Higgs mechanism, phase transitions, and anomalous Hall effect in three-dimensional topological superconductors

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We demonstrate that the Higgs mechanism in three-dimensional topological superconductors exhibits unique features with experimentally observable consequences. The Higgs model we discuss has two superconducting components and an axionlike magnetoelectric term with the phase difference of the superconducting order parameters playing the role of the axion field. Due to this additional term, quantum electromagnetic and phase fluctuations lead to a robust topologically nontrivial state that holds also in the presence of interactions. In this sense, we show that the renormalization flow of the topologically nontrivial phase cannot be continuously deformed into a topologically nontrivial one. One consequence of our analysis of quantum critical fluctuations is the possibility of having a first-order phase transition in the bulk and a second-order phase transition on the surface. We also explore another consequence of the axionic Higgs electrodynamics, namely, the anomalous Hall effect. In the low-frequency London regime an anomalous Hall effect is induced in the presence of an applied electric field parallel to the surface. This anomalous Hall current is induced by a Lorentz-like force arising from the axion term, and it involves the relative superfluid velocity of the superconducting components. The anomalous Hall current has a negative sign, a situation reminiscent of but quite distinct in physical origin from the anomalous Hall effect observed in high-T_c superconductors. In contrast to the latter, the anomalous Hall effect in topological superconductors is nondissipative and occurs in the absence of vortices.

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I. INTRODUCTION

Topological solid states of matter [1,2] have bulk states that are gapped, while states at the boundaries are gapless and protected by some discrete quantum symmetry. The topological aspect emerges when considering the transport properties of the boundary states, where the transport current happens to also be a topological current. The most well-established topological solid states of matter are topological insulators (TIs), which are gapped in the bulk and have helical (or chiral) gapless states at the boundaries which are protected by time-reversal symmetry. Helical here means that the electronic spin is locked to momentum due to strong spin-orbit coupling. Thus, the boundary states have a helicity determined by the eigenvalues of $\boldsymbol{\sigma} \cdot \mathbf{k}/k$ at each boundary. Topological insulators have been predicted to exist [3] and confirmed experimentally in subsequent papers [4]. Although many of the materials investigated experimentally are not perfect insulators in the bulk [5], the observed boundary helical states are robust features of these materials.

There exists another predicted topological solid state of matter, namely topological superconductors (TSCs) [1,2], where the experimental situation is less clear. TSCs follow a symmetry classification scheme closely related to TIs, as far as Hamiltonians of the Bogoliubov–de Gennes type are concerned [6]. Just like TIs, TSCs have gapped states in the bulk and symmetry-protected gapless states at the boundaries. Unlike TIs, in TSCs the $U(1)$ symmetry is broken, either spontaneously or by proximity effect. The gapless boundary states are Majorana fermions, which are fermionic particles that are their own antiparticles. In order to support such states at the boundaries, the topological superconductivity must feature a p-wave type of gap. Particle-hole is the underlying symmetry protecting the boundary Majorana states. In one dimension, a paradigmatic simple model for topological superconductivity has been proposed by Kitaev [7] where the Majorana zero-energy states live at the ends of a quantum wire. An experimental way of realizing a superconducting state in a quantum wire is by proximity effect. In this case a semiconducting wire with strong spin-orbit coupling is deposited on the surface of an s-wave superconductor in the presence of an external perpendicular magnetic field. Then by proximity effect p-wave-like superconductivity is induced on the wire for a certain range of parameters [8,9]. There are some experimental signatures of Majorana modes in indium antimonide nanowires in contact with normal and superconducting electrodes [10]. More recently, strong support for Majorana boundary zero modes has been reported in an experiment with a ferromagnetic chain of iron fabricated on the surface of lead [11].

Three-dimensional (3D) TSCs have also been discussed theoretically, in particular focusing on vortex physics [1,2] and possible topological phase transitions [12]. A distinctive feature of both three-dimensional TIs and TSCs with respect to their nontopological counterparts is the topological magnetoelectric response induced by a mechanism similar to the so-called chiral anomaly [13]. When fermions in topological materials interact with the electromagnetic field, a Berry phase mixing electric and magnetic fields is induced [14,15]. In TIs this occurs due to strong spin-orbit coupling that locks spin to momentum. The resulting Berry phases combine in the form of a so-called axion term, which is a magnetoelectric term $\sim\mathbf{E} \cdot \mathbf{B}$ with a periodic field, θ, appearing as a coefficient [14]. This coefficient corresponds to a topological invariant implied by the Chern number of the band structure. If TR symmetry is preserved, there are only two possible values for θ in a 3D insulator, namely, $\theta = 0$ and π, the former value corresponding to a topologically trivial insulator [14]. In the case of TSCs a topological magnetoelectric contribution also arises, but now θ corresponds to the phase difference between order parameters of opposite chirality [15]. Recently, axionic superconductivity has been also discussed in the context of doped narrow-gap semiconductors [16].

The 3D TSC constructed by Qi *et al.* [15] should actually correspond to an interacting topological state of matter, going beyond the the symmetry classification of topological noninteracting theories for insulators and superconductors [6]. According to the standard symmetry classification and an argument following from the gravitational anomaly [17], 3D TSCs having TR invariance are classified by an integer topological invariant belonging to the DIII class in the free fermion classification table [6]. Thus, unlike TR-invariant TIs, the electromagnetic axionic response of 3D TSCs would involve an axion field having values $\theta = 0$ or π, mod 2π. However, as pointed out in Ref. [15], the axion in TSCs is a dynamical phase variable associated with the superconducting order parameter, or Higgs field, in the language of quantum field theory. Furthermore, the Meissner effect gives a mass to the photon via the Higgs mechanism. This is an important difference with respect to the axion electrodynamics of TIs, where no $U(1)$ symmetry is broken and the gauge field remains gapless. Furthermore, nontrivial Chern numbers are associated with the different phases. For the simpler case featuring two Weyl fermions with opposite helicity, we have opposite Chern numbers, leading to a topological invariant given by the sign of the gap amplitude [15]. Thus, the arguments in Ref. [15] seems to point to a Z_2 classification. However, since the relevant symmetry in the problem is $U(1) \times Z_2$, it has been shown recently [18,19] that in topological superconductors with TR symmetry the $\mathbb{Z}$ classification is reduced to Z_{16}.

