Bound collective modes in nonuniform superconductors

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We study the dynamics of a superconducting condensate in the presence of a domain wall defect in the order parameter. We find that broken translation and reflection symmetries result in collective excitations, bound to the domain wall region. Two additional amplitude/Higgs modes lie below the bulk pair-breaking edge 2Δ; one of them is a Goldstone mode with vanishing excitation energy. The spectrum of bound collective modes is related to the topological structure and stability of the domain wall. The "unbound" bulk collective modes and transverse gauge field mostly propagate across the domain wall, but the longitudinal component of the gauge field is completely reflected. Softening of the amplitude mode suggests reduced damping and a possible route to its detection in geometrically confined superfluids or in superconductor-ferromagnetic heterostructures.

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I. INTRODUCTION

The observation of the Higgs particle at the Large Hadron Collider (LHC) [1] has emphasized the connection between high-energy and condensed matter physics through collective modes [2,3]. These excitations are the normal modes of order parameter (OP) fluctuations, reflecting the symmetry and structure of the OP's potential landscape. In a singlet isotropic superconductor with a complex order parameter $\Delta(\mathbf{r},t) = \psi(\mathbf{r},t)\exp[i\varphi(\mathbf{r},t)]$, a gapless Bogoliubov-Anderson $\varphi(\mathbf{r},t)$-phase mode [4–6] is a result of spontaneously broken $U(1)$ symmetry [7,8]. Interaction with an electromagnetic gauge field shifts this mode up to plasma frequency [9]. Fluctuations of the other degree of freedom, $\psi(\mathbf{r},t)$, represent the amplitude mode, often called the Higgs mode, due to the close analogy to its particle counterpart [10].

Detection of the amplitude mode in condensed matter systems has been a long-standing challenge. The original discovery of this mode in the charge-density-wave material $NbSe_2$ [11,12] highlights the main difficulty associated with the fact that its energy is $2|\Delta|$, leading to its quick decay into two-particle excitations. This search is continuing due to its fundamental importance and intriguing possibility of insight into the Standard Model from low-energy experiments [2,13]. Recently, the amplitude mode near a quantum critical point was investigated theoretically [14] and experimentally in a neutral superfluid of cold atoms [15]. Another report of amplitude mode detection in disordered superconductors [16] was questioned in Ref. [17] due to an expected strong mixing of the amplitude and phase modes.

In this paper we show that nonuniform superfluids or superconductors may provide a different avenue to investigate the amplitude/Higgs mode. We consider a general problem of a domain wall that breaks extra symmetries beside $U(1)$, *translation and reflection*, as shown in Fig. 1. In the region of the domain wall, additional amplitude modes exist below the pair-breaking edge, including one with a gapless spectrum. While a free-standing domain wall is not likely, their evolution and dynamics are interesting from the point of view of frozen topological defects of the early Universe [18–20]. In superconductors, domain wall structures appear in Fulde-Ferrell-Larkin-Ovchinnikov states (FFLOs) [21], or in thin films [22]. Half-domain walls are more common and appear as OP suppression in the boundary regions of unconventional superconductors [23,24], or when a singlet superconductor is in contact with a strong ferromagnet [25]. Collective modes in unconventional superconductors with broken momentum-space symmetries have been studied in d-wave materials [26], such as UPt_3 and UBe_{13} [27–29] and Sr_2RuO_4 [30,31]. Superfluid ^{3}He features many collective modes [32–35]. In particular, several modes in the ^{3}He-B phase [36] are easily detectable by ultrasound [37], and have evolved into a tool that can distinguish details of the pairing interactions on a few percent scale [38]. Distinct characteristics of bound collective modes can be used in the detection of nonuniform superconducting states. Below, we investigate both a neutral

