Enhancement of spin accumulation in ballistic transport regime

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The conventional spin-diffusion equation, based on the presence of spin-split local chemical potentials, has successfully described spin accumulation attendant to diffusive transport in spintronics. A recent experiment shows that spin accumulation far exceeds the limit set by such spin-diffusive theory when the mean free path is longer than the spin dephasing length. By introducing the momentum and spin dependent chemical potential, we develop a generalized spin transport equation that is capable of addressing spin transport in systems where ballistic processes are embedded in the overall diffusive conductor. We find that the ballistic spin injection through a barrier into a diffusive nonmagnetic layer with strong spin-orbit coupling can enhance spin accumulation by an order of magnitude when compared to the conventional theory.

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I. INTRODUCTION

Spin accumulation (SA), a nonequilibrium spin density created by external sources such as spin injection across a tunnel barrier, and spin current (SC), the difference between the electric currents carried by spin-up and -down electrons, play central roles in spintronics. At present, the macroscopic description of SA and SC relies on the spin-diffusion equation in which the spatial and temporal dependence of SA, $\mathbf{m}(\mathbf{r},t)$, satisfies

$$\frac{\partial \mathbf{m}}{\partial t} = D\nabla^2\mathbf{m} - \frac{\mathbf{m}}{\tau_{sf}} \tag{1}$$

and the diffusion SC is given by the spin dependent Ohm's law $\mathbf{j}_s = -D\nabla\mathbf{m}$ where D is the diffusion constant, τ_{sf} is the spin-relaxation time, and $\lambda_{sd} \equiv \sqrt{D\tau_{sf}}$ is the spin-diffusion length (SDL). It is understood that the above equation can be used to determine the local SA and SC at length scales larger than the mean free path (MFP) [1,2]. While Eq. (1) has been successfully applied to explain and predict spin transport phenomena in almost all spintronic devices [3], a recent experimental result has challenged the validity of this theory: the SA could be much larger than that predicted by Eq. (1) when the distance between the spin injector and detector in a nonlocal spin-valve geometry is less than the mean free path [4,5]. This finding calls for a theory beyond the conventional spin-diffusion equation. Several earlier attempts [6–9] by incorporating quantum and ballistic effects have not been able to predict an enhanced spin accumulation in the ballistic regime.

Recall that the above spin-diffusive equation was established based on the assumption that the SDL is much larger than the MFP; it would inevitably fail in the opposite limit: the spin dependent local chemical potential (LCP) without specifying the direction of electron momentum becomes meaningless. In ballistic transport in which the relevant spatial length is shorter than the mean free path, the LCP is ill-defined since the "chemical potential" (CP) of electrons at a given spatial point depends on the direction of electron momentum. If the entire system is ballistic, the standard mesoscopic transport assumption would be that the left-going (right-going) electron has a CP of the right (left) reservoir [10]. To address the SA within the length scale of the mean free path, the ballistic nature of the transport must be included. One attempt would be completely giving up the concept of chemical potentials and instead directly solving the distribution function from the generalized semiclassical integrodifferential Boltzmann equation. Such an approach is numerically complicated in general, and the obtained numerical results may not give rise to significant physical insight. Alternatively, we derive a set of useful macroscopic spin transport equations, similar to Eq. (1), but take into account ballistic processes embedded in a diffusive conductor. The key component is to introduce the *spin and directional dependence of the LCP*; namely, the left-going and right-going electrons have different CPs, in addition to the spin dependent CP. We find that macroscopic equations of these CPs can be established by approximately solving the spinor Boltzmann equation in the presence of spin-orbit coupling (SOC), e.g., of the Dresselhaus form. The spin ballistic-diffusion equations are solved for the spin injection from a magnetic tunnel junction to a nonmagnetic (NM) layer. We show that the SA in the NM layer can largely exceed the classical limit set by the conventional spin-diffusion theory when the mean free path is longer than the spin-orbit-coupling induced spin dephasing length. Our calculated results successfully explain recent experimental observations [4].

The paper is organized as follows. In Sec. II, we start from the Boltzmann equation and derive the spin transport equations within a two-dimensional electron gas (2DEG) in the presence of Dresselhaus SOC. In Sec. III, by utilizing our spin transport equations together with appropriate boundary conditions, we study the spin injection into a 2DEG across a tunnel barrier. Compared to the spin injection into a diffusive material, we find that there is an enhancement of the spin accumulation when the mean free path is larger than the spin relaxation length within the 2DEG. Section IV comments on the results and discusses the results obtained in this paper. Detailed derivations are included in the Appendices.

II. SPIN TRANSPORT EQUATIONS IN 2DEG WITH SOC

We start by considering a simple bilayer structure shown in Fig. 1 where a ferromagnetic (FM) layer injects spin-polarized

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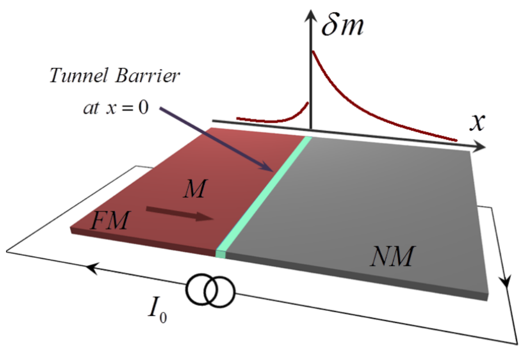

FIG. 1. (Color online) A model bilayer consists of a ferromagnet and a nonmagnetic metallic layer separated by a tunnel barrier at $x = 0$. The spin dependent transmission and reflection coefficients are T_σ and R_σ. The spin accumulation at both sides of the barrier is also shown and will be quantitatively calculated.

electrons into a NM 2DEG through a tunnel barrier and we determine the SA in the NM 2DEG. An example of this layered structure is $Ga_{1-x}Mn_xAs$ as the FM layer, the (Al,Ga)As/GaAs interface as the NM layer, and the tunnel barrier between them can be either a Schottky barrier or an insulator film spacer. Note that the actual experimental geometry in Ref. [4] involves a nonlocal spin valve for the measurement of the SA. The steady-state spinor distribution $\hat{F}(x,\mathbf{k})$ in the NM layer satisfies the Boltzmann equation [11]

$$\hat{v}_x \frac{\partial \hat{F}}{\partial x} + \frac{eE}{m}\frac{\partial \hat{F}}{\partial v_x} - \frac{1}{i\hbar}[\hat{H}_{\mathrm{SO}}, \hat{F}] = -\frac{\hat{F} - \bar{F}}{\tau_m} - \frac{\bar{F} - (1/2)\hat{I}\mathrm{Tr}(\bar{F})}{\tau_{sf}} \tag{2}$$

