Crossover between localized states and pinned Wigner crystal in high-mobility n-GaAs/AlGaAs heterostructures near filling factor $\nu = 1$

I. L. Drichko,[1] I. Yu. Smirnov,[1,*] A. V. Suslov,[2] L. N. Pfeiffer,[3] K. W. West,[3] and Y. M. Galperin[1,4]

[1]*A. F. Ioffe Physico-Technical Institute of Russian Academy of Sciences, 194021 St. Petersburg, Russia*

[2]*National High Magnetic Field Laboratory, Tallahassee, Florida 32310, USA*

[3]*Department of Electrical Engineering, Princeton University, Princeton, New Jersey 08544, USA*

[4]*Department of Physics, University of Oslo, 0316 Oslo, Norway*

(Received 26 September 2015; revised manuscript received 12 November 2015; published 30 November 2015)

We have measured magnetic field dependences of the attenuation and velocity of surface acoustic waves in a high-mobility n-GaAs/AlGaAs structure with a wide quantum well. The results allowed us to find the complex conductance $\sigma(\omega)$ of the heterostructure for different frequencies, temperatures, and magnetic fields near filling factors $\nu = 1,2$. Observed behavior of $\sigma(\omega)$ versus magnetic field outside close vicinities of integer fillings reveals an oscillation pattern indicative of the rich fractional quantum Hall effect. Our result is that in very close vicinities of integer filling factors the AC response of a high-mobility two-dimensional structures behaves as that of a two-dimensional system of localized electrons. Namely, both real and imaginary parts of the complex AC conductance at low temperatures agree with the predictions for the two-site model for a two-dimensional hopping system. Another result is the specific temperature dependences of $\sigma(\omega)$, which are extremely sensitive to the filling factor value. These dependences indicate a sharp crossover between the localized modes and a pinned Wigner crystal.

DOI: 10.1103/PhysRevB.92.205313 PACS number(s): 73.63.Hs, 73.50.Rb

I. INTRODUCTION

The nature of the ground state of a two-dimensional electron system (2DES) in a large perpendicular magnetic field B has attracted great interest. At small filling factors, $\nu = 2\pi\hbar n/eB$, where n is the 2DES density and e is the electronic charge, the ground state in the absence of disorder is expected to be the Wigner crystal (WC) [1–5]. Another known ground states is the fractional quantum Hall effect (FQHE) one [6,7]. Both states are induced by electron-electron interaction. It turns out that the Laughlin FQHE liquid states at $\nu = p/q$ (where p and q are integers) are particularly robust and have ground state energies which are lower than the WC state energy, at least for $\nu > 1/5$ [8]. Theoretical calculations predict that, in an ideal 2DES, the WC should be the ground state for $\nu \lesssim 1/6$. However, the WC state may win as the filling deviates slightly from 1/5. It is possible therefore to have a WC which is reentrant around a FQHE liquid state, see Fig. 9 in Ref. [9]. This would rationalize the general current belief that the insulating phase observed around $\nu = 1/5$ in very high quality n-GaAs/AlGaAs structures is the signature of the WC state pinned by a disorder potential. This conclusion has been confirmed using various experimental methods. The magnetic-field-induced WC problem in 2DESs has been studied extensively since the late 1980's [10,11].

In 2D systems, along with direct current (DC) measurements of the components of the magnetoresistance tensor, a few research groups study alternating current (AC) conductance $\sigma(\omega)$. The radio-frequency electric field can be excited using the coplanar-wave-guide technique [12]; this method was successfully employed for studies of the FQHE in Ref. [13] and other works.

Another probeless method of studying AC conductance uses a traveling electric field created by a surface acoustic wave (SAW). In connection with the integer QHE structures, it was implemented in Refs. [14,15] and subsequent works; the FQHE was studied using this method in Refs. [16,17].

AC methods are complementary to the DC ones. In particular, specific resonances in the AC response allow one to identify the nature of insulating states observed at specific values of the filling factor.

Interestingly, the magnetic field dependences of ρ_{xx} and σ_{xx} in high-mobility structures show sharp peaks (called "wings" in Ref. [18]) around integer values of the filling factor [11,19–22]. These sharp wings have not ever been observed in low-mobility systems. Numerous microwave studies of the real part $\sigma_1(\omega)$ of the complex conductance $\sigma(\omega) \equiv \sigma_1(\omega) - i\sigma_2(\omega)$ close to integer ν revealed resonances at frequencies of 0.4–3 GHz, which were ascribed to pinned modes of the Wigner crystal [9,10,18,23–28].

