Nonuniform superconducting state in an antiferromagnetic superconductor

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We consider the superconducting state in a clean crystal with antiferromagnetic (AF) structure of localized magnetic moments taking into account the exchange interaction between magnetic moments and conduction electrons. We assume that the localized moments order at the Neel temperature T_N due to the RKKY interaction predominantly. In such crystals, the periodic exchange field acting on conducting electrons results in the formation of an insulating gap on the Fermi surface for electrons moving in directions that depend on the orientation of the wave vector of AF ordering. We assume a scenario in which the Cooper pairing occurs in the open parts of the Fermi surface at the temperature T_c. We show that at high amplitudes of exchange field $h_e > T_c/\mu_B$, the structure of superconducting state just below the temperature T_c depends on the relation between T_c and T_N. At low ratio T_c/T_N, a nonuniform superconducting state, like the Fulde-Farrell-Larkin-Ovchinnikov high-field phase, should exist, while at bigger ratio superconducting order parameter is uniform. The nonuniform structure of superconducting state may be probed by tunneling measurements.

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I. INTRODUCTION

The question posed by V. Ginzburg [1] in 1956 whether antiferromagnetic (AF) ordering and superconductivity can coexist was answered positively both theoretically and experimentally long ago. It was argued [1–6] that in antiferromagnets the effect of magnetic moments on Cooper pairing produced by modulated exchange and magnetic fields is not so destructive as that produced by uniform fields. Indeed, the periodic exchange field is weakened by averaging in volumes of a coherence length size $\xi = v_F/(2\pi T_c)$ since the moments are separated by the interatomic length $a \ll \xi$. Here, v_F is the electron Fermi velocity. The electromagnetic field induced by magnetic moments is averaged even more effectively because the London penetration length $\lambda_L \gg \xi$ in type-II superconductors. Let us denote h_e the amplitude of the AF exchange field and $\mathbf{q}$ the wave vector of antiferromagnetic ordering whose magnitude we assume to be of the order $2\pi/a$. The effect of an AF exchange field on the superconducting pairing is controlled not only by the small parameter $1/(\xi q)$, characterizing exchange field averaging, but also by the parameter $\gamma = \mu_B h_e/(\hbar v_F q)$ characterizing the strength of the exchange field, which is also small even if $\mu_B h_e$ is strong in comparison with T_c. In the framework of the BCS model Machida *et al.* [5] showed that an antiferromagnetic exchange field suppresses the superconducting critical temperature insignificantly, by the factor $(1 - \gamma/2)$ at $\gamma \ll 1$. The magnetic scattering of electrons near T_c and the scattering by spin waves at low temperatures are also weak if the Neel temperature $T_N \gg T_c$. It is also weak in crystals with high values of magnetic moments or in crystals with a high magnetic anisotropy [6–8].

The next question, which should be addressed, is whether antiferromagnetic ordering may affect significantly superconducting pairing due to the presence of a periodic exchange field in the antiferromagnetically ordered phase. The effective exchange field acting on pairing electrons is $h_e\gamma = \mu_B h_e^2/(\hbar v_F q)$, and we argue that only if the energy of one electron in an effective exchange field, $\mu_B h_e^2/(\hbar v_F q)$, is comparable with T_c, the superconducting order parameter may be modified by the presence of the AF ordering. In the following we discuss, in the framework of the BCS model, how AF ordering affects superconducting pairing if such condition is fulfilled.

We consider the systems with localized magnetic moments such as $M\mathrm{Rh_4B_4}$, $M\mathrm{Mo_6S_8}$, $M\mathrm{Ni_2B_2C}$ and κ-$\mathrm{(BETS)_2}M\mathrm{Br_4}$, where M stands for the ion with a magnetic moment (rare-earth or element with d electrons) [9–14]. The coexistence of the AF ordering with superconductivity was confirmed experimentally in all these systems. In all of them, the magnetic ordering is due to direct magnetic dipole-dipole interaction and the indirect RKKY interaction caused by polarization of the conducting electrons induced by the exchange field of localized moments. We assume the latter dominates and thus AF critical temperature T_N is of the order of $\mu_B^2 h_e^2/\epsilon_F$, where ϵ_F is the Fermi energy. As $v_F q$ is of the order of ϵ_F, the energy of electron in the effective exchange field is of the order of T_N and one can anticipate a significant effect of an AF exchange field on the superconducting structure when $T_c \lesssim T_N$.

Usually, γ is small and in most rare-earth superconductors with AF ordering, T_c is either bigger or comparable with T_N. Hence a generally accepted opinion emerged that there is nothing especially interesting in the coexistence of superconductivity and the AF order, unlike the effect of a constant magnetic field that favors a nonuniform Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state [16,17]. However, in crystals with $T_c < T_N$, such as λ-$\mathrm{(BETS)_2FeBr_4}$ ($T_c = 1.4$ K and $T_N = 2.5$ K) and $\mathrm{Tb_2Mo_3Si_4}$ with $T_N = 19$ and $T_c \approx 0.8$ K [15], one can anticipate a nontrivial coexistence of AF order and Cooper pairing.

The first theoretical indication that such coexistence may not be completely trivial was presented in Ref. [18]. It was noticed that a helix exchange field $h_e \gg T_c/\mu_B$ results in the suppression of superconducting gap for quasiparticles moving along the direction nearly perpendicular to the helix wave vector $\mathbf{q}$ [18]. Later Overhouser and Daemen [3] found similar suppression of superconducting gap in SDW antiferromagnets along the direction $\perp \mathbf{q}$ since SDW gap suppresses the coupling with phonons for electrons moving in this direction. One can anticipate that in the presence of partially vanishing superconducting pairing the superconducting order parameter itself may become nonuniform, as in the FFLO state. The argument of continuity with respect to an increasing q starting from $q = 0$ also tells us that the nonuniform superconducting state would exist at some $q \neq 0$. The question then is what is the area in the parameter space $\mu_B h,(\hbar v_F q),T_c$ for such a state. The possibility of a FFLO state in the coexistence phase in antiferromagnets was discussed earlier by Machida *et al.* [5], who suggested that in the presence of an exchange field with a wave vector q the optimal Cooper pair net momentum could be q rather than the standard $q = 0$ BCS state. However, Nass *et al.* [4] showed that the approximation used in Ref. [5] to find the minimum energy, was not self-consistent.