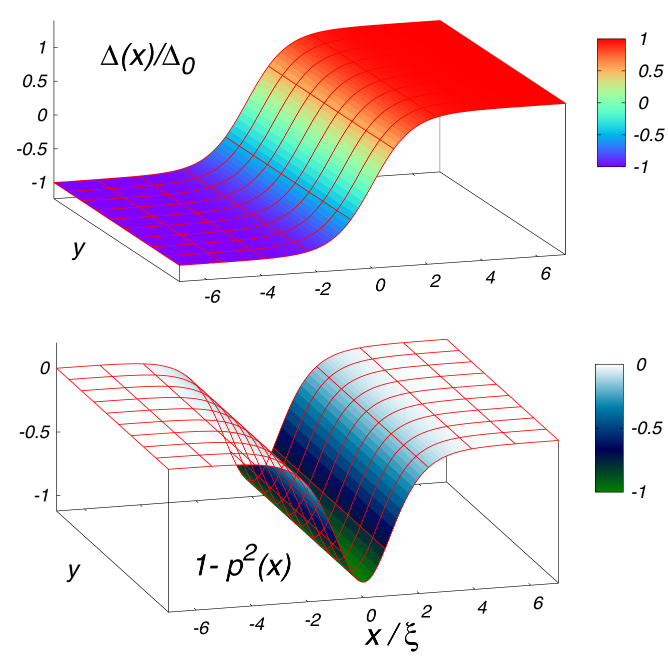

FIG. 1. Domain wall with profile $\Delta(x)/\Delta_0 \equiv p(x) = \tanh(x/\sqrt{2}\xi)$ separates two degenerate values of the order parameter $\Delta = \pm\Delta_0$ (top). The dynamics of the order parameter perturbations is described by the Schrödinger equation with $1/\cosh^2(x/\sqrt{2}\xi)$ potential well (bottom).

superfluid and charged superconductor coupled to the gauge field.

II. MODEL

We consider the time-dependent Ginzburg-Landau (TDGL) Lagrangian, where the order parameter field $\Delta(\mathbf{r},t)$ is minimally coupled to the electromagnetic gauge field $[\Phi(\mathbf{r},t),\mathbf{A}(\mathbf{r},t)]$,

$$\mathscr{L} = -\gamma|(i\hbar\partial_t - 2e\Phi)\Delta|^2 + \kappa\left|\left(\frac{\hbar}{i}\nabla - \frac{2e}{c}\mathbf{A}\right)\Delta\right|^2 - \alpha\left(|\Delta|^2 - \frac{1}{2\Delta_0^2}|\Delta|^4\right) + \frac{\mathbf{B}^2-\mathbf{E}^2}{8\pi}. \quad (1)$$

Here, $\mathbf{B} = \nabla\times\mathbf{A}$, $\mathbf{E} = -\nabla\Phi - (1/c)\partial_t\mathbf{A}$ are magnetic and electric fields, and we put $\hbar = 1$ from now on. In the superconducting state below T_c we take $\alpha > 0$, and Δ_0 is the real amplitude of the uniform solution to GL equations without fields. In relativistic Lorentz-invariant theories, $\gamma = \kappa$. This particular choice of $\mathscr{L}$ agrees with microscopically derived equations of motion for the OP, which are of the wave type at low temperatures [39,40]. From Ref. [40] we can extract low-T phenomenological coefficients $\gamma = N_f/8\Delta_0^2$, $\alpha = N_f/4 = 2\gamma\Delta_0^2$, $\kappa = N_f v_f^2/24\Delta_0^2 = n/8m\Delta_0^2$, where N_f is the density of states at the Fermi level for two spin projections, v_f is the Fermi velocity, and $n = N_f m v_f^2/3$ is the uniform electronic density. We define the wave speed $v^2 = \kappa/\gamma = v_f^2/3$, and the coherence length $\xi^2 = \kappa\hbar^2/\alpha = \hbar^2 v^2/2\Delta_0^2$.

