) kept at the same temperature but, in general, at different chemical potentials. For FQH anyons, there is no simple description of the beam splitter in terms of a scattering matrix. Therefore we perform the calculation by resorting to nonequilibrium Keldysh formalism. In particular, we realize the beam splitter as a quantum point contact (QPC), which allows for LQP tunneling between the edges. Besides weak interedge tunneling, no other approximation is involved in our calculation. As a result, while in the "shot-noise" regime, $|eV| \gg k_B T$ (V being the voltage bias between the edges and k_B being the Boltzmann constant), we recover that correlations are proportional to the tunneling current, with the constant of proportionality being equal to q, in the "thermal" regime $|eV| \ll k_B T$ the constant of proportionality is renormalized by a purely statistics dependent function $\gamma(\nu) = (6/\pi^2)\partial_z^2 \ln \Gamma(z)|_{z=\nu}$ [$\Gamma(z)$ being Euler Gamma function], which can be directly measured by looking at the current-current correlation probed in the appropriate regime.

The paper is organized as follows. In Sec. II, we introduce the model for a beam splitter realized with edge channels of an FQH system. In Sec. III, we calculate the correlation of currents measured at different drains as a function of the voltage bias V and the temperature T. In Sec. IV, we show how fractional statistics can be probed from current-current correlation normalized to the tunneling current. In Sec. V, we discuss and summarize our results and give an outlook of the possible implications of our work. Mathematical details and a review of the noninteracting case ($\nu = 1$) are provided in the appendices.

II. THE MODEL

In this section we introduce the model for the edge channels that we use in the calculation of the current-current correlation. Throughout this paper, we limit our analysis to Laughlin's states at filling ν, which are characterized by only one branch of chiral excitations per edge [5]. This is not a potential limitation, as we outline in the concluding section. Our analysis, indeed, is expected to be generalizable to non-Laughlin FQH states, e.g., $\nu = 2/3$ and $\nu = 5/2$, as a possible tool to investigate the properties of these more exotic FHQ states.

The device we discuss here has four edge channels (cfr. Fig. 2), we only need to focus onto the ones labeled e_1 and e_2. In order to realize a beam splitter, we assume that a QPC is obtained between the two channels by means of electric gates, allowing for quasiparticle tunneling between e_1 and e_2. Finally, it is worth stressing that we choose our geometry to allow for independent tuning of the chemical potentials at e_1 and e_2, respectively, μ_1 and μ_2.

Edge excitations of Laughlin's FQH states are described within the chiral Luttinger liquid (CLL) framework [5]. In the two-edge model, the Hamiltonian for the edges is given by

$$H_0 = \frac{\hbar v}{4\pi} \sum_{k=1,2} \int dx (\partial_x \phi_k(x))^2, \tag{4}$$

with v the plasmonic velocity. The chiral bosonic fields $\{\phi_1(x),\phi_2(x)\}$ obey the commutation relations

$$[\phi_k(x),\phi_l(x')] = i\pi \delta_{k,l} \text{sgn}(x - x'). \tag{5}$$

With the normalizations in Eqs. (4) and (5), the density operator at edge k ($k = 1,2$), $\rho_k(x)$ is given by

$$\rho_k(x) = -\frac{\sqrt{\nu}}{2\pi} \partial_x \phi_k(x), \tag{6}$$

while, because of the chiral propagation along the edges, the electric current density operator is $i_k(x) = ev\rho_k(x)$.

The Hamiltonian operator describing the tunneling of a charge-q LQP at the QPC is constructed in terms of the quasiparticle creation and annihilation operators at edge k.

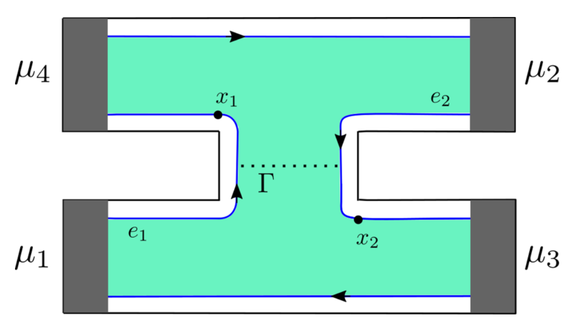

FIG. 2. Schematic representation of the device used for the proposed measurement. The green colored area represents the incompressible electron liquid due to a strong perpendicular magnetic field. The boundary of the electron liquid (blue lines) are the edge channels supporting gapless excitations. As discussed in the main text, we only need to focus on edges e_1 and e_2, they originate respectively from reservoirs at chemical potentials μ_1 and μ_2. The dotted line represents tunneling between the two edges due to a quantum point contact. Currents are measured at points x_1 and x_2.

Within CLL framework, these are realized in terms of vertex operators, respectively, given by

$$V_k^\dagger(x) = F_k^\dagger e^{i\sqrt{\nu}\phi_k(x)},$$
$$V_k(x) = F_k e^{-i\sqrt{\nu}\phi_k(x)}, \tag{7}$$

with $\{F_k, F_k^\dagger\}$ being the Klein factors that one has to introduce in order to recover the correct commutation relations between operators belonging to different edges. Choosing the x coordinates so that the QPC is located at $x = 0$, we take the tunneling Hamiltonian to be

$$H_T = \Gamma V_1^\dagger(0) V_2(0) + \text{H.c.} \tag{8}$$

We have assumed to work in a temperature/voltage regime such that terms that are less relevant in the renormalization group sense [40,41] can be disregarded. These terms, indeed, correspond to tunneling of quasiparticles with the charge being an integer multiple of q.

