RAPID COMMUNICATIONS

PHYSICAL REVIEW B **92**, 180414(R) (2015)

Heat transport between antiferromagnetic insulators and normal metals

Arne Brataas,[*] Hans Skarsvåg, Erlend G. Tveten, and Eirik Løhaugen Fjærbu

Department of Physics, Norwegian University of Science and Technology, NO-7491 Trondheim, Norway

(Received 22 June 2015; revised manuscript received 6 November 2015; published 30 November 2015)

Antiferromagnetic insulators can become active spintronics components by controlling and detecting their dynamics via spin currents in adjacent metals. This cross talk occurs via spin transfer and spin pumping, phenomena that have been predicted to be as strong in antiferromagnets as in ferromagnets. Here, we demonstrate that a temperature gradient drives a significant heat flow from magnons in antiferromagnetic insulators to electrons in adjacent normal metals. The same coefficients as in the spin-transfer and spin-pumping processes also determine the thermal conductance. However, in contrast to ferromagnets, the heat is not transferred via a spin Seebeck effect which is absent in antiferromagnetic insulator-normal metal systems. Instead, the heat is proportional to a large staggered spin Seebeck effect.

DOI: 10.1103/PhysRevB.92.180414 PACS number(s): 72.25.Mk, 72.10.Di, 72.20.Pa, 73.50.Lw

In spintronics, the properties which make antiferromagnets markedly different from ferromagnets also make them attractive in a more dynamic role. Antiferromagnets operate at much higher frequencies and may empower terahertz circuits. They also have no magnetic stray fields, which therefore enables denser spintronics circuits. Antiferromagnets are usually passive spintronics components. However, they can play a role as active components despite their lack of a macroscopic magnetic moment [1–13] and even when they are insulating [10,12,13].

We demonstrate that the thermal coupling between antiferromagnetic insulators (AFIs) and normal metals is relatively strong. The strong thermal coupling facilitates several outcomes: The interface coupling can lead to efficient cooling of antiferromagnetic spintronics devices, might function as heat sensors, and can reveal valuable information about the high-frequency spin excitations in dc measurements that are complicated to extract with other techniques.

Antiferromagnets can produce pure spin currents as large as those produced by ferromagnets. We recently showed that spin pumping may be as operative from antiferromagnets as from ferromagnets [13], in apparent contradiction to naive intuition. Furthermore, the efficiency of spin pumping from antiferromagnets to normal metals implies, via Onsager reciprocity relations, that there is a considerable spin-transfer torque on antiferromagnets from a spin accumulation in adjacent normal metals. However, in the absence of external magnetic fields, the spin Seebeck effect in antiferromagnet-normal metal systems vanishes [14]. This fact seems to indicate that spins in antiferromagnets decouple from, or are only weakly connected to, heat currents and temperature gradients in adjacent normal metals.

To the contrary, we find that the thermal coupling constant is orders of magnitude stronger than its ferromagnetic counterpart. This radical difference is caused by the large exchange field in antiferromagnets that governs the heat transfer rather than the much smaller anisotropy fields or external magnetic fields in ferromagnets. The thermal coupling between antiferromagnetic insulators and normal metals is associated with a staggered spin Seebeck effect rather than via the spin Seebeck effect.

Spin caloritronics determines how spins are coupled to currents and temperature gradients [15]. Measurements of important thermoelectric properties in ferromagnetic insulators, such as the spin Seebeck effect [16], are central to this field. In the spin Seebeck effect, a temperature gradient transfers a magnon spin current in a ferromagnet into an itinerant spin current in a normal metal [17,18]. This process is active even in insulating ferromagnets [19]. The spin Peltier effect is reciprocal to the spin Seebeck effect; a heat current generates a spin accumulation [20,21]. These fascinating thermoelectric properties can be useful to control the heat flow in spintronics devices and in devices that recycle waste heat.