Model (1) is an adequate first step to investigate general relations between collective modes, topology, and broken spatial symmetry. However, its main limitation is the lack of coupling to fermionic quasiparticles that would contribute to the damping of collective modes. This is in part due to the absence of first-order time derivative terms (diffusion), dominant near T_c [39], which is also an indication of complete particle-hole symmetry that results in full decoupling of the amplitude and phase dynamics [3,41]. The domain wall region hosts a high density of Andreev bound states that interact with collective modes and limit their lifetime. One might expect that the bound states' damping effects are similar to those of low-energy quasiparticles in uniform nodal superconductors. For example, in the ^{3}He-A phase, collective modes are damped [42] but still detectable [43]. It is then plausible that in some frequency range, depending on the availability of the excitation phase space, the collective modes near a domain wall will not be overdamped [44]. The complete treatment of the dynamics of coupled order parameter modes, excitations, and charge density will require future fully microscopic calculations.

In terms of the OP amplitude and phase, this model is

$$\mathscr{L} = -\gamma[(\partial_t\psi)^2 + \psi^2(\partial_t\varphi + 2e\Phi)^2] + \frac{\mathbf{B}^2-\mathbf{E}^2}{8\pi} + \kappa\left[(\nabla\psi)^2 + \psi^2\left(\nabla\varphi - \frac{2e}{c}\mathbf{A}\right)^2\right] - \alpha\left(\psi^2 - \frac{\psi^4}{2\Delta_0^2}\right). \quad (2)$$

Finding the extrema of the action $\mathcal{S} = \int d\mathbf{r}\int dt\,\mathscr{L}$ with respect to amplitude ψ, and field potentials $\mathbf{A}$ and Φ, gives the dynamics of the order parameter

$$\gamma\frac{\partial^2}{\partial t^2}\psi - \kappa\,\nabla^2\psi - \alpha\psi\left(1 - \frac{\psi^2}{\Delta_0^2}\right) - \gamma\psi(\partial_t\varphi + 2e\Phi)^2 + \kappa\psi\left(\nabla\varphi - \frac{2e}{c}\mathbf{A}\right)^2 = 0, \quad (3)$$

and that of the gauge field,

$$\nabla\times\mathbf{B} - \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t} = \frac{4\pi}{c}\mathbf{j}, \quad \nabla\cdot\mathbf{E} = 4\pi\rho,$$
$$\mathbf{j} = 4e\kappa\psi^2\left(\nabla\varphi - \frac{2e}{c}\mathbf{A}\right), \quad \rho = -4e\gamma\psi^2(\partial_t\varphi + 2e\Phi). \quad (4)$$

Minimization with respect to the phase of the order parameter φ results in a statement of charge conservation, $\partial_t\rho + \nabla\cdot\mathbf{j} = 0$, which also follows from Eqs. (4) as a consequence of the gauge symmetry [45].

A real-valued domain wall $\psi_0(x)$ in the absence of the fields is a solution to $-\kappa\psi'' - \alpha\psi(1 - \psi^2/\Delta_0^2) = 0$:

$$p(x) \equiv \frac{\psi_0(x)}{\Delta_0} = \tanh\frac{x}{\sqrt{2}\xi}. \quad (5)$$

A free-standing kink extends from $-\infty < x < \infty$ (Fig. 1). Half of the domain wall $0 \leqslant x < \infty$ can be pinned by an interface with $\Delta(x=0) = 0$.

A. Neutral condensate

First, consider a neutral superconductor $e = 0$, where the condensate is not coupled to the gauge field. The field equations $\nabla^2\mathbf{A} - \partial_t^2\mathbf{A}/c^2 = 0$ give the electromagnetic (EM) wave with two transverse polarizations $\omega = ck$, $\mathbf{k}\cdot\mathbf{A}_\mathbf{k} = 0$, propagating with the speed of light. The dynamics of the order parameter perturbation around the domain wall solution $[\psi_0(x),\varphi_0 = 0]$ follows from (1) with the substitution $\Delta(\mathbf{r},t) = \psi_0(x) + D(\mathbf{r},t)$. One introduces $D_\pm = [D(\mathbf{r},t) \pm D(\mathbf{r},t)^*]/2$, related to amplitude and phase fluctuations in linearized theory: $D_+(\mathbf{r},t) = \delta\psi(\mathbf{r},t)$ and $D_-(\mathbf{r},t) = i\psi_0(x)\delta\varphi(\mathbf{r},t)$. The equations for the amplitude and phase are

$$\frac{1}{v^2}\frac{\partial^2}{\partial t^2}D_+ - \nabla^2 D_+ - \frac{3}{\xi^2}[1 - p^2(x)]D_+ = -\frac{2}{\xi^2}D_+,$$
$$\frac{1}{v^2}\frac{\partial^2}{\partial t^2}D_- - \nabla^2 D_- - \frac{1}{\xi^2}[1 - p^2(x)]D_- = 0. \quad (6)$$