In this paper, we investigate the Higgs mechanism and anomalous Hall effect of three-dimensional TSCs within the model introduced recently in Ref. [15]. In the simplest case, the model features two superconducting order parameters associated with left and right fermion chiralities interacting with the electromagnetic field, and a topological magnetoelectric term. We show that quantum fluctuations in such a superconductor (SC) imply an interacting topologically nontrivial phase that cannot be continuously deformed into the interacting topologically trivial one. Our claim is substantiated by a renormalization group analysis. This result does not follow from the classical Lagrangian of the system derived earlier in Ref. [15] and is a purely quantum effect involving the interaction between photons and Higgs fields. We show that due to this behavior, the type I regime of the TSC features a first-order phase transition in the bulk and a second-order phase transition on the surface. This distinguishes a TSC from a topologically trivial superconductor in the type I regime. The latter would exhibit a first-order phase transition in the bulk as well as on the surface, provided quantum fluctuations are accounted for [20] (this point will be discussed in detail in Sec. III).

Another consequence of the topological magnetoelectric term is the occurrence of an anomalous Hall effect when an electric field is applied parallel to the surface of a TSC. Due to the magnetoelectric effect, a transverse current is generated from a Lorentz-like force involving the relative superfluid velocity $\sim\nabla\theta$ and the applied electric field. The generated transverse current is negative, a situation reminiscent of the anomalous Hall effect in superconductors [21] and observed high-T_c cuprate superconductors [22]. However, in the latter case the anomalous Hall effect occurs due to vortex motion induced by the Faraday law and is typically a very small effect. Furthermore, in this case the Lorentz force acts directly on the vortex core, and therefore on the normal components of the superconductor. For this reason, it automatically leads to dissipation. In the case of three-dimensional TSCs, on the other hand, the anomalous Hall effect occurs even in absence of vortices and is induced solely by an external electric field via the magnetoelectric effect. Thus, a *dissipationless anomalous Hall current is generated on the surface*.

The plan of the paper is as follows. In Sec. II we discuss how the Higgs mechanism works in a topological superconductor. We show that after the phases are integrated out, interactions between the photons automatically arise due to the axion term. This is in contrast to the ordinary Higg mechanism, where the phases can be trivially integrated out by a gauge transformation. In Sec. III we discuss the role of quantum fluctuations and derive the effective potential on the surface and the renormalization group (RG) equations. We show that the RG equations of the topological superconductor cannot be connected to the ones of a topologically trivial superconductor. This is shown to occur as a direct consequence of the axion term. Finally, in Sec. IV we discuss the dissipationless variant of the anomalous Hall effect using the London limit of topological superconductors.

II. HIGGS MECHANISM IN THREE-DIMENSIONAL TOPOLOGICAL SUPERCONDUCTORS

The effective Lagrangian for a three-dimensional TSC featuring two Fermi surfaces is given by [15]

$$\mathcal{L}_{\text{eff}} = \frac{e^2\theta}{32\pi^2}\epsilon^{\mu\nu\sigma\tau}F_{\mu\nu}F_{\sigma\tau} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \sum_{i=L,R}[|(\partial_\mu - qA_\mu)\phi_i|^2 - m^2|\phi_i|^2] - \frac{u}{2}(|\phi_L|^2 + |\phi_R|^2)^2 + 2J(\phi_L^*\phi_R + \phi_R^*\phi_L), \quad (1)$$

where $q = 2e$ is the charge of the condensate. The Greek indices run from 0 to 3 and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, with $(A_\mu) = (A_0,\mathbf{A})$. The Lagrangian (1) corresponds to an Abelian Higgs model with a two-component scalar field (ϕ_L,ϕ_R), where R,L denote right and left chiralities of the two components of the scalar condensate matter field, minimally coupled to the gauge field (A_μ). In contrast to the standard Higgs model, Eq. (1) features a so-called axion term [23,24], which is the first term appearing in the Lagrangian above. The term is topological in nature and contains a scalar field (the axion) $\theta = \theta_L - \theta_R$, where θ_L and θ_R are the phases of ϕ_L and ϕ_R, respectively, i.e., $\phi_i = |\phi_i|\exp(i\theta_i)$. In terms of electric and magnetic field components, the toplogical term reads $e^2\theta\mathbf{E}\cdot\mathbf{B}/(4\pi^2)$, which is precisely the magnetoelectric form mentioned in the introductory paragraphs. A Josephson coupling term $\propto \phi_L^*\phi_R + \phi_R^*\phi_L$ accounts for the interference between the two superconducting order parameter fields. This is a characteristic feature in superconductors with two or more components of the order parameter and is absent only if prohibited by symmetries of the problem [25,26]. Below, we show that the Josephson coupling is generated by fluctuations, and therefore it is legitimate to include such a term from the very beginning in the Lagrangian. Furthermore, the Josephson coupling is important for tuning between topologically trivial and nontrivial phases. In fact, a simple mean-field analysis shows that for $J < 0$ the Josephson coupling implies $\theta = 0$, yielding a topologically trivial superconductor. For $J > 0$, on the other hand, θ is locked to π, thus leading to a topologically nontrivial superconducting ground state. Since θ is periodic, $\theta = \pi$ corresponds to a situation where the time-reversal (TR) symmetry is preserved [15]. Thus, at the mean-field level, $J = 0$ separates a topologically trivial ground state from a nontrivial one. Thus, varying J from positive to negative values induces a topological quantum phase transition.