In the following, we find the structure of the superconducting order parameter just below T_c in the presence of a helix AF ordering established along one (x) and two (x,y) directions for $T_N > T_c$. We call them the single and the double helix cases, respectively. We consider the BCS Hamiltonian with a two-dimensional electron motion in x,y plane. We assume s-wave pairing in the framework of clean limit for conducting electrons, in which the impurity and magnetic scattering rate $\hbar/\tau \ll T_c$. We show that even if the parameters γ and $T_c/(v_F q)$ are small and thus the suppression of T_c is weak, a nonuniform superconducting state, similar to that of the FFLO [16,17], should exist in the helical AF just below T_c, provided $T_c < T_c^*$, where $T_c^* = 0.26\mu_B^2 h_e^2/(\hbar v_F q)$ in a helix with AF ordering in one direction and $3.55\mu_B^2 h_e^2/(\hbar v_F q)$ in a spiral with AF ordering in two directions. We show also that if $T_c > T_c^*$, the superconducting order parameter is uniform but nevertheless the superconducting gap vanishes at $\mu_B h_e > T_c$ in one and two directions in single and double helix, respectively. This result is a consequence of insulating "magnetic" gaps in a single electron spectrum induced by the periodic exchange field on the sectors of the Fermi surface along the y direction of electron motion in a single helix and along x and y directions of motion in a double helix. Those "magnetic" gaps do not lead to a net insulating state because the main part of the Fermi surface remains open, but they prevent the formation of Cooper pairs on the parts of the Fermi surface they cover [3].

II. THE HAMILTONIAN AND THE STRUCTURE OF THE SUPERCONDUCTING ORDER PARAMETER AT T_c

We consider the BCS Hamiltonian

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_{\rm BCS} + \int d^3\mathbf{r}\hat{\psi}_\alpha^+(\mathbf{r})h_{e,i}(\mathbf{r})(\sigma_i)_{\alpha\beta}\hat{\psi}_\beta(\mathbf{r}), \quad (1)$$

$$\hat{\mathcal{H}}_{\rm BCS} = \int d^3\mathbf{r}\Big[\hat{\psi}_\alpha^+(\mathbf{r})(\epsilon(\hat{\mathbf{p}}) - \epsilon_F)\hat{\psi}_\alpha(\mathbf{r}) - \frac{1}{2}g\hat{\psi}_\alpha^+(\mathbf{r})\hat{\psi}_\beta^+(\mathbf{r})\hat{\psi}_\beta(\mathbf{r})\hat{\psi}_\alpha(\mathbf{r})\Big], \quad (2)$$

$$\epsilon(\mathbf{p}) = \frac{\mathbf{p}^2}{2m}, \quad (3)$$

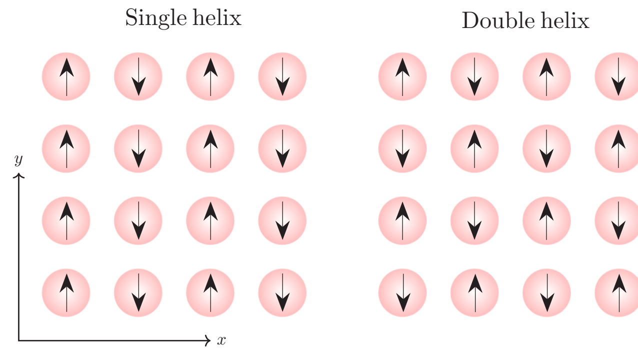

FIG. 1. (Left) The single-wave-vector helix structure with an alternating exchange field along the x axis and a constant exchange field along the y axis, helix wave vector is $\mathbf{q} = (\pi/a,0,0)$. (Right) Double helix with alternating exchange field in both x and y directions, $\mathbf{q} = (\pi/a,\pi/a,0)$.

in a superconductor with a helix magnetic ordering resulting in the exchange field

$$\mathbf{h}_e(\mathbf{r}) = h_e[\cos(\mathbf{q}\cdot\mathbf{r}), \sin(\mathbf{q}\cdot\mathbf{r}),0]. \quad (4)$$

Here, $g > 0$ describes electron attraction due to phonon or spin wave exchange, a summation over repeated indices is implied, σ_i with $i = x,y,z$ are the Pauli matrices, and $\alpha,\beta = 1,2$ are spin indices. In the single helix with a wave vector $\boldsymbol{q} = (\pi/a,0,0)$, the exchange field oscillates with a period $2a$ along the x axis and remains constant along the y axis, while for a double helix with $\mathbf{q} = (\pi/a,\pi/a,0)$ the exchange field oscillates in both x and y directions, see Fig. 1. The Hamiltonian $\mathcal{H}$ results in the RKKY interaction of localized magnetic moments. We assume that this is the predominant interaction and that it leads to the helical ordering displayed in Fig. 1, below the Neel temperature T_N.

In Eq. (4), the magnitude of the exchange field may be presented as $h_e = h_{e0}|\langle\mathbf{S}\rangle|$, where h_{e0} is the material parameter, while the quantum-mechanical and thermodynamic average value of a localized spin on the site, $\langle\mathbf{S}\rangle$, depends on the spin value S and the relative temperature T/T_N. The important point is that the structure of the superconducting order parameter may be found at a given $h_e(T)$ because the energy of superconducting pairing, of the order of T_c^2/ϵ_F per spin, is much smaller than the magnetic energy, of the order of T_N per spin. In the following, we assume that this condition is fulfilled. The temperature dependence in h_e may be neglected if $T \ll T_N$ or $\gamma \ll 1$. Hence superconducting ordering does not affect the magnetic ordering so much, and we will consider h_e as a fixed temperature independent parameter.