In fact, as our device contains only one QPC, the Klein factors can be dropped from the tunneling Hamiltonian H_T. Following Ref. [42], the commutation rules between Klein factors must be assigned so that the vertex operators corresponding to different edges must obey the same commutation relations as the vertex operators corresponding to the same edge, that is, $e^{i\sqrt{\nu}\phi_k(x_1)}e^{i\sqrt{\nu}\phi_{k'}(x_2)} = e^{i\pi\nu\text{sgn}(x_1-x_2)}e^{i\sqrt{\nu}\phi_{k'}(x_2)}e^{i\sqrt{\nu}\phi_k(x_1)}$. As a result, they have to satisfy the relations $F_i^\dagger F_i = F_i F_i^\dagger = 1$, $F_1F_2 = e^{i\nu\pi}F_2F_1$, and $F_1^\dagger F_2 = e^{-i\nu\pi}F_2F_1^\dagger$. Taking into account these commutation relations, it is easy to check that the commutator between H_T in the interaction representation computed at different times, that is, $[H_T(t_1), H_T(t_2)]$, is the same whether one introduces or does not introduce the Klein factors in the vertex operators in Eq. (7). Therefore they can be safely disregarded, without affecting the validity of our derivation. The tunneling Hamiltonian H_T can be simplified to

$$H_T = \Gamma e^{i\sqrt{\nu}(\phi_1(0)-\phi_2(0))} + \text{H.c.} \tag{9}$$

The key quantity we consider in the following is the correlation function between $i_1(x_1,t_1)$ and $i_2(x_2,t_2)$, where $i_k(x,t)$ is the current operator at edge k in Heisenberg representation, $x_1 \in e_1$, $x_2 \in e_2$, and both points x_1 and x_2 are situated after the QPC (in the sense of the propagation direction defined on each edge). Within the CLL formalism, the correlation functions can be derived in a perturbative expansion in Γ, as we present in the next section.

III. CURRENT-CURRENT CORRELATION

In this section, we illustrate the details of our calculations of the correlation of currents measured at the points x_1 and x_2 as a function of the temperature and of the chemical potentials μ_1 and μ_2. The finite frequency current-current correlation reads

$$S(\Omega;x_1,x_2) = \frac{1}{2}\int_{-\infty}^{+\infty} d(t_1-t_2)\langle\langle \hat{i}_1(t_1,x_1)\hat{i}_2(t_2,x_2) + \hat{i}_2(t_2,x_2)\hat{i}_1(t_1,x_1)\rangle\rangle e^{i\Omega(t_1-t_2)}. \tag{10}$$

Similar current-current correlation has been studied in the context of a quantum spin Hall system [43]. Henceforth, operators with a "hat" are to be understood in the Heisenberg representation. We first evaluate the finite frequency correlation $S(\Omega;x_1,x_2)$, later, taking the limit for $\Omega \to 0$, correctly calculate $S(0)$ as it will be clear from the discussion below.

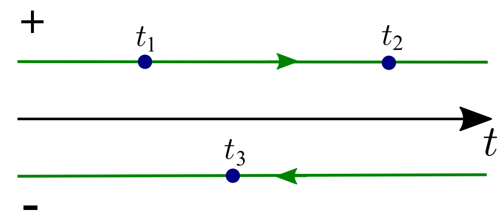

FIG. 3. Keldysh contour, the direction of the arrows indicates the ordering of times along the contour. Here + and − indicate the upper and the lower branches, respectively, and will be used to define the four components of the Keldysh Green's function. As an example of time ordering on the contour we have t_2 at later time than t_1 but at earlier time with respect to t_3.

Introducing the Keldysh time contour (see Fig. 3), and the Keldysh time ordering operator T_K we can rewrite the previous expression as

$$S(\Omega;x_1,x_2) = \frac{1}{2}\sum_{\eta=\pm1}\int_{-\infty}^{+\infty} d(t_1-t_2)e^{i\Omega(t_1-t_2)} \times \langle\langle T_K i_1(t_1,x_1,\eta)i_2(t_2,x_2,-\eta)\rangle\rangle e^{i\Omega(t_1-t_2)}. \tag{11}$$

Notice that in the above equation, we have introduced an index $\eta = \pm 1$, which specifies the upper and the lower part of the Keldysh contour.

We assume that the tunneling H_T is adiabatically turned on at $t = -\infty$. In order to evaluate Eq. (10), we move to the interaction representation with respect to H_0 and rewrite Eq. (10) as

$$S(\Omega;x_1,x_2) = \frac{1}{2}\sum_{\eta=\pm1}\int_{-\infty}^{+\infty} d(t_1-t_2)e^{i\Omega(t_1-t_2)} \times \langle\langle T_K i_1(t_1,x_1,\eta)i_2(t_2,x_2,-\eta)S_K\rangle\rangle, \tag{12}$$

where $S_K = T_K \exp\{-\frac{i}{\hbar}\int_K H_T(\tau)d\tau\}$ with K labeling the Keldysh contour. Notice that operators without the "hat" are to be understood in the interaction representation with respect to H_0. Expanding S_K to the lowest nonvanishing order in the tunneling Hamiltonian, we have

$$S(\Omega;x_1,x_2) = -\frac{1}{4\hbar^2}\sum_{\eta,\eta_1,\eta_2=\pm1}\eta_1\eta_2 \times \int_{-\infty}^{+\infty} ds_1 \int_{-\infty}^{+\infty} ds_2 \int_{-\infty}^{+\infty} d(t_1-t_2)e^{i\Omega(t_1-t_2)} \times \langle\langle T_K i_1(t_1,x_1,\eta)i_2(t_2,x_2,-\eta)H_T(s_1,\eta_1)H_T(s_2,\eta_2)\rangle\rangle. \tag{13}$$

Keeping only connected contributions and dropping terms that are trivially zero by Keldysh integration we may rewrite the previous expression as

$$S(\Omega;x_1,x_2) = -\frac{|\Gamma|^2e^2\nu v^2}{16\pi^2\hbar^2}\sum_{\eta,\eta_1,\eta_2=\pm1}\eta_1\eta_2\int_{-\infty}^{+\infty}ds_1\int_{-\infty}^{+\infty}ds_2\int_{-\infty}^{+\infty}d(t_1-t_2)e^{i\Omega(t_1-t_2)}$$
$$\times\{\langle T_K\partial_x\phi_1(x_1,t_1,\eta)e^{i\sqrt{\nu}\phi_1(0,s_1,\eta_1)}e^{-i\sqrt{\nu}\phi_1(0,s_2,\eta_2)}\rangle\langle T_K\partial_x\phi_2(x_2,t_2,-\eta)e^{-i\sqrt{\nu}\phi_2(0,s_1,\eta_1)}e^{i\sqrt{\nu}\phi_2(0,s_2,\eta_2)}\rangle$$
$$+\langle T_K\partial_x\phi_1(x_1,t_1,\eta)e^{-i\sqrt{\nu}\phi_1(0,s_1,\eta_1)}e^{i\sqrt{\nu}\phi_1(0,s_2,\eta_2)}\rangle\langle T_K\partial_x\phi_2(x_2,t_2,-\eta)e^{i\sqrt{\nu}\phi_2(0,s_1,\eta_1)}e^{-i\sqrt{\nu}\phi_2(0,s_2,\eta_2)}\rangle\}. \quad (14)$$