In explaining our calculations, we interpret the theories on the spin Seebeck effect [15–18,22] as a combination of three mechanisms. First, a precessing magnetization can pump a spin current across a ferromagnet-normal metal junction [23–25]. Spin pumping gives rise to an increased magnetization dissipation rate [23,26,27]. Second, the enhanced dissipation implies that there is also an enhanced spin current noise in terms of a fluctuating spin-transfer torque [28]. At equilibrium, there is no thermal bias and the dc spin current vanishes because the temperature-driven spin pumping and a fluctuating spin-transfer torque exactly compensate each other. Third, a temperature difference alters this balance and causes a net spin current [17,18,22].

In this picture, to compute the heat transfer between AFIs and normal metals, we first establish the fluctuating spin transfer and staggered spin transfer in such hybrid systems. Both quantum *and* thermal fluctuations are required to determine the magnon occupations. Subsequently, we use these results to define the thermal gradient-driven (staggered) spin currents, which we then use to evaluate the heat current from the AFI to the normal metal. We focus on insulating antiferromagnets where the transport properties are magnon driven. Generalizations to conducting antiferromagnets are straightforward.

We model the AFI as a two sublattice system with spatiotemporal magnetizations $\mathbf{M}_1$ and $\mathbf{M}_2$. The dynamics are described by the staggered magnetizations $\mathbf{L} = \mathbf{M}_1 - \mathbf{M}_2 = L\mathbf{n}$ and the magnetization $\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2 = L\mathbf{m}$. These fields satisfy the constraints $\mathbf{n}^2 + \mathbf{m}^2 = 1$ and $\mathbf{n} \cdot \mathbf{m} = 0$. At

[*]Arne.Brataas@ntnu.no

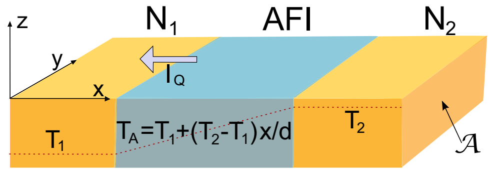

FIG. 1. (Color online) An AFI sandwiched between two normal metals N_1 and N_2. The left normal metal (N_1) is a good spin sink. The electrons in the right normal metal (N_2) are decoupled from the magnons, e.g., the interface coupling is weak and/or there is no spin-memory loss. A heat current between the normal metals flows in response to an applied temperature gradient across the AFI. The cross section is $\mathcal{A}$ and d is the AFI thickness. The heat flow I_Q is along the longitudinal coordinate x.

equilibrium, the staggered field is homogeneous and constant in time, $|\mathbf{L}| = L$, and the magnetization vanishes, $\mathbf{M} = 0$, i.e., $\mathbf{n}^2 = 1$ and $\mathbf{m} = 0$. We consider an easy-axis AFI that is described by the energy $E = \int d\mathbf{r}[\varepsilon(\mathbf{r}) + \varepsilon_s(\mathbf{r})]$, where the energy density is

$$\varepsilon = \frac{L}{\gamma}\left[\frac{1}{2}\omega_E(\mathbf{m}^2 - \mathbf{n}^2) - \frac{1}{2}\omega_A\left(m_z^2 + n_z^2\right)\right], \tag{1}$$

with the exchange frequency ω_E and the smaller anisotropy frequency ω_A, $\omega_A \ll \omega_E$. When $\mathbf{n}$ and $\mathbf{m}$ spatially vary, the stiffness contributions are

$$\varepsilon_s = \frac{L}{2\gamma}\omega_A \sum_{i=x,y,z} [(\lambda_n \partial_i \mathbf{n})^2 + (\lambda_m \partial_i \mathbf{m})^2], \tag{2}$$

where λ_n and λ_m are exchange lengths associated with $\mathbf{n}$ and $\mathbf{m}$, respectively. The dynamic equations are

$$\dot{\mathbf{n}} = \boldsymbol{\omega}_m \times \mathbf{n} + \boldsymbol{\omega}_n \times \mathbf{m} + \boldsymbol{\tau}_n, \tag{3a}$$