The superconducting pairing is described by the Green function $G_{\alpha,\beta}(\mathbf{r},\mathbf{r}') = -\langle T\hat{\psi}_\alpha(\mathbf{r})\hat{\psi}_\beta^+(\mathbf{r}')\rangle$ and by the anomalous Gor'kov function $F_{\alpha,\beta}^+(\mathbf{r},\mathbf{r}') = \langle T\hat{\psi}_\alpha^+(\mathbf{r})\hat{\psi}_\beta^+(\mathbf{r}')\rangle$. They obey the equations

$$[i\omega_n - \xi(\hat{\mathbf{p}}) - \hat{V}(\mathbf{r})]\hat{G}(\mathbf{r},\mathbf{r}') + \Delta\hat{I}\hat{F}^+(\mathbf{r},\mathbf{r}') = \delta(\mathbf{r}-\mathbf{r}'), \quad (5)$$

$$[i\omega_n + \xi(\hat{\mathbf{p}}) + \hat{V}(\mathbf{r})]\hat{F}^+(\mathbf{r},\mathbf{r}') - \Delta^*\hat{I}\hat{G}(\mathbf{r},\mathbf{r}') = 0. \quad (6)$$

Here, $V_{kk}=0$, $V_{12}=\mu_B h_e \exp(i\mathbf{qr})=V_{21}^*$, $\xi(\mathbf{p})=\epsilon(\mathbf{p})-\epsilon_F$, Matsubara frequencies are ω_n, while $\hat{G}$, $\hat{I}$, and $\hat{V}$ are matrices in the spin-1/2 space.

To find the second-order phase transition line N-S between the normal and the superconducting states, it is sufficient to know the normal state Green functions $\hat{G}(\mathbf{r},\mathbf{r}')$ and to solve the Bethe-Salpeter equation for the vortex (see, e.g., Refs. [17,19]). The equation for $G_{\alpha,\beta}(\mathbf{r},\mathbf{r}')$ in the normal sate is Eq. (5) with $\Delta=0$. In Fourier representation, we then have

$$\begin{aligned}(i\omega-\xi_+)G_{11}(\mathbf{p}_+,\mathbf{p}')+h_e G_{21}(\mathbf{p}_-,\mathbf{p}')&=\delta(\mathbf{p}'-\mathbf{p}),\\ (i\omega-\xi_+)G_{12}(\mathbf{p}_+,\mathbf{p}')+h_e G_{22}(\mathbf{p}_-,\mathbf{p}')&=0,\end{aligned} \tag{7}$$

where $\mathbf{p}_\pm=\mathbf{p}\pm\mathbf{q}/2$ and $\xi_\pm=\xi(\mathbf{p}_\pm)$. In the equations for G_{22} and G_{21}, we need to replace vice/versa $+$ by $-$.

A. The single helix, the AF ordering in one direction

The solutions for a single helix are [18]

$$\begin{aligned}G_{11}(\mathbf{p}_+,\mathbf{p}')&=\frac{i\omega_n-\xi_-}{(i\omega-\xi_+)(i\omega_n-\xi_-)-h_e^2}\delta(\mathbf{p}'-\mathbf{p}),\\ G_{12}(\mathbf{p}_+,\mathbf{p}')&=-\frac{h_e}{(i\omega_n-\xi_+)(i\omega_n-\xi_-)-h_e^2}\delta(\mathbf{p}'-\mathbf{p}).\end{aligned} \tag{8}$$

Here and in the following, h_e is the Zeeman electron energy in the exchange field, $v=v_F$ and $\hbar=1$. It is seen that G_{11} depends on the momenta $\mathbf{p}\pm\mathbf{q}$ so that $G_{11}(\mathbf{r},\mathbf{r}')$ describes an anisotropic electron motion as it depends not only on $|\mathbf{r}-\mathbf{r}'|$ but also on the direction of $(\mathbf{r}-\mathbf{r}')$ relative to $\mathbf{q}$.

In the case of strongly anisotropic (2D) electron spectrum with Fermi velocities $v_x,v_y\gg v_z$, all three vectors $\mathbf{q},\mathbf{p}$, and $\mathbf{R}=\mathbf{r}-\mathbf{r}'$ are essentially in the x,y plane. We obtain

$$G_{11}(\mathbf{R})=\int\frac{d\mathbf{p}}{(2\pi)^2}\frac{(i\omega-\epsilon+\delta)\exp(i\mathbf{p}\cdot\mathbf{R})}{(i\omega-\epsilon-\delta)(i\omega-\epsilon+\delta)-h_e^2}, \tag{9}$$

with $\mathbf{p}\cdot\mathbf{R}=(p_0+\epsilon/v)R\cos(\theta-\phi)$, where p_0 is the Fermi momentum, θ, ϕ_q and ϕ are the angles of $\mathbf{R}$, $\mathbf{q}$, and $\mathbf{p}$ with respect to the x axis. Here we denote $\epsilon=(1/2)[\xi(\mathbf{p}_+)+\xi(\mathbf{p}_-)]$ and $\delta=(1/2)[\xi(\mathbf{p}_+)-\xi(\mathbf{p}_-)]=\mathbf{v}\cdot\mathbf{p}/2$. We replace the integration over $\mathbf{p}$ by the integration over variables ϵ and ϕ. The denominator of G_{11} is $(\epsilon-i\omega-A)(\epsilon-i\omega+A)$ with

$$A=\left(\delta^2+h_e^2\right)^{1/2},\quad \delta=vq\cos(\phi-\phi_q). \tag{10}$$

The quasiparticle spectrum is given as $E(\epsilon,\delta)=\epsilon\pm A$. It consists of two branches corresponding to two different superpositions of up and down spins. The pieces of the Fermi surface are $\epsilon=\pm A$. There are insulating magnetic gaps on the Fermi surface in the direction of $\mathbf{p}$ along the y axis $(\theta=\pi/2,3\pi/2)$, see Fig. 2. These gaps prevent the Cooper pairing on the Fermi surface patches destroyed by magnetic gaps, as was noticed in the case of SDW by Overhauser and Daemen [3]. As a result, superconducting electrons cannot move in the direction along the y axis even when the superconducting order parameter is uniform. [18]. However, the development of such magnetic pseudogaps below T_N does not result in the overall insulation behavior at small γ because the most part of the Fermi surface remains open for both electron motion and Cooper pairing.

