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Chiral Bogoliubov excitations in nonlinear bosonic systems

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We present a versatile scheme for creating topological Bogoliubov excitations in weakly interacting bosonic systems. Our proposal relies on a background stationary field that consists of a kagome vortex lattice, which breaks time-reversal symmetry and induces a periodic potential for Bogoliubov excitations. In analogy to the Haldane model, no external magnetic field or net flux is required. We construct a generic model based on the two-dimensional nonlinear Schrödinger equation and demonstrate the emergence of topological gaps crossed by chiral Bogoliubov edge modes. Our scheme can be realized in a wide variety of physical systems ranging from nonlinear optical systems to exciton-polariton condensates.

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Introduction. The quantum Hall effect is one of the most celebrated results of modern condensed matter physics [1]. The robustness of the Hall conductance can be traced back to the nontrivial topology of the underlying electronic band structure [2], which ensures the existence of chiral edge states and thus eliminates backscattering. Recently there was a surge of interest in the possibility to exploit such topology to create chiral bosonic modes in driven-dissipative systems—with possible applications to one-way transport of photons [3–13], polaritons [14–16], excitons [16,17], magnons [18,19], and phonons [20,21]. A common thread through these seemingly diverse ideas has been to induce topology by external manipulations of a single-particle band structure, with interactions playing a negligible role. Exceptions from this noninteracting paradigm are proposals that combine strong interactions with externally induced artificial gauge fields to create nonequilibrium analogs of bosonic fractional quantum Hall states [22–25].

Here, we take a new perspective and consider (bosonic) Bogoliubov excitations ("Bogoliubons") where *weak interactions* induce a nontrivial topology [26]. We demonstrate that topological Bogoliubons naturally occur on top of a condensate that exhibits a lattice of vortex-antivortex pairs, with no net flux required. Interactions are key to harness the time-reversal (TR) symmetry breaking induced by the condensate vortices. From the viewpoint of Bogoliubov excitations, they generate nontrivial "hopping" phases which lead to an analog of the Haldane lattice model [27]. The corresponding lattice can be defined by a periodic potential introduced either externally or via interactions with the condensate.

Although our scheme can be applied to any system described by a two-dimensional (2D) nonlinear Schrödinger equation (Gross-Pitaevskii equation or analog thereof), our analysis focuses on systems of weakly interacting bosons that have a light component, where the required vortex lattice can readily be obtained from the interference of several coherent optical fields (see Fig. 1). In this setting, the phase-imprinting mechanism allowing for nontrivial topology is analogous to that proposed a few years ago in the context of optomechanical systems [28]. The same mechanism was recently applied to create topological phonons using photons that remain trivial [20]. Here we demonstrate that vortex lattices can be exploited in a broader variety of systems where, in contrast to previous works, topology does not emerge from the coupling to a different bosonic species, but is intrinsically granted by interactions.

Our analysis starts with the 2D nonlinear Schrödinger equation commonly used to describe weakly nonlinear dispersive physical systems:

$$i\partial_t\psi(\mathbf{x},t) = [\omega_d(-i\nabla) + \alpha|\psi(\mathbf{x},t)|^2 + V(\mathbf{x})]\psi(\mathbf{x},t), \quad (1)$$

where $\psi(\mathbf{x},t)$ is a complex field (or "wave function") describing the amplitude and phase of a coherent field, and $\omega_d(-i\nabla)$ is a function of momentum ($-i\nabla \equiv \mathbf{k}$) which describes the