The aim of the present paper is a detailed investigation of low-temperature complex conductivity of high-mobility 2DES in the vicinities of integer filling factors. We use an acoustic, namely SAW, technique to address properties of high-mobility n-GaAs/AlGaAs wide quantum well at low temperatures close to integer values of the filling factor $\nu = 1$ and 2. Our main task is the study of the crossover from the electronic state at an integer ν to a pinned mode of the WC at some deviation from an integer ν. We will focus on ν close to 1, but similar behavior is also observed close to $\nu = 2$.

We will show that in high-mobility structures at low temperatures and exactly at filling factors $\nu = 1,2$ the AC response behaves as that for localized electronic states, whereas at small deviations from the integer values its behavior crosses over: first to that of the Wigner crystal and then to that of the FQHE states. This conclusion is based on experimental studies of absorption and velocity of surface acoustic waves.

*ivan.smirnov@mail.ioffe.ru

 ©2015 American Physical Society

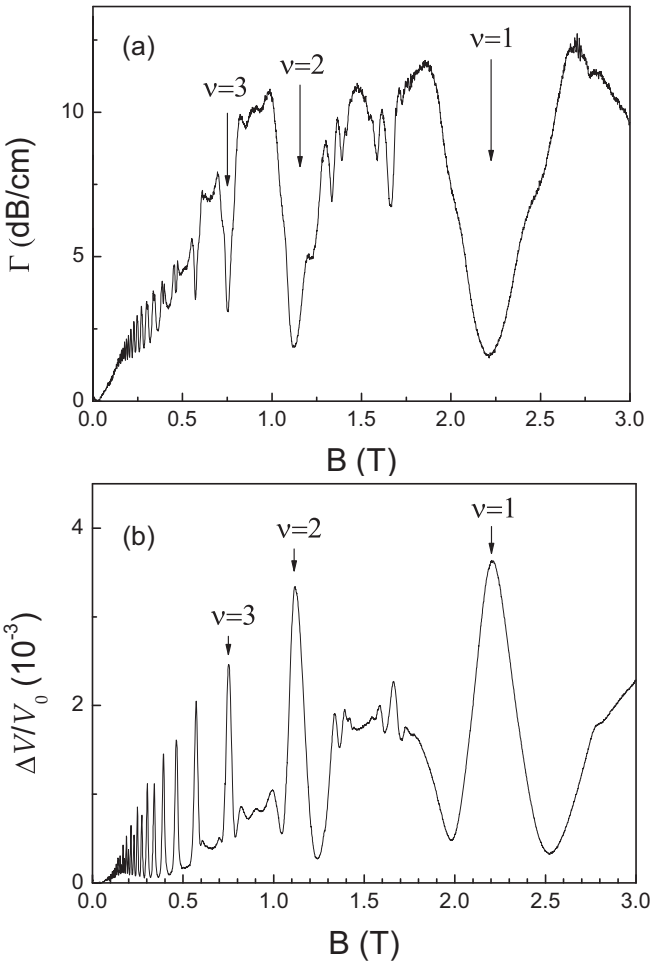

FIG. 1. Magnetic field dependences of (a) the attenuation $\Gamma(B)$ and (b) SAW velocity $\Delta V(B)/V_0$ at frequency $\omega/2\pi \equiv f = 86$ MHz at temperature $T = 40$ mK.

II. EXPERIMENTAL METHOD

We use the so-called hybrid method for determining complex $\sigma(\omega)$ from experimentally measured attenuation and velocity of a SAW excited by interdigital transducers and propagating along a surface of a piezoelectric crystal ($LiNbO_3$) in a perpendicular magnetic field. A sample containing a 2DES is mounted at the surface of the piezoelectric crystal and is pressed to this surface by springs. The traveling wave of electric field generated by the SAW penetrates the 2DES causing a magnetic-field-dependent attenuation of the SAW and a change of its velocity. The method is described in detail in Refs. [15,29]; in the last paper it is sketched in Fig. 1 (left panel).

We study multilayered n-GaAlAs/GaAs/GaAlAs structures with a wide (65 nm) GaAs quantum well (QW). The QW is δ doped from both sides and is located at the depth $d = 845$ nm from the surface. The electron density is $n = 5 \times 10^{10}$ cm^{-2}, and the mobility is $\mu_{0.3\,\mathrm{K}} = 8 \times 10^6$ cm^2/V s. Studies show that at the given electron density only the lowest band of transverse quantization should be occupied [21].