The integration over ϕ from 0 to 2π in Eq. (9) can be done from $\theta-\pi/2$ to $\theta+\pi/2$, where $\cos(\theta-\phi)>0$, and from $\theta+\pi/2$ to $\theta+3\pi/2$, where $\cos(\theta-\phi)<0$. Integrating over ϵ by residues we choose the integration contour in the upper half-plane in the first integral for $\omega>0$ and in the lower half-plane at $\omega<0$. We obtain for $\omega>0$,

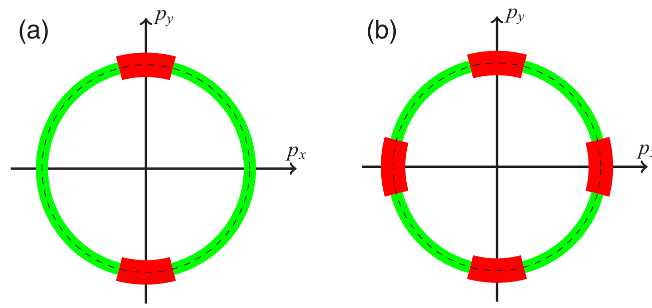

FIG. 2. The magnetic (red) and superconducting (green) gaps on the Fermi surface.

$$G_{11,\omega}(\mathbf{R})=\frac{im}{\pi}\int_0^\pi d\phi\sum_\pm\left\{\frac{(\pm\delta-A)}{2A}\times\exp\left[i\left(p_0+\frac{i\omega\pm A}{v}\right)R\cos(\theta-\phi)\right]\right\}. \tag{11}$$

For $\omega<0$, it suffices to take the complex conjugated expression. In our calculations of T_c, following Eq. (13), we need values of $R\gg\xi$. For large R, the main contribution to the integral over ϕ comes from $\phi\approx\theta$. Hence we take $\phi=\theta$ in $\delta(\phi)$ when integrating over ϕ near θ. The result for $\omega>0$ is

$$\begin{aligned}G_{11,\omega}(\mathbf{R})&=-\frac{im}{\pi\sqrt{p_0R}}\exp\left(ip_0R-\frac{|\omega|R}{v}\right)\\&\quad\times\left\{\frac{\delta-A}{2A}\exp\left[iR\frac{\delta+A}{v}\right]-\frac{\delta+A}{2A}\exp\left[iR\frac{\delta-A}{v}\right]\right\},\\ G_{12,\omega}(\mathbf{R})&=\frac{mh_e}{\pi A\sqrt{p_0R}}\exp\left(ip_0R-\frac{(|\omega|+\delta)R}{v}\right)\sin\frac{AR}{v},\end{aligned} \tag{12}$$

and now $\delta=(vq/2)\cos(\theta-\phi_q)$. For $\omega<0$, again one takes the complex conjugate of this result. In a similar way, we derive the other components of the Green functions in coordinate space. We see explicitly that the Green functions $\hat{G}(\mathbf{r})$ depend on $R=|\mathbf{R}|$ and, via $\delta(\theta)$, on the direction of the electron motion described by the angle θ counted from the direction of $\mathbf{q}$. At $h_e\neq0$, there is an additional periodic coordinate dependence in $G_{11,\omega}$ due to the exchange field $A-\delta\approx h_e^2/(vq)$. It is this additional coordinate dependence, which results in a nonuniform structure of the superconducting order parameter at T_c, when T_c is low enough.

Next, we find the critical temperature T_c of the second-order phase transition N-S solving the linear equation for the order parameter $\Delta(\mathbf{r})$ [19],

$$\begin{aligned}\Delta(\mathbf{r})&=\int d\mathbf{l}K(\mathbf{r}-\mathbf{l})\Delta(\mathbf{l}),\\ K(\mathbf{r})&=\frac{g}{2}T\sum_{|\omega|<\omega_D}\mathrm{Tr}(i\sigma_y)\hat{G}_\omega(\mathbf{r})(i\sigma_y)\hat{G}_{-\omega}(\mathbf{r}),\end{aligned} \tag{13}$$

where ω_D is the Debye frequency for a phonon mechanism or the maximum spin wave frequency of the order of T_N for a spin-wave exchange mechanism. We consider the pairing of electrons with single particle energies close to either $\epsilon = \delta + A$ or $\epsilon = \delta - A$. Thus we account for pairs of electrons with energies $A + \delta$ for $\delta < 0$ and $A - \delta$ for $\delta > 0$. We readily get with $u = \cos\theta$

$$K(\mathbf{r}) = T \sum_{|\omega|<\omega_D} \frac{gm^2}{\pi^2 p_0 R} \mathcal{B}(r,\theta) \exp(-2|\omega|R/v),$$
$$\mathcal{B}(r,\theta) = S(u)\cos[2\Omega(u)r] + \frac{\gamma^2}{u^2+\gamma^2}[\cos((2\Omega(u)+u)r) - 2\sin^2(qru)],$$
$$S(u) = \frac{(\sqrt{u^2+\gamma^2}+u)^2}{2(u^2+\gamma^2)}, \quad \Omega(u) = \sqrt{u^2+\gamma^2} - u. \quad (14)$$