In a uniform superconductor we put $p(x) = 1$ and obtain an amplitude (Higgs) mode $\omega_+^2 = v^2k^2 + 2v^2/\xi^2 = v^2k^2 + 4\Delta_0^2$, with "mass" $2\Delta_0$ [12], and the massless Bogoliubov-Anderson phase mode $\omega_- = vk = (v_f/\sqrt{3})k$ [6].

In the presence of a domain wall we look for collective modes that are localized in the x direction, and propagate along y, $D_\pm(\mathbf{r},t) = D_\pm(x)e^{-i\omega t + ik_y y}$. For $D_\pm(x)$ prefactors from Eq. (6) we obtain

$$-D_+'' - \frac{3/\xi^2}{\cosh^2(x/\sqrt{2}\xi)}D_+ = \left(\frac{\omega^2}{v^2} - k_y^2 - \frac{2}{\xi^2}\right)D_+,$$
$$-D_-'' - \frac{1/\xi^2}{\cosh^2(x/\sqrt{2}\xi)}D_- = \left(\frac{\omega^2}{v^2} - k_y^2\right)D_-. \quad (7)$$

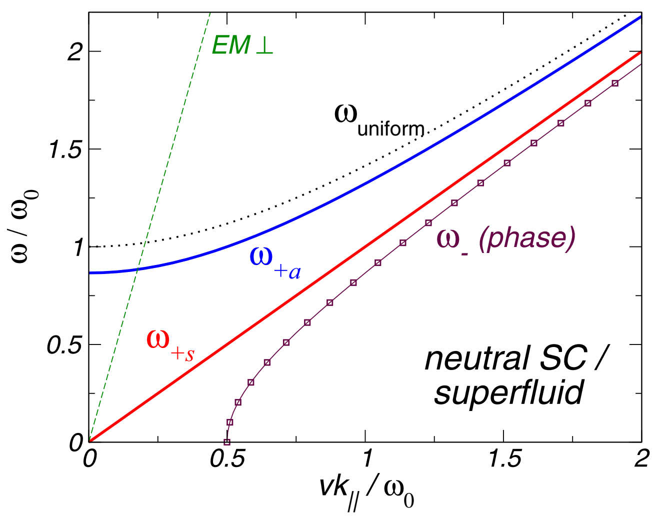

FIG. 2. Dispersion of order parameter modes propagating along the domain wall. $\omega_0 = 2\Delta_0$. The solid lines are the modes bound to the domain wall with $\omega_+ < \omega_{\text{uniform}}$ (dotted line). The phase mode ω_- is unstable for long wavelengths $k < 1/\sqrt{2}\xi$. The transverse EM modes are decoupled from the order parameter dynamics. We use exaggerated $v/c = 0.2$.

These equations are similar to the Schrödinger equation for eigenstates of a particle in a one-dimensional Eckart potential $-U_0[1-\tanh^2(x/w)] = -U_0/\cosh^2(x/w)$, shown in Fig. 1. The energies of the bound states are $E_n = -(s-n)^2/w^2$ with $n < s = -1/2 + \sqrt{1/4 + U_0 w^2}$ [46]. Even/odd n give symmetric/asymmetric eigenfunctions $D(-x) = \pm D(x)$. The OP amplitude has two bound eigenmodes ($U_0 = 3/\xi^2$, $w = \sqrt{2}\xi$, $s = 2$, and $n = 0,1$) $\omega_+^2/v^2 - k_y^2 - 2/\xi^2 = -(2-n)^2/2\xi^2$, resulting in the dispersion relations