In the $U(1)$ Higgs mechanism the phases disappear from the spectrum due to spontaneous breaking of the local $U(1)$ symmetry, being transmuted into the longitudinal mode for the photon, which becomes gapped. Thus, only amplitude modes remain in the spectrum of scalar particles. The Higgs mechanism is equivalent to integrating out the phase degrees of freedom, which in the case of the Higgs model automatically leads to a massive gauge particle. This point of view of integrating out the phases is particularly appealing in the case where a Josephson coupling is present. However, additional nonlinearities arise in the presence of the axion term. To see this, let us first consider the Higgs mechanism in Eq. (1) for the case where the axion term is absent. In this case we can simply write $\phi_j = \rho_j e^{i\theta_j}/\sqrt{2}$, $j = L,R$ and make the shift

$$A_\mu \to A_\mu + \frac{1}{q}\left(\frac{\rho_L^2\partial_\mu\theta_L + \rho_R^2\partial_\mu\theta_R}{\rho_L^2+\rho_R^2}\right), \tag{2}$$

which yields

$$\begin{aligned}\mathcal{L}_{\rm eff} = {}& \frac{e^2\theta}{32\pi^2}\epsilon^{\mu\nu\sigma\tau}F_{\mu\nu}F_{\sigma\tau} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{q^2(\rho_L^2+\rho_R^2)}{2}A_\mu^2 \\ &+ \frac{\rho_L^2\rho_R^2}{2(\rho_L^2+\rho_R^2)}(\partial_\mu\theta)^2 + J\rho_L\rho_R\cos\theta \\ &- \frac{m^2}{2}(\rho_L^2+\rho_R^2) - \frac{u}{8}(\rho_L^2+\rho_R^2)^2.\end{aligned} \tag{3}$$

Note that in the absence of the axion term and for the particular situation of a single scalar field component, i.e., $\rho_L = \rho$ and $\rho_R = 0$, the above effective Lagrangian trivially reduces to the usual Lagrangian for the Higgs model in the unitary gauge. In this particular case the Lagrangian is independent of the phase. For the case relevant to us here, where two scalar fields are present, there is a term $\sim(\partial_\mu\theta)^2$ remaining. Thus, in the absence of Josephson coupling there is still a massless (Goldstone) mode present in the spectrum. This occurs because there are two Higgs fields and one Abelian gauge field. Thus, it is only possible to gauge away one phase degree of freedom. The phase θ would not couple directly to the gauge field in the absence of the axion term. Due to the axion term, integrating out the phases generates direct interactions between photons, even if the amplitudes are assumed to be uniform. For the topological phase of the system occurring for $J > 0$, we have to integrate out the lowest order Gaussian phase fluctuations around $\theta = \pi$. In general, this renders the induced photon-photon interaction nonlocal. If the amplitudes are uniform, we obtain the effective Lagrangian,

$$\begin{aligned}\mathcal{L}_{\rm Higgs}|_{J>0} = {}& -\frac{1}{4}F^2 + \frac{q^2(\rho_L^2+\rho_R^2)}{2}A^2 \\ &+ \frac{e^2}{32\pi}\epsilon^{\mu\nu\sigma\tau}F_{\mu\nu}F_{\sigma\tau} - \frac{1}{2}\left(\frac{e^2}{16\pi^2}\right)^2\left(\frac{\rho_L^2+\rho_R^2}{\rho_L^2\rho_R^2}\right) \\ &\times \int d^4x' V(x-x')\epsilon_{\mu\nu\lambda\rho}\epsilon^{\alpha\beta\gamma\delta}F^{\mu\nu}(x) \\ &\times F^{\lambda\rho}(x)F_{\alpha\beta}(x')F_{\gamma\delta}(x') - J\rho_L\rho_R \\ &- \frac{m^2}{2}(\rho_L^2+\rho_R^2) - \frac{u}{8}(\rho_L^2+\rho_R^2)^2,\end{aligned} \tag{4}$$

where

$$V(x) = \int \frac{d^4p}{(2\pi)^4}\frac{e^{ip\cdot x}}{p^2+m_\theta^2}, \tag{5}$$

with $m_\theta^2 = J(\rho_L/\rho_R + \rho_R/\rho_L)$. In the long-wavelength regime the induced photon interaction is strongly screened by the axion. Hence, we have $V(x-x') \approx m_\theta^{-2}\delta^4(x-x')$. The resulting photon-photon interaction simplifies and we obtain [27]

$$\begin{aligned}\mathcal{L}_{\rm Higgs}|_{J>0} = {}& -\frac{1}{4}F^2 + \frac{q^2(\rho_L^2+\rho_R^2)}{2}A^2 + \frac{e^2}{32\pi}\epsilon^{\mu\nu\sigma\tau}F_{\mu\nu}F_{\sigma\tau} \\ &+ \frac{1}{2J\rho_L\rho_R}\left(\frac{e^2}{16\pi^2}\right)^2\det(F_{\mu\nu}) - J\rho_L\rho_R \\ &- \frac{m^2}{2}(\rho_L^2+\rho_R^2) - \frac{u}{8}(\rho_L^2+\rho_R^2)^2,\end{aligned} \tag{6}$$

where $\det(F_{\mu\nu}) = (\mathbf{E}\cdot\mathbf{B})^2$.

III. QUANTUM FLUCTUATIONS

A. Important vanishing of a Feynman diagram

We now turn to a crucial aspect of the topological phase with respect to the surface states. It turns out that in the presence of quantum fluctuations, the topological surface states cannot be continuously deformed into topologically trivial ones in the long wavelength limit when crossing the critical point. To see this, let us assume that θ is uniform on the surfaces, with $\theta = \pi$ for the TR invariant case. Note that θ_L and θ_R are still allowed to fluctuate, with $\theta_L = \theta/2 + \delta\theta_L$ and $\theta_R = -\theta/2 + \delta\theta_R$. Since $\epsilon^{\mu\nu\sigma\tau}F_{\mu\nu}F_{\sigma\tau} = 2\partial^\mu(\epsilon_{\mu\nu\lambda\rho}A^\nu F^{\lambda\rho})$, each surface contains a Chern-Simons (CS) term [28]. Thus, assuming two surfaces perpendicular to the z axis, we find that the imaginary time photon propagator at any surface is given by

$$\Delta_{\mu\nu}^{\pm}(p) = \frac{p^2+m_A^2}{\Delta(p^2)}\left[\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \pm \frac{M_\theta}{p^2+m_A^2}\epsilon_{\mu\nu\lambda}p^\lambda\right], \tag{7}$$

where $m_A^2 = q^2(\rho_L^2+\rho_R^2)$, $M_\theta = e^2\theta/(8\pi^2)$, $\Delta(p^2) = (p^2+m_A^2)^2 + M_\theta^2 p^2$, and the $\pm$ sign is chosen depending on which surface one is referring to. In Eq. (7) the transverse