In order to explicitly compute the multiple correlators at finite μ_1 and μ_2 entering Eq. (14), we recall that, adding a nonzero chemical potential μ to a chiral Luttinger liquid described by the Hamiltonian of Eq. (4) is equivalent (apart for an overall constant contribution to the ground-state energy) to the replacement $\partial_x\phi(x)\to\partial_x\phi(x)-\frac{\mu\sqrt{\nu}}{v}$. Therefore denoting with $\langle\dots\rangle_0$ the averages computed at $\mu_1=\mu_2=0$, we obtain

$$\langle T_K\partial_x\phi_k(x,t,\eta)e^{i\sqrt{\nu}\phi_k(0,t_k,\eta_1)}e^{-i\sqrt{\nu}\phi_k(0,t_2,\eta_2)}\rangle_{\mu_k}$$
$$=\left\{\langle T_K\partial_x\phi_k(x,t,\eta)e^{i\sqrt{\nu}\phi_k(0,t_1,\eta_1)}e^{-i\sqrt{\nu}\phi_k(0,t_2,\eta_2)}\rangle_0-\frac{\mu_k\sqrt{\nu}}{v}\langle T_Ke^{i\sqrt{\nu}\phi_k(0,t_1,\eta_1)}e^{-i\sqrt{\nu}\phi_k(0,t_2,\eta_2)}\rangle_0\right\}e^{i\nu\mu_k(t_1-t_2)}, \quad (15)$$

and similarly for the conjugate expression. Notice that contributions to Eq. (14) proportional to the chemical potentials $\{\mu_k\}$ vanish identically after integration over the Keldysh contour. In order to complete the calculation, we can use the following identity:

$$\langle T_K\partial_x\phi_k(x,t,\eta)e^{i\sqrt{\nu}\phi_k(0,t_1,\eta_1)}e^{-i\sqrt{\nu}\phi_k(0,t_2,\eta_2)}\rangle=-i\,\partial_x\lim_{\lambda\to0}\partial_\lambda\langle T_Ke^{i\lambda\phi_k(x,t,\eta)}e^{i\sqrt{\nu}\phi_k(0,t_1,\eta_1)}e^{-i(\sqrt{\nu}+\lambda)\phi_k(0,t_2,\eta_2)}\rangle. \quad (16)$$

We finally obtain

$$\langle T_K\partial_x\phi_k(x,t,\eta)e^{i\sqrt{\nu}\phi_k(0,t_1,\eta_1)}e^{-i\sqrt{\nu}\phi_k(0,t_2,\eta_2)}\rangle_0=\frac{\sqrt{\nu}\pi}{\hbar\beta v}\left(\cot\left\{\frac{\pi}{\hbar\beta}[i(t-t_1-x/v)+\tau_c\,\sigma_{\eta,\eta_1}(t-t_1)]\right\}\right.$$
$$\left.-\cot\left\{\frac{\pi}{\hbar\beta}[i(t-t_2-x/v)+\tau_c\,\sigma_{\eta,\eta_2}(t-t_2)]\right\}\right)G^{(\nu)}_{\eta_1,\eta_2}(t_1-t_2). \quad (17)$$

In Eq. (17), we have set $\beta=(k_BT)^{-1}$. Also, we have defined $\sigma_{\eta,\eta'}(t-t')=[(\eta+\eta')\text{sgn}(t-t')+\eta'-\eta]/2$ and have introduced the cutoff time $\tau_c=l_c/v$, with l_c being a short-distance cutoff length. Moreover, we have introduced the Keldysh Green function $G^{(\nu)}_{\eta_1,\eta_2}(t_1-t_2)=\langle T_Ke^{i\sqrt{\nu}\phi_k(0,t_1,\eta_1)}e^{-i\sqrt{\nu}\phi_k(0,t_2,\eta_2)}\rangle_0$, given by

$$G^{(\nu)}_{\eta_1,\eta_2}(t_1-t_2)=l_c^{\nu}\left(\frac{\hbar\beta v}{\pi}\sin\left\{\frac{\pi}{\hbar\beta}[i(t_1-t_2)\sigma_{\eta_1,\eta_2}(t_1-t_2)+\tau_c]\right\}\right)^{-\nu}. \quad (18)$$

The cutoff-dependent contribution to the argument of the cotangent functions at the second and at the third line of Eq. (17) is effective only when $t-t_1-x/v\sim0$ (second line), or when $t-t_1-x/v\sim0$ (third line). This enables us to set $\sigma_{\eta,\eta_1}(t-t_1)=\sigma_{\eta,\eta_1}(x/v)=\eta_1$ (second line), and $\sigma_{\eta,\eta_2}(t-t_2)=\sigma_{\eta,\eta_2}(x/v)=\eta_2$ (third line). As a result, we may eventually rewrite Eq. (17) as

$$\langle T_K\partial_x\phi_k(x,t,\eta)e^{i\sqrt{\nu}\phi_k(0,t_1,\eta_1)}e^{-i\sqrt{\nu}\phi_k(0,t_2,\eta_2)}\rangle_0=\frac{\sqrt{\nu}}{v}\left[\xi_{\eta_1}\left(t-t_1-\frac{x}{v}\right)-\xi_{\eta_2}\left(t-t_2-\frac{x}{v}\right)\right]G^{(\nu)}_{\eta_1,\eta_2}(t_1-t_2), \quad (19)$$

with

$$\xi_\eta(t)=\frac{\pi}{\hbar\beta}\cot\left[\frac{\pi}{\hbar\beta}(it+\eta\tau_c)\right]. \quad (20)$$