$$\dot{\mathbf{m}} = \boldsymbol{\omega}_n \times \mathbf{n} + \boldsymbol{\omega}_m \times \mathbf{m} + \boldsymbol{\tau}_m, \tag{3b}$$

where the effective fields are $\boldsymbol{\omega}_n = -(\gamma/L)\delta\varepsilon/\delta\mathbf{n}$ and $\boldsymbol{\omega}_m = -(\gamma/L)\delta\varepsilon/\delta\mathbf{m}$. In Eqs. (3a) and (3b), the dissipation and fluctuation torques $\boldsymbol{\tau}_m$ and $\boldsymbol{\tau}_n$ are essential to describe spin caloritronics effects.

We consider a thin-film AFI of thickness d sandwiched between two normal metals, the left one of which is a good spin sink (e.g., Pt), and the right is only weakly coupled or has little or no spin-memory loss (see Fig. 1). We assume planar AFI-normal metal interfaces of cross section $\mathcal{A}$. The coordinate $\mathbf{r} = (x,\boldsymbol{\rho})$ is decomposed into a perpendicular coordinate x ($0 \leqslant x \leqslant d$) and the two-dimensional (2D) in-plane coordinates $\boldsymbol{\rho}$. The fluctuation-dissipation torques have bulk and (spin-pumping-induced) interface contributions, $\tau_\nu(\mathbf{r}) = \tau_\nu^{(b)}(\mathbf{r}) + \delta(x - x_I)\tau_\nu^{(p)}(\boldsymbol{\rho})$, where $x_I = 0^+$ is infinitesimally near the interface on the AFI side close to the spin sink and the subscript ν denotes the product of either the subindex n or m and a Cartesian component x, y, or z. The bulk torques arise from the magnon-phonon interaction. At the AFI-N interface, the torques are governed by spin pumping induced by the coupling of the magnetic moments to itinerant electrons in adjacent normal metals. In finding the torques, we introduce dissipation in a similar way as in Ref. [7] and further generalize this description to include quantum and thermal fluctuations. The resulting fluctuation-dissipation torques are

$$\boldsymbol{\tau}_n = (\mathbf{h}_m - \alpha\dot{\mathbf{m}}) \times \mathbf{n} + (\mathbf{h}_n - \alpha\dot{\mathbf{n}}) \times \mathbf{m}, \tag{4a}$$

$$\boldsymbol{\tau}_m = (\mathbf{h}_n - \alpha\dot{\mathbf{n}}) \times \mathbf{n} + (\mathbf{h}_m - \alpha\dot{\mathbf{m}}) \times \mathbf{m}, \tag{4b}$$

for both bulk and interface contributions where we have suppressed the superscript [(b) or (p)] in $\boldsymbol{\tau}$, $\mathbf{h}$, and α. The bulk Gilbert damping constant is $\alpha^{(b)}$. $\alpha^{(p)}$ is a measure of the spin-pumping-induced enhanced dissipation; for homogeneous macrospin excitations the enhanced damping constant is $\alpha^{(b)} + \alpha^{(p)}/d$ [25].

The fluctuation-dissipation theorem implies the existence of the fluctuating forces $\mathbf{h}_m$ and $\mathbf{h}_n$. The average of the independent fluctuating forces $\mathbf{h}_m$ and $\mathbf{h}_n$ and the cross correlations between different fields vanish, but the variance is

$$\left\langle h_\nu^{(p)}(\boldsymbol{\rho}t)h_\nu^{(p)}(\boldsymbol{\rho}'t')\right\rangle = \frac{\gamma\alpha^{(p)}R(t-t',T_1)}{L\pi}\delta(\boldsymbol{\rho}-\boldsymbol{\rho}'), \tag{5a}$$

$$\left\langle h_\nu^{(b)}(\mathbf{r}t)h_\nu^{(b)}(\mathbf{r}'t')\right\rangle = \frac{\gamma\alpha^{(b)}R(t-t',T_A)}{L\pi}\delta(\mathbf{r}-\mathbf{r}'). \tag{5b}$$