III. RESULTS AND DISCUSSION

Magnetic field dependences of the attenuation $\Gamma(B)$ and SAW velocity $\Delta V(B)/V_0$ were measured at temperatures of 40–380 mK and SAW frequencies of 28.5–306 MHz in magnetic fields up to 18 T, although the analysis throughout the paper is limited by 3 T. Shown in Fig. 1 are the results obtained at $T = 40$ mK and $f = 86$ MHz. These and similar dependences were used to calculate the complex AC conductance $\sigma(\omega)$. The procedure of extracting $\sigma(\omega) \equiv \sigma_1(\omega) - i\sigma_2(\omega)$ from the data on Γ and $\Delta V/V_0$ is described in detail in Ref. [15] and references therein.

Magnetic field dependences of σ_1 and $|\sigma_2|$ for $f = 28.5$ MHz and $T = 40$ mK are shown in the upper panel of Fig. 2. The sign of $\sigma_2(\omega)$ will be discussed later, see the discussion related to Fig. 4. Shown in the middle panel are the temperature dependences of σ_1 and $|\sigma_2|$ for the same frequency and $\nu = 1$. In the lower panel, the magnetic field dependences of σ_1 and $|\sigma_2|$ for ν close to 1 at different temperatures are shown.

One can see that integer values of ν correspond to the minima in $\sigma_1(\omega)$. Close to the minima $|\sigma_2(\omega)| > \sigma_1(\omega)$. Between these minima rich oscillation patterns typical for the FQHE were observed. Interestingly, the minima and the FQHE regions are separated by sharp maxima ("wings")—each minimum is surrounded by two wings. Outside close vicinities of integer ν, $|\sigma_2(\omega)| < \sigma_1(\omega)$. The behavior described above is observed only in high-mobility samples. In the low-mobility samples, pronounced maxima of $\sigma_1(\omega)$ at half-integer ν are observed rather than the FQHE related oscillations. Consequently, the wings around integer ν values are absent.

Let us now consider the behavior of $\sigma(\omega)$ at "exactly" integer ν. We focus on temperature dependences of σ_1 and $|\sigma_2|$ at $\nu = 1$ shown in the middle panel of Fig. 2. One observes that at $T < 400$ mK σ_1 *increases* with temperature, and in this temperature domain $|\sigma_2| > \sigma_1$ in agreement with Ref. [30]. The temperature dependence of σ_1 at $\nu = 2$ is similar.

The frequency dependence of σ_1 in the above temperature domain is weak: As the frequency changes by factor 11, σ_1 changes only by 20%. The values of σ_1 at different minima corresponding to an integer ν decrease with magnetic field $\propto B^{-1.8}$ as it follows from the analysis for $\nu = 2,4,6,8$, and 10. This dependence is compatible with theoretical prediction $\sigma_1(\omega) \propto B^{-2}$ based on the two-site model for absorption by localized states [31].

Now let us recall that in low (medium) mobility systems showing only an integer quantum Hall effect behavior of σ_1 is well described by the one-electron picture involving electrons trapped by a random potential. According to this picture, at integer ν the Fermi level is located in the middle of the distance between the Landau levels, the electron states are localized by disorder, and low-temperature DC conductance σ_{DC} is exponentially small. The AC conductance is determined by electron hops between nearest potential minima resulting in $\sigma_1(\omega) \gg \sigma_{\mathrm{DC}}$. In this case, the AC response can be explained by the two-site model; for a review see Refs. [32,33] and references therein. According to this model, a pair of the electron energy minima is described as a two-level tunneling system (TLS) with diagonal splitting Δ and tunneling splitting $\Lambda(r)$, the interlevel spacing being $E = \sqrt{\Delta^2 + \Lambda^2}$.