Taking into account the result in Eq. (19), it is now possible to explicitly compute $S(\Omega;x_1,x_2)$. Introducing the Fourier transform of $G^{(\nu)}_{\eta_1,\eta_2}$ and of ξ_η (see Appendix A for details), we obtain

$$S(\Omega;x_1,x_2)=\frac{|\Gamma|^2e^2\nu^2}{4\pi^2\hbar^2}e^{i\Omega(x_1-x_2)}\sum_{\eta_1,\eta_2}\eta_1\eta_2\big\{[\xi_{\eta_1}(\Omega)\xi_{\eta_1}(-\Omega)+\xi_{\eta_2}(\Omega)\xi_{\eta_2}(-\Omega)]G^{(2\nu)}_{\eta_1,\eta_2}(\nu\Delta\mu)$$
$$-\xi_{\eta_1}(\Omega)\xi_{\eta_2}(-\Omega)G^{(2\nu)}_{\eta_1,\eta_2}(\nu\Delta\mu+\Omega)-\xi_{\eta_1}(-\Omega)\xi_{\eta_2}(\Omega)G^{(2\nu)}_{\eta_1,\eta_2}(\nu\Delta\mu-\Omega)\big\}, \quad (21)$$

with $\Delta\mu=\mu_1-\mu_2$. Using the explicit formulas for $G^{(\nu)}_{\eta_1\eta_2}$ and $\xi_\eta(\Omega)$ [see Eqs. (A9), (A10), (A11), (A12), and (A14)], we perform the sum over the Keldysh indices. Taking the limit for $\Omega\to0$ eventually, we obtain

$$S(0)=2i\left(\frac{\hbar\beta}{2\pi}\right)^{1-2\nu}\frac{e^2\nu^2\tau_c^{2\nu}|\Gamma|^2}{\hbar^2\pi\,\Gamma(2\nu)}\sinh\left(\frac{\beta\nu\Delta\mu}{2}\right)\left|\Gamma\left(\nu+\frac{i\beta\nu\Delta\mu}{2\pi}\right)\right|^2\left[\psi\left(\nu+\frac{i\beta\nu\Delta\mu}{2\pi}\right)-\psi\left(\nu-\frac{i\beta\nu\Delta\mu}{2\pi}\right)\right]. \quad (22)$$

To recover a compact notation, in Eq. (22), we have expressed $S(0)$ in terms of Euler Gamma function $\Gamma(z)$ and of its logarithmic derivative, the digamma function $\psi(z)=\partial_z[\ln\Gamma(z)]$. As anticipated, in order to obtain the correct result for $S(0)$, one has to first perform the calculation of $S(\Omega;x_1,x_2)$ at finite Ω and then take the limit $\Omega \to 0$ at the end of the calculation, thus avoiding problems related to $\xi_\eta(\Omega)$ being ill-defined as $\Omega \to 0$ (see Appendix A for details). Taking $\nu = 1$ in Eq. (22) reproduces the known result for noninteracting electrons [44], which we discuss in detail in Appendix B.

In the next section, we will look at the ratio $S(0)/i_T$, with i_T being the tunneling current across the QPC. For the reader's convenience, we report below the standard result [45]

$$i_T = \frac{2q}{\hbar^2\Gamma(2\nu)}|\Gamma|^2\tau_c^{2\nu}\left(\frac{\hbar\beta}{2\pi}\right)^{1-2\nu}\sinh\left(\frac{\beta\nu\Delta\mu}{2}\right)\left|\Gamma\left(\nu+i\frac{\beta\Delta\mu\nu}{2\pi}\right)\right|^2. \tag{23}$$

The generalization of Eq. (23), beyond perturbative expansion, was calculated exactly by Bethe ansatz in Refs. [46–48].

IV. FRACTIONAL STATISTICS DETECTION FROM CURRENT-CURRENT CORRELATION

Equation (22) is the main result of this paper, in this section, we discuss its consequences. As a first comment, we notice that in analogy with the result in Eq. (3), we find that $S(0) = 0$ for $\mu_1 = \mu_2$. This is consistent with Büttiker's result of Ref. [44] for the noninteracting case ($\nu = 1$), and it is now generalized to the case of Laughlin fractions. Such a result for $\nu = 1$ is easily shown to be in agreement with the fluctuation-dissipation theorem. Indeed, for noninteracting electrons, the equilibrium correlation function between currents measured at drains α and β, i_α, and i_β, satisfies the relation

$$\int_{-\infty}^{+\infty}\langle\langle\{i_\alpha(t),i_\beta(0)\}\rangle\rangle dt = 2(\mathbf{G}_{\alpha,\beta}+\mathbf{G}_{\beta,\alpha})k_BT, \tag{24}$$

with $\mathbf{G}_{\alpha,\beta}$ being the dc conductance between terminals α and β. In fact, in the particular geometry we are considering here, no electric current can flow between x_1 and x_2, because of the chiral propagation along the edge channels. Therefore we have shown that a result similar to that of Ref. [44] also applies to currents at the edges of a FQH liquid. Moreover, Eq. (3) shows that, for any Laughlin filling ν, the current-current correlation is negative, suggesting that the beam splitter geometry we consider highlights the exclusion statistics character of Laughlin's quasiparticles [49]. This result agrees with Ref. [32] for their case $\nu = 1/3$ but not for $\nu \leqslant 1/5$, and it is in contrast with Refs. [34,36,37]. We suspect that this is somehow related to the different geometries involved, and we will investigate this issue in future works. We also notice that negative correlations are found in Ref. [50] where the current-current correlation is studied for a beam of diluted anyons impinging on a beam splitter, an analysis complementary to the study reported here.

Besides the results outlined above, our most important finding is that, combining together Eqs. (22) and (23), it is possible to propose a way to directly measure the fractional statistics of LQPs. A key observation is now that, by setting $\Delta\mu = eV$, where V is the voltage bias between e_1 and e_2, the argument of the Gamma and the digamma functions in Eqs. (22) and (23) can be rewritten as $\zeta = \nu + i\frac{\beta qV}{2\pi}$. Roughly speaking, one might say that $\Re e(\zeta)$ carries information about the fractional statistics, while $\Im m(\zeta)$ carries information about the fractional charge. Therefore one might expect that either information can be extracted, according to whether one considers the formulas in the limit $|\Re e(\zeta)/\Im m(\zeta)| \gg 1$, or $|\Re e(\zeta)/\Im m(\zeta)| \ll 1$.