$$\omega_{+s}^2 = v^2 k_y^2, \quad \omega_{+a}^2 = v^2 k_y^2 + 3\Delta_0^2, \tag{8}$$

shown in Fig. 2. The symmetric, $n = 0$, Higgs mode is massless. Its eigenfunction is $D_{+s}(x,y) \propto \exp(ik_y y)/\cosh^2(x/w)$, which can be written as $\tanh(x/w)|_x^{x+\delta x_0 \exp(ik_y y)}$—a ripple of the domain wall plane. For $k_y = 0$ it is a uniform lateral shift of the entire domain wall plane without energy cost—a consequence of spontaneously broken translational symmetry. Thus, the amplitude Higgs mode became a Goldstone mode, propagating along the defect with speed $v = v_f/\sqrt{3}$. The $n = 1$ mode, in addition to translations, breaks the discrete reflection symmetry $x \to -x$ and corresponds to an excited state of the domain wall condensate; it has a minimal energy $\sqrt{3}\Delta_0 = \omega_0\sqrt{3/4}$. Analogous results appear in the extended-hadron model in field theory [47], and for the dynamics of domain walls in structurally unstable lattices [48]. Low-energy modes associated with the dynamics of periodic latticelike FFLO structures were explored in superconductors [49] and in cold atoms [50].

The phase mode ($U_0 = 1/\xi^2$, $w = \sqrt{2}\xi$, and $s = 1$) has only one eigenvalue with $n = 0$, $\omega^2/v^2 - k_y^2 = -(1-n)^2/2\xi^2$, and dispersion

$$\omega_-^2 = v^2 k_y^2 - \Delta_0^2. \tag{9}$$

For a free-standing kink this indicates an "imaginary" mass and instability at wave vectors $k_y < 1/\sqrt{2}\xi$, resulting in the decay of the domain wall, which we address later. For a half kink pinned at the surface, the symmetric solutions $n = 0$ are excluded by the boundary condition on the order parameter $\Delta(0) = 0$, and only the asymmetric amplitude mode propagates.

B. Charged superconductor

If $e \neq 0$, the phase degree of freedom is no longer independent, and is absorbed into potentials $(\Phi, \mathbf{A})$. $\mathbf{A} \to \mathbf{A} - (c/2e)\nabla\varphi$, $\Phi \to \Phi + (1/2e)\partial_t \varphi$. This is the unitary gauge with a real order parameter $\varphi(\mathbf{r},t) = 0$. We assume no topological defects in the phase (vortices) that in this gauge would represent themselves as nonphysical singularities in the gauge field [e.g., a superconducting vortex $\varphi(r,\phi) \propto \phi$ gives $A_\phi \sim 1/r$ [51]]. We linearize Eqs. (3) and (4) around the zero-field domain wall solution $\psi_0(x) = \Delta_0 p(x)$, $\Phi_0 = \mathbf{A}_0 = 0$. The equation for the amplitude mode does not change from the neutral case, and the dispersion relations Eq. (8) remain the same.

Combining the continuity equation with the Ampère law in (4), we eliminate Φ and obtain a single equation for the vector potential:

$$-\nabla^2\mathbf{A} + \frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} + \nabla\left(\text{div}\mathbf{A} - \frac{v^2}{c^2}\frac{1}{p^2(x)}\text{div}[p^2(x)\mathbf{A}]\right) - \frac{1}{\lambda^2}[1-p^2(x)]\mathbf{A} = -\frac{1}{\lambda^2}\mathbf{A}. \tag{10}$$