where E is the electric field in the x direction, $\hat{H}_{\mathrm{SO}} = (\hbar/2)\Omega_{\mathbf{k}} \cdot \hat{\boldsymbol{\sigma}}$ is the Hamiltonian for the spin-orbit coupling, the bar over F indicates an average over the momentum, τ_m and τ_{sf} are momentum and spin relaxation times due to impurity scattering, and $\hat{I}$ is the 2×2 unit matrix. In the presence of SOC, the velocity is a spinor $\hat{v}_x = \hbar k_x/m_e \hat{I} + \partial \hat{H}_{\mathrm{SO}}/\hbar\partial k_x$ which leads to spin-charge transport coupling; namely, the charge density and charge current are dependent on the spin density and spin current, and vice versa. In Eq. (2), $\hat{v}_x \frac{\partial \hat{F}}{\partial x}$ stands for the anticommutator, $\frac{1}{2}\{\hat{v}_x, \frac{\partial \hat{F}}{\partial x}\}$. In Refs. [12] and [13], a set of coupled spin and charge transport equations in a diffusive conductor have been derived. In the present work, we first simply take $v_x = \hbar k_x/m_e$ in Eq. (2) such that the spin and charge currents are not coupled. We show in Appendix A that the spinor velocity could be included but the resulting transport equations are far more cumbersome. Since our present focus is on the ballistic contribution to spin accumulation and spin current, we consider the limit that spin-orbit coupling remains small compared to the Fermi energy, i.e., $E_{\mathrm{SO}}/E_F \ll 1$ such that the charge and spin transport are separated, as shown in Appendix A.

One may explicitly separate the equilibrium F_0 and nonequilibrium parts of the distribution function

$$\hat{F} = F_0\hat{I} + \left(-\frac{\partial F_0}{\partial \varepsilon}\right)(f_0\hat{I} + \boldsymbol{f}_1 \cdot \boldsymbol{\sigma}) \tag{3}$$

where f_0 and $\boldsymbol{f}_1$ characterize the spin-independent and spin dependent parts of the nonequilibrium distributions. By placing Eq. (3) into Eq. (2), and only keeping the term linear in the electric field, one finds

$$v_x \frac{\partial f_0}{\partial x} + eEv_x = -\frac{f_0 - \bar{f}_0}{\tau_m}, \tag{4}$$

$$v_x \frac{\partial \boldsymbol{f}_1}{\partial x} - \Omega_{\mathbf{k}} \times \boldsymbol{f}_1 = -\frac{\boldsymbol{f}_1 - \bar{\boldsymbol{f}}_1}{\tau_m} - \frac{\bar{\boldsymbol{f}}_1}{\tau_{sf}}. \tag{5}$$

To establish macroscopic equations for SA and SC from the above integrodifferential equation, Eq. (5), for arbitrary ratios of the mean free path and spin dephasing length, we introduce left-going and right-going CPs for spin and charge:

$$f_0 = \mu_0^>(x)\theta(k_x) + \mu_0^<(x)\theta(-k_x) - g_0(k_x,x), \tag{6}$$

$$\boldsymbol{f}_1 = \boldsymbol{\mu}_1^>(x)\theta(k_x) + \boldsymbol{\mu}_1^<(x)\theta(-k_x) - \boldsymbol{g}_1(k_x,x) \tag{7}$$

where $\theta(k_x)$ is a step function; thus we identify $\mu_c \equiv (\mu_0^> + \mu_0^<)/2$ as the CP of charge and $\mu_{cb} \equiv \mu_0^> - \mu_0^<$ is the ballistic component of the CP. Similarly, we define that $\boldsymbol{\mu}_s \equiv (\boldsymbol{\mu}_1^> + \boldsymbol{\mu}_1^<)/2$ is the spin CP and $\boldsymbol{\mu}_b \equiv \boldsymbol{\mu}_1^> - \boldsymbol{\mu}_1^<$ is its ballistic part (see Appendix A). Next, we specify the spin-orbit coupling $\Omega_{\mathbf{k}}$. For systems with either structure-inversion or bulk-inversion symmetry breaking, $\Omega_{\mathbf{k}}$ is usually taken as linear with respect to the momentum $\mathbf{k}$, i.e., $\Omega_{\mathbf{k}} = 2\alpha/\hbar(\mathbf{k} \times \hat{\mathbf{e}}_z)$ for the Rashba Hamiltonian, and $\Omega_{\mathbf{k}} = 2\alpha/\hbar(k_x, -k_y, 0)$ for the Dresselhaus Hamiltonian where α is the Rashba or Dresselhaus constant. By placing the above definitions in Eqs. (4) and (5), we have shown in Appendix A that the following macroscopic equations can be obtained for the Dresselhaus SOC (similarly for Rashba SOC):

$$\frac{d^2}{dx^2}\boldsymbol{\mu}_b - \frac{2}{\lambda}\frac{d}{dx}(\hat{\mathbf{e}}_x \times \boldsymbol{\mu}_b) = \frac{1}{l_{\mathrm{eff}}^2}\boldsymbol{\mu}_b + \frac{1}{\lambda^2}(\hat{\mathbf{e}}_z \cdot \boldsymbol{\mu}_b)\hat{\mathbf{e}}_z \tag{8}$$

and

$$\frac{d^2}{dx^2}\boldsymbol{\mu}_s - \frac{2}{\lambda}\frac{d}{dx}(\hat{\mathbf{e}}_x \times \boldsymbol{\mu}_s) = \frac{1}{\lambda^2}[\boldsymbol{\mu}_s + (\hat{\mathbf{e}}_z \cdot \boldsymbol{\mu}_s)\hat{\mathbf{e}}_z] + \frac{1}{2l_0}\frac{d}{dx}\boldsymbol{\mu}_b - \frac{1}{2l_0\lambda}\hat{\mathbf{e}}_x \times \boldsymbol{\mu}_b \tag{9}$$

where $l_0 = v_F\tau_m/\sqrt{2}$ is the mean free path, $\lambda \equiv \hbar^2/(2m_e\alpha)$ is the spin dephasing length due to spin-orbit coupling, and $l_{\mathrm{eff}} = (1/l_0^2 + 1/\lambda^2)^{-1/2}$ is the effective mean free path (EMFP).

One immediately notes from Eq. (8) that the ballistic spin dependent potential $\boldsymbol{\mu}_b$ has a length scale determined by the EMFP. In the weak spin-orbit coupling limit where $l_0 \ll \lambda$, or $l_{\mathrm{eff}} \approx l_0$, the ballistic CP approaches zero beyond the length scale of l_0 while the spin CP or spin diffusion survives up to a larger scale of the order of λ. This is the conventional scenario. In the opposite limit where $l_0 \gg \lambda$, the ballistic CP and the spin CP have a common length scale, $l_{\mathrm{eff}} \approx \lambda$.

The salient feature of the spin ballistic-diffusion equation, Eq. (9), is that the spin CP $\boldsymbol{\mu}_s$ depends on the ballistic components of the chemical potential $\boldsymbol{\mu}_b$. In addition to the precession term, Eq. (9) contains the gradient of $\boldsymbol{\mu}_b$, indicating that the SA within the length scale of l_{eff} could differ from that of the conventional theory.