Let us discuss first the case $|\Re e(\zeta)/\Im m(\zeta)| \ll 1$, corresponding to $|eV| \gg k_BT$. In this regime, an appropriate approximation for Eqs. (22) and (23) can be derived by using Stirling's formula for the Γ functions, that is,

$$\Gamma(z) \approx \sqrt{2\pi}\,(z-1)^{z-\frac{1}{2}}\,e^{-z+1}, \tag{25}$$

valid for $|z| \gg 1$. Using Eq. (25), one finds the following asymptotic expansions for $S(0)$ and i_T (assuming $V > 0$):

$$\begin{aligned} S(0) &\approx -\frac{2\pi q^2|\Gamma|^2\tau_c^{2\nu}}{\hbar^{2\nu+1}\Gamma[2\nu]}(qV)^{2\nu-1},\\ i_T &\approx \frac{2\pi q|\Gamma|^2\tau_c^{2\nu}}{\hbar^{2\nu+1}\Gamma[2\nu]}(qV)^{2\nu-1}. \end{aligned} \tag{26}$$

Equations (26) suggest that, for $|eV| \gg k_BT$, the fractional charge q can be directly probed by looking at the ratio $q = |S(0)|/i_T$ between two directly measurable quantities such as $S(0)$ and i_T, which is the main idea typically implemented in shot-noise based measurements of the fractional charge (notice that here, instead, we look at correlation between currents at different drains).

In the complementary limit, in order to directly access information on the fractional statistics, one has rather to consider the thermal regime, namely, $|eV| \ll k_BT$. In this regime, the limiting formulas for Eqs. (22) and (23) can be recovered by expanding the Gamma and the digamma functions to leading order in $\Im m(\zeta)/\Re e(\zeta)$, obtaining

$$\begin{aligned} S(0) &\approx -\frac{q^2|\Gamma|^2\tau_c^{2\nu}}{\hbar^{2\nu+1}\Gamma[2\nu]}\left(\frac{\beta}{2\pi}\right)^{1-2\nu}(\beta qV)^2\{\Gamma''[\nu]\Gamma[\nu]-(\Gamma'[\nu])^2\},\\ i_T &\approx \frac{q|\Gamma|^2\tau_c^{2\nu}\Gamma^2[\nu]}{\hbar^{2\nu+1}\Gamma[2\nu]}\left(\frac{\beta}{2\pi}\right)^{1-2\nu}(\beta qV). \end{aligned} \tag{27}$$

From Eq. (27), one therefore obtains

$$\frac{|S(0)|}{i_T} = \left(\frac{q^2\pi^2 V}{6k_BT}\right)\gamma(\nu), \tag{28}$$

with $\gamma(\nu) = (6/\pi^2)\partial_z^2 \ln\Gamma(z)|_{z=\nu}$. Except for the factor $\gamma(\nu)$, the result in Eq. (28) is the same one would obtain for noninteracting electrons ($\nu = 1$) by simply replacing e with q. Therefore the additional factor $\gamma(\nu)$ is not a feature simply related to the fractional charge of LQPs—it is a clear signature of the quasiparticle *fractional statistics* which, as we propose, can be directly measured by looking at current correlations probed in the appropriate thermal regime. We report here, for the convenience of readers, some numerical values of $\gamma(\nu)$, $\gamma(1) = 1$, $\gamma(1/3) \simeq 6.18$, and $\gamma(1/5) \simeq 15.97$.

As a final remark, we notice that the reason to look at the correlation of currents measured at different drains lies in the fact that, if one considers noise of the tunneling current, i.e.,

$S_{i_T} = (1/2)\int dt\langle\langle\{i_T(0),i_T(t)\}\rangle\rangle$, one would obtain [41]

$$S_{i_T} = \frac{2q^2}{\hbar^2\Gamma(2\nu)}|\Gamma|^2\tau_c^{2\nu}\left(\frac{\hbar\beta}{2\pi}\right)^{1-2\nu} \times \cosh\left(\frac{\beta\nu\Delta\mu}{2}\right)\left|\Gamma\left(\nu+i\frac{\beta\Delta\mu\nu}{2\pi}\right)\right|^2. \quad (29)$$

Such a quantity normalized to the tunneling current i_T of Eq. (23), i.e., the Fano factor, only carries information about the quasiparticles' charge but not their statistics.

V. SUMMARY AND OUTLOOK

We have discussed the correlation of currents measured at separate drains in a beam splitterlike geometry for fractional quantum Hall systems at Laughlin filling factors. Because of the chiral propagation of LQPs along the edge channels, we have proved (within perturbation theory) that the equilibrium correlation, i.e., for $\mu_1 = \mu_2$, is zero, as it was found for chiral fermions ($\nu = 1$). Using Keldysh technique, we have also obtained expressions for the stationary out of equilibrium case and show how correlation measurements carry information about the fractional statistics. Our findings suggest an antibunching character of the LQPs.

In perspective, our result might also provide a useful tool to investigate more exotic filling fractions like for instance $\nu = 2/3$ where neutral counterpropagating modes have been predicted [51,52] and recently observed [53–55], but still need a thorough characterization.

In such systems, even for both the measuring points x_1 and x_2 situated after the QPC, due to the counterpropagating modes and their interaction with the charge modes, one might expect a signal propagation between these two points—giving rise to a nonzero *equilibrium* correlation. We will investigate this possibility as a tool to study counterpropagating modes in future works. In addition, we also plan to expand our work to analyze the relation between correlations and fractional statistics-related interactions among particles with fractionalized quantum numbers [56–58].