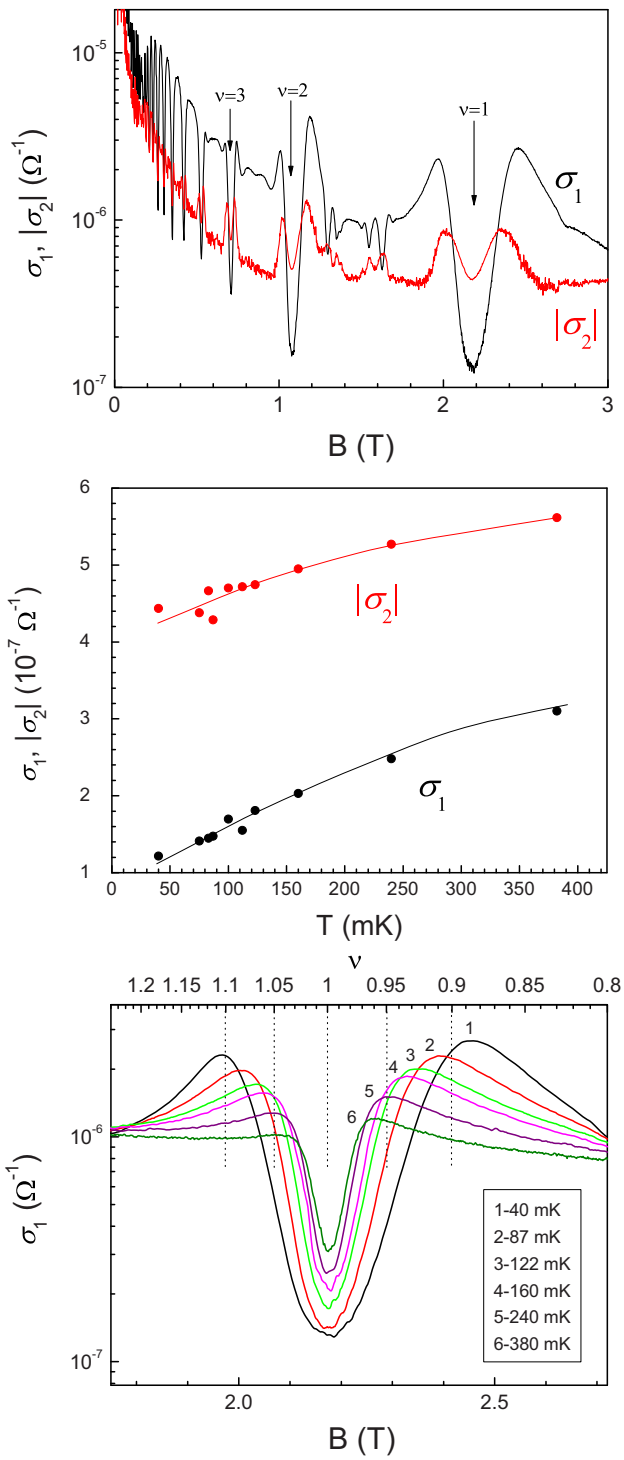

FIG. 2. (Color online) Upper panel: Magnetic field dependences of σ_1 and absolute value of σ_2 for $f = 28.5$ MHz and $T = 40$ mK. Middle panel: Temperature dependences of σ_1 and $|\sigma_2|$ for the same frequency and $\nu = 1$. The lines are guides to the eye. Lower panel: Magnetic field dependences of σ_1 for ν close to 1 at various temperatures.

At sufficiently low frequencies the AC response is due to relaxation of the nonequilibrium populations of the minima. The corresponding relaxation rate can be expressed as, cf. Ref. [33],

$$\frac{1}{\tau(E,r)} = \frac{1}{\tau_0(T)} F\left(\frac{E}{kT}\right)\left(\frac{\Lambda(r)}{E}\right)^2. \tag{1}$$

Here k is the Boltzmann constant. Equation (1) assumes that the levels' populations relax due to interaction between localized electrons and phonons. The interaction matrix element contains, therefore, the electron-phonon coupling constant as well as the tunneling coupling between the sites $\Lambda(r)$, which exponentially decays with the distance r between the minima. The corresponding relaxation rate $\tau^{-1}(E,r)$ is, therefore, proportional to $\Lambda^2(r)$ that is taken into account by the last factor in Eq. (1). In addition to the coupling constant squared, the rate is proportional to the phonon density of states at frequency $E/\hbar$. However, only the configurations with $E \lesssim kT$ are important since the configurations with $E \gg kT$ are frozen in their ground states. Therefore, we split the rate into the factor $\tau_0^{-1}(T)$ (corresponding to the systems with $E = kT$) and dimensionless function F, which depends on the details of the electron-phonon interaction [33]. It is normalized in order to have $F(1) = 1$. Since $\Lambda(r) \leqslant E$, the time τ_0 has a meaning of the *minimal* relaxation time for a TLS with the level splitting $E = kT$.

