# Enhanced Casimir effect for doped graphene

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We analyze the Casimir interaction of doped graphene. To this end we derive a simple expression for the finite-temperature polarization tensor with a chemical potential. It is found that doping leads to a strong enhancement of the Casimir force, reaching almost 60% in quite realistic situations. This result should be important for planning and interpreting Casimir measurements, especially taking into account that the Casimir interaction of undoped graphene is rather weak.

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# I. INTRODUCTION

Graphene, which is a two-dimensional sheet of carbon atoms, possesses many unusual properties and attracts a lot of attention. Particular excitement among the theoreticians is caused by the fact that the spectrum of quasiparticles in graphene is described by the quasirelativistic Dirac model with an effective propagation speed of about 300 times less than the speed of light. This continuous model turned out to be very successful in describing a broad range of effects [1], for instance, the optical properties of graphene such as the absorption of light [2] and the (giant) Faraday effect [3], to mention a few.

In recent years, the Casimir effect for pristine graphene was studied for both zero [4–6] and finite [7,8] temperatures. For temperatures that are not too high (compared with the inverse distance between the interaction sheets), the effect between a graphene monolayer and an ideal metal is defined by the fine-structure constant  $\alpha \simeq 1/137$  and is roughly 2.5% of the one between two ideal metal plates. Such small forces are at the limit of the sensitivity of modern experimental techniques. For high temperatures (or separations) the effect is hugely reinforced [8], but measurements under such conditions are a separate, quite challenging task, which is not completely solved yet even for metals (see, e.g., [9]). It is not surprising therefore that just a single experiment has been performed until now [10]. That experiment revealed [11] a good agreement with the theory [12]. Possibilities opened by doping, were not, however, explored there.

Although it has been approached in some papers [13-17], until now the Casimir interaction of doped graphene has not been the principal subject of a study. The methods used in the above-cited papers applied various approximations, with the most important one being restriction to the longitudinal component of the conductivity in the description of graphene layers, as in [13-15] and [17]. This is equivalent to considering the isotropic approximation of the graphene conductivity despite the presence of spatial dispersion [16]. Moreover, the results in Refs. [13] and [14] and Ref. [15] are mutually contradictory as pointed out in the Conclusions in Ref. [14]. Therefore, after experimental confirmation of the approach to the Casimir energy of undoped graphene based on the full polarization tensor [10,11], we find it important to extend this approach to doped graphene as well. Though we agree with some of the qualitative predictions in the previous publications (like the enhancement of the Casimir interaction with doping [14,16]), we disagree on many important details, like the precise form of curves and the behavior at high doping.

In this article we consider the Casimir effect at finite temperature, chemical potential, and mass and find a substantial enhancement of the effect in graphene-metal systems which potentially permits us to avoid the above-mentioned experimental difficulties. Our findings show that for relatively highly (but still feasibly) doped graphene monolayers the effect gets stronger by approximately 60%. In the (formal) limit of an infinite chemical potential the Casimir interaction becomes one-half that for the ideal metal. Our calculation is based on a complete representation of the polarization tensor of the fermionic quasiparticles in graphene at finite temperature, chemical potential, and mass gap. The obtained result is surprisingly simple and can be easily analytically continued to the whole complex frequency plane, including, importantly, real optical frequencies. It is based on the QED formalism applied to graphene-like systems in [18] and [19] and generalizes the results in [4,8] and [20] and those in [21], which also allows continuation to the whole plane of complex frequencies, to the case of the simultaneous presence of a finite chemical potential, finite temperature, and nonzero mass gap. We also describe a very precise approximation scheme that considerably simplifies the computation of the Casimir interaction with graphene.