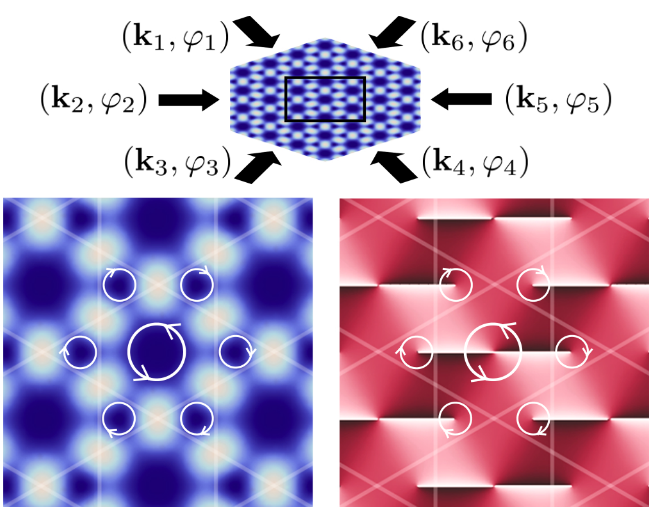

FIG. 1. Top: Optical pumping scheme allowing one to create a suitable condensate for topological Bogoliubons, with incident field composed of six equal-frequency plane-wave components with wave vectors $\mathbf{k}_n$ (depicted by arrows) and phases chosen as $\varphi_n = 0$ except for $\varphi_4 = 2\pi/3$ and $\varphi_6 = -2\pi/3$. Bottom: Intensity (left) and phase (right) pattern of the resulting field. Intensity maxima form a kagome structure (white lines) with vortex-antivortex pairs located such that each hexagonal plaquette is threaded by a flux $2\Phi_0 = 4\pi$, with smaller triangular plaquettes threaded by $-\Phi_0$. Although fluxes cancel out over the whole system, a well-defined chirality emerges, defined by the sign of Φ_0. Bogoliubov excitations of the condensate experience nontrivial fluxes due to interactions, leading to TR symmetry breaking and ultimately to topological states.

energy dispersion of the system. Nonlinearities are characterized by a parameter α which may be positive (repulsive/self-defocusing interactions) or negative (attractive/self-focusing interactions). We also allow for a spatially dependent potential $V(\mathbf{x})$ although, as we will show, this is only essential when $\alpha > 0$. Equation (1) is applicable to a wide variety of physical systems ranging from light propagation through Kerr nonlinear media [29] to exciton-polariton systems [30] and Bose-Einstein condensates [31]. Below we present a generic mechanism for creating topological Bogoliubov excitations when the underlying bosonic fields have a light component. We then discuss specific implementations.

Theoretical scheme. The first and principal ingredient of our scheme consists of a stationary field $\psi_0(\mathbf{x},t)$ which exhibits vortex-antivortex pairs and intensity maxima that form a kagome lattice pattern, as illustrated in Fig. 1. In bosonic systems with a photonic component, such a kagome "vortex lattice" can be directly imprinted onto the field $\psi(\mathbf{x},t)$ using an optical coherent field (or "pump") composed of six plane-wave components, as depicted in Fig. 1 (see also Supplemental Material [32]). Denoting by $\hat{\Psi}_L(\mathbf{x})$ the field operator associated with the light component, such coherent pumping can be described by a Hamiltonian term of the form $\int d\mathbf{x} f(\mathbf{x})e^{-i\omega_0 t}\hat{\Psi}_L^\dagger(\mathbf{x}) + \text{H.c.}$, where $f(\mathbf{x})$ and ω_0, respectively, denote the pump spatial profile and frequency. In the mean-field limit where Eq. (1) applies, this results in an additional pumping term proportional to $f(\mathbf{x})e^{-i\omega_0 t}$ (see, e.g., Ref. [33]). The required pumping field is readily obtainable with a spatial light modulator [34] or by passing light through a mask [35].

Remarkably, the vortex-antivortex pairs found in the above kagome vortex lattice are located in such a way that each elementary hexagonal plaquette is threaded by a flux $2\Phi_0 = 4\pi$ (i.e., contains two vortices), while smaller triangular plaquettes are threaded by $-\Phi_0$ (i.e., contain a single antivortex) (see Fig. 1). Since the incident pumping field has no net orbital angular momentum to transfer to the internal field, vortices and antivortices necessarily come in pairs and fluxes cancel out over the whole system. Nevertheless, the field $\psi_0(\mathbf{x},t)$ breaks time-reversal symmetry, as revealed by the "chirality" defined by the sign of Φ_0 (set by the phases of the pumping field [32] and invariant under rotations or translations in the plane that leave the kagome lattice of intensities unchanged [36]).

The fact that $\psi_0(\mathbf{x},t)$ exhibits a staggered flux pattern with no net flux but a well-defined chirality is very reminiscent of Haldane's seminal proposal for quantum Hall physics with no external magnetic field [27]. Here, however, the phase accumulated by an excitation around any plaquette of the kagome lattice (or around any loop in the plane) seems to be an integer multiple of 2π (equivalent to no phase at all), which would naively indicate that TR symmetry is preserved and topological states are out of reach. Remarkably, however, we demonstrate that Bogoliubov excitations, *through interactions*, do experience the TR symmetry breaking of the pumping field. This will allow us to access a *nonequilibrium interaction-induced* analog of Haldane's model.