The theory predicts that (with logarithmic accuracy) [33]

$$\sigma_1(\omega) \propto \min\left\{\omega, \tau_0^{-1}(T)\right\}, \quad |\sigma_2(\omega)| \gtrsim \sigma_1(\omega). \tag{2}$$

The first of the above expressions allows a simple qualitative interpretation. Let us consider TLSs with $E \approx kT$, which plays the main role. The contribution of a TLS to the dissipation depends on the product $\omega\tau$. Very "fast" systems with $\tau \ll \omega^{-1}$ do not essentially contribute, because their populations almost adiabatically follow the AC electric field. On the other hand, very "slow" systems having $\tau \gg \omega^{-1}$ also do not contribute since their populations have not enough time to follow the AC field. Therefore, the optimal ones are those having $\tau \sim \omega^{-1}$.

On the other hand, since $\Lambda(r)$ exponentially decreases with r there exists an exponentially broad set of systems having relaxation times longer than $\tau_0(T)$. Therefore, if $\omega\tau_0 \ll 1$ the optimal pairs with $\omega\tau \sim 1$ can always be found, and it is those pairs that provide the main contribution to the absorption. On the contrary, at $\omega\tau_0 \gg 1$ the optimal pairs are absent, and the absorption is dominated by the pairs with $\tau \sim \tau_0$.

Looking at our data we conclude that at $\nu = 1$ the behavior of σ_1 and σ_2 in our sample is compatible with the picture of relaxation absorption of SAW by localized electrons under condition $\omega \gg \tau_0^{-1}$, see Eq. (2). Indeed, estimates based on Eq. (1) show that the main contribution to the relaxation rate τ_0^{-1} is due to piezoelectric interaction between localized electrons and phonons. In this case, see, e.g., Ref. [33], $\tau_0^{-1}(T)$ is roughly proportional to T, and at 40 mK $\tau_0 = 1.4 \times 10^{-8}$ s.

Now let us discuss the magnetic field dependences of σ_1 in the vicinity of $\nu = 1$, the behavior of σ_1 around $\nu = 2$ being similar. As is seen in the lower panel of Fig. 2, the minimum of σ_1 corresponding to $\nu = 1$ is surrounded by maxima heights, and locations of these maxima depend on temperature.

Temperature dependences of σ_1 at frequency 28.5 MHz for different values of the filling factor in the range $0.9 \leqslant \nu \leqslant 1.1$ are shown in Fig. 3. The dependences obtained for other investigated SAW frequencies are similar. Let us first consider

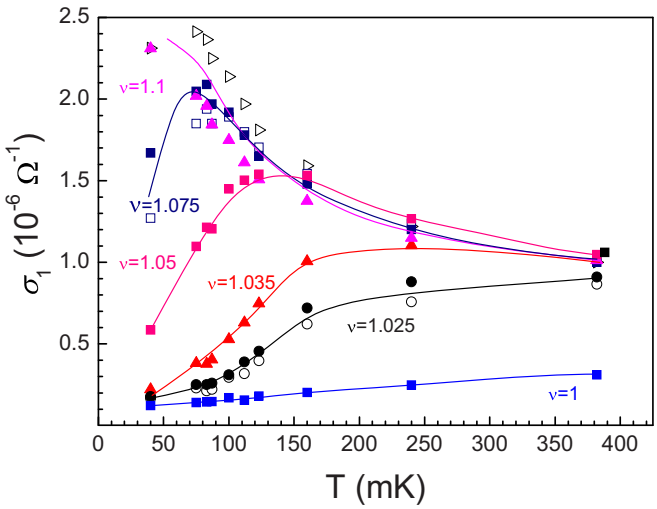

FIG. 3. (Color online) Temperature dependences of σ_1 (28.5 MHz) for different values of ν (shown near the curves). Mainly, the results for $\nu \geqslant 1$ are demonstrated (the filled symbols); the picture is almost symmetric for $\nu < 1$ as partially shown by open symbols ▷ ($\nu = 0.9$), □ ($\nu = 0.925$), and ◯ ($\nu = 0.975$). The lines are guides to the eye.

these temperature dependences at the range ends, i.e., at $\nu = 1.1$ and 0.9. We conclude that the electronic state at $\nu = 1.1$ (0.9) is indeed the WC. This conclusion came from (i) dramatic increase of the conductance as against $\nu = 1$, (ii) different temperature dependences of σ_1 at $\nu = 1$ and $\nu = 1.1$ (0.9) (these dependences at $=1.1$ and 0.9 are similar to those at $\nu = 0.19$ and 0.21, whereas the formation of Wigner solids in the latter cases were proved by various authors who used a number of experimental techniques [8,11]), and (iii) the frequency dependence of $\sigma_2(\omega)$ shown in Fig. 4 for different values of the filling factor, which demonstrates zero crossing of the $\sigma_2(\omega)$ at $\nu = 1.1$ at some frequency that is typical for the Wigner crystal. According to Ref. [28], the frequency corresponding to the zero crossing is equal to the pinning frequency of the Wigner solid. This zero crossing in the frequency dependence of the imaginary part $\sigma_2(\omega)$ should be accompanied by a maximum in the frequency dependence of the real part of $\sigma_1(\omega)$. We did not observe such maxima due to precision limitations of our equipment.