## **II. MODEL**

The theoretical description of the electronic properties of graphene is based on the continuous Dirac model with a 2 + 1-dimensional action. In the notation of [22] it reads

$$S_{\rm D} = \int d^3x \, \bar{\psi}(\tilde{\gamma}^l(i\partial_l - eA_l) - m)\psi, \qquad (1)$$

where l = 0, 1, 2 and  $x = (x^0, x^1, x^2)$ . The gamma matrices  $\tilde{\gamma}^l$  are rescaled:  $\tilde{\gamma}^0 \equiv \gamma^0$ ,  $\tilde{\gamma}^{1,2} \equiv v_F \gamma^{1,2}$ ,  $\gamma_0^2 = -(\gamma^1)^2 = -(\gamma^2)^2 = 1$ . We use natural units  $\hbar = c = k_B = 1$ , and the Fermi velocity is  $v_F \simeq (300)^{-1}$ . Therefore we assume that the graphene monolayer is placed at the  $(x^1, x^2)$  plane. The

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electromagnetic potential  $A_{\mu}$  is normalized in such a way that  $e^2 \equiv 4\pi \alpha = \frac{4\pi}{137}$ .

Reflecting the spin and valley degeneracy in graphene, the gamma matrices,  $\gamma^l$ , are 8×8, being the direct sum of four 2×2 representations (with two copies of each of the two inequivalent ones). The value of the mass-gap parameter *m* and mechanisms of its generation are under discussion [18,19].

As shown in many previous works (see references in [22]), the electronic properties of graphene in the formalism of quantum-field theory can be described by the one-loop polarization operator [33]. In Minkowski momentum space it is given by

$$\Pi^{mn}(p) = ie^2 \int \frac{d^3q}{(2\pi)^3} \operatorname{tr}(\hat{S}(q)\tilde{\gamma}^m \hat{S}(q-p)\tilde{\gamma}^n), \quad (2)$$

where  $p = (p_0, p_1, p_2)$ ,  $q = (q_0, q_1, q_2)$ , and  $\hat{S}$  is the causal (Feynman) propagator of the quasiparticles in graphene:

$$\hat{S}(q_0, \mathbf{q}_{\parallel}) = -\frac{(q_0 + \mu)\gamma_0 - v_F \mathbf{q}_{\parallel} - m}{(q_0 + \mu + i\epsilon q_0)^2 - v_F^2 \mathbf{q}_{\parallel}^2 - m^2}$$
(3)

 $(\epsilon > 0)$ . Note that due to the quasirelativistic nature of excitations in graphene,  $\hat{S}$  also depends on the Fermi velocity  $v_F$ . Further notations are  $\mathbf{q}_{\parallel} = (q^1, q^2)$ ,  $\mathbf{q}_{\parallel} = \gamma^1 q_1 + \gamma^2 q_2$ , and  $\mu$  is the chemical potential.

The temperature is introduced using the Matsubara formalism. In the  $\gamma$  trace in (2), which can be calculated immediately (see, e.g., Eq. (A20) in [20]), one has to substitute the integral with a sum,

$$i\int dq_0 \to -2\pi T \sum_{k=-\infty}^{\infty}, \quad q_0 \to 2\pi i T(k+1/2), \quad (4)$$

where k is an integer. The external frequency of the polarization operator is bosonic,  $p_0 \rightarrow i p_4 = 2\pi i T n, n = 0, 1, 2, ...$ 

## III. CALCULATION OF THE POLARIZATION OPERATOR FOR FINITE TEMPERATURE, MASS GAP, AND CHEMICAL POTENTIAL

All components of the polarization tensor can be expressed via two scalar quantities (form factors), for instance,  $\Pi_{tr} \equiv \Pi_{00} - \Pi_{11} - \Pi_{22}$  and  $\Pi_{00}$  [8,23]. As in the QED/QCD cases these quantities consist of the vacuum part and a part carrying the dependence on *T* and  $\mu$ ,

$$\Pi_{\rm xx}(p;\mu,T) = \Pi_{\rm xx}^{\rm (vac)}(p) + \Delta \Pi_{\rm xx}(p;\mu,T), \tag{5}$$