To derive the spectrum of (Bogoliubov) excitations on top of $\psi_0(\mathbf{x},t)$, we perform a linearization of Eq. (1). We define the slowly varying field $\phi(\mathbf{x},t) \equiv \psi(\mathbf{x},t)e^{i\omega_0 t}$, where ω_0 is the pump frequency, and consider weak perturbations (or "fluctuations") of the form [37]

$$\phi(\mathbf{x},t) = \phi_0(\mathbf{x}) + u(\mathbf{x})e^{-i\omega t} + v^*(\mathbf{x})e^{i\omega^* t}, \tag{2}$$

where $\phi_0(\mathbf{x})$ denotes the rotating-frame counterpart of the stationary field $\psi_0(\mathbf{x},t) \equiv \phi_0(\mathbf{x})e^{-i\omega_0 t}$, $u(\mathbf{x})$ and $v(\mathbf{x})$ are (in general complex) functions determining the spatial form of the fluctuations, and ω is the frequency of the perturbations, which is kept complex in order to capture potential instabilities [37]. Plugging the above expression into Eq. (1) and neglecting second-order terms in $u(\mathbf{x})$ and $v(\mathbf{x})$ yields the Bogoliubov equation

$$\begin{pmatrix} \omega'(\mathbf{x}) & \alpha\phi_0(\mathbf{x})^2 \\ -\alpha\phi_0^*(\mathbf{x})^2 & -\omega'(\mathbf{x}) \end{pmatrix}\begin{pmatrix} u(\mathbf{x}) \\ v(\mathbf{x}) \end{pmatrix} = \omega\begin{pmatrix} u(\mathbf{x}) \\ v(\mathbf{x}) \end{pmatrix}, \tag{3}$$

where $\omega'(\mathbf{x}) \equiv \omega_d(-i\nabla) - \omega_0 + 2\alpha|\phi_0(\mathbf{x})|^2 + V(\mathbf{x})$. This shows that Bogoliubov excitations experience both the intensity and phase pattern of the underlying field $\phi_0(\mathbf{x})$.

In the case of repulsive interactions ($\alpha > 0$), the effective potential term $2\alpha|\phi_0(\mathbf{x})|^2$ tends to localize the excitations at the vortex/antivortex points of the lattice (see Fig. 1). To allow for topological states, however, it is crucial for excitations to remain localized away from vortices/antivortices so as to pick up a phase when hopping around them. This is achieved, e.g., by introducing an external potential $V(\mathbf{x})$ with minima that coincide with the maxima of $|\phi_0(\mathbf{x})|^2$. In the case of attractive interactions ($\alpha < 0$), the effective potential $2\alpha|\phi_0(\mathbf{x})|^2$ automatically localizes the excitations at the desired points, thus obviating the need for an external potential.

According to Eqs. (2) and (3), fluctuations described by $u(\mathbf{x})$ and $v(\mathbf{x})$ can be viewed as "particle-hole" analogs of each other: $u(\mathbf{x})$ as a "particle" excitation with dispersion $\omega'(\mathbf{x})$, and $v(\mathbf{x})$ as a "hole" excitation with opposite dispersion $-\omega'(\mathbf{x})$. The pump frequency ω_0 sets the relative energy between the two. Interestingly, the minus sign present on the off-diagonal of Eq. (3) (a hallmark of bosonic Bogoliubov excitations [31]) makes the corresponding matrix non-Hermitian. The stability of the system is then assessed by the imaginary part of ω: if $\text{Im}(\omega) > 0$, fluctuations grow exponentially and $\phi_0(\mathbf{x})$ is an unstable solution of Eq. (1).

The emergence of topological Bogoliubov excitations in our scheme relies on four essential ingredients: (i) a well-defined lattice structure [controlled by the effective potential $2\alpha|\phi_0(\mathbf{x})|^2 + V(\mathbf{x})$]; (ii) a nontrivial flux pattern in the corresponding unit cell, such that elementary excitations pick up a nonzero phase when hopping some closed loops on the lattice; and (iii) a well-defined chirality, which arises from time-reversal symmetry breaking. Finally, (iv) interactions are crucially required: as we demonstrate below, the nonzero phase picked up by excitations around loops on the lattice only becomes nontrivial in the presence of the off-diagonal coupling in Eq. (3). Therefore, interactions are necessary to harness the TR symmetry breaking encoded in the condensate.