The correlation function $R(t,T)$ depends on the (local) temperature. As demonstrated for ferromagnets in Ref. [28], the spin-current fluctuations associated with spin pumping depend on the temperature in the normal metal close to the interface, T_1. We posit that the one-to-one correspondence between spin pumping in ferromagnets and antiferromagnets [13] implies that the spin-current fluctuations in antiferromagnets obeys the same relationship, as in Eq. (5a). In the bulk of the AFI, the phonon-induced fluctuations associated with the bulk Gilbert damping depend on the temperature profile in the antiferromagnet $T_A(x)$, as in Eq. (5b). The correlation function only describes white noise in the (classical) high-temperature limit, $R(t,T) \approx 2\pi k_B T \delta(t)$. However, for the purpose of computing the heat current, we need to take into account the quantum behavior of the fluctuations, which we describe after Eq. (15).

The effective fields determined by Eqs. (1) and (2) are

$$\boldsymbol{\omega}_n = \omega_E\mathbf{n} + \omega_A(\mathbf{n}\cdot\hat{z})\hat{z} + \omega_A(\lambda_n\boldsymbol{\nabla})^2\mathbf{n}, \tag{6a}$$

$$\boldsymbol{\omega}_m = -\omega_E\mathbf{m} + \omega_A(\mathbf{m}\cdot\hat{z})\hat{z} + \omega_A(\lambda_m\boldsymbol{\nabla})^2\mathbf{m}. \tag{6b}$$

In the absence of bulk (electron-magnon) Gilbert damping represented by α, the energy current density $\mathbf{j}_E$ is defined via the continuity equation $\langle\partial_t(\varepsilon + \varepsilon_s)\rangle + \boldsymbol{\nabla}\cdot\mathbf{j}_E = 0$. Because there is no change in external parameters (e.g., spin accumulation) in the system, the energy current can be identified as the heat current. From this continuity equation and by using the dynamic equations (3), with the interface surface normal $\hat{x}$, we find that the total heat current $I_Q = \int d\boldsymbol{\rho}(-\hat{x}\cdot\mathbf{j}_E)$ across the normal-metal–AFI interface is

$$I_Q = \frac{L}{\gamma}\left\langle\omega_A\lambda_n^2\partial_x\mathbf{n}\cdot\partial_t\mathbf{n} + \omega_A\lambda_m^2\partial_x\mathbf{m}\cdot\partial_t\mathbf{m}\right\rangle\Big|_{x=0}. \tag{7}$$

I_Q contains products of the deviations from equilibrium of the staggered field and the magnetization. It is therefore sufficient to carry out the computation of $\mathbf{n}$ and $\mathbf{m}$ in linear response.

We use a circular basis so that $n_\pm = n_x \pm in_y$ and $m_\pm = m_x \pm im_y$ are first-order corrections with respect to the equilibrium configuration $\mathbf{n} = \hat{z}$ and $\mathbf{m} = 0$. Next, we Fourier transform in the transverse coordinate $\boldsymbol{\rho}$ and time t so that any function $c(x,\boldsymbol{\rho},t) = \sum_{\mathbf{q}} \int d\omega \tilde{c}(x,\mathbf{q},\omega) \exp i(\omega t - \mathbf{q} \cdot \boldsymbol{\rho})$. Using Eq. (3), the linearized dynamic equations of motion become

$$\{i\alpha^{(b)}\omega + \omega_A[1 + \lambda_m^2(\mathbf{q}^2 - \partial_x^2)] + 2\omega_E\}\tilde{m}_\pm = \pm\omega\tilde{n}_\pm + \tilde{h}^{(b)}_{m\pm}(x), \tag{8a}$$

$$\{i\alpha^{(b)}\omega + \omega_A[1 + \lambda_n^2(\mathbf{q}^2 - \partial_x^2)]\}\tilde{n}_\pm = \pm\omega\tilde{m}_\pm + \tilde{h}^{(b)}_{n\pm}(x). \tag{8b}$$