where xx stands for either "tr" or "00." The vacuum part,  $\Pi_{xx}^{(vac)}(p)$ , corresponds to  $\mu = T = 0$ . While such decomposition is a well-known feature of the polarization tensor in different theories (see, e.g., [24]), its realization in particular cases and the derivation of simple transparent formulas may be a challenging task. One transforms the sum over the Matsubara frequencies, (4), into a contour integral consisting of three parts; one gives the original integral over the continuous (Euclidean) momenta  $q_4$  and the other two can be taken explicitly by the Cauchy theorem. The remaining integral over the in-plane momenta,  $\mathbf{q}_{\parallel} = (q_1, q_2)$ , can be further simplified by performing angular integration. Recently this procedure was applied to graphene at  $\mu = 0$ ,  $T \neq 0$  in [21] and T = 0,  $\mu \neq 0$  in [20]. Omitting the technicalities we arrive at

$$\Delta \Pi_{xx} = \frac{8\alpha}{v_F^2} \int_m^\infty d\varkappa \left( 1 + \operatorname{Re} \frac{M_{xx}}{\sqrt{Q^2 - 4p_{\parallel}^2(\varkappa^2 - m^2)}} \right) \Xi(\varkappa).$$
(6)

Here the distribution function,  $\Xi \equiv (e^{(\alpha+\mu)/T} + 1)^{-1} + (e^{(\alpha-\mu)/T} + 1)^{-1}$ , carries the dependence on *T* and  $\mu$ . Further notations in (6) are

$$\begin{split} M_{00} &= -\tilde{p}^2 + 4ip_4\varkappa + 4\varkappa^2, \\ M_{\rm tr} &= -(2 - v_F^2)(4m^2 - \tilde{p}^2) + 4(1 - v_F^2)(p_4\varkappa + \varkappa^2 - m^2), \\ Q &= \tilde{p}^2 - 2ip_4\varkappa, \quad \tilde{p}^2 \equiv p_4^2 + v_F^2 p_{\parallel}^2, \quad p_{\parallel} = |\mathbf{p}_{\parallel}|. \end{split}$$

Note that  $\Delta \Pi_{xx}$ , (6), does not have UV singularities.

For the vacuum part,  $\Pi^{(vac)}$ , one can directly use the well-known expressions [4] valid for graphene,

$$\Pi_{00}^{(\text{vac})} = \frac{\alpha \Phi p_{\parallel}^2}{\tilde{p}^2}, \quad \Pi_{\text{tr}}^{(\text{vac})} = \frac{\alpha \Phi (p^2 + \tilde{p}^2)}{\tilde{p}^2}, \tag{7}$$

where  $\Phi = 4[m + \frac{\tilde{p}^2 - 4m^2}{2\tilde{p}} \arctan(\frac{\tilde{p}}{2m})]$ . Henceforth we set m = 0 (gapless graphene) unless otherwise stated.

One of the advantages of the decomposition, (5), is the absence of the summation over Matsubara frequencies, which permits relatively easy derivation of the limiting cases. In particular, in the limit of zero temperature (but not zero chemical potential) we obtain

$$\Delta \Pi_{tr} = 8\alpha \tag{8}$$

$$\left(\frac{\mu}{v_F^2} - \frac{\tilde{p}^2 + p^2}{4\tilde{p}} \operatorname{Im}(\log[x + \sqrt{x^2 + 1}] - \frac{\tilde{p}x}{2}\sqrt{x^2 + 1})\right),$$
  
$$\Delta \Pi_{00} = 8\alpha \left(\frac{\mu}{v_F^2} - \frac{p_{\parallel}^2}{4\tilde{p}} \operatorname{Im}(x\sqrt{x^2 + 1} + \log[x + \sqrt{x^2 + 1}])\right),$$
  
(9)

with  $x = \frac{2i\mu - p_4}{v_F p_{\parallel}}$ . The formulas above are the analog of the (B2) [20] taken at Matsubara frequencies, and one can check that in the appropriate limits they reproduce the results of other authors [4,8,18–21]. Similarly to the results in [21], the representation (5) with (6) directly permits continuation to real frequencies and, thus, can be applied for investigation of the optical properties, surface plasmons, and other effects in graphene at a finite temperature and chemical potential.