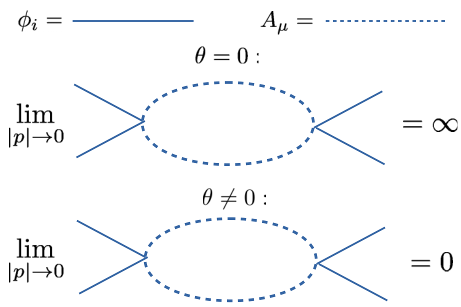

FIG. 1. (Color online) Difference in behavior for surface photon-mediated Higgs scattering at the critical point in the long-wavelength limit. For the topologically trivial case ($\theta = 0$) the corresponding Feynman diagram diverges. On the other hand, for the topologically nontrivial case ($\theta \neq 0$), the same diagram vanishes. This shows beyond the noninteracting regime that a topological superconductor cannot be continuously deformed into a topologically nontrivial one in the long wavelength limit.

gauge has been fixed. At the phase transition to the normal state where $m_A^2 \to 0$, the Feynman diagram shown in Fig. 1, which is associated with Higgs scattering mediated by photons, behaves very differently at long wavelengths ($|p| \to 0$), depending on whether $\theta = 0$ (topologically trivial) or $\theta \neq 0$ (topologically nontrivial). Namely, for all $\theta \neq 0$ we have

$$\lim_{|p|\to 0} \lim_{m_A^2 \to 0} \int \frac{d^3q}{(2\pi)^3} \Delta^{\pm}_{\mu\nu}(p+q)\Delta^{\pm}_{\nu\mu}(q) = 0, \tag{8}$$

whereas the result is divergent for $\theta = 0$. Thus, after other one-loop scattering amplitudes are taken into account to obtain the full four-Higgs vertex, we see that at the critical point the topological field theory cannot be continuously deformed into a topologically trivial one. This statement holds trivially for topological Bogoliubov–de Gennes superconductors. Here, we have shown that it also holds in the presence of quantum fluctuations in an interacting system, beyond the Bogoliubov–de Gennes picture. This occurs because the photon is topologically gapped, despite the vanishing of the Meissner gap ($m_A = 0$). Note that this result is due to the topological character of the axion term and not due to a symmetry protection. Indeed, since the diagram of Fig. 1 vanishes for any $\theta \neq 0$, TR invariance is not required. Thus, in this context the topologically nontrivial phase simply corresponds to the case where the axion term is nonzero.

A continuous deformation to the topologically trivial phase can be done in the Higgs phase, where there are no gapless modes. At the critical point such a continuous deformation is not possible. Thus, quantum critical fluctuations in this system will govern topologically stable universal behavior in physical quantities. This has important implications for critical exponents and amplitude ratios. Indeed, as we will see, the vanishing of the diagram shown in Fig. 1 changes significantly the renormalization group (RG) β function associated to the interaction vertex between scalar fields.

B. One-loop effective action on the surface

As is well known, the one-loop effective action is more easily obtained by integrating out the quadratic fluctuations of the scalar fields and gauge fields [29]. We assume that the magnitudes of both ϕ_L and ϕ_R have the same expectation value in the broken symmetry phase. Thus, we write $\phi_L = e^{i\theta/2}\varphi + \tilde{\phi}_L$ and $\phi_R = e^{-i\theta/2}\varphi + \tilde{\phi}_R$, where φ is uniform and $\tilde{\phi}_i$ represents the fluctuation around $\langle\phi_i\rangle$. The effective action is therefore written in the form

$$S_{\rm eff}^{1-{\rm loop}} = S_{\rm eff}^0 + \frac{1}{2}\int d^3x \int d^3x' [\Phi^\dagger(x) M(x-x';\varphi)\Phi(x') + A_\mu(x) M_{\mu\nu}(x-x';\varphi)A_\nu(x')], \tag{9}$$

where

$$S_{\rm eff}^0 = 2V[(m^2 - 2J\cos\theta)|\varphi|^2 + u|\varphi|^4], \tag{10}$$

with V being the (infinite) volume of three-dimensional spacetime, and $\Phi^\dagger = [\tilde{\phi}_L^* \; \tilde{\phi}_R^* \; \tilde{\phi}_L \; \tilde{\phi}_R]$. The matrices $M(x-x';\varphi)$ and $M_{\mu\nu}(x-x';\varphi)$ are given in momentum space by

$$M(p,\varphi) = \begin{bmatrix} (p^2 + m^2 + 3u|\varphi|^2)I + (u|\varphi|^2 e^{i\sigma_z\theta} - 2J)\sigma_x & u|\varphi|^2(e^{i\theta\sigma_z} + \sigma_x) \\ u|\varphi|^2(e^{-i\theta\sigma_z} + \sigma_x) & (p^2 + m^2 + 3u|\varphi|^2)I + (u|\varphi|^2 e^{-i\sigma_z\theta} - 2J)\sigma_x \end{bmatrix}, \tag{11}$$

where I is a 2×2 identity matrix, while σ_x and σ_z are Pauli matrices, and

$$M_{\mu\nu}(p,\varphi) = (p^2 + 4q^2|\varphi|^2)\delta_{\mu\nu} - p_\mu p_\nu - M_\theta \epsilon_{\mu\nu\lambda} p_\lambda. \tag{12}$$

From Eq. (11), we see that a correction to the Josephson coupling has been generated by fluctuations. It has the form, $u|\varphi|^2(e^{i\theta}\tilde{\phi}_L^*\tilde{\phi}_R + e^{-i\theta}\tilde{\phi}_R^*\tilde{\phi}_L)$. This term is generated even if $J = 0$, which means that including a Josephson coupling in the Lagrangian is a physically reasonable assumption. This should be expected, since fluctuations will necessarily lead to an overlap between the two complex field components. Note that this result is valid in general for any two-component superconductor and is not restricted to the topological one being considered here.