Tight-binding analysis. To unveil the generic mechanism leading to topological states, we now strip our model from topologically irrelevant details and consider the tight-binding limit of Eq. (3), which captures the low-energy behavior obtained in a deep potential $2\alpha|\phi_0(\mathbf{x})|^2 + V(\mathbf{x}) - \omega_0$ whose minima form a kagome lattice. We denote the corresponding on-site energy by $\Omega(-\Omega)$ in the sector $u(v)$, and assume that excitations "hop" with amplitude $t(-t)$ between nearest-neighboring sites. We then model the off-diagonal terms in Eq. (3) (connecting u and v) by couplings of the form $\pm g e^{\pm i\varphi_j}$ on each site j, choosing phases φ_j so as to reproduce the main features of the phase pattern of $\phi_0(\mathbf{x})^2$ [so that each triangular unit cell of the kagome lattice carries a single vortex; see Fig. 2(b)].

The mechanism giving rise to TR symmetry breaking (and, in turn, to topological states) appears most clearly in the limit $2\Omega \gg g$ where the sectors u and v of Eq. (3) are coupled off-resonantly and weakly. In that case, virtual transitions between the latter lead to a renormalization of the hopping amplitude and phase. In a perturbative picture [38], a u-like Bogoliubov excitation can be turned into a v-like excitation, hop to a neighboring site, and transform back into a u-like excitation [see Fig. 2(c)]. Since the u-v coupling involves a site-dependent phase, excitations pick up an overall phase $e^{\pm i(\varphi_j-\varphi_i)}$ ["+(−)" if $u(v)$-like] [39], which lead to an effective hopping

$$t_{\rm eff} = \pm t\left[1 + \left(\frac{g}{2\Omega}\right)^2 e^{\pm 2\pi i/3}\right] \tag{4}$$

in counterclockwise direction around the triangular unit cell. The corresponding flux per triangular plaquette is given by $\Phi = \pm 3\arctan[\sqrt{3}g^2/(8\Omega^2 - g^2)] \neq 0$.

Equation (4) illustrates one of the key aspects of our proposal: the fact that interactions generically lead to a nontrivial flux Φ (not a multiple of π), which extends the TR symmetry breaking of the condensate to Bogoliubov excitations. In the above off-resonant tight-binding regime, u- and v-like excitations are both individually described by an effective kagome lattice model with staggered flux pattern as depicted in Fig. 1. If the flux Φ was trivial (for $g = 0$), the positive or negative part of the Bogoliubov spectrum would exhibit three bands touching at high-symmetry points of the Brillouin zone (Dirac cones at K and K', and a quadratic band touching at Γ). The nontrivial flux Φ induced by interactions gaps out these degeneracies and leads to topological bands with nonzero Chern number [40] (see Fig. 2). To second order in g, the size of the topological gaps is given by [32]

$$\Delta_K = \frac{3tg^2}{4(\Omega - t)(\Omega + t/2)}, \quad \Delta_\Gamma = \frac{3tg^2}{2(\Omega - t)(\Omega + 2t)}. \tag{5}$$

The Bogoliubov spectrum is stable in this off-resonant regime, since it only exhibits real eigenvalues.

Exciton-polariton systems. To realize our scheme, one can consider exciton-polaritons in semiconductor microcavities, which are renowned for their strong nonlinearities [30]. The Bogoliubov spectrum of polariton condensates has been observed in photoluminescence [41,42] and four-wave mixing [43] experiments. A suitable amplitude and phase pattern can be imposed by optical means, as illustrated in Fig. 1 [44].

Since polariton-polariton interactions are repulsive, an external (kagome) periodic potential $V(\mathbf{x})$ is required [see discussion below Eq. (1)]. Such potential can be realized, e.g., by depositing thin metal films on the structure surface [45,46] [as illustrated in Fig. 3(b)], by applying surface acoustic waves [47], or by etching micropillar arrays [48]. Alternatively, one can engineer the desired potential optically [49] by taking