The presence of the ballistic CP also modifies the spin dependent Ohm's law. The SC $\boldsymbol{j}_s$ and the SA $\delta\boldsymbol{m}$ can be expressed in terms of these CPs (see Appendix A):

$$\boldsymbol{j}_s\rho = -\frac{d\boldsymbol{\mu}_s}{dx} + \frac{\boldsymbol{\mu}_b}{2l_0} + \frac{1}{\lambda}\hat{\mathbf{e}}_x \times \boldsymbol{\mu}_s, \tag{10}$$

$$\delta\boldsymbol{m} = \boldsymbol{\mu}_s - \frac{l_0}{2}\frac{d\boldsymbol{\mu}_b}{dx} + \frac{l_0}{2\lambda}\hat{\mathbf{e}}_x \times \boldsymbol{\mu}_b \tag{11}$$

where ρ is the Drude resistivity.

Equations (8) and (9), along with Eqs. (10) and (11), are generalizations of the macroscopic spin-diffusion and spin dependent Ohm's law. The crucial ingredient is the existence of the ballistic CPs. If one places $\boldsymbol{\mu}_b = 0$, these equations are identical to the conventional results. How and when does $\boldsymbol{\mu}_b$ becomes nonzero? For a mesoscopic system where the sample size is smaller than the mean free path, $\boldsymbol{\mu}_b$ naturally exists since there is no scattering to establish a well-defined local potential in the sample, i.e., the left-going electrons have the CP of the right reservoir while the right-going electrons have the CP of the left reservoir. For a diffusive conductor, such as magnetic metals, $\boldsymbol{\mu}_b$ is identically zero inside the sample. For magnetic tunnel junctions where a tunnel barrier is imbedded in the diffusive layers, as shown in Fig. 1, $\boldsymbol{\mu}_b$ is nonzero in the vicinity of the tunnel barrier because the transport across the barrier is governed by the quantum (ballistic) tunneling rather than diffusive scattering. To determine $\boldsymbol{\mu}_b$ in this bilayer system, a set of boundary conditions is needed.

III. SPIN INJECTION INTO A 2DEG

In this section, we study the spin injection into a 2DEG across a tunnel barrier characterized by the spin dependent transmission and reflection coefficients T_σ and R_σ where $\sigma = \uparrow,\downarrow$ and $T_\sigma + R_\sigma = 1$. In principle, these coefficients are momentum dependent as well. For our macroscopic description, we simply consider them as their average values. Within the ballistic picture, the CPs for the incoming and outgoing electrons are related by these coefficients:

$$\boldsymbol{\mu}_\sigma^>(+0) = T_\sigma\boldsymbol{\mu}_\sigma^>(-0) + R_\sigma\boldsymbol{\mu}_\sigma^<(+0) \tag{12}$$

and

$$\boldsymbol{\mu}_\sigma^<(-0) = T_\sigma\boldsymbol{\mu}_\sigma^<(+0) + R_\sigma\boldsymbol{\mu}_\sigma^>(-0). \tag{13}$$

The next boundary condition involves the definition of contact resistance at the interface that connects the spin current to the CPs between the left and right sides of the interface:

$$\boldsymbol{\mu}_\sigma^>(-0) - \boldsymbol{\mu}_\sigma^<(+0) = R_J^\sigma \boldsymbol{j}_\sigma(0) \tag{14}$$

where $R_J^\sigma = (h/Ne^2)(R_\sigma/T_\sigma)$ is the interface resistance of spin channel σ, and N is the number of modes within the layer per unit cross-section area [10,14,15]. By combining Eqs. (12)–(14), we immediately find

$$\boldsymbol{\mu}_\sigma^>(-0) - \boldsymbol{\mu}_\sigma^<(-0) = \boldsymbol{\mu}_\sigma^>(+0) - \boldsymbol{\mu}_\sigma^<(+0) = j_\sigma\frac{h(1-T_\sigma)}{e^2N}. \tag{15}$$

From the definition $\boldsymbol{\mu}_b = (\boldsymbol{\mu}_\uparrow^> - \boldsymbol{\mu}_\downarrow^>) - (\boldsymbol{\mu}_\uparrow^< - \boldsymbol{\mu}_\downarrow^<)$, we have

$$\boldsymbol{\mu}_b(+0) = \boldsymbol{\mu}_b(-0) = j_e p_{\text{eff}}\left(\frac{h}{e^2N}\right), \tag{16}$$

where we have defined the effective spin polarization $p_{\text{eff}} \equiv [(1-T_\uparrow)j_\uparrow - (1-T_\downarrow)j_\downarrow]/(j_\uparrow + j_\downarrow)$.

The above boundary conditions result in three interesting consequences.

(1) The ballistic CP is continuous across the junction which is in direct contrast with the diffusive CP which has a jump if there is interface roughness scattering or if the interface is treated as a diffusive resistor.

(2) The ballistic CP is zero if $T_\sigma = 1$, i.e., if there is no tunnel barrier; this is evident since the entire bilayer is diffusive.

(3) If T_σ is small, the ballistic CP is always nonzero, indicating the fundamental difference between tunneling and diffusive scattering.

These boundary conditions together with the continuity of current and the spin ballistic-diffusion equations, Eqs. (8)–(11), completely determine the position dependence of the CP, spin accumulation, and spin current. To gain further insight on the roles of ballistic CP, we present the detailed solution for a simple case where the magnetization of the FM layer is parallel to $\hat{\mathbf{e}}_x$ such that the precession terms (the terms with cross products) in Eqs. (8)–(11) vanish and $\boldsymbol{\mu}_s = \mu_s\hat{\mathbf{e}}_x$ and $\boldsymbol{\mu}_b = \mu_b\hat{\mathbf{e}}_x$:

$$\frac{d^2\mu_b}{dx^2} = \frac{\mu_b}{l_{\text{eff}}^2}, \tag{17}$$

and

$$\frac{d^2\mu_s}{dx^2} = \frac{\mu_s}{\lambda^2} + \frac{1}{2l_0}\frac{d\mu_b}{dx}. \tag{18}$$

The solutions are

$$\begin{aligned} \mu_b &= A\exp\left(-\frac{x}{l_{\text{eff}}}\right), \\ \mu_s &= -\frac{l_0}{2l_{\text{eff}}}A\exp\left(-\frac{x}{l_{\text{eff}}}\right) + A'\exp\left(-\frac{x}{\lambda}\right), \end{aligned} \tag{19}$$

where A and A' are integration constants determined by the boundary conditions. The general expressions for spin accumulation, spin current, and CPs for arbitrary parameters are given in Appendix B. Here, we illustrate some limiting cases. In Fig. 2, we show four CPs (spin up and down with the momentum right and left going) as well as μ_s, μ_b and δm at $x > 0$ where we have chosen a small ratio of the mean free path to the spin-diffusion length, $l_0/\lambda = 0.2$; this is the case valid for the conventional spin-diffusion equation. The left-going and right-going CP merges to a single value after $x = l_0$, but a spin-split CP exists up to λ. Equivalently, μ_b approaches zero for $x > l_0$ and μ_s survives between l_0 and λ. For $l_0 < x < \lambda$, δm arises purely from μ_s. Thus, we conclude that the conventional spin-diffusion equation, Eq. (1), describes the transport well in this limit, even though the ballistic transmission through the tunnel barrier is not a diffusive process.