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APPENDIX A: GREEN'S FUNCTIONS

In this Appendix, we provide the details of the calculation of the Green's functions we use to compute the current-current correlation, and their corresponding Fourier transforms. The first quantities we need are the correlation functions of vertex operators evaluated on branches η_1,η_2 of the Keldysh path,

$$G^{(\nu)}_{\eta_1,\eta_2}(t_1,t_2) = \left\langle T_K e^{i\sqrt{\nu}\phi_k(t_1,\eta_1)}e^{-i\sqrt{\nu}\phi_k(t_2,\eta_2)}\right\rangle_0. \quad (A1)$$

A standard bosonization calculation yields the result in Eq. (18),

$$G^{(\nu)}_{\eta_1,\eta_2}(t_1,t_2) = l_c^\nu\left(\frac{\hbar\beta v}{\pi}\right)^{-\nu}\sin^{-\nu}\left[\frac{\pi}{\hbar\beta}(it\,\sigma_{\eta_1,\eta_2}(t_1-t_2)+\tau_c)\right]. \quad (A2)$$

For the sake of clarity, we list the Keldysh Green functions corresponding to the four possible choices of the Keldysh indices:

$$\begin{aligned}
G^{(\nu)}_{+,+}(t) &= l_c^\nu\left(\frac{\hbar\beta v}{\pi}\right)^{-\nu}\ \sin^{-\nu}\left[\frac{\pi}{\hbar\beta}(i|t|+\tau_c)\right],\\
G^{(\nu)}_{-,-}(t) &= l_c^\nu\left(\frac{\hbar\beta v}{\pi}\right)^{-\nu}\ \sin^{-\nu}\left[\frac{\pi}{\hbar\beta}(-i|t|+\tau_c)\right],\\
G^{(\nu)}_{-,+}(t) &= l_c^\nu\left(\frac{\hbar\beta v}{\pi}\right)^{-\nu}\ \sin^{-\nu}\left[\frac{\pi}{\hbar\beta}(it+\tau_c)\right],\\
G^{(\nu)}_{+,-}(t) &= l_c^\nu\left(\frac{\hbar\beta v}{\pi}\right)^{-\nu}\ \sin^{-\nu}\left[\frac{\pi}{\hbar\beta}(-it+\tau_c)\right].
\end{aligned} \quad (A3)$$

Next, we compute the Fourier transform of Eq. (A3) defined as

$$G^{(\nu)}_{\eta_1,\eta_2}(\omega) = \int_{-\infty}^{+\infty} dt\, e^{i\omega t}\, G^{(\nu)}_{\eta_1,\eta_2}(t).$$

In computing $G^{(\nu)}_{\eta_1,\eta_2}(\omega)$, it is useful to start with $G^{(\nu)}_{-,+}(\omega)$ and with $G^{(\nu)}_{+,-}(\omega)$. Moreover, in view of the identity $G^{(\nu)}_{-,+}(\omega) = G^{(\nu)}_{+,-}(-\omega)$ (which is readily proved from the definition of the Keldysh Green functions), one concludes that it is enough to just compute $G^{(\nu)}_{-,+}(\omega)$. In order to do so, we notice that the branch points of $G^{(\nu)}_{-,+}(t)$ are located at $t_n = i(\tau_c + \hbar\beta n)$, with $n = 0, \pm 1, \ldots$. Therefore to make sure that no branch cuts intersect the real axis in computing $G^{(\nu)}_{-,+}(\omega)$, we chose the phase branch so that $-\pi \leqslant \arg(it) < \pi$ and, accordingly, the branch cuts are all horizontal. Having stated this, $G^{(\nu)}_{-,+}(\omega)$ takes the following integral representation:

$$\begin{aligned}
G^{(\nu)}_{-,+}(\omega)
&= l_c^\nu\left(\frac{\hbar\beta v}{2\pi}\right)^{-\nu}\int_{-\infty}^{\infty} dt\, e^{i\omega t}[(-i)e^{-\frac{\pi t}{\hbar\beta}}e^{i\delta} - (-i)e^{\frac{\pi t}{\hbar\beta}}e^{-i\delta}]^{-\nu}\\
&= \frac{2l_c^\nu}{v^\nu}\left(\frac{\hbar\beta}{2\pi}\right)^{1-\nu}\int_0^{\infty} du\, e^{i\frac{\hbar\omega\beta}{\pi}u}[(-i)e^{-u}e^{i\delta} - (-i)e^{u}e^{-i\delta}]^{-\nu}\\
&\quad + \frac{2l_c^\nu}{v^\nu}\left(\frac{\hbar\beta}{2\pi}\right)^{1-\nu}\int_{-\infty}^{0} du\, e^{i\frac{\hbar\omega\beta}{\pi}u}[(-i)e^{-u}e^{i\delta} - (-i)e^{u}e^{-i\delta}]^{-\nu}\\
&= \frac{2l_c^\nu}{v^\nu}\left(\frac{\hbar\beta}{2\pi}\right)^{1-\nu} e^{i\pi\nu/2}\int_0^{\infty} du\, e^{\left(i\frac{\hbar\omega\beta}{\pi}-\nu\right)u}[1-e^{-2u}]^{-\nu}\\
&\quad + \frac{2l_c^\nu}{v^\nu}\left(\frac{\hbar\beta}{2\pi}\right)^{1-\nu} e^{-i\pi\nu/2}\int_{-\infty}^{0} du\, e^{\left(i\frac{\hbar\omega\beta}{\pi}+\nu\right)u}[1-e^{2u}]^{-\nu}.
\end{aligned} \quad (A4)$$

In Eq. (A4), we have set $\delta = \pi\tau_c/\hbar\beta$ and have taken advantage of the fact that, in the last two lines, it was possible to drop the terms depending on the regularizator δ. Going through straightforward manipulation we can readily trade Eq. (A4) for a known integral representation of the Beta function, that is,

$$B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)} = \int_0^1 dw\, w^{x-1}(1-w)^{y-1}. \tag{A5}$$

To do so, we resort to the integration variable $w = e^{-2u}$ ($w = e^{2u}$) in the first (second) integral of Eq. (A4), so that we eventually obtain

$$G^{(\nu)}_{-,+}(\omega) = \left(\frac{l_c}{v}\right)^{\nu}\left(\frac{\hbar\beta}{2\pi}\right)^{1-\nu} e^{-i\pi\nu/2}\int_0^1 dw\, w^{i\frac{\hbar\omega\beta}{2\pi}+\frac{\nu}{2}-1}(1-w)^{-\nu} + \left(\frac{l_c}{v}\right)^{\nu}\left(\frac{\hbar\beta}{2\pi}\right)^{1-\nu} e^{i\pi\nu/2}\int_0^1 dw\, w^{-i\frac{\hbar\omega\beta}{2\pi}+\frac{\nu}{2}-1}(1-w)^{-\nu}. \tag{A6}$$