We drop in the following the term proportional to γ^2. In $\mathcal{B}(r,\theta)$, we expressed distances in units of ξ so that $r = R/\xi$, where $\xi = v_F/(2\pi T_{c0})$ and T_{c0} at $\gamma \ll 1$ satisfies the equation

$$\frac{\lambda}{2\pi} \sum_n \int_0^\pi d\theta\, S(\cos\theta) \frac{\omega_n}{\omega_n^2 + \Omega^2(\cos\theta)} = 1. \quad (15)$$

This equation was obtained assuming a uniform superconducting order parameter by the integration over r of Eqs. (13) and (14), with $\omega_n = \pi\rho_0(2n+1)$ with $\rho_0 = T_{c0}/(2\pi vq)$ and λ being the dimensionless parameter associated with the electron coupling for frequencies $|\omega_n| < \omega_D/(2vq)$. Here and in the following, we introduce a dimensionless temperature parameter

$$\rho = T/(2vq).$$

The periodic space dependence, $\cos[2\Omega(u)r]$, in the $\mathcal{B}(r,\theta)$ leads to the suppression of the formation of Cooper pairs moving in the direction defined by the angle θ. This factor results in the suppression of T_c in comparison with T_c at $h_e = 0$. The most interesting case for real antiferromagnets is $\gamma\rho^{-1} \gg 1$, $\gamma \ll 1$, while $\rho^{-1} \gg 1$. In this situation, Cooper pairs moving in the directions perpendicular to $\mathbf{q}$ would be destroyed by the strong exchange field $h_e \gg T_c$ if not supported, due to a proximity effect, by pairs moving in other directions.

Clearly, the oscillatory space dependence of the superconducting order parameter may compensate the destructive effect of the oscillating exchange field resulting in an increase of T_c in comparison with that given by Eq. (15). The order parameter with oscillations along y turns out to be the most favorable. For a nonuniform state with $\Delta(y) = \Delta_0\cos(iky)$ we obtain the condition of the second-order phase transition N-S_k from normal to nonuniform superconducting state with a nonzero Cooper pair center-of-mass k, as

$$\frac{\lambda}{2\pi}[\mathcal{K}(k) + \mathcal{K}(-k)] = 1,$$
$$\mathcal{K}(k) = \sum_n \int_0^\pi d\theta\, S(\cos\theta) \frac{\omega_n}{\omega_n^2 + [\Omega(\cos\theta) - k\cos\theta]^2}. \quad (16)$$

The parameter k (the wave vector of the nonuniform state in units $1/\xi$) should be determined by the condition of maximum

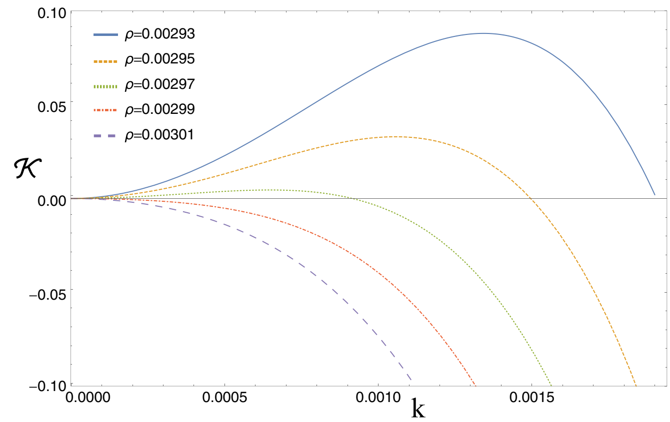

FIG. 3. The function $\mathcal{K}(k)$ that defines the parameter k of nonuniformity of superconducting state in a single helix AF for different transition temperatures $\rho = T/vq$ at $\gamma = 0.1$. The formation of $k \neq 0$ peak as ρ decreases below $\rho^* = 0.002\,97$ signals the nonuniform state below the second-order phase transition from the normal state to the superconducting state.

T_c. To find the optimal parameter k, we maximized numerically the function $\mathcal{K}(k) + \mathcal{K}(-k) - 2\mathcal{K}(0)$ over k at a given γ and the critical temperature parameter ρ. The function $\mathcal{K}(k)$ at $\gamma = 0.1$ for different ρ is shown in Fig. 3. We see that at ρ above $0.002\,97$ there is only one peak at $k = 0$ corresponding to the uniform superconducting state. But at ρ below this critical value ρ_c^*, a peak at nonzero k develops, signaling the normal state instability with respect to the transition to a nonuniform superconducting state.

The plot $\rho_c^*(\gamma)$ is shown in Fig. 4. The parameter ρ_c^* increases with γ as $\rho_c^* \approx 0.26\gamma^2$. The value of optimal k, $k_{\rm opt}$, increases as ρ drops. Thus the formation of the nonuniform state becomes less favorable at high temperatures, as in the case of FFLO at constant exchange field, where a nonuniform FFLO state develops only at temperatures $T_c(h_e)$ below $T_c^* = 0.55T_c(h_e = 0)$ [16,17]. Similarly, in helix antiferromagnets, the nonuniform state develops when the second-order phase transition temperature $T_c(h_e, vq)$ satisfies the condition $\rho = T_c/(2vq) < \rho_c^*$. Accounting for $\rho_c^* \approx 0.26\gamma^2$, we obtain the