Next, we consider the case, $l_0 > \lambda$. Within the conventional spin diffusion theory, the SA in the nonmagnetic layer is $\delta m_0 = p_J j_e \rho\lambda\exp(-x/\lambda)$, where p_J is the spin polarization at the interface, j_e is the electric current density, and ρ is the resistivity [16,17]. In Fig. 3(a) we show four CPs and the corresponding μ_s, μ_b; in Fig. 3(b) we show the SA.

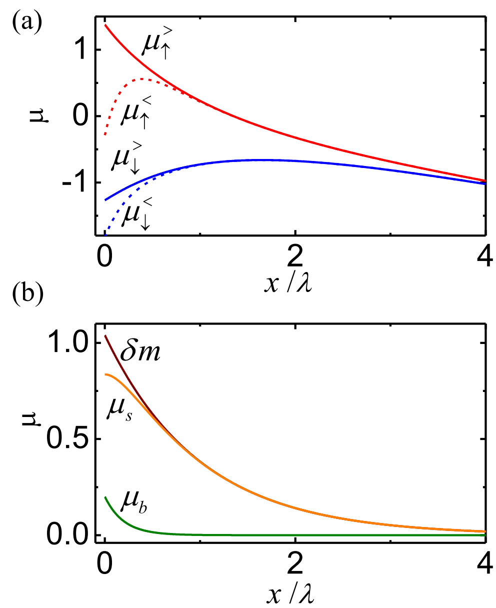

FIG. 2. (Color online) Spin injection through a tunnel barrier ($T_\sigma \ll 1$) into a diffusive NM layer, $l_0/\lambda = 0.2$; the polarization of the barrier resistance is $p_J = 0.5$. We plot CPs and spin accumulation in units of $\rho j_e p_J \lambda$. (a) The directional and spin dependence of chemical potentials in the NM layer for left-going, right-going, spin-up, and spin-down electrons. (b) The spin dependent CP μ_s and its ballistic part μ_b, as well as the spin accumulation δm derived from CPs shown in the top panel.

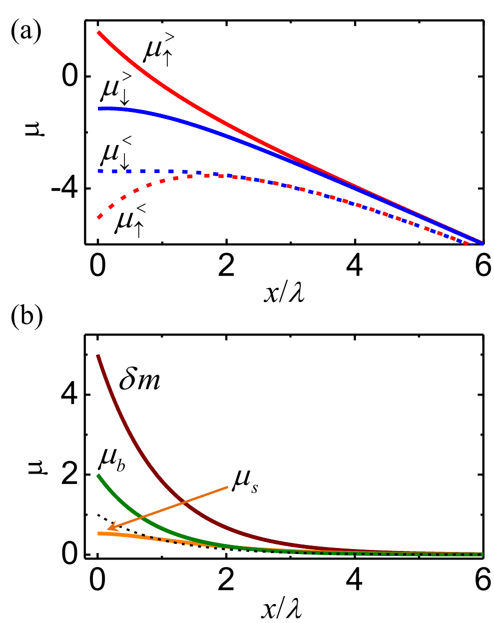

FIG. 3. (Color online) Spin injection through a tunnel barrier into a weak scattering NM layer where we choose $l_0/\lambda = 2$ and $p_J = 0.5$. All CPs and spin accumulation are plotted in units of $\rho j_e p_J \lambda$. (a) The directional and spin dependence of chemical potentials in the NM layer for left-going, right-going, spin-up, and spin-down electrons. (b) The spin dependent CP μ_s and its ballistic part μ_b, as well as the spin accumulation δm. The dotted line shows $\delta m_0 = \rho j_e p_J \lambda e^{-x/\lambda}$.

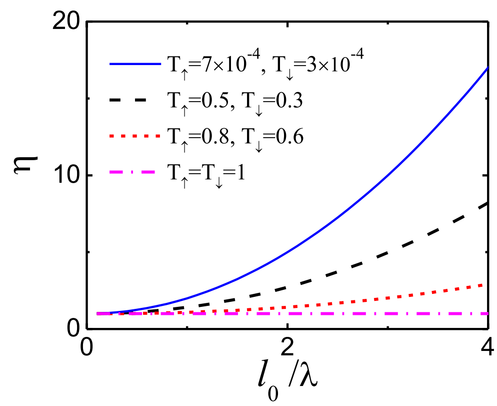

FIG. 4. (Color online) The enhancement factor of spin accumulation as a function of the ratio of the mean free path and spin dephasing length for various transmission coefficients. The other parameters are $\rho_F\lambda_F = 5$, $\rho_{F\sigma} l_{F\sigma} = 2$, $p_F = 0.5$, $\rho l_0 = 1$, $(h/e^2N) = 2$, where $\rho_{F\sigma}(l_{F\sigma})$ is the resistivity (mean free path) in the FM layer for spin channel σ, $p_F = (\rho_{F\downarrow} - \rho_{F\uparrow})/(\rho_{F\downarrow} + \rho_{F\uparrow})$, λ_F is the spin diffusion length of the FM layer, and $\rho_F = \rho_{F\uparrow}\rho_{F\downarrow}/(\rho_{F\downarrow} + \rho_{F\uparrow})$.

On comparison with the conventional result for δm_0 one immediately sees that the SA is greatly enhanced.

The enhancement of the SA originates from the existence of the ballistic CP, i.e., the second term of Eq. (11). By recalling that $\boldsymbol{\mu}_b$ characterizes the difference in the number of electron spins moving to the left and right, we may loosely consider $\boldsymbol{\mu}_b$ as a source of spin current. The divergence of the spin current generates a spin accumulation, therefore $-l_0 d\boldsymbol{\mu}_b/dx \propto (l_0/l_{\mathrm{eff}})\boldsymbol{\mu}_b$ is the ballistic contribution to SA. More quantitatively, when we carry out the detailed algebra in the limit of a large tunnel resistance (see Appendix B) we find

$$\delta m = \left(1 + \frac{l_0^2}{\lambda^2}\right) j_e p_J \rho \lambda \exp(-x/\lambda). \tag{20}$$

Thus the enhancement factor of the SA, which is defined as the ratio of SA to the conventional one, $\eta \equiv \delta m/\delta m_0$, in the limiting case $T_\sigma \ll 1$ is

$$\eta = 1 + l_0^2/\lambda^2. \tag{21}$$

In Fig. 4, we show the SA enhancement factor as a function of the ratio l_0/λ for various tunnel transmission coefficients. When $l_0/\lambda \ll 1$, there is no enhancement, $\eta = 1$ for all transmission coefficients; as l_0/λ increases, the enhancement depends on the transmission coefficient. As T increases η decreases. Thus we conclude that the large enhancement must simultaneously satisfy two conditions: a spin dependent barrier resistance that dominates over the bulk resistance, and a long mean free path compared to the spin dephasing length. Our results are consistent with experimental results [4]: η could be as large as 6 when the temperature is lowered such that the mean free path exceeds the spin dephasing length in the 2DEG at a (Al, Ga)As/GaAs interface when a spin current is injected through a tunnel barrier.