#### IV. ENHANCEMENT OF THE CASIMIR EFFECT

The Casimir energy density (per unit area) for two parallel interfaces separated by the distance *a* is given by the Lifshitz formula [25] in terms of the reflection coefficients,  $r_{\text{TE,TM}}^{(1)}$  and  $r_{\text{TE,TM}}^{(2)}$ , of the TE and TM electromagnetic modes on the two interfaces,

$$\mathcal{E} = k_B T \sum_{n=-\infty}^{\infty} \int \frac{d^2 \mathbf{p}_{\parallel}}{8\pi^2} \sum_{X=\text{TE,TM}} \ln\left(1 - e^{-2p_{\parallel}a} r_X^{(1)} r_X^{(2)}\right), \quad (10)$$

where  $p = \sqrt{p_4^2 + p_{\parallel}^2}$ , and  $p_4 = 2\pi nT$  are the Matsubara frequencies [34]. The reflection coefficients are taken at Euclidean momenta  $r = r(p_4, \mathbf{p}_{\parallel})$ . They were derived in [8] in terms of the polarization operator components  $\Pi_{00,\text{tr}}$ ,

$$r_{\rm TM} = \frac{p\Pi_{00}}{p\Pi_{00} + 2p_{\parallel}^2}, \quad r_{\rm TE} = -\frac{p^2\Pi_{00} - p_{\parallel}^2\Pi_{\rm tr}}{p^2\Pi_{00} - p_{\parallel}^2(\Pi_{\rm tr} + 2p)}$$
(11)

(and rederived in numerous papers afterwards). For better comparison with previous works we also give the reflection coefficients in terms of the components of the anisotropic graphene conductivity taken at Euclidian momenta,  $\sigma_{ab}(p_4) = \frac{\prod_{ab}(p_4)}{p_4}$ , a, b = 1,2 (see [22] for more details),

$$r_{\rm TM} = \frac{p\sigma_{\rm TM}}{2p_4 + p\sigma_{\rm TM}}, \quad r_{\rm TE} = -\frac{p_4\sigma_{\rm TE}}{2p + p_4\sigma_{\rm TE}},$$
 (12)

where

$$\sigma_{\rm TM} = \frac{p_1^2 \sigma_{11} - p_2^2 \sigma_{22}}{p_1^2 - p_2^2}, \quad \sigma_{\rm TE} = \frac{p_1^2 \sigma_{22} - p_2^2 \sigma_{11}}{p_1^2 - p_2^2}.$$

This is to be compared with (8) in [16] (see also [26]), where the corresponding quantities are called longitudinal ( $\sigma_{TM}$ ) and transversal ( $\sigma_{TE}$ ) conductivities.

transversal ( $\sigma_{\text{TE}}$ ) conductivities. For the perfect conductor,  $r_{\text{TM}}^{(2)} = 1$ ,  $r_{\text{TE}}^{(2)} = -1$ . Combining (10) with (11) and using (5) and (6) for the polarization operator at finite temperature and chemical potential, we are able to calculate the Casimir energy density,  $\mathcal{E}$ , the Casimir pressure,  $\mathcal{F} = -\partial \mathcal{E}/\partial a$ , and its gradient,  $\mathcal{G} = \partial \mathcal{F}/\partial a \equiv -\partial^2 \mathcal{E}/\partial a^2$ , between a doped graphene layer and an ideal metal plate.

It is instructive to consider the case of very large  $\mu$  first. In the formal limit  $\mu \to \infty$ , both  $\Pi_{00}$  and  $\Pi_{tr}$  have identical asymptotics,

$$\Pi_{\rm xx} \underset{\mu \to \infty}{\simeq} \frac{8\alpha}{v_F^2} \mu + \cdots, \qquad (13)$$

which can be deduced from (6). Thus the electronic properties of graphene would be expected to become closer to those of an ideal metal. At a finite temperature, contributions of higher Matsubara frequencies to the Casimir energy, (10), are suppressed by the exponential factor  $e^{-2p_{\parallel}a}$ . Let us suppose that *a* is large enough to neglect all terms with  $n \neq 0$  in (10). For a zero Matsubara frequency, due to the specific structure of the reflection coefficients, (11), only the contribution of the TM mode to the Casimir interaction survives in the limit of large  $\mu$ . Thus, at  $\mu \rightarrow \infty$ , the Casimir interaction reaches a value that is one-half the ideal metal–ideal metal one,

$$\mathcal{E}_{\mu \to \infty} = \frac{1}{2} \mathcal{E}_{id} = -\frac{k_B T \zeta(3)}{16\pi a^2}.$$
 (14)