Integrating out the fluctuations in Eq. (9), we obtain

$$e^{-VU(|\varphi|,\theta)} = \int \mathcal{D}A_\mu \mathcal{D}\Phi^\dagger \mathcal{D}\Phi e^{-S_{\rm eff}^{1-{\rm loop}}(A_\mu,\Phi^\dagger,\Phi)}, \tag{13}$$

where V is the three-dimensional spacetime volume and $U(|\varphi|,\theta)$ is the effective potential given by

$$U(|\varphi|,\theta) = \frac{1}{2V}[{\rm Tr}\ln M_{\mu\nu} + {\rm Tr}\ln M]. \tag{14}$$

The trace logs can be written in more explicit form:

$$U(|\varphi|,\theta) = 2[(m^2 - 2J\cos\theta)|\varphi|^2 + u|\varphi|^4] + \frac{1}{2}\sum_{\sigma=\pm}\int \frac{d^3p}{(2\pi)^3}\{\ln[p^2 + M_\sigma^2(|\varphi|,\theta)] + \ln[p^2 + M_{1\sigma}^2(|\varphi|,\theta)] + \ln[p^2 + M_{2\sigma}^2(|\varphi|,\theta)]\}, \quad (15)$$

where

$$M_\pm^2(|\varphi|,\theta) = 2q^2|\varphi|^2 + \frac{M_\theta^2}{2} \pm \frac{|M_\theta|}{2}\sqrt{8q^2|\varphi|^2 + M_\theta^2}, \quad (16)$$

$$M_{1\pm}^2(|\varphi|,\theta) = m^2 + 2u|\varphi|^2 \pm 2J, \quad (17)$$

$$M_{2\pm}^2(|\varphi|,\theta) = m^2 + 4u|\varphi|^2 \pm 2\sqrt{J^2 + u^2|\varphi|^4 - 2Ju\cos\theta|\varphi|^2}. \quad (18)$$

From this mass spectrum underlying the effective potential, we recognize $p^2 + M_{1\pm}^2$ and $p^2 + M_{2-}^2$ as the would-be Goldstone modes from the regime $J = 0$, corresponding to the absence of the Josephson coupling. Note that the corresponding uncharged system in absence of Josephson coupling features three Goldstone modes. This is to be expected, since the system in this case would be $O(4)$ invariant. As usual, in the charged system there are only gapped modes, even in absence of Josephson coupling, as required by the Higgs mechanism.

The momentum integral in Eq. (15) can be easily evaluated using an ultraviolet cutoff Λ,

$$U(|\varphi|,\theta) = U_0 + 2[(m^2 - 2J\cos\theta)|\varphi|^2 + u|\varphi|^4] - \frac{1}{12\pi}\sum_{\sigma=\pm}[|M_\sigma(|\varphi|,\theta)|^3 + |M_{1\sigma}(|\varphi|,\theta)|^3 + |M_{2\sigma}(|\varphi|,\theta)|^3], \quad (19)$$

where U_0 is a field-independent (vacuum) contribution. We have absorbed a term $\Lambda(3u + q^2)/(2\pi^2)$ in m^2, since this contribution just corresponds to tadpole diagrams. Note that this simple renormalization does not actually affect the other terms, since the error committed by doing this is of higher order than one loop. From the effective potential above we see that the diagram of Fig. 1 can only arise from the sum $\sum_{\sigma=\pm}|M_\sigma(|\varphi|,\theta)|^3$, with $M_\sigma^2(|\varphi|,\theta)$ given in Eq. (16). An expansion in powers of $|\varphi|$ for $\theta \neq 0$ clearly shows that the diagram shown in the figure indeed vanishes when $\theta \neq 0$. However, the contributions $q^{2n}|\varphi|^{2n}$ for $n \geqslant 4$ become singular *for all* θ. We emphasize that this is only true when the topological magnetoelectric term is nonzero. On the other hand, if $\theta = 0$, we obtain

$$\sum_{\sigma=\pm}|M_\sigma(|\varphi|,\theta = 0)|^3 = 8q^3|\varphi|^3, \quad (20)$$

meaning that the effective potential is nonanalytic as a function of $q^2|\varphi|^2$. Albeit simple, this is actually a nonperturbative result, since it cannot be obtained as a power series in q^2. The absence of a power series involving terms $q^{2n}|\varphi|^{2n}$ reflects the divergence of the diagram shown in Fig. 1 when $\theta = 0$ for vanishing external momenta. In this case a meaningful perturbative evaluation of the vertex function can only be done for nonzero external momenta, a fact related to the infrared divergence arising from a massless photon. Note that in contrast to the $\theta \neq 0$ case, the expansion is singular in the infrared for all $n \geqslant 2$, rather than for $n \geqslant 4$. However, the singularities in the case of a topologically massive photon is not a problem, since they correspond to interactions that are irrelevant in an RG sense. For instance, this type of infrared singularity in higher order vertices would also occur in a simple φ^4 Landau theory. Having $\theta \neq 0$ turns the photon topologically massive without spoiling gauge invariance [28]. Thus, we can interpret the cubic contribution arising in the limit of vanishing θ as a consequence of resumming up all the one-loop infrared divergent diagrams containing only internal photon lines. This leads to a nonanalytic contribution to the effective potential. As is well known, summing up these contributions in $d = 3 + 1$ in the case of standard scalar electrodynamics leads to a logarithmic term $\sim q^4|\varphi|^4 \ln|\varphi|^2$ in the effective potential [30], while the cubic term has been obtained in the context of Ginzburg-Landau superconductors for $d = 3 + 0$ (i.e., scalar electrodynamics in $d = 2 + 1$ and imaginary time) [20]. In both cases these one-loop photon contributions lead to a fluctuation-induced first-order phase transition. However, it was later shown that this result is valid only in the type I regime, while in the type II regime a second-order phase transition in the so-called inverted 3D XY universality class arises [31].

In view of the above discussion, it is of interest to investigate the character of the superconducting phase transition on the surface of a topological superconductor. The theory features two ingredients that are not present in the previous analysis of fluctuation-induced first-order phase transitions by Halperin *et al.* [20]. These are the Josephson coupling between the scalar field components and the CS term. The case with Josephson coupling in the absence of the CS term has been examined in the London limit (i.e., in the strong type II regime) in Ref. [26]. For this case it has been shown by means of exact duality arguments that a two-component superconductor with Josephson coupling exhibits a phase transition in the 3D XY universality class [26]. On the other hand, when the Josephson coupling is absent, it has been shown using renormalization group (RG) methods in Ref. [32] that for a large enough CS coupling (i.e., M_θ in our notation), the first-order phase transition is turned into a second-order one. Regarding the result in the London limit without CS term, we note that our analysis in this section is being done in the type I regime, since amplitude fluctuations play an important role in the calculations above. Indeed, the theory with $\theta = 0$ yields a fluctuation induced first-order phase transition.