Finally, we wish to emphasize a few points on the role of the ratio of the mean free path relative to the spin dephasing length. First, in quantum wells, the D'yakonov-Perel' relaxation [18,19] has been well studied theoretically and experimentally in both strong and weak scattering limits [20–23]. One might ask whether the ballistic components have to be considered in the weak scattering limit as well. The answer relies on the initial or boundary conditions; if the SA is optically injected over a large spatial region, which is the case for most experiments on semiconductors, the ballistic chemical potentials remain zero even if $l_0/\lambda > 1$ because there is no mechanism to introduce a nonzero μ_b. Second, the spin-orbit coupling has various forms due to different growth directions of quantum wells [24,25] or the coexistence of Rashba and Dresselhaus SOC [23,24], therefore the resulting spin ballistic-diffusions, Eqs. (8) and (9), would be modified. In these cases, the solutions become rather tedious and complex. However, the physics on the spin accumulation enhancement from the ballistic transport remains the same.

IV. CONCLUSIONS

In summary, we have extended the conventional spin-diffusion equation to one with both ballistic and diffusion scattering. In addition to spin dependent chemical potentials the key physics is the presence of nonzero directional-dependent chemical potentials. When the spin-diffusion or spin dephasing length is shorter than the impurity mean free path, and spin injection is achieved through quantum-mechanical tunneling, the ballistic component of the chemical potentials can significantly contribute to the spin accumulation so that the SA is much larger than that derived from conventional spin-diffusion theory.

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APPENDIX A

In this Appendix, we show the derivation of Eqs. (8)–(11) starting from the Boltzmann equation, Eq. (2). We use the Dresselhaus Hamiltonian to model the SOC. Thus, $\hat{v}_x = v_x\hat{I} + \alpha\hat{\sigma}_x/\hbar$, then Eq. (2) reads

$$v_x\frac{\partial \hat{F}}{\partial x} + \frac{1}{2}\frac{\alpha}{\hbar}\left\{\hat{\sigma}_x,\frac{\partial \hat{F}}{\partial x}\right\} + \frac{eE}{m}\frac{\partial \hat{F}}{\partial v_x} - \frac{1}{i\hbar}[\hat{H}_{\mathrm{SO}},\hat{F}] = -\frac{\hat{F}-\bar{F}}{\tau_m} - \frac{\bar{F}-(1/2)\hat{I}\mathrm{Tr}(\bar{F})}{\tau_{sf}}. \tag{A1}$$

Substituting the distribution function with equilibrium and nonequilibrium parts defined in Eq. (3), we get the equations for the spin and charge parts distribution functions:

$$v_x\frac{\partial f_0}{\partial x} + eEv_x + \frac{\alpha}{\hbar}\frac{\partial(\boldsymbol{f}_1\cdot\hat{e}_x)}{\partial x} = -\frac{f_0-\bar{f}_0}{\tau_m}, \tag{A2}$$

$$v_x\frac{\partial \boldsymbol{f}_1}{\partial x} - \Omega_{\mathbf{k}}\times\boldsymbol{f}_1 + \frac{\alpha}{\hbar}\frac{\partial f_0}{\partial x}\hat{e}_x = -\frac{\boldsymbol{f}_1-\bar{\boldsymbol{f}}_1}{\tau_m} - \frac{\bar{\boldsymbol{f}}_1}{\tau_{sf}}. \tag{A3}$$

where the third terms on the left-hand side are the spin charge couplings (SCCs). The same equations have been derived in Ref. [12] except that we have taken the distribution function to be uniform along the $\hat{y}$ direction. We then neglect the spin-flip term and assume the spin relaxation is dominated by the spin-orbit coupling.

Inserting the left and right split CPs defined in Eqs. (6) and (7), we start to derive our spin transport equations in the presence of SOC. We only show the detailed derivation for the spin part, Eq. (A3). For the charge part, the derivation is similar. With $\boldsymbol{f}_1$ substituted with CPs, Eq. (A3) now reads

$$\begin{aligned} v_x\frac{\partial}{\partial x}&[\boldsymbol{\mu}_1^>\theta(k_x)+\boldsymbol{\mu}_1^<\theta(-k_x)-\boldsymbol{g}_1(k_x,x)]\\ &-\Omega_{\mathbf{k}}\times[\boldsymbol{\mu}_1^>\theta(k_x)+\boldsymbol{\mu}_1^<\theta(-k_x)-\boldsymbol{g}_1(k_x,x)]\\ &+\frac{\alpha}{\hbar}\frac{\partial}{\partial x}[\mu_0^>(x)\theta(k_x)+\mu_0^<(x)\theta(-k_x)-g_0(k_x,x)]\\ &=-\frac{\boldsymbol{\mu}_1^>\theta(k_x)+\boldsymbol{\mu}_1^<\theta(-k_x)-\boldsymbol{g}_1(k_x,x)}{\tau_m}\\ &+\frac{\boldsymbol{\mu}_1^>+\boldsymbol{\mu}_1^<-2\bar{\boldsymbol{g}}_1}{2\tau_m} \end{aligned} \tag{A4}$$

where $\Omega_{\mathbf{k}} = 2\alpha/\hbar(k_x,-k_y,0)$.

Following the conventional protocol to establish the corresponding macroscopic equation from the Boltzmann equation, one needs to relate $\boldsymbol{g}_1(k_x,x)$ to $\boldsymbol{\mu}_1^>$ and $\boldsymbol{\mu}_1^<$. The common choice is

$$\begin{aligned} g_0(k_x,x) &= v_x\tau_m\frac{\partial}{\partial x}[\mu_0^>\theta(k_x)+\mu_0^<\theta(-k_x)],\\ \boldsymbol{g}_1(k_x,x) &= v_x\tau_m\frac{\partial}{\partial x}[\boldsymbol{\mu}_1^>\theta(k_x)+\boldsymbol{\mu}_1^<\theta(-k_x)]\\ &\quad-\tau_m\Omega_{\mathbf{k}}\times[\boldsymbol{\mu}_1^>\theta(k_x)+\boldsymbol{\mu}_1^<\theta(-k_x)]. \end{aligned} \tag{A5}$$

The average over the Fermi circle is

$$\begin{aligned} \bar{\boldsymbol{g}}_1 &\approx \frac{l_0}{2}\left[\frac{\partial}{\partial x}(\boldsymbol{\mu}_1^>-\boldsymbol{\mu}_1^<)-\frac{1}{\lambda}\hat{e}_x\times(\boldsymbol{\mu}_1^>-\boldsymbol{\mu}_1^<)\right]\\ &= \frac{l_0}{2}\left(\frac{\partial}{\partial x}\boldsymbol{\mu}_b-\frac{1}{\lambda}\hat{e}_x\times\boldsymbol{\mu}_b\right) \end{aligned} \tag{A6}$$

where $l_0 = \sqrt{\bar{v_x^2}}\tau_m = v_F\tau_m/\sqrt{2}$ is the mean free path, and we have approximated $|\bar{v_x}| \approx \sqrt{\bar{v_x^2}} = v_F/\sqrt{2}$ to simplify the notation without changing essential results obtained below. We have also introduced the definition of the ballistic spin CP, $\boldsymbol{\mu}_b \equiv \boldsymbol{\mu}_1^> - \boldsymbol{\mu}_1^<$, as explained in the main text.