The same result was obtained for the high-temperature limit [8]. Equation (14) gives a very rough idea of how far the enhancement of the Casimir effect with  $\mu$  might go. Practically, it hardly makes sense to consider  $|\mu|$  exceeding a couple of electron volts in the framework of the Dirac model.

In [16], based on an isotropic approximation of the graphene conductivity, the force for doped graphene layers was predicted to grow without limits as  $\sqrt{\mu}$  for  $\mu \to \infty$  at T = 0. Although in the derivation of (14) it was important to assume that T > 0



FIG. 1. Ratios of the Casimir energy density (solid lines), the pressure (dashed lines), and its gradient (dotted lines) at  $\mu = 0.1, 0.5$ , and 0.8 eV (black, blue, and red lines, respectively) and at  $\mu = 0$ , between a perfect metal plate and doped graphene, as a function of the distance *a* (in nm).

(which makes an immediate comparison impossible), we find it quite unnatural that the Casimir interaction between some materials may eventually become stronger than that between two ideal metals. This indicates certain limitations of the approximation used in [16].

Our numerical analysis shows that at distances of about 100–300 nm the Casimir effect between a perfect metal plate and doped graphene is highly enhanced even for relatively moderate values of the chemical potential. In Fig. 1 we compare the behavior of the ratios of the energy, pressure, and pressure gradient at given values of the chemical potential to the corresponding values for pristine graphene, as functions of the distance. For  $\mu = 0.1$  eV, the interaction force in the case of doped graphene is only 3.5% higher than in the case of pristine graphene. However, already for  $\mu = 0.5$  eV their ratio has a maximum of 32% at approximately 70 nm, and for  $\mu = 0.8$  eV the force between a doped graphene layer and an ideal metal is almost 60% higher then that for a pristine one. These simulations are performed at T = 300 K and m = 0.

One further notes that the effect is more pronounced the more derivatives of the energy we calculate. Thus, the ratio of the energy density at  $\mu = 0.8$  eV to its pristine value is 1.52 at maximum, for the pressure  $\mathcal{F}$  it is 1.54, and for the pressure gradient  $\mathcal{G}$  it reaches 1.56. Moreover, at larger distances (400–1000 nm), the enhancement effect for the gradient diminishes much more slowly than that for the energy, which suggests a preference for gradient force experiments.

Reaching values of the chemical potential of 0.8 eV and higher might be a challenging task and would require the preparation of special samples. Without special treatment, the chemical potential in epitaxial graphene layers stays low up to the level of 0.3–0.4 eV or less [27,28]. However, Fermi energy shifts of the order of 0.8 eV are achievable in epitaxial single-layer graphene due to molecular doping [29]. In certain circumstances, doping may lead to considerable inhomogeneities in the charge distribution (see, e.g., [30]), which may give rise to additional forces in Casimir experiments. Due to the strong charge density dependence on the nature of the acceptor/donor mechanism, these forces should be treated individually for each particular experiment.

The distance dependence of the Casimir energy for doped graphene will be altered compared to the pristine one as is usual when an additional dimensionful parameter is introduced. However, its detailed study is beyond the scope of the present paper.

## V. INFLUENCE OF THE MASS GAP

From the physical point of view, the larger the mass parameter in (1), the lower should be the conductivity of quasiparticles and, consequently, the smaller the Casimir effect. This was shown explicitly in [4] for  $T = \mu = 0$ . One can also show that the influence of mass is negligible as far as  $m \ll \mu$ . In particular, in the formal limit, (13), any dependence on the mass disappears. Our numerical simulation shows (see Fig. 2) that for  $\mu = 0.8$  eV and m = 0.1 eV, doping gives an up to 70% enhancement of the Casimir energy density (red line; to be compared with the solid red line in Fig. 1). In Fig. 2 it is also shown that the influence of the mass gap on the value of the energy for doped graphene (dotted blue line) is almost negligible, while the energy for pristine graphene gets lower by about 15% (solid blue line). Therefore, doping becomes even more important for gapped graphene.