Studies based on microwave spectroscopy, see, e.g., Ref. [18] and references therein, also lead to the conclusion that for $\nu = 1.1$ and 0.9 the electron state in a similar sample can be understood as a Wigner solid. This conclusion was based on the observation of resonances in the frequency dependence of σ_1.

Let us now analyze the temperature dependences of σ_1 inside the filling factor range $0.9 \leqslant \nu \leqslant 1.1$. For all filling factor values at low temperatures, conductivity σ_1 initially rises as the temperature increases. However, as the conductivity reaches the value corresponding to $\nu = 1.1$ (0.9) its temperature dependence changes, and the conductivity begins to decrease as the temperature rises. Notice that the characteristic temperature of this crossover in the temperature dependence of conductivity decreases with increasing deviation of the filing factor from the unity, $|\nu - 1|$.

We assume that the observed initial increase of σ_1 with rising temperature at all values $\nu < 1.1$ is associated with the hopping nature of this conductivity at low temperatures. Assuming that the conductance of the localized phase can be represented by the single-electron expression (2) at $\omega\tau_0 \gg 1$, we expect that $\sigma_1(\omega) \propto \tau_0^{-1}(T)$ is an increasing function of temperature, with the slope proportional to the squared single-electron density of states g^2 [33]. Therefore, as the value $|\nu - 1|$ increases in the range $|\nu - 1| \leqslant 0.1$, both the single-electron density and the slope rise.

As the conductivity is increasing, conditions favorable for formation of the Wigner crystal are emerging, and at some temperature the crystal gets formed. The characteristic temperature of this crossover is decreasing with increasing deviation of the filling factor from an integer value, in this case $|\nu - 1|$.

We have also conducted measurements of the SAW intensity impact on the ac conductance near $\nu = 1$. In Fig. 5 we present the dependence of the real part of AC conductivity σ_1 on the electric field, accompanying the SAW of frequency

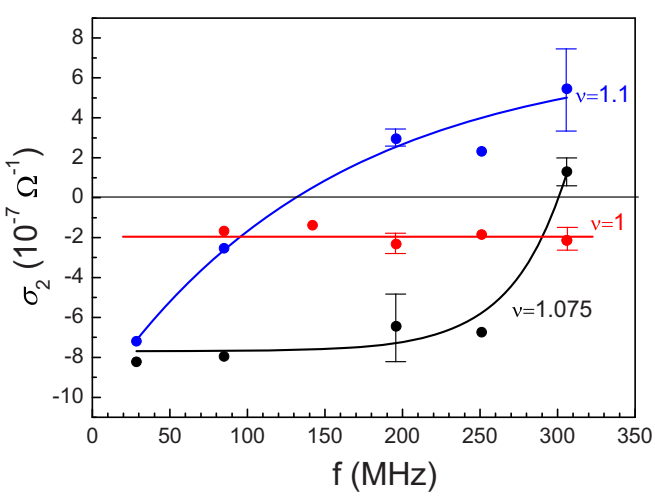

FIG. 4. (Color online) Frequency dependences of σ_2 for different ν. $T = 40$ mK.