Inserting the above expression of $\boldsymbol{g}_1(k_x,x)$ and $\bar{\boldsymbol{g}}_1$ into Eq. (A4) and averaging over left ($k_x < 0$) and right ($k_x > 0$) half Fermi circles separately, we get two equations:

$$\begin{aligned} &\frac{\partial^2\boldsymbol{\mu}_1^>}{\partial x^2}-\frac{2}{\lambda}\hat{e}_x\times\frac{\partial\boldsymbol{\mu}_1^>}{\partial x}-\Delta\left[\frac{1}{l_0}\frac{\partial\mu_0^>}{\partial x}-\frac{\partial^2\mu_0^>}{\partial x^2}\right]\hat{e}_x\\ &\quad=\frac{\boldsymbol{\mu}_1^>}{l_0^2}-\frac{1}{l_0^2}\left[\boldsymbol{\mu}_s-\frac{l_0}{2}\left(\frac{\partial}{\partial x}\boldsymbol{\mu}_b-\frac{1}{\lambda}\hat{e}_x\times\boldsymbol{\mu}_b\right)\right]+\Gamma\boldsymbol{\mu}_1^>, \end{aligned} \tag{A7}$$

$$\begin{aligned} &\frac{\partial^2\boldsymbol{\mu}_1^<}{\partial x^2}-\frac{2}{\lambda}\hat{e}_x\times\frac{\partial\boldsymbol{\mu}_1^<}{\partial x}-\Delta\left[\frac{1}{l_0}\frac{\partial\mu_0^<}{\partial x}+\frac{\partial^2\mu_0^<}{\partial x^2}\right]\hat{e}_x\\ &\quad=\frac{\boldsymbol{\mu}_1^<}{l_0^2}-\frac{1}{l_0^2}\left[\boldsymbol{\mu}_s-\frac{l_0}{2}\left(\frac{\partial}{\partial x}\boldsymbol{\mu}_b-\frac{1}{\lambda}\hat{e}_x\times\boldsymbol{\mu}_b\right)\right]+\Gamma\boldsymbol{\mu}_1^< \end{aligned} \tag{A8}$$

where $\Delta = E_{SO}/E_F$ denotes the strength of SCC and Γ is a matrix which describes the anisotropic spin relaxation due to Dresselhaus spin-orbit coupling and

$$\Gamma = \begin{pmatrix} \lambda^{-2} & 0 & 0 \\ 0 & \lambda^{-2} & 0 \\ 0 & 0 & 2\lambda^{-2} \end{pmatrix}.$$

Linear combination of the above two equations leads to the following two differential equations:

$$\frac{\partial^2}{\partial x^2}\boldsymbol{\mu}_b - \frac{2}{\lambda}\hat{e}_x \times \frac{\partial}{\partial x}\boldsymbol{\mu}_b - \Delta\left[\frac{\partial}{l_0\partial x}\mu_{cb} - 2\frac{\partial^2}{\partial x^2}\mu_c\right]\hat{e}_x = \left(\frac{1}{l_0^2} + \Gamma\right)\boldsymbol{\mu}_b, \tag{A9}$$

$$\frac{\partial^2}{\partial x^2}\boldsymbol{\mu}_s - \frac{2}{\lambda}\hat{e}_x \times \frac{\partial}{\partial x}\boldsymbol{\mu}_s - \Delta\left[\frac{\partial}{l_0\partial x}\mu_c - \frac{1}{2}\frac{\partial^2}{\partial x^2}\mu_{cb}\right]\hat{e}_x = \frac{1}{2l_0}\left(\frac{\partial}{\partial x}\boldsymbol{\mu}_b - \frac{1}{\lambda}\hat{e}_x \times \boldsymbol{\mu}_b\right) + \Gamma\boldsymbol{\mu}_s. \tag{A10}$$

The above equations are equivalent to Eqs. (8) and (9) in the main text when the SCC is neglected ($\Delta = 0$). Similarly, one can get

$$\frac{\partial^2}{\partial x^2}\mu_{cb} - \Delta\left[\frac{\partial}{l_0\partial x}\boldsymbol{\mu}_b \cdot \hat{e}_x - 2\frac{\partial^2}{\partial x^2}\boldsymbol{\mu}_s \cdot \hat{e}_x\right] = \frac{\mu_{cb}}{l_0^2}, \tag{A11}$$

$$\frac{\partial^2}{\partial x^2}\mu_c - \Delta\left[\frac{\partial}{l_0\partial x}\boldsymbol{\mu}_s \cdot \hat{e}_x - \frac{1}{2}\frac{\partial^2}{\partial x^2}\boldsymbol{\mu}_b \cdot \hat{e}_x\right] = \frac{1}{2l_0}\frac{\partial}{\partial x}\mu_{cb}. \tag{A12}$$

The spinor current density is defined as $\hat{j} = \frac{e}{2}\int\{\hat{v}_x, \hat{F}\}d^2k$, where $\hat{F}$ is the spinor distribution function. By separating the current density into the charge and spin and parts, $\hat{j} = j_e\hat{I} + \boldsymbol{j}_s \cdot \hat{\boldsymbol{\sigma}}$, and by utilizing $\hat{F}$ defined in Eqs. (7) and (A5), with $\hat{v}_x = \hbar k_x/m_e\hat{I} + \alpha\hat{\sigma}_x/\hbar$, we obtain the expression for spin current $\boldsymbol{j}_s$:

$$\boldsymbol{j}_s\rho = -\frac{\partial\boldsymbol{\mu}_s}{\partial x} + \frac{\boldsymbol{\mu}_b}{2l_0} + \frac{1}{\lambda}\hat{\boldsymbol{e}}_x \times \boldsymbol{\mu}_s + \frac{\Delta}{l_0}\delta n\hat{e}_x. \tag{A13}$$

Similarly for the charge current, j_e,

$$j_e\rho = -\frac{\partial\mu_c}{\partial x} + \frac{\mu_{cb}}{2l_0} + \frac{\Delta}{l_0}\delta\mathbf{m} \cdot \hat{e}_x \tag{A14}$$

where ρ is the Drude conductivity for a 2DEG. Similarly, we can derive the expression of the charge and spin accumulation by using the relation $\delta n\hat{I} + \delta\boldsymbol{m} \cdot \hat{\boldsymbol{\sigma}} = \int \hat{F}d^2k$; we find

$$\delta\boldsymbol{m} = \boldsymbol{\mu}_s - \frac{l_0}{2}\frac{\partial}{\partial x}\boldsymbol{\mu}_b + \frac{l_0}{2\lambda}\hat{\boldsymbol{e}}_x \times \boldsymbol{\mu}_b, \tag{A15}$$

$$\delta n = \mu_c - \frac{l_0}{2}\frac{\partial}{\partial x}\mu_{cb}. \tag{A16}$$

Equations (A13) and (A15) are the spin dependent Ohm's law used in the main text, Eqs. (10) and (11).