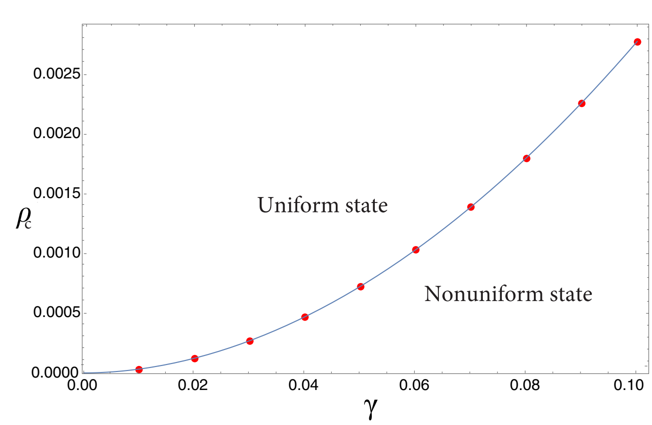

FIG. 4. The critical dimensionless temperature $\rho_c^* = T_c/vq$, separating transitions to the uniform and the nonuniform states, as a function of the parameter γ in a single helix antiferromagnet. The fit for $\rho_c^*(\gamma)$ is $\rho_c^* \approx 0.26\gamma^2$.

condition of nonuniform state below T_c as

$$T_c(h_e, vq) < T_c^* = 0.26h_e^2/vq. \quad (17)$$

When the dominant mechanism of the AF ordering is the RKKY interaction with $T_N \approx h_e^2/\epsilon_F$ and when $vq \lesssim \epsilon_F$, we get this condition as $T_c^* = 0.26T_N$.

As was mentioned before, the Cooper pairing is suppressed in the sectors of the Fermi surface in which a magnetic pseudogap exists. Correspondingly, at low critical temperatures $T_c < T_c^*$, the superconducting order parameter becomes nonuniform and vanishing in the coordinate space at lines where $\cos(ky) = 0$. At higher critical temperatures, $T_c > T_c^*$, the thermal motion of an electron averages over all directions (effect similar to the proximity effect) and the superconducting order parameter becomes uniform since the pairing in other directions, where a magnetic gap is absent, supports a total nonzero order parameter.

B. The double helix, the AF ordering in two dimension

For a double helix with $\mathbf{q} = (\pi/a, \pi/a, 0)$, Eq. (7) results in eight different equations for $G_{11}(p_x, p_y)$, $G_{11}(p_x, p_y + \pi/a)$, $G_{11}(p_x + \pi/a, p_y)$, $G_{11}(p_x + \pi/a, p_y + \pi/a)$, and $G_{12}(p_x, p_y)$, $G_{12}(p_x, p_y + \pi/a)$, $G_{12}(p_x + \pi/a, p_y)$, and $G_{12}(p_x + \pi/a, p_y + \pi/a)$. The solution for G_{11} is

$$G_{11}(\mathbf{p}, \mathbf{p}') = \frac{(i\omega - \xi_{0+})(i\omega - \xi_{+0})(i\omega - \xi_{++})\delta(\mathbf{p} - \mathbf{p}')}{\mathcal{P} - h_e^2 Q},$$
$$\mathcal{P} = (i\omega - \xi_{0+})(i\omega - \xi_{+0})(i\omega - \xi_{++})(i\omega - \xi_{00}).$$
$$Q = (2i\omega - \xi_{00} - \xi_{++})(2i\omega - \xi_{0+} - \xi_{+0}). \quad (18)$$

Here, $\xi_{+0} = \xi(p_x + \pi/a, p_y)$, $\xi_{0+} = \xi(p_x, p_y + \pi/a)$ and similar for others energies ξ. From the denominator $\mathcal{P} - h_e^2 Q$, we obtain four branches of quasiparticle energies in the normal state, $i = 1,2,3,4$:

$$E_{0,i}(\mathbf{p}) = \epsilon \pm \left[(v_F q \cos\theta)^2 + h_e^2\right]^{1/2}$$
$$\pm\left[(v_F q \sin\theta)^2 + h_e^2\right]^{1/2}. \quad (19)$$

Hence we get two gaps $2h_e$ on the Fermi surface in the directions of $\mathbf{p}$ along x and y axes, see Fig. 2. Next, we find the Green functions in the coordinate space. They have oscillatory dependence on both coordinates x and y with the wave vectors

$$\Omega_x(\theta) = (\cos^2\theta + 4\gamma^2)^{1/2} - \cos\theta, \quad (20)$$

$$\Omega_y(\theta) = (\sin^2\theta + 4\gamma^2)^{1/2} - \sin\theta. \quad (21)$$

We obtain for $\mathcal{B}(r,\theta)$ with $s = \sin\theta$

$$\mathcal{B}(r,\theta) = \frac{\mathcal{A}}{8(u + \Omega_x)^2(s + \Omega_y)^2},$$
$$\mathcal{A} = \mathcal{A}_1 \cos[(2u + 2s + \Omega_x + \Omega_y)r]$$
$$+ \mathcal{A}_4 \cos[\Omega_x + \Omega_y)r] + \mathcal{A}_2 \cos[(2u + \Omega_x - \Omega_y)r]$$
$$+ \mathcal{A}_3 \cos[(2s - \Omega_x + \Omega_y)r],$$
$$\mathcal{A}_1 = (2u + s + \Omega_x + \Omega_y)^2(u + 2s + \Omega_x + \Omega_y)^2$$
$$\mathcal{A}_2 = (2u - s + \Omega_x - \Omega_y)^2(u + \Omega_x - \Omega_y)^2,$$
$$\mathcal{A}_3 = (s - \Omega_x + \Omega_y)^2(2s - u - \Omega_x + \Omega_y)^2,$$
$$\mathcal{A}_4 = (u + \Omega_x + \Omega_y)^2(s + \Omega_x + \Omega_y)^2. \quad (22)$$

At small superconducting coupling $\lambda \ll 1$ the main contribution to T_c comes from slowly spatially varying term $\mathcal{A}_4 \cos[(\Omega_x + \Omega_y)r]$ in $\mathcal{B}(r)$, while terms with $\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3$ as well as Ω_x and Ω_y in the denominator of $\mathcal{B}(r)$ may be dropped. We obtain for the optimal nonuniform superconducting order parameter $\Delta(\mathbf{r}) = \Delta_0 \cos[k(u + s)]$ or $\Delta(\mathbf{r}) = \Delta_0 \cos[k(u - s)]$ (due to symmetry of x and y coordinates). Optimizing with respect to k we get $\rho_c^* \approx 3.55\gamma^2$ and $k_{\text{opt}}(\gamma, T_c^*) \approx 4.23\gamma^{2.43}$. They are both represented for the double and single helix in Figs. 5 and 6. As T drops below T_c, the value of k_{opt} increases, but to find it we need to solve the nonlinear equations (5) and (6).