C. Renormalization group analysis

It is a well-known fact that the CS term does not renormalize [33]. This is a consequence of the topological nature of the CS term. Indeed, since it is independent of the metric, it does not change under scale transformations. Thus, using this result and the invariance of the effective action under renormalization, we obtain

$$M_\theta\epsilon_{\mu\nu\lambda}A^\mu\partial^\nu A^\lambda = M_{\theta,r}\epsilon_{\mu\nu\lambda}A_r^\mu\partial^\nu A_r^\lambda, \quad (21)$$

where $A_r^\mu = Z_A^{-1/2} A^\mu$ is the renormalized gauge field with the corresponding wave function renormalization, Z_A, and $M_{\theta,r}$ is the renormalized topological mass. From the above equation it follows that $M_{\theta,r} = Z_A M_\theta$. Since gauge invariance implies the renormalization $q_r^2 = Z_A q^2$ [20,30], which is easily obtained from the vacuum polarization, it follows that θ is a renormalization group invariant,

$$\frac{d\theta}{dl} = 0, \tag{22}$$

where $l = \ln(m_r/\Lambda)$ is a logarithmic renormalization scale defined in terms of the renormalized mass, m_r, yielding the inverse correlation length. Equation (22) is an important result, since it allows one to study the critical topological behavior with a vanishing renormalized Josephson coupling J_r, while still having $\theta \neq 0$, corresponding to the critical point of the topological phase transition.

Combining both CS terms stemming from the two surfaces, we obtain that the axion term in the bulk does not get renormalized either, so the result (22) also holds in the bulk. An interesting consequence of this analysis is that the RG flow of the bulk theory does not differ significantly from the one where the axion term is absent, which is just given by the well-known analysis of Coleman and Weinberg [30]. In this case, the phase transition is known to be of first order, irrespective of the superconducting regime being of type I or type II. However, this is not the case for the phase transition on the surfaces. Indeed, the vanishing of Feynman graph of Fig. 1 when $\theta \neq 0$ implies that a contribution $\sim\hat{q}^4$ is absent in the RG β function of $\hat{u}$. Here are $\hat{q} = q_r/m_r$ and $\hat{u} = u_r/m_r$ renormalized dimensionless couplings defined on the surface of a TSC. The one-loop RG functions for a superconductor in 2+1 dimensions with a CS term, and N complex order parameter fields, are obtained in a way similar to that in Ref. [20], except that we use the propagator (7) with $m_A = 0$ in the Feynman diagrams involving photon lines. The result is

$$\frac{d\hat{q}^2}{dl} = \left(\frac{N\hat{q}^2}{24\pi} - 1\right)\hat{q}^2, \tag{23}$$

$$\frac{d\hat{u}}{dl} = -\left[1 + \frac{4}{3\pi}\frac{\hat{q}^2}{\left(1+\frac{\hat{q}^2|\theta|}{8\pi^2}\right)^2}\right]\hat{u} + \frac{(N+4)}{8\pi}\hat{u}^2, \tag{24}$$

where we now have

$$\theta = \sum_{j=1}^{N} C_{1j}\theta_j, \tag{25}$$

where C_{1j} are the Chern numbers associated to the helicity of the N Fermi surfaces involved [15]. Note that we have not expanded $1/(1+\hat{q}^2|\theta|/8\pi^2)$ in powers of $\hat{q}$, because θ can also be very large, so that the product $\hat{q}^2|\theta|$ is not necessarily small.

It is easily seen that the above RG equations have an infrared stable fixed point for all N, in contrast to the analysis by Halperin *et al.* for the nontopological superconductor, where infrared fixed points are only found for $N > 183$ [34]. However, there is a stability condition involving θ that has to be fulfilled in the case of a TSC. It is obtained by considering the critical correlation function of the superconducting order field components,

$$\langle \phi_i(x)\phi_j^*(0)\rangle \sim \frac{\delta_{ij}}{|x|^{1+\eta}}, \tag{26}$$

where at one loop

$$\eta = -\frac{16}{N\left(1+\frac{3|\theta|}{\pi N}\right)^2}, \tag{27}$$

which implies the inequality $\eta > -1$. The latter inequality is fulfilled provided

$$|\theta| > \frac{\pi}{3}(4\sqrt{N} - N), \tag{28}$$

and we see that for $N \geqslant 16$ a quantum critical point is obtained for all values θ, showing that at least sixteen Weyl fermions are necessary to have a quantum critical point. Values of θ violating the inequality $\eta > -1$ correspond to a situation where a continuum limit cannot be defined and is therefore unphysical. Thus, in order to have a physically meaningful phase transition, the inequality (28) has to be satisfied. We note that the lower bound for $|\theta|$ is larger than π when $N = 2$. Therefore, the TR symmetric value $\theta = \pi$ obtained at the mean-field level does not produce a second-order phase transition when quantum fluctuations are accounted for. The one-loop RG predicts that a second-order phase transition occurs for $\theta = \pi$ only if $N \geqslant 10$.

It is tempting to relate the critical value of N to the Z_{16} classification [19]. However, at this stage it would be too speculative, since higher order results may affect the values of N for which critical points obey the inequality $\eta > -1$.