1. The effects of SCC on spin transport

In the main text, we discard the SCC in all equations, which is valid when $\Delta \ll 1$. When Δ cannot be neglected, the spin injection into the 2DEG with SOC can still be evaluated using the above differential equations, Eqs. (A9)–(A12), and boundary conditions mentioned in the main text, Eqs. (12)–(14). We redo the calculation and keep up to the second order of Δ. We find the SCC reduces the spin-relaxation length:

$$\frac{1}{\lambda'} = \frac{1}{\lambda} + \Delta^2\left(\frac{1}{\lambda} + \frac{\lambda}{l_0^2}\right) \tag{A17}$$

where λ is the spin-relaxation length defined in the main text which is merely determined by the spin-orbit coupling. When injecting spin into a 2DEG across a tunnel barrier, the SCC modifies the spin accumulation enhancement in the ballistic regime ($l_0 \gg \lambda$):

$$\delta m = \left[1 + (1 + 3\Delta^2)l_0^2/\lambda^2\right]\rho j_e p_J \lambda' e^{-x/\lambda'}. \tag{A18}$$

APPENDIX B

In Appendix B, we first show the solution for a simple case where the polarization of the spin current is solely determined by the tunneling barrier between the FM and NM layer. We then solve the equation for general cases where the resistance of the layers is comparable to the tunnel resistance.

1. Resistance dominated by the tunnel barrier

If the resistance due to tunneling is much larger than the impurity scattering induced resistance in the layers, the injected current density and its spin polarization across the interface will be entirely determined by tunnel parameters, independent of the resistance in the layers, i.e.,

$$j_s(0) = j_e p_J \tag{B1}$$

where

$$p_J = \frac{R_J^{\downarrow} - R_J^{\uparrow}}{R_J^{\downarrow} + R_J^{\uparrow}} \tag{B2}$$

and $R_J^{\sigma} = (h/Ne^2)(R_\sigma/T_\sigma) \approx (h/Ne^2)(1/T_\sigma)$. From the boundary condition, Eq. (16),

$$\mu_b(+0) = j_e p_J R' \tag{B3}$$

where $R' = h/e^2N$, we may directly obtain the solution of δm by using Eqs. (10), (11), and (19):

$$\delta m = j_e p_J e^{-x/\lambda}\left[\frac{R'}{2}\frac{l_0\lambda}{l_{\text{eff}}^2} + \rho\lambda\left(1 - \frac{R'}{2\rho l_0}\right)\right]. \tag{B4}$$

The above expression can be simplified by relating the number of channels to the bulk resistivity and mean free path as we show below.

For an ideal conductor with N modes per unit cross-section area which connects two reservoirs, the current density flowing through the conductor carried by one spin channel is given by the Landauer formula:

$$j_e = \frac{e^2N}{h}(\mu_L - \mu_R) \tag{B5}$$

where $\mu^{R/L}$ is the chemical potential of the left or right reservoir. In our case, the current density is given by

$$j_\sigma = \frac{1}{\rho_\sigma}\left[-\frac{\partial\mu_\sigma^>}{\partial x} - \frac{\partial\mu_\sigma^<}{\partial x} + \frac{1}{l_0}(\mu_\sigma^> - \mu_\sigma^<)\right] \tag{B6}$$

where the last term describes in the same way as the contact potential from Eq. (B5). Thus, we can easily identify

$$\frac{1}{\rho_\sigma l_\sigma} = \frac{e^2 N}{h}. \tag{B7}$$

Inserting this relation into Eq. (B4) and taking $\rho_\sigma = 2\rho, l_\sigma = l_0$ for the NM layer, we find

$$\delta m = \frac{l_0^2}{l_{\rm eff}^2} j_e \rho \lambda p_J e^{-x/\lambda}, \tag{B8}$$

which is the same as Eq. (20) from the main text.

2. General solution and exact calculation

When the tunnel resistance is not much larger than that of the bulk, or equivalently when the transmission coefficient is not small (note that $T_\sigma = 1$ describes the transparent barrier or no barrier), one must solve the CPs for the entire bilayer, including the ferromagnetic layer. In this case, the spin polarization and spin accumulation depend on the detailed parameters of all layers in addition to the barrier transmission coefficients. We first write down the general solution of CPs according to Eqs. (A9)–(A12) (while the SCC terms are neglected) and then determine the coefficients by using the boundary conditions from the main text.

In the NM layer ($x > 0$),

$$\mu_\uparrow^>(x) = \gamma_0 + \gamma_1 z + 2ce^{-x/\lambda} + ae^{-x/l_{\rm eff}}, \tag{B9}$$

$$\mu_\uparrow^<(x) = \gamma_0 + \gamma_1 z - ge^{-x/l_0} + 2ce^{-x/\lambda} + be^{-x/l_{\rm eff}}, \tag{B10}$$

$$\mu_\downarrow^>(x) = \gamma_0 + \gamma_1 z - 2ce^{-x/\lambda} - ae^{-x/l_{\rm eff}}, \tag{B11}$$

$$\mu_\downarrow^<(x) = \gamma_0 + \gamma_1 z - ge^{-x/l_0} - 2ce^{-x/\lambda} - be^{-x/l_{\rm eff}} \tag{B12}$$

where $\mu_\sigma^> = \mu_0^> \pm \boldsymbol{\mu}_1^> \cdot \hat{e}_x$, $\sigma = \uparrow,\downarrow$. Equation (A10) also requires

$$a + b = -\frac{l_0}{l_{\rm eff}}(a - b). \tag{B13}$$

For notation simplicity, we assume that the effective mean free path is the same as the mean free path and the spin-diffusion length is much longer in the FM layer ($x < 0$). The general solution in the FM layer is

$$\mu_\uparrow^>(x) = \gamma_0' + \gamma_1' z + \frac{\rho_F^\uparrow}{\rho_F} c' e^{x/\lambda_F} + a' e^{x/l_{F\uparrow}}, \tag{B14}$$

$$\mu_\uparrow^<(x) = \gamma_0' + \gamma_1' z + \frac{\rho_F^\uparrow}{\rho_F} c' e^{x/\lambda_F}, \tag{B15}$$

$$\mu_\downarrow^>(x) = \gamma_0' + \gamma_1' z - \frac{\rho_F^\downarrow}{\rho_F} c' e^{x/\lambda_F} + b' e^{x/l_{F\downarrow}}, \tag{B16}$$

$$\mu_\downarrow^<(x) = \gamma_0' + \gamma_1' z - \frac{\rho_F^\downarrow}{\rho_F} c' e^{x/\lambda_F} \tag{B17}$$

where $\rho_F = \rho_{F\uparrow}\rho_{F\downarrow}/(\rho_{F\uparrow} + \rho_{F\downarrow})$, $\rho_{F\sigma} = \frac{h}{e^2}\frac{\sqrt{2}\pi}{k_F l_{F\sigma}}$, and the polarization of the conductivity is

$$p_F = (\rho_F^\downarrow - \rho_F^\uparrow)/(\rho_F^\downarrow + \rho_F^\uparrow) = (l_{F\uparrow} - l_{F\downarrow})/(l_{F\uparrow} + l_{F\downarrow}). \tag{B18}$$