The magnetic penetration length is $\lambda^{-2} = 32\pi e^2\kappa\Delta_0^2/c^2 = 4\pi e^2 n/c^2 m = \omega_p^2/c^2$, with a plasma frequency $\omega_p^2 = 4\pi e^2 n/m$. In a uniform superconductor this equation gives a dispersion $\omega^2 = c^2k^2 + \omega_p^2$ for two transverse ($\mathbf{k}\mathbf{A} = 0$) modes, and $\omega^2 = v^2k^2 + \omega_p^2$ for a longitudinal ($\mathbf{k}\mathbf{A}_\ell = kA_\ell$) mode that couples phase oscillations with the motion of the electric charge. For bound waves propagating along the domain wall, $\mathbf{A}(\mathbf{r},t) = \mathbf{A}(x)e^{ik_y y - i\omega t}$, we find several solutions. A transverse wave with z polarization $\hat{\mathbf{z}}A_z(x)$ satisfies an equation similar to (7), with an Eckart potential amplitude $U_0 = 1/\lambda^2$ and eigenvalues $\omega^2/c^2 - k_y^2 - 1/\lambda^2 = -(s-n)^2/2\xi^2$ ($n < s = -1/2 + \sqrt{1/4 + 2\xi^2/\lambda^2}$) producing

$$\omega^2 = \omega_p^2(s,n) + c^2k_y^2, \tag{11}$$

with a lowered plasma frequency $\omega_p^2(s,n) = \omega_p^2[1 - \lambda^2(s-n)^2/2\xi^2]$. For $\lambda \geqslant \xi$ there is only one bound solution $n = 0$, while for $\lambda < \xi$ one has $s > 1$ and multiple branches of the plasmon mode. Other modes satisfy coupled differential equations for $A_x(x)$ and $A_y(x)$, which we solve numerically. The dispersion relations and structure of these modes for $\lambda = \xi$ are shown in Fig. 3. These modes have a resemblance to the plasmon polariton modes that are bound to the interface regions between two different dielectrics, for example.

We close this discussion by mentioning the reflection properties of the domain wall. The traveling wave solution

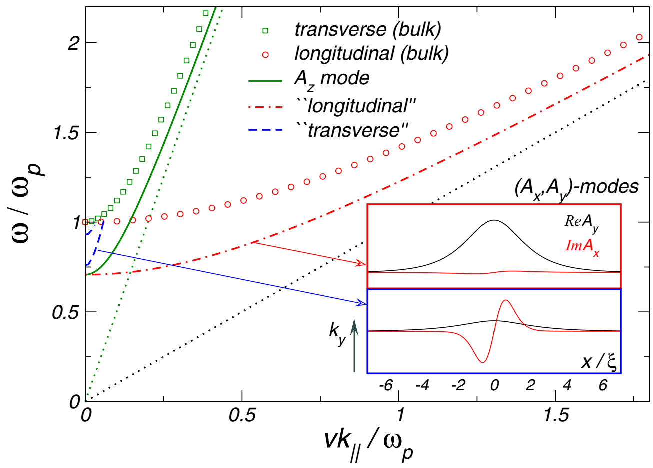

FIG. 3. EM modes in a superconductor. Uniform superconductor modes are gapped with plasma frequency (open symbols). Modes bound to the domain wall are a transverse $A_z(x)$ mode, and $[A_x(x), A_y(x)]$ coupled modes. For the chosen parameters, $c/v = 5$, $\lambda/\xi = 1$, and examples of profiles for "longitudinal" and one of the "transverse" modes are shown in the inset.

$D_\pm(\mathbf{r},t) = D_\pm(x)\exp(-i\omega t)$ to Eqs. (6), with boundary conditions on the far left/right,

$$D_\pm(-\infty) \sim e^{ik_x x} + R_\pm e^{-ik_x x}, \quad D_\pm(+\infty) \sim T_\pm e^{ik_x x},$$

is known [46]. The transmission is determined by a combinations of Γ functions,

$$T_\pm = \frac{\Gamma(-s_\pm - ik_x w)\Gamma(s_\pm + 1 - ik_x w)}{\Gamma(-ik_x w)\Gamma(1 - ik_x w)}, \qquad (12)$$