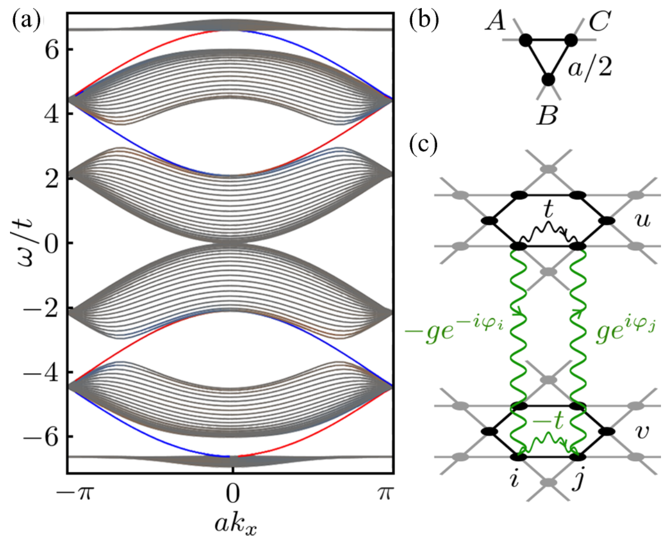

FIG. 2. (a) Bogoliubov spectrum in the off-resonant tight-binding limit (in strip geometry): The positive part of the spectrum exhibits three bands with topological gaps crossed by pairs of counterpropagating chiral edge states, reminiscent of a kagome lattice model with staggered fluxes [40]. The red (blue) coloring indicates the degree of localization of states at the lower (upper) edge (as measured by their total intensity on the lower (upper) half of the system). Parameters were chosen as $\Omega/t = 6$ and $g/\Omega = 2/3$, in a strip geometry with periodic boundary conditions in the x direction (see Supplemental Material [32] for details). (b) Unit cell used in the tight-binding model, with phases for the on-site coupling chosen as $(0, 2\pi/3, -2\pi/3)$ in the basis (A,B,C). (c) TR symmetry-breaking mechanism: The off-resonant coupling between u- and v-like excitations [see Eq. (3)] leads to virtual transitions which renormalize the hopping term t.

advantage of the fact that polaritons have two possible circular polarizations (spin projections along the structure growth axis), and that polaritons with opposite spins typically interact attractively [50,51]. Specifically, one can consider Bogoliubov excitations with spin $\sigma = \pm 1$ and use a component with spin $-\sigma$ to induce the desired potential. In practice, this can be achieved by pumping both components simultaneously with an elliptically polarized incident field.

Here, we follow this all-optical approach and consider Bogoliubov excitations with spin σ on top of a polariton condensate with spin components $\phi_{0,\sigma}(\mathbf{x})$ and $\phi_{0,-\sigma}(\mathbf{x})$ [defined as in Eq. (2)]. The relevant Bogoliubov equation is then given by Eq. (3) with $\alpha \equiv \alpha_1$ and $V(\mathbf{x}) = \alpha_2|\phi_{0,-\sigma}(\mathbf{x})|^2$, where α_1 and α_2 denote the strength of interactions between polaritons with parallel and opposite spins, respectively [32]. To provide a reliable estimate of the size of the topological gaps achievable in practice, we compute the stationary fields $\phi_{0,\pm\sigma}(\mathbf{x})$ and the spectrum of Bogoliubov excitations with spin σ in an exact and self-consistent way, without relying on any tight-binding approximation (see Supplemental Material [32]). Our results are illustrated in Fig. 3(a).

In accordance with our tight-binding analysis, the positive (u-like) part of the low-energy Bogoliubov spectrum exhibits three kagome-like bands with a clear topological gap between the lowest two bands [52]. With conservative parameters from existing experiments [53], the topological gap reaches about 0.06 meV, which exceeds typical polariton linewidths of the order of μeV [54]. Dissipation, which is an important feature of exciton-polaritons, can be treated by introducing a decay term into the u and v dispersions: $\pm\omega_d(-i\nabla) \to \pm\omega_d(-i\nabla) - i/(2\tau)$, where τ is the polariton lifetime [37].

In practice, topological Bogoliubov excitations can be created by illuminating the system at an edge with an additional weak coherent field whose frequency lies within the topological gap. The chiral propagation of excitations then provides a smoking-gun signature of the topological nature of the system. We present numerical simulations of such an experiment in the Supplemental Material [32].