Comparing Eq. (A6) to Eq. (A5), we eventually find

$$G^{(\nu)}_{-,+}(\omega) = \left(\frac{l_c}{v}\right)^{\nu}\left(\frac{\hbar\beta}{2\pi}\right)^{1-\nu}\Gamma(1-\nu)\left[e^{i\pi\nu/2}\frac{\Gamma\left(\frac{\nu}{2}-i\frac{\hbar\omega\beta}{2\pi}\right)}{\Gamma\left(1-\frac{\nu}{2}-i\frac{\hbar\omega\beta}{2\pi}\right)} + e^{-i\pi\nu/2}\frac{\Gamma\left(\frac{\nu}{2}+i\frac{\hbar\omega\beta}{2\pi}\right)}{\Gamma\left(1-\frac{\nu}{2}+i\frac{\hbar\omega\beta}{2\pi}\right)}\right]. \tag{A7}$$

Finally, using the identity

$$\Gamma(z)\Gamma(1-z) = \frac{\pi}{\sin(\pi z)}, \tag{A8}$$

we can recast Eq. (A7) into the form

$$G^{(\nu)}_{-,+}(\omega) = \frac{l_c^{\nu}}{v^{\nu}\Gamma(\nu)}\left(\frac{\hbar\beta}{2\pi}\right)^{1-\nu} e^{\frac{\hbar\omega\beta}{2}}\left|\Gamma\left(\frac{\nu}{2}+i\frac{\hbar\omega\beta}{2\pi}\right)\right|^2. \tag{A9}$$

Equation (A9) also implies

$$G^{(\nu)}_{+,-}(\omega) = \frac{l_c^{\nu}}{v^{\nu}\Gamma(\nu)}\left(\frac{\hbar\beta}{2\pi}\right)^{1-\nu} e^{-\frac{\hbar\omega\beta}{2}}\left|\Gamma\left(\frac{\nu}{2}+i\frac{\hbar\omega\beta}{2\pi}\right)\right|^2. \tag{A10}$$

Following exactly the same strategy of splitting the integral over t into an integral from $-\infty$ to 0 plus and integral from 0 to ∞ and separately manipulating the two integrals as we have done before, one eventually finds

$$G^{(\nu)}_{+,+}(\omega) = \frac{l_c^{\nu}e^{-i\pi\nu/2}}{v^{\nu}\Gamma(\nu)\cos\left(\frac{\pi\nu}{2}\right)}\left(\frac{\hbar\beta}{2\pi}\right)^{1-\nu} \times \cosh\left(\frac{\hbar\omega\beta}{2}\right)\left|\Gamma\left(\frac{\nu}{2}+i\frac{\hbar\omega\beta}{2\pi}\right)\right|^2, \tag{A11}$$

and

$$G^{(\nu)}_{-,-}(\omega) = \frac{l_c^{\nu}e^{i\pi\nu/2}}{v^{\nu}\Gamma(\nu)\cos\left(\frac{\pi\nu}{2}\right)}\left(\frac{\hbar\beta}{2\pi}\right)^{1-\nu} \times \cosh\left(\frac{\hbar\omega\beta}{2}\right)\left|\Gamma\left(\frac{\nu}{2}+i\frac{\hbar\omega\beta}{2\pi}\right)\right|^2. \tag{A12}$$

Equations (A9)–(A12) provide us with the Fourier transforms of the Keldysh Green functions, which we used in the main text to compute the current correlations. To perform the calculation, one needs an additional function, the Fourier transform of $\xi_\pm(t)$. These are given by

$$\xi_\eta(\omega) = \int_{-\infty}^{\infty} dt\, e^{i\omega t}\xi_\eta(t) = i\int_{-\infty}^{\infty} du\, e^{i\frac{\hbar\omega\beta}{\pi}u}\left(\frac{e^u e^{-i\eta\frac{\pi\tau_c}{\hbar\beta}} + e^{-u}e^{i\eta\frac{\pi\tau_c}{\hbar\beta}}}{e^u e^{-i\eta\frac{\pi\tau_c}{\hbar\beta}} - e^{-u}e^{i\eta\frac{\pi\tau_c}{\hbar\beta}}}\right), \tag{A13}$$

where we have set $u \equiv t\pi/\hbar\beta$. When $\omega \neq 0$, a straightforward application of residue theorem gives

$$\xi_+(\omega) = \int_{-\infty}^{\infty} d\omega\, e^{i\omega t}\xi_+(t) = \frac{2\pi e^{-\omega\tau_c}}{1-e^{-\hbar\beta\omega}},$$
$$\xi_-(\omega) = \int_{-\infty}^{\infty} d\omega\, e^{i\omega t}\xi_-(t) = \frac{2\pi e^{\omega\tau_c}}{e^{\hbar\beta\omega}-1}. \tag{A14}$$

The integrals in Eq. (A13) are ill-defined if $\omega = 0$, this motivates the need of first computing the current-current correlation in Fourier space at finite frequency Ω, and then only afterwards take $\Omega \to 0$.

APPENDIX B: CURRENT-CURRENT CORRELATION FOR CHIRAL FERMIONS

In this appendix, to check the consistency of the formulas we derived in Sec. III with the standard results obtained by Büttiker in the noninteracting case, we derive the current-current correlation in the case of the integer quantum Hall (IQH) effect at filling $\nu = 1$. At $\nu = 1$, IQH edges e_1 and e_2 (see Fig. 2) are described by the noninteracting chiral fermion Hamiltonian

$$H_0 = -i\hbar v\sum_{j=1}^{2}\int dx\, :\psi_j^\dagger(x)\partial_x\psi_j(x): , \tag{B1}$$

with v being the Fermi velocity. In the momentum basis, the chiral fermionic fields $\psi_i(x)$ take the mode expansion

$$\psi_j(x) = \frac{1}{\sqrt{\mathcal{L}_j}}\sum_{k_j} e^{ik_j x}c_{k_j,j}. \tag{B2}$$

In Eq. (B2), we use $\mathcal{L}_i$ to denote the length of the edge i, which we assume to be large enough to be irrelevant for our final result. The double columns : : denote normal ordering with respect to the ground state $|GS\rangle = \prod_{i=1,2;\epsilon(k_j)\leqslant 0} c^\dagger_{k_j,i}|0\rangle$. $c_{k_j,i}$ is the electron annihilation operator for a state with momentum k_j on edge i. The creation and annihilation operators in the momentum basis satisfy the standard fermionic anticommutations rules, $\{c^\dagger_{k_j,j},c_{k'_{j'},j'}\} = \delta_{k_j,k'_{j'}}\delta_{j,j'}$, $\{c_{k_j,j},c_{k'_{j'},j'}\} = 0$. To account for the chemical potential bias between the edges, we assume that, in the absence of tunneling, each edge i is at equilibrium with a reservoir at chemical potential μ_i.