There are many constants to be determined. γ_0 and γ_0' are the voltages on two sides of the interface of which the difference addresses the voltage drop due to contact resistance. The total charge current density can be obtained from Eq. (A14):

$$-\gamma_1/\rho_F = -\gamma_1/\rho = j_e/2 \tag{B19}$$

where j_e is the injected charge current density and ρ_F, ρ are the resistivity of the FM and NM layer, respectively. In Eq. (B19), we have assumed the cross-section area of the FM and NM layer to be the same for simplicity. With the boundary conditions from the main text, one can determine all the coefficients straightforwardly. The final results for the spin accumulation and spin polarization are

$$\delta \boldsymbol{m} = j_e \rho \lambda e^{-x/\lambda} \frac{\frac{2p_F R_F}{1-p_F^2} + 2R'(T_\downarrow^{-1} - T_\uparrow^{-1}) - \frac{R_F}{1-p_F^2}(1-\beta^2)T(p_F + p_J) + 2R'(1-\beta^2)\frac{T_\downarrow^2 - T_\uparrow^2}{T_\uparrow T_\downarrow} + (T_\uparrow - T_\downarrow)R'(2 - \beta - 5\beta^2)}{\frac{2\beta^2 R_F}{1-p_F^2} + 2\beta^2 R'(T_\downarrow^{-1} + T_\uparrow^{-1}) + R_N[2 - (1-\beta^2)(T)] + \beta(1+3\beta)R'T - 2\beta(1+5\beta)R'} \tag{B20}$$

$$p_{\rm inj} = \frac{\frac{2\beta^2 p_F R_F}{1-p_F^2} + 2\beta^2 R'(T_\downarrow^{-1} - T_\uparrow^{-1}) + [(1-\beta^2)R_N - \beta(1+3\beta)R'](T_\uparrow - T_\downarrow)}{\frac{2\beta^2 R_F}{1-p_F^2} + 2\beta^2 R'(T_\downarrow^{-1} + T_\uparrow^{-1}) + R_N[2 - (1-\beta^2)(T)] + \beta(1+3\beta)R'T - 2\beta(1+5\beta)R'} \tag{B21}$$

where $T = T_\uparrow + T_\downarrow$, $R_F \equiv \rho_F \lambda_F$, $R_N \equiv \rho\lambda$, $R' = 2\rho l_0$, and $\beta \equiv l_{\rm eff}/l_0$. Below we show the results for two limiting cases.

a. Transparent interface

By placing $T_\uparrow = T_\downarrow = 1$ into Eqs. (B20) and (B21), we have

$$p_{\rm inj} = \frac{\frac{p_F R_F}{1-p_F^2}}{\frac{R_F}{1-p_F^2} + R_N}, \tag{B22}$$

$$\delta m = j_e \rho \lambda p_J e^{-x/\lambda}. \tag{B23}$$

Thus, there is no SA enhancement; this is because the transport is purely diffusive and the conventional spin-diffusion theory applies.

b. Tunneling dominated interface

For the resistance dominated by the tunneling interface, we take $T_\sigma \ll 1$, $R_J^\sigma \approx \frac{h}{e^2 N}\frac{1}{T_\sigma}$, and $R_J^\downarrow, R_J^\uparrow \gg R_N, R', R_F$. Equations (B20) and (B21) are reduced to

$$p_{\text{inj}} = p_J = \frac{R_J^\downarrow - R_J^\uparrow}{R_J^\downarrow + R_J^\uparrow}, \tag{B24}$$

$$\delta m = j_e \rho \lambda p_J e^{-x/\lambda}\left(1 + l_0^2/\lambda^2\right) \tag{B25}$$

where we used the definition of l_{eff} and $\beta^{-2} = l_0^2/l_{\text{eff}}^2 = 1 + l_0^2/\lambda^2$.

[1] P. C. van Son, H. van Kempen, and P. Wyder, Phys. Rev. Lett. **58**, 2271 (1987).
[2] T. Valet and A. Fert, Phys. Rev. B **48**, 7099 (1993).
[3] A. Fert, Rev. Mod. Phys. **80**, 1517 (2008).
[4] M. Oltscher, M. Ciorga, M. Utz, D. Schuh, D. Bougeard, and D. Weiss, Phys. Rev. Lett. **113**, 236602 (2014).
[5] H. Jaffrès, Physics **7**, 123 (2014).
[6] V. S. Rychkov, S. Borlenghi, H. Jaffrès, A. Fert, and X. Waintal, Phys. Rev. Lett. **103**, 066602 (2009).
[7] D. Culcer, J. Sinova, N. A. Sinitsyn, T. Jungwirth, A. H. MacDonald, and Q. Niu, Phys. Rev. Lett. **93**, 046602 (2004).
[8] V. Y. Kravchenko and E. I. Rashba, Phys. Rev. B **67**, 121310(R) (2003).
[9] Y.-N. Qi and S. Zhang, Phys. Rev. B **65**, 214407 (2002).
[10] S. Datta, *Electronic Transport in Mesoscopic Systems*, 1st ed. (Cambridge University, Cambridge, England, 1997).
[11] Y. Qi and S. Zhang, Phys. Rev. B **67**, 052407 (2003).
[12] E. G. Mishchenko and B. I. Halperin, Phys. Rev. B **68**, 045317 (2003).
[13] A. A. Burkov, Alvaro S. Núñez, and A. H. MacDonald, Phys. Rev. B **70**, 155308 (2004).
[14] P. W. Anderson, D. J. Thouless, E. Abrahams, and D. S. Fisher, Phys. Rev. B **22**, 3519 (1980).
[15] R. Landauer, J. Phys. Condens. Matter **1**, 8099 (1989).
[16] E. I. Rashba, Phys. Rev. B **62**, R16267(R) (2000).
[17] A. Fert and H. Jaffrès, Phys. Rev. B **64**, 184420 (2001).
[18] M. I. D'yakonov and V. I. Perel', Zh. Eksp. Teor. Fiz. **60**, 1954 (1971) [Sov. Phys. JETP **33**, 1053 (1971)].
[19] M. I. D'yakonov and V. I. Perel', Fiz. Tverd. Tela (Leningrad) **13**, 3581 (1971) [Sov. Phys. Solid State **13**, 3023 (1972)].
[20] M. A. Brand, A. Malinowski, O. Z. Karimov, P. A. Marsden, R. T. Harley, A. J. Shields, D. Sanvitto, D. A. Ritchie, and M. Y. Simmons, Phys. Rev. Lett. **89**, 236601 (2002).
[21] W. J. H. Leyland, R. T. Harley, M. Henini, A. J. Shields, I. Farrer, and D. A. Ritchie, Phys. Rev. B **76**, 195305 (2007).
[22] A. V. Poshakinskiy and S. A. Tarasenko, Phys. Rev. B **84**, 155326 (2011).
[23] A. V. Poshakinskiy and S. A. Tarasenko, Phys. Rev. B **87**, 235301 (2013).
[24] I. Žutić, J. Fabian, and S. Das Sarma, Rev. Mod. Phys. **76**, 323 (2004).
[25] X. Cartoixà, D. Z.-Y. Ting, and Y.-C. Chang, Phys. Rev. B **71**, 045313 (2005).