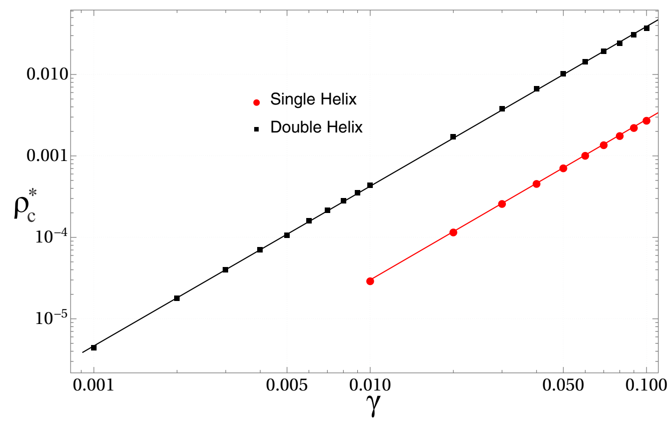

FIG. 5. The dependence of the critical dimensionless temperature ρ_c^* on the parameter γ. At $T_c < T_c^*$, a nonuniform superconducting state establishes below T_c, while at $T_c > T_c^*$ the system undergoes a transition from a normal to a uniform superconducting state. The upper (black) curve is for $\mathbf{q} = (\pi/a, \pi/a, 0)$ helix. The solid line is $\ln\rho_c^* = \ln a + 2\ln\gamma$ with $a = 3.55$. The lower (red) curve is for a helix $\mathbf{q} = (\pi/a, 0, 0,)$. The solid line is $\ln\rho_c^* = \ln b + 2\ln\gamma$ with $b = 0.26$.

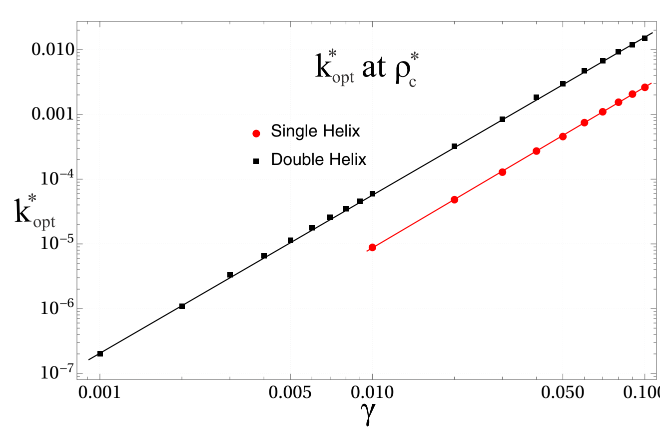

FIG. 6. The dependence of the optimal wave vector k_{opt}^* at the critical temperature $T_c < T_c^*$ on the parameter γ. The upper (black) curve is for a $\mathbf{q} = (\pi/a, \pi/a, 0)$ helix, the solid line is $\ln k_{\text{opt}}^* = \ln c + 2.43\ln\gamma$ with $c = 4.23$. The lower (red) curve is for the helix $\mathbf{q} = (\pi/a, 0, 0,)$. The solid line is $\ln k_{\text{opt}}^* = \ln g + 2.48\ln\gamma$ with $g = 0.81$.

C. The type of superconducting transition in helix antiferromagnets

In the case of the FFLO transition, in a constant exchange field, the temperature T_c^* actually separates the second-order phase transition N-S into the uniform superconducting state from the first-order phase transition into the nonuniform state. We check now the order of the N-S transition in a helix AF by calculating the third-order term in $\Delta(\mathbf{r})$ in Eq. (13). This cubic term has the form [19]

$$-\frac{g}{2}T\sum_{\omega}\int \hat{G}_{\omega}(\mathbf{l},\mathbf{m})\Delta(\mathbf{m})\hat{G}_{\omega}(\mathbf{s},\mathbf{r})\Delta^*(\mathbf{s})\hat{G}_{-\omega}(\mathbf{s},\mathbf{m}) \times\Delta^*(\mathbf{l})\hat{G}_{-\omega}(\mathbf{l},\mathbf{r})d\mathbf{l}d\mathbf{m}d\mathbf{s}. \tag{23}$$

The Green functions drop exponentially with $|\mathbf{l}-\mathbf{r}|/\xi$, while at $\gamma \ll 1$ their additional coordinate dependence and the spatial dependence of the order parameter are much weaker and as a result they can be neglected altogether. Then we have the additional standard negative cubic term [19] in the right-hand side of Eq. (13). Hence the transition N-S$_k$ in a helix AF with the small $\gamma \ll 1$ is a second-order phase transition in contrast to the FFLO transition in a uniform exchange field [20].