with $k_x^2 = (\omega^2 - \omega_0^2)/v^2$, $s_+ = 2$ for amplitude, and $k_x^2 = \omega^2/v^2$, $s_- = 1$ for phase, modes. For the integer parameter s there is no reflected wave $R_\pm \propto 1/\Gamma(-s) = 0$ [48]. Similarly, for transverse EM waves at normal incidence, $A_{y,z}$, Eq. (10) reduces again to one with a $-(1/\lambda^2)/\cosh^2(x/w)$ potential. The transmission amplitude is given by (12) with $k_x = \sqrt{\omega^2 - \omega_p^2}/c$ and $s = -1/2 + \sqrt{1/4 + \xi^2/\lambda^2}$. For frequencies such that $k_x w \gg 1$ or $s \ll 1$, $|T_\perp| \sim 1$. The longitudinal component A_x is entirely reflected, $T_{||} = 0$, due to the divergent term $1/p^2(x)$.

C. Topology connection

Finally, we interpret the collective mode frequencies in terms of the topological properties of the order parameter space and stability of the domain wall. The $\omega_-^2 < 0$ frequency of the imaginary component (9) in a neutral superfluid indicates that the real-valued domain wall is not stable. Indeed, the kink has energy $\alpha(4\sqrt{2}/3)\Delta_0^2\xi$ over the uniform configuration; it is represented by the red line on the left of Fig. 4. An alternative solution to a hard domain wall is a long-wavelength "soft texture" of phase variation $\Delta(x)/\Delta_0 = e^{i\varphi(x)}$, $\varphi = \pi \to 0$

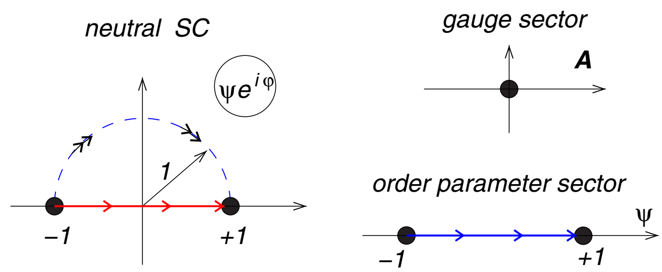

FIG. 4. In a neutral condensate the real-valued domain wall (red line) is unstable with respect to deformations towards the phase texture (dashed semicircle), which can be continuously deformed into a trivial uniform configuration. After coupling to the EM potentials, the field/phase sector separates from the amplitude sector, and the degeneracy space of the real OP amplitude becomes disconnected (±1), stabilizing the real-valued kink.

along the connected $U(1)$ degeneracy manifold, denoted by the dashed semicircle. This configuration has the energy of a trivial uniform state, and can be continuously deformed into one, due to the gapless nature of the phase fluctuations [18]. $|\mathrm{Im}\,\omega_-|$ gives the decay rate of the hard domain wall towards the topologically trivial texture. In a charged superconductor, the phase degree of freedom is absorbed into the gauge field sector, gapped with plasma frequency. The manifold of the degenerate states of the real order parameter becomes disconnected, containing just two points $\pm\Delta_0$, which stabilizes the topological kink. This manifold has $\mathbb{Z}_2$ symmetry: the kink and antikink are unstable and will continuously deform into a lower-energy uniform configuration [52]. This also follows from the Schrödinger equation (6) for D_+ with two potential wells separated by L. In the WKB solution, the zero-frequency mode $\omega_+^2(k_y = 0) = 0$ is split, and one of the frequencies becomes imaginary, $\omega^2 \sim -\exp(-L/\xi)$, signifying the instability of the double domain wall configuration.

III. CONCLUSIONS

In summary, a region of strongly varying condensate, such as a domain wall or a pair-breaking interface, hosts additional bound collective modes of the order parameter. For a single-component complex order parameter we find two additional amplitude modes below the bulk pair-breaking edge 2Δ. One mode lies at 1.73Δ, and the other has zero excitation mass, due to broken translational symmetry (Fig. 2). The nonuniform region supports extra bound gauge field modes as well (Fig. 3). The domain wall completely reflects the longitudinal component of the field and is transparent to others, perfectly transmitting bulk amplitude modes.

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