In order to allow for electrons to tunnel between the two edges, we consider the tunneling Hamiltonian H_T given by

$$H_T = \Gamma_e\psi_1^\dagger(0)\psi_2(0) + \text{H.c.} \tag{B3}$$

The current density operator at site x of edge i is given by $i_i(x) = ev : \psi^\dagger(x)\psi(x) :$. The current correlation function between point x_1 on e_1 and point x_2 on e_2 (cfr Fig. 2) is defined as in Eq. (10) and, resorting again to the Keldysh formalism we use in Sec. III, we readily find that the corresponding zero-frequency limit, $S(0)$, is given by

$$S(0) = \frac{e^2 v^2}{2} \sum_{\eta=\pm1} \int_{-\infty}^{+\infty} d(t_1 - t_2) \langle\langle T_K : \hat{\psi}_1^\dagger(x_1,t_1 + \eta\, 0^+,\eta)\hat{\psi}_1(x_1,t_1,\eta) :: \hat{\psi}_2^\dagger(x_2,t_2 - \eta\, 0^+, -\eta)\hat{\psi}_2(x_2,t_2,-\eta) :\rangle\rangle. \quad \text{(B4)}$$

[Note that, in order to preserve the correct ordering of the fermionic operators under the action of T_K, in Eq. (B4), we introduced the infinitesimal positive quantity 0^+ as a regularizator.] Assuming a weak tunneling rate between the edges to recover consistency with the analysis of Sec. III, we compute $S(0)$ to second order in H_T, obtaining

$$\begin{aligned} S(0) = &-\frac{|\Gamma_e|^2 e^2 v^2}{4\hbar^2} \sum_{\eta,\eta_1,\eta_2=\pm1} \eta_1\eta_2 \int_{-\infty}^{+\infty} d(t_1 - t_2) \int_{-\infty}^{+\infty} ds_1 \int_{-\infty}^{+\infty} ds_2 \\ &\times \big[\mathcal{G}^{(1)}_{\eta_2,\eta}(-x_1,s_2 - t_1)\mathcal{G}^{(1)}_{\eta,\eta_1}(x_1,t_1 - s_1)\mathcal{G}^{(2)}_{\eta_1,-\eta}(-x_2,s_1 - t_2)\mathcal{G}^{(2)}_{-\eta,\eta_2}(x_2,t_2 - s_2) \\ &+ \mathcal{G}^{(1)}_{\eta_1,\eta}(-x_1,s_1 - t_1)\mathcal{G}^{(1)}_{\eta,\eta_2}(x_1,t_1 - s_2)\mathcal{G}^{(2)}_{\eta_2,-\eta}(-x_2,s_2 - t_2)\mathcal{G}^{(2)}_{-\eta,\eta_1}(x_2,t_2 - s_1)\big], \end{aligned} \quad \text{(B5)}$$

with the fermionic Keldysh Green function $\mathcal{G}^{(i)}_{\eta_1,\eta_2}(x_1 - x_2,t_1 - t_2) = -i\langle T_K \psi_i(x_1,t_1,\eta_1)\psi_i^\dagger(x_2,t_2,\eta_2)\rangle$, with $\psi_i(x,t)$ being the fermion fields in the interaction representation with respect the Hamiltonian H_0. Moving to Fourier space, we may rewrite Eq. (B5) as

$$S(0) = \frac{|\Gamma_e|^2 e^2 v^2}{2\hbar^2} \sum_{\eta,\eta_1,\eta_2=\pm1} \eta_1\eta_2 \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \big[\mathcal{G}^{(1)}_{\eta_2,\eta}(-x_1,\omega)\mathcal{G}^{(1)}_{\eta,\eta_1}(x_1,\omega)\mathcal{G}^{(2)}_{\eta_1,-\eta}(-x_2,\omega)\mathcal{G}^{(2)}_{-\eta,\eta_2}(x_2,\omega)\big], \quad \text{(B6)}$$

with the single-fermion Keldysh Green functions in Fourier space given by [37]

$$\mathcal{G}^{(i)}_{++}(x,\omega) = \frac{i}{v} e^{i\omega x/v}[f(\hbar\omega - \mu_i) - \Theta(x)], \quad \text{(B7)}$$

$$\mathcal{G}^{(i)}_{+-}(x,\omega) = \frac{i}{v} e^{i\omega x/v} f(\hbar\omega - \mu_i), \quad \text{(B8)}$$

$$\mathcal{G}^{(i)}_{-+}(x,\omega) = -\frac{i}{v} e^{i\omega x/v}[1 - f(\hbar\omega - \mu_i)], \quad \text{(B9)}$$

$$\mathcal{G}^{(i)}_{--}(x,\omega) = \frac{i}{v} e^{i\omega x/v}[f(\hbar\omega - \mu_i) - \Theta(-x)]. \quad \text{(B10)}$$

In Eq. (B10), $f(\omega)$ denotes the Fermi-Dirac distribution function $f(\omega) = [1 + \exp(\beta\hbar\omega)]^{-1}$, while $\Theta(x)$ is the Heaviside step function regularized so that $\Theta(0) = 1/2$.

Performing the sum over the Keldysh indices, using Eq. (B10) for both x_1 and $x_2 > 0$, we obtain

$$S(0) = -\frac{e^2|\Gamma_e|^2}{2\pi v^2\hbar^2} \int_{-\infty}^{+\infty} d\omega [f(\hbar\omega - \mu_1) - f(\hbar\omega - \mu_2)]^2. \quad \text{(B11)}$$

Notice that in Eq. (9) the tunneling amplitude Γ has the dimension of an energy, while Γ_e has the dimensions of an energy times a length. Equation (22) evaluated for $\nu = 1$ reproduces Eq. (B11) by taking $\Gamma = \Gamma_e/(2\pi l_c)$, which is indeed consistent with the bosonization identity [59] $\psi_i(x) = e^{-i\phi_i(x)}/\sqrt{2\pi l_c}$.

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