D. Quasiparticle gaps in superconducting state at $T_c > T_c^*$

For a single helix, the quasiparticle spectrum $E(\mathbf{p})$ was found in Ref. [18]. At $h_e \gg \Delta$, it has the form

$$E_{1,2}^2 = \left[\epsilon \pm \left(\delta^2 + h_e^2\right)^{1/2}\right]^2 + \frac{\delta^2}{\delta^2+h_e^2}\Delta^2. \tag{24}$$

The first term describes an insulating magnetic gap $2h_e$ on the part of the Fermi surface at angles $\cos\theta \lesssim \gamma$ around the p_y axis, see Fig. 2. The second term describes the superconducting gap, which vanishes in the same angular interval. For a helix with AF ordering along x and y directions, we solve Eqs. (5) and (6) for a coordinate independent Δ and expand the determinant of these equations, $\mathcal{D}(\Delta)$, in powers of Δ^2:

$$\mathcal{D}(i\omega,\mathbf{p}) = \prod_{i=1}^{4}(i\omega-\epsilon-E_{0,i})^2 + \frac{1}{2}\sin^2(2\theta) \times[1+(\cos^2\theta+\gamma^2)^{1/2} \times(\sin^2\theta+\gamma^2)^{1/2}]\Delta^2 + o(\Delta^2). \tag{25}$$

Hence the quasiparticle spectrum is

$$E_i^2(\mathbf{p}) = E_{0,i}^2(\mathbf{p}) + \frac{\sin^2(2\theta)\Delta^2}{4(\cos^2\theta+\gamma^2)(\sin^2\theta+\gamma^2)}. \tag{26}$$

Now the magnetic gaps are around p_x and p_y axes, and there the superconducting gap is absent. The rest of the Fermi surface is covered by the superconducting gap Δ.

III. DISCUSSION

We consider now the electric properties in crystals with a double helix exchange field (see, Fig. 1). The organic superconductor with AF ordering and almost two-dimensional electron motion λ-(BETS)$_2$FeBr$_4$ belongs to this type of crystals. Above T_N, the magnetic moments affect the electron properties due to magnetic scattering only. The situation changes below the Neel temperature T_N. In the temperature interval $T_c < T < T_N$, an insulating gap for electrons moving along the x and y axes results in the suppression of electrical and thermal conductivity along these directions, though the main part of the Fermi surface remains open and as a result the overall nature of this system is metallic. Thus the electron transport becomes anisotropic, but this anisotropy is not strong in crystals with small γ. Radical change in transport properties occurs below the superconducting transition at T_c. If $T_c > T_c^*$, a uniform superconducting order parameter develops, resulting in a superconducting gap on the main part of the Fermi surface coexisting with insulating magnetic gaps near the momentum directions p_x and p_y. The anisotropy of electron transport in response to external electromagnetic fields, such as magnetic field screening, becomes stronger below T_c. Electron thermal transport vanishes due to a combination of both magnetic and superconducting gaps. The tunneling I-V characteristics in this phase depend on whether a normal or a superconducting tip is used, as well as on the tip orientation.

The nonuniform state below $T_c < T_c^*$ shows the most interesting electron transport properties. The superconducting order parameter in this phase becomes periodic with the wave vector $k > k_{\rm opt}^*$ corresponding to the period $d < 2\pi/k_{\rm opt}^*$. An important point is that that due to the small amplitude of the effective exchange field, $h_e^2/(v_F q)$, the value of the period d in the nonuniform state of AF system is much bigger than ξ, while in a FFLO state in high magnetic fields it is of the order of ξ. Thus relatively big and quite clean crystals are needed to observe a nonuniform state in AF systems. in fact, such a state may be observed only in crystals with size $L > d$ and electron mean free path $\ell > d$. This puts strict conditions on the lower boundary of γ for a nonuniform phase observation:

$$\gamma^{2.43} > \frac{0.24\hbar v_F}{[\min(L,\ell)]T_c}, \tag{27}$$

or on the lower boundary of T_N because $T_N \approx \gamma^2\epsilon_F$. Taking $T_c = 1$ K and $[\min(L,\ell)] = 0.1$ cm, we obtain the condition $\gamma^2 > 10^{-3}$ corresponding to $T_N > 0.001\epsilon_F$. We note that to fulfill conditions (27) on a crystal of size L and electron mean free path ℓ, indeed very clean crystals with high T_N are needed. These conditions are not fulfilled in the compound λ-(BETS)$_2$FeBr$_4$ with $h_e \approx 14$ T from H_{c2} measurements in fields parallel to the layers and with $T_N = 2.5$ K [22]. In such crystals, the superconducting state below T_c should be uniform. If conditions (27) would be met, a nonuniform superconducting phase may be checked by tunneling experiments, which may reveal the periodicity of the quasiparticle density of states. The change of sign of superconducting order parameter at different crystal edges in a nonuniform state may be revealed by measurements similar to those that showed the change of sign in cuprate d-wave superconductors [26]. In a nonuniform state, the Meissner screening and the thermal conductivity will be both anisotropic and anisotropy will be more pronounced than in the uniform state.

Recently, in relation to Fe-based superconductors, much attention is payed to the spin density wave (SDW) state, which may coexist with superconductivity in the case of the s-wave pairing [21,23,24] or in the case of the anisotropic pairing [25]. SDW ordering introduces an exchange field acting on the superconducting electrons and at a first glance the problem of coexistence of SDW and superconducting states is similar to that of the antiferromagnetism of localized magnetic moments (f or d electrons) and superconductivity (mainly of s electrons). However, actually, there is an important difference since both SDW and superconducting pairing occur on the same Fermi surface of conducting electrons. The energies involved in the transitions with a comparable T_c and SDW transition temperature T_{SDW} are also of the same magnitude and one cannot treat that superconducting transition as occurring in a given exchange field, i.e., in the approach used throughout this paper. Only in the limit $T_c \ll T_N$, such a treatment is correct and all our results obtained for the magnetic pseudogaps and for the nonuniform state may be applied to the SDW superconducting system as well.

In conclusion, in the framework of BCS model, we establish the condition $\rho_c = T_c/vq < \rho_c^*(\gamma)$ for the transition into a nonuniform FFLO-like superconducting state for a single- and in a double-wave-vector helical AF system with s-wave pairing. At $\rho_c > \rho_c^*$ below T_c, the superconducting order parameter is uniform even if magnetic pseudogaps are also present on some sectors of the Fermi surface.

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