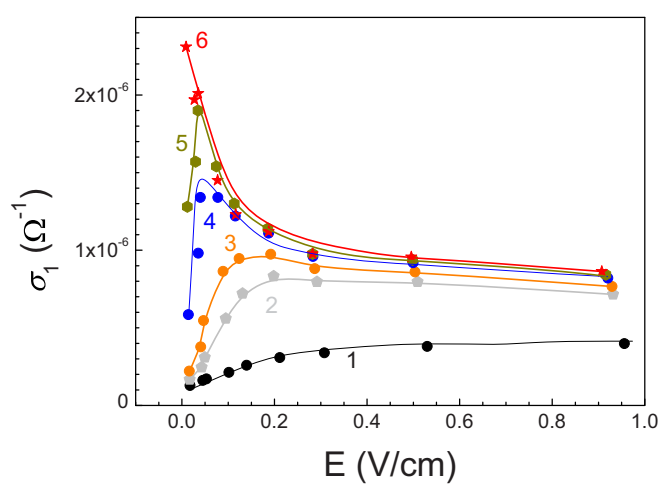

FIG. 5. (Color online) Dependence of the real part of the conductivity σ_1 on the SAW electric field near $\nu = 1$: 1—$\nu = 1.0$, 2—$\nu = 1.025$, 3—$\nu = 1.035$, 4—$\nu = 1.05$, 5—$\nu = 1.075$, 6—$\nu = 1.1$; $T = 40$ mK, $f = 28.5$ MHz. The lines are guides to the eye.

28.5 MHz at temperature 40 mK. The dependences obtained at other frequencies are similar. The electric field was calculated from values of SAW intensity using Eq. (6) of Ref. [22]. The dependences $\sigma_1(E)$ are qualitatively similar to the dependences $\sigma_1(T)$ shown in Fig. 3. Therefore, increase of SAW intensity acts as an increase of temperature—the SAW heats the electron system. This mechanism differs from the predicted one based on nonlinear sliding of the WC [34]. Assuming that the electron system can be characterized by an effective temperature T_e and performing an analysis similar to the one described in Ref. [22] for $\nu = 1.1$, we conclude that the heat released is proportional to $(T_e^3 - T^3)$. It is worth noting that when the temperature (or the SAW intensity) increases, the behavior of the complex conductance can be interpreted as a manifestation of WC melting.

For $\nu = 1$, the dependence $\sigma_1(E)$ is increasing but rather weak. Therefore, it is hard to identify the underlying mechanism of nonlinearity. We attribute the observed increase of AC conductivity with E to a combination of electron heating and field-induced ionization of the electron states localized in shallow wells of random potential.

In this way we arrive at the following scenario. In high-mobility 2DES the electronic states at small integer filling factors 1 and 2 are localized. However, very small deviation from integer ν leads to delocalization facilitating formation of a collective mode—a pinned WC—due to pronounced electron-electron interaction. Further deviation from integer ν results in formation of FQHE states.

IV. CONCLUSION

In this paper, we have measured magnetic field dependences of the attenuation and velocity of a SAW in high-mobility n-GaAs/AlGaAs structure. The results allowed us to find complex conductance, $\sigma(\omega) \equiv \sigma_1(\omega) - i\sigma_2(\omega)$, for different frequencies, temperatures, and magnetic fields. We found that at a small exact integer filling factor ($\nu = 1$ and 2) 2D electrons were localized. The observed AC conductivity is of the hopping nature, and it agrees with the two-site model provided that $\omega\tau_0 > 1$. We also found that at filling factor $\nu = 1.1$ a pinned Wigner solid was formed, and that at small deviations of the filling factor from exact integers sharp crossovers between the localized states and Wigner solids were observed in the temperature dependences of σ_1.

ACKNOWLEDGMENTS

I.L.D. is grateful for support from the Russian Foundation for Basic Research via Grant No. 14-02-00232. The authors would like to thank E. Palm, T. Murphy, J.-H. Park, and G. Jones for technical assistance. NHMFL is supported by National Science Foundation Cooperative Agreement No. DMR-1157490 and the State of Florida. The work at Princeton was partially funded by the Gordon and Betty Moore Foundation through Grant No. GBMF2719 and by the National Science Foundation MRSEC-DMR-0819860 at the Princeton Center for Complex Materials.