Discussion. Other bosonic systems would be suitable to realize our proposal. The nonlinear Schrödinger equation (1) provides, in particular, a direct representation of Maxwell's equations for light propagating through a nonlinear medium with polarization transverse to the propagation direction z [where $t \to z$ in Eq. (1) and $\mathbf{x} = (x,y)$ defines the lateral coordinates] [29]. In that case the potential $V(\mathbf{x})$ can also be induced optically. In particular, in nonlinear media with a strong electro-optic anisotropy, polarizations can be chosen such that a suitably polarized incident field experiences negligible self-nonlinearity and induces a stable potential $V(\mathbf{x})$ for optical fields with opposite (orthogonal) polarization, which experience a strong nonlinearity [56–58]. Alternatively, $V(\mathbf{x})$ can be realized by refractive-index modulation, e.g., in arrays of coupled waveguides [59,60] as depicted in Fig. 3(c). In this context, nonlinearities $|\alpha\phi_0|^2 > 0.2\ \mathrm{cm}^{-1}$ with significantly weaker losses $< 0.04\ \mathrm{cm}^{-1}$ have been demonstrated for fused silica waveguides [61].

Equation (1) is also relevant for the description of cold atom condensates. Despite available methods for optical flux lattices [62,63] and optical phase imprinting [64–66], realizing a kagome vortex lattice could be more challenging in this context. An alternative would be to consider Abrikosov-type lattices of identical vortices with winding number $+$ or -1, which can be realized, e.g., by stirring an atomic condensate with rotating laser beams [67]. Although this would be suitable for generating topological Bogoliubov modes, a net transfer of orbital angular momentum would occur in that case, unlike in our scheme (see also Ref. [21]).

Conclusion. We have proposed a generic scheme to create topological Bogoliubov excitations in systems described by a 2D nonlinear Schrödinger equation—a universal equation governing systems ranging from exciton-polariton condensates to nonlinear optical media. The key to our proposal is a background mean field exhibiting vortex-antivortex pairs and a kagome intensity pattern, which can typically be generated by optical means. By virtue of weak interactions, Bogoliubov excitations propagating on top of this condensate acquire nontrivial phases which break TR symmetry and grant access to topological states. The resulting topological Bogoliubons manifest as robust chiral edge states, as we have demonstrated using both tight-binding and full wave-expansion methods. Our scheme does not require any external magnetic field or net transfer of orbital angular momentum, which allows one to generate chiral edge modes while keeping spin/polarization degrees of freedom degenerate and simultaneously accessible.

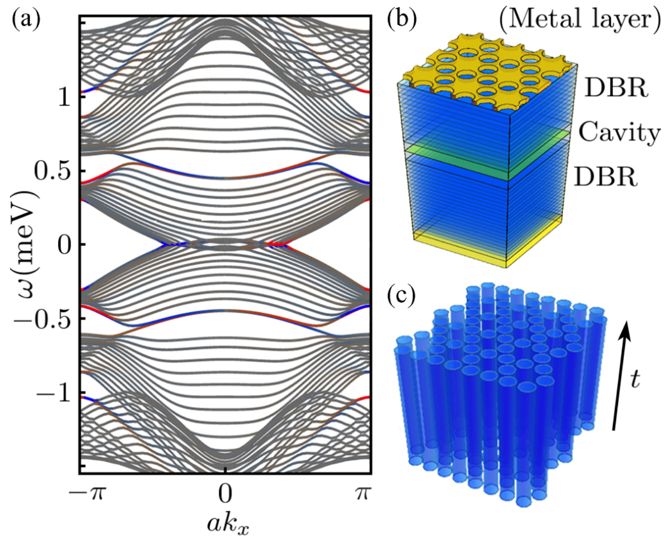

FIG. 3. (a) Bogoliubov spectrum of exciton-polaritons (in strip geometry), exhibiting qualitatively similar low-energy features as the tight-binding spectrum of Fig. 2(a). Differences stem from longer-range hopping terms and weak repulsive interactions within the condensate which are neglected in tight-binding approximation (e.g., the upper topological gap vanishes because of the modified dispersion). Although u- and v-like excitations [see Eq. (3)] overlap in energy (around $\omega = 0$), all eigenvalues are real, indicating that the system is stable. Parameters were taken from Ref. [53], giving a topological gap of about 0.06 meV [32]. (b) Practical realization in semiconductor microcavities composed of distributed Bragg reflectors (DBRs) [45], with periodic potential $V(\mathbf{x})$ induced, e.g., via metal surface patterning (other mechanisms are discussed in the text). (c) Realization of the effective potential $V(\mathbf{x})$ in a nonlinear optical system, using coupled waveguides [55].

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