Notation used in Fig. 2 is self-explanatory. For example,  $\mathcal{E}_{m,0}$  denotes the Casimir energy density for a given value of the mass gap, m = 0.1 eV, and  $\mu = 0$ .

The large mass limit,  $m \gg T/2$ , was considered in [17], where a strong enhancement of the Casimir force with the chemical potential was reported. That paper used an isotropic



FIG. 2. Ratios of the Casimir energies between a perfect metal plate and graphene, as a function of the distance a (in nm). m = 0.1 eV,  $\mu = 0.8$  eV. See text for notation.



FIG. 3. Ratios of the Casimir energy to its approximation according to [8],  $\mathcal{E}/\mathcal{E}_{app}$ , for  $\mu = 0, 0.5$ , and 0.8 eV (solid, dashed, and dotted lines, respectively) between a perfect metal plate and doped graphene, as a function of the distance *a* (in nm).

model for the conductivity of graphene. It would be interesting to reconsider this case in the formalism based on the full conductivity tensor.

## VI. APPROXIMATING THE CASIMIR ENERGY

For a finite temperature, the numerical calculation of the Casimir energy in a realistic setup requires summing a large number of contributions to the sum over the Matsubara frequencies. Following the ideas in [8] (which were later developed in [31]), one might significantly facilitate this calculation by considering the T = 0 approximation, (8) and (9), for the polarization operator in all terms of the summation, (10), except in the zeroth one. The comparison of exact results for the Casimir energy  $\mathcal{E}$  with such an approximation,  $\mathcal{E}_{app}$ , is given in Fig. 3. As one can see, the error is less then 0.5% for the pristine graphene and one order of magnitude lower for the doped one. This confirms once again, at an even better level, the asymptotic considerations reported in [8].

### VII. SUMMARY

In this paper we have calculated the polarization operator for quasiparticles in graphene at nonzero temperature, chemical potential, and mass gap applicable at all complex frequencies without the need for any special procedure for analytical continuation. This result can be used in a variety of physical problems, including investigation of TE surface plasmons in graphene [20], quantum reflection [32], Casimir interaction, etc.

Based on these results and the Lifshitz formula, we numerically simulated the Casimir interaction between a doped graphene monolayer and an ideal metal. For high but feasible doping we predict enhancement of the Casimir effect compared to the case of pristine graphene by up to 54% for the Casimir force and by up to 56% for the force gradient. High doping of graphene is shown to bring significant enhancement of the values of the force gradient at a wide range of separations, which should facilitate future experimental measurements. At such levels of doping the influence of the mass gap is not important. We saw also that even moderate values of the chemical potential have a nonnegligible effect on the Casimir force and thus should be taken into account in realistic descriptions of experiments together with real material

properties and a finite temperature and mass-gap parameter, if present.

All calculations in the present paper were performed using the fully retarded approach valid at all distances. It may be interesting to study whether the nonretarded approach might deliver a good approximation at some separations. With regard to the graphene properties, we used the full tensor structure of conductivity following from the polarization tensor, in contrast to previous works [13–17]. Therefore, although we agree qualitatively with the predictionsin, e.g., [16], we obtain different numbers and a somewhat different shape of the curves, especially at small separations.

Finally, we note that the considerations given in this paper are concerned with the proper graphene properties and the enhancement of the Casimir interaction is invoked by the change in its conductivity. Thus, we can conclude that even in experiments involving a real metal and/or graphene on a substrate the enhancement effect must be present, though its particular value may differ from the one presented here. The good concordance between the force gradient measurements [10] and the theoretical considerations presented in [11] shows that the graphene samples used in [10] were rather pristine.

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