As a final remark on the RG analysis, let us comment on another possible renormalization scheme allowing us to continuously connect the cases $\theta = 0$ and $\theta \neq 0$. The infrared divergences stemming from the photon propagator for $\theta = 0$ requires defining the renormalized four-point vertex at external nonzero momenta. This yields a renormalization scale μ that replaces m_r in the RG flow. Since the diagram of Fig. 1 does not vanish for nonzero external momenta, it turns out that a $\hat{q}^4$ term would be generated in the RG β function of $\hat{u}$, even when $\theta \neq 0$. This $\hat{q}^4$ term would have a θ-dependent coefficient allowing to smoothly connect the result to the known RG equations of a topologically trivial superconductor in the limit $\theta \to 0$. We would find once more that a second-order phase transition occurs for large enough θ and a stability criterion would follow from the inequality $\eta > -1$ [32]. The latter inequality would yield in this case values of θ leading to a *negative* coefficient of the $\hat{q}^4$ term, with values of θ yielding a positive coefficient of $\hat{q}^4$ violating the condition $\eta > -1$. This implies that the RG β function of $\hat{u}$ can actually not be continuously connected to the $\theta = 0$ regime, since to this end it would be necessary to enter a regime where the continuum limit is not even defined.

D. Summary of the phase structure

There are two important consequences of quantum fluctuations as unveiled by the analysis in this section. First, we note that it is not possible to reach the topologically trivial phase from the topologically nontrivial one within the RG. Simply taking the limit $\theta \to 0$ does not recover the RG flow of topologically trivial superconductors, while this limit can be realized classically. Second, and most importantly, the phase transition in the bulk is always a first-order one, while a second-order phase transition is possible on the surface. This is not the case for the topologically trivial superconductor, where a second-order phase transition on the surface is only obtained for sufficiently large N.

IV. VORTEX-FREE ANOMALOUS HALL EFFECT

We next turn to the Meissner effect aspects of a TSC, which as we now show implies an anomalous Hall effect even in the absence of vortices. This is more conveniently done by rewriting the Lagrangian in a London limit exhibiting explicitly electric and magnetic fields, i.e.,

$$\mathcal{L}_{\rm eff} = \frac{1}{2}(\mathbf{E}^2 - \mathbf{B}^2) + \frac{e^2\theta}{4\pi^2}\mathbf{E}\cdot\mathbf{B} + \frac{1}{2}\sum_{i=L,R}\rho_i^2(\partial_\mu\theta_i - qA_\mu)^2 + J\rho_L\rho_R\cos\theta - \frac{m^2}{2}(\rho_L^2+\rho_R^2) - \frac{u}{8}(\rho_L^2+\rho_R^2)^2. \quad (29)$$

From the effective Lagrangian we obtain that the electric displacement and magnetic fields are given respectively by $\mathbf{D} = \mathbf{E} + e^2\theta\mathbf{B}/\pi$ and $\mathbf{H} = \mathbf{B} - e^2\theta\mathbf{E}/\pi$, while the superconducting current is given by

$$\mathbf{j} = q\left[\rho_L^2(\nabla\theta_L - q\mathbf{A}) + \rho_R^2(\nabla\theta_R - q\mathbf{A})\right]. \quad (30)$$

From Eq. (30) we obtain the usual London equation in absence of vortices, $\nabla\times\mathbf{j} = -(1/\lambda^2)\mathbf{B}$, where $\lambda^2 = 1/m_A^2$ is the square of the penetration depth. Thus, the Maxwell equation in the presence of the axion field,

$$\nabla\times\mathbf{B} = \mathbf{j} + \partial_t\mathbf{E} + \frac{e^2}{\pi}(\nabla\theta\times\mathbf{E} + \partial_t\theta\mathbf{B}), \quad (31)$$

yields the equation determining the London electrodynamics of the TSC in the form

$$\partial_t^2\mathbf{B} - \nabla^2\mathbf{B} + m_A^2\mathbf{B} = \frac{e^2}{\pi}[\nabla\times(\nabla\theta\times\mathbf{E}) + \nabla\times(\partial_t\theta\mathbf{B})]. \quad (32)$$

For the axion field we obtain the equation of motion,

$$\partial_t^2\theta - \nabla^2\theta + m_\theta^2\sin\theta = \frac{e^2}{4\pi^2}\left(\frac{1}{\rho_L^2}+\frac{1}{\rho_R^2}\right)\mathbf{E}\cdot\mathbf{B}. \quad (33)$$

In the low-frequency regime and in the absence of vortices, the London equation (32) simplifies to

$$-\nabla^2\mathbf{B} + m_A^2\mathbf{B} = \frac{e^2}{\pi}[\nabla\times(\nabla\theta\times\mathbf{E})], \quad (34)$$

while the current satisfies

$$-\nabla^2\mathbf{j} + m_A^2\mathbf{j} = -\frac{e^2m_A^2}{\pi}(\nabla\theta\times\mathbf{E}). \quad (35)$$

The London equation for the electric field is unaffected by the axion term, retaining its traditional form, $-\nabla^2\mathbf{E} + m_A^2\mathbf{E} = 0$. This result is closely related to the fact that the electromagnetic energy density does not contain a magnetoelectric term.

We now consider a solution with a simple geometry, namely, a semi-infinite TSC ($z \geqslant 0$) with a surface at $z = 0$ at an external electric field $\mathbf{E}_0 = E_0\hat{\mathbf{x}}$ parallel to the surface. We obtain

$$-\frac{d^2\theta}{dz^2} + m_\theta^2\sin\theta = \frac{e^2}{4\pi^2}\left(\frac{1}{\rho_L^2}+\frac{1}{\rho_R^2}\right)E_x(z)B_x(z), \quad (36)$$

where $E_x(z) = E_0e^{-m_Az}$, and

$$-\frac{d^2B_x}{dz^2} + m_A^2B_x = -\frac{e^2}{\pi}\frac{d}{dz}\left[E_x(z)\frac{d\theta}{dz}\right], \quad (37)$$

$$-\frac{d^2j_y}{dz^2} + m_A^2j_y = -\frac{e^2m_A^2}{\pi}E_x(z)\frac{d\theta}{dz}. \quad (38)$$