[1] Y. E. Lozovik and V. I. Yudson, Pis'ma v Zh. Eksp. Teor. Fiz. **22**, 26 (1975) [JETP Lett. **22**, 11 (1975)].
[2] D. Yoshioka and H. Fukuyama, J. Phys. Soc. Jpn. **47**, 394 (1979).
[3] D. S. Fisher, Phys. Rev. B **26**, 5009 (1982).
[4] D. Yoshioka and P. A. Lee, Phys. Rev. B **27**, 4986 (1983).
[5] P. K. Lam and S. M. Girvin, Phys. Rev. B **30**, 473 (1984); **31**, 613 (1985).
[6] D. C. Tsui, H. L. Stormer, and A. C. Gossard, Phys. Rev. Lett. **48**, 1559 (1982).
[7] R. B. Laughlin, Phys. Rev. Lett. **50**, 1395 (1983).
[8] T. Sajoto, Y. P. Li, L. W. Engel, D. C. Tsui, and M. Shayegan, Phys. Rev. Lett. **70**, 2321 (1993).
[9] M. Shayegan, *Flatland Electrons in High Magnetic Fields*, Vol. 3 of High Magnetic Fields: Science and Technology (World Scientific Co, Singapore, 2006), pp. 31–60.
[10] M. Shayegan, *Perspectives in Quantum Hall Effects* (Wiley, New York, 1997), Chap. 9.
[11] W. Pan, H. L. Stormer, D. C. Tsui, L. N. Pfeiffer, K. W. Baldwin, and K. W. West, Phys. Rev. Lett. **88**, 176802 (2002).
[12] C. P. Wen, IEEE Trans. Microwave Theory Tech. **17**, 1087 (1969).
[13] L. W. Engel, D. Shahar, C. Kurdak, and D. C. Tsui, Phys. Rev. Lett. **71**, 2638 (1993).
[14] A. Wixforth, J. P. Kotthaus, and G. Weimann, Phys. Rev. Lett. **56**, 2104 (1986).
[15] I. L. Drichko, A. M. Diakonov, I. Y. Smirnov, Y. M. Galperin, and A. I. Toropov, Phys. Rev. B **62**, 7470 (2000).
[16] M. A. Paalanen, R. L. Willett, P. B. Littlewood, R. R. Ruel, K. W. West, L. N. Pfeiffer, and D. J. Bishop, Phys. Rev. B **45**, 11342 (1992).
[17] M. A. Paalanen, R. L. Willett, R. R. Ruel, P. B. Littlewood, K. W. West, and L. N. Pfeiffer, Phys. Rev. B **45**, 13784(R) (1992).
[18] Y. P. Chen, R. M. Lewis, L. W. Engel, D. C. Tsui, P. D. Ye, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **91**, 016801 (2003).
[19] H. W. Jiang, H. L. Stormer, D. C. Tsui, L. N. Pfeiffer, and K. W. West, Phys. Rev. B **40**, 12013 (1989).
[20] T. Sajoto, Y. W. Suen, L. W. Engel, M. B. Santos, and M. Shayegan, Phys. Rev. B **41**, 8449 (1990).
[21] H. C. Manoharan, Y. W. Suen, M. B. Santos, and M. Shayegan, Phys. Rev. Lett. **77**, 1813 (1996).
[22] I. L. Drichko, I. Yu. Smirnov, A. V. Suslov, and D. R. Leadley, Phys. Rev. B **83**, 235318 (2011).
[23] H. A. Fertig, Phys. Rev. B **59**, 2120 (1999).
[24] H. Yi and H. A. Fertig, Phys. Rev. B **61**, 5311 (2000).
[25] M. M. Fogler and D. A. Huse, Phys. Rev. B **62**, 7553 (2000).
[26] R. Chitra, T. Giamarchi and P. Le Doussal, Phys. Rev. B **65**, 035312 (2001).
[27] M. M. Fogler, Physica E **22**, 98 (2004).
[28] A. T. Hatke, Y. Liu, B. A. Magill, B. H. Moon, L. W. Engel, M. Shayegan, L. N. Pfeiffer, K. W. West, and K. W. Baldwin, Nat. Commun. **5**, 4154 (2014).
[29] I. L. Drichko, A. M. Diakonov, V. A. Malysh, I. Yu. Smirnov, Y. M. Galperin, N. D. Ilyinskaya, A. A. Usikova, M. Kummer, and H. von Känel, J. Appl. Phys. **116**, 154309 (2014).

[30] A. L. Efros, Zh. Eksp. Teor. Fiz. **89**, 1834 (1985) [Sov. Phys. JETP **62**, 1057 (1985)].

[31] Y. M. Galperin and E. Y. Priev, Fiz. Tverd. Tela **28**, 692 (1986) [Sov. Phys. Solid. State **28**, 385 (1986)].

[32] A. L. Efros and B. I. Shklovskii, *Electron-Electron Interaction in Disordered Systems* (Elsevier Science Publishers B. V., Amsterdam, 1985), Chap. 5.

[33] Y. M. Galperin, V. L. Gurevich, and D. A. Parshin, *Hopping Transport in Solids* (Elsevier Science Publishers B. V., Amsterdam, 1991), Chap. 3.

[34] B. G. A. Normand, P. B. Littlewood, and A. J. Millis, Phys. Rev. B **46**, 3920 (1992); Xuejun Zhu, P. B. Littlewood, and A. J. Millis, *ibid.* **50**, 4600 (1994).