The solution for Eq. (38) in terms of the axion is

$$j_y(z) = \frac{e^2m_A^2E_0}{\pi}\left[\frac{\theta(0)}{2m_A}e^{-m_Az} - e^{m_Az}\int_z^\infty dz'e^{-2m_Az'}\theta(z')\right], \quad (39)$$

where $\theta(0) = \pi$. Since $\mathbf{E}\cdot(\nabla\times\mathbf{j}) = -\lambda^{-2}\mathbf{E}\cdot\mathbf{B}$, we obtain the following relation:

$$\frac{1}{\lambda^2}B_x(z) = m_Aj_y(z) + \frac{e^2m_A^2E_0}{\pi}e^{-m_Az}[\theta(z)-\theta(0)], \quad (40)$$

which implies $m_AB_x(0) = j_y(0)$. Thus, we find that the usual boundary condition of the London theory, $dj_y/dz|_{z=0} = m_Aj_y(0)$, is obviously fulfilled by the solution (39) in the presence of the axion field. However, the Maxwell equation (31) in the static regime implies a boundary condition that deviates from the standard one in the London theory of nontopological superconductors,

$$\left.\frac{dB_x}{dz}\right|_{z=0} = m_AB_x(0) + \frac{e^2E_0}{\pi}\left.\frac{d\theta}{dz}\right|_{z=0}. \quad (41)$$

From Eqs. (39) and (40) we see that an approximate solution can be obtained by considering terms proportional to e^4 as being of higher order, which amounts to approximating Eq. (36) as being homogeneous. In this case we can use the domain wall solution $\theta(z) = \pi + 2\arcsin[\tanh(m_\theta z)]$ in Eq. (39), which yields $j_y(z)$ explicitly. The explicit solution for $j_y(z)$ with $m_\theta \neq m_A$ in terms of hypergeometric and Lerch transcedents is not very illuminating. Instead, we plot it in Fig. 2 for four different values of the ratio m_θ/m_A. It has a negative sign, just like in the case of the anomalous Hall effect in high-T_c superconductors [22]. As emphasized in the introductory paragraphs, the anomalous Hall effect in nontopological superconductors has a quite different origin from the one discussed here. In three-dimensional TSCs the anomalous Hall current arises independently of vortex motion and is associated with a dissipationless current.

For $m_\theta = m_A$ (blue [gray] curve in Fig. 2), the expression for $j_y(z)$ does not involve special functions, reading

$$j_y(z) = \frac{e^2m_AE_0}{\pi}\{(\pi/2)e^{-m_Az} - 2 - 2[e^{-m_Az}\arctan(e^{m_Az}) - e^{m_Az}\arctan(e^{-m_Az})]\}. \quad (42)$$

In order to make connection with the quantization of Hall conductivity in the normal state, it is instructive to integrate the

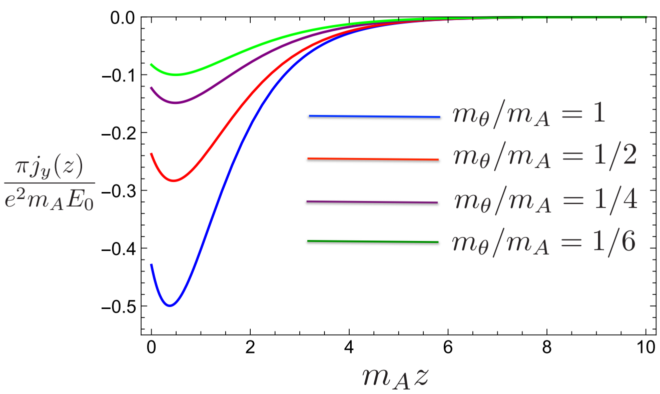

FIG. 2. (Color online) Induced anomalous Hall current $j_y(z)$ for $m_\theta/m_A = 1, 1/2, 1/4, 1/6$.

current density over $z \in [0,\infty)$ to obtain the surface current density,

$$j_y^{\rm surf} = \int_0^\infty dz j_y(z) = \frac{e^2}{2\pi}E_0\theta(0) - \frac{e^2 E_0 m_A}{\pi}\int_0^\infty dz(e^{-m_A z} - e^{-2m_A z})\theta(z). \tag{43}$$

We obtain

$$j_y^{\rm surf} = \frac{e^2 E_0}{2\pi}\left[2\psi\left(\frac{m_\theta + m_A}{4m_\theta}\right) - \psi\left(\frac{m_\theta + 2m_A}{4m_\theta}\right) + \psi\left(\frac{3m_\theta + 2m_A}{4m_\theta}\right) - 2\psi\left(\frac{3m_\theta + m_A}{4m_\theta}\right)\right], \tag{44}$$

where $\psi(z) = \Gamma'(z)/\Gamma(z)$ is the $\digamma$ function. The normal state corresponds to $m_A/m_\theta \to 0$, which yields $j_y^{\rm surf} = -(e^2/2)E_0$, i.e., the half-quantum of the quantized Hall conductivity.

V. CONCLUSION

In conclusion, we have shown that due to quantum electromagnetic fluctuations the Higgs mechanism in three-dimensional TSCs implies a robust topological state of matter, since its RG flow cannot be continuously deformed into the RG flow of a topologically trivial one. This is an example of a topological state that is protected due to the coupling of phase and electromagnetic fluctuations via the axion term, with TR symmetry not being required. In fact, TR can be spontaneously broken by quantum fluctuations. In this context, we have also shown that a second-order quantum phase transition happens on the surface of a TSC, while its bulk undergoes a first-order phase transition. Without the axion term a first-order phase transition would happen both in the bulk and on the surface, provided the superconductor is in the type I regime.

Another aspect of the Higgs mechanism we have studied is the influence of the axion term in the Meissner effect. We have found that the gradient of the axion field on the surface induces a transverse supercurrent. In the low-frequency limit this implies a London regime leading to the generation of an anomalous Hall current with a negative sign. This anomalous Hall current is dissipationless and is the consequence of a Lorentz-like force involving the relative superfluid velocity, which is simply given by the gradient of the phase difference between the chiral superconducting components.

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$$-\epsilon^{\mu\nu\lambda\rho}\epsilon_{\alpha\beta\gamma\delta} = \begin{vmatrix} \delta^\mu_\alpha & \delta^\mu_\beta & \delta^\mu_\gamma & \delta^\mu_\delta \\ \delta^\nu_\alpha & \delta^\nu_\beta & \delta^\nu_\gamma & \delta^\nu_\delta \\ \delta^\lambda_\alpha & \delta^\lambda_\beta & \delta^\lambda_\gamma & \delta^\lambda_\delta \\ \delta^\rho_\alpha & \delta^\rho_\beta & \delta^\rho_\gamma & \delta^\rho_\delta \end{vmatrix}.$$

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