In the coupled dynamic equations (8), the stiffness contributions (2) can be interpreted as arising from the continuity equations for the staggered field and the magnetizations, $(\partial_t \mathbf{n})_s + \sum_i \partial_i \mathbf{j}_{n,i} = 0$ and $(\partial_t \mathbf{m})_s + \sum_i \partial_i \mathbf{j}_{m,i} = 0$. In linear response, the staggered spin current and spin current along the x direction are $\mathbf{j}_{n,x} = \omega_A \lambda_m^2 \hat{z} \times \partial_x \mathbf{m}$ and $\mathbf{j}_{m,x} = \omega_A \lambda_n^2 \hat{z} \times \partial_x \mathbf{n}$. The boundary conditions for the linearized equation of motion (8) are obtained by integrating the dynamic equations (3) across the AFI-N interface. This results in the continuity of the staggered spin and spin currents in linearized forms at $x = 0$:

$$\omega_A \lambda_m^2 \frac{\partial \tilde{m}_\pm}{\partial x} = i\omega\alpha^{(p)}\tilde{m}_\pm - \tilde{h}^{(p)}_{m,\pm}, \tag{9a}$$

$$\omega_A \lambda_n^2 \frac{\partial \tilde{n}_\pm}{\partial x} = i\omega\alpha^{(p)}\tilde{n}_\pm - \tilde{h}^{(p)}_{n,\pm}. \tag{9b}$$

Similarly, at $x = d$, there is no loss of currents and the boundary conditions are $\omega_A \lambda_m^2 \partial \tilde{m}_\pm/\partial x = 0$ and $\omega_A \lambda_n^2 \partial \tilde{n}_\pm/\partial x = 0$.

In typical antiferromagnets, ω_E is much larger than all other energy scales and we may employ the so-called exchange approximation. This implies that we may disregard smaller terms in the equation of motion (8a) so that it greatly simplifies to $\tilde{m}_\pm = \omega\tilde{n}_\pm/2\omega_E$. By inserting this relation into Eq. (8b), we find the equation of motion in the exchange approximation

$$\lambda_n^2(q_x^2 + \partial_x^2)\tilde{n}_\pm = -\frac{\tilde{h}^{(b)}_{n\pm}}{\omega_A}, \tag{10}$$

which can be solved with the boundary conditions of Eq. (9b). In the exchange approximation, to the lowest order in the dissipation, we have introduced the longitudinal wave number q_x. The complex wave number q_x is implicitly defined via the relation $\omega = \omega_R + i/t^{(b)}$, where the bulk resonance frequency and the bulk lifetime are determined by

$$\omega_R^2 = 2\omega_A\omega_E[1 + \lambda_n^2(q_x^2 + q_y^2 + q_z^2)], \tag{11a}$$

$$1/t^{(b)} = \alpha^{(b)}\omega_E. \tag{11b}$$

The central results we will obtain can be interpreted in terms of the eigenstates with the associated eigenfrequencies and lifetimes in a thin-film antiferromagnet. The eigenstates are determined by expressing $\tilde{n}_\pm = A_\pm \exp(iq_x x) + B_\pm \exp(-iq_x x)$ in Eq. (10) when the right-hand side (the fluctuations) vanishes. The only nontrivial solution that satisfies both the boundary conditions of Eq. (9b) at $x = 0$ (with no fluctuations) and $\partial\tilde{n}_\pm/\partial x = 0$ at $x = d$ is determined by the secular equation $s(q_x) = 0$, where

$$s(q_x) = \frac{q_x \lambda_n^2 \omega_A}{d\omega} \tan(q_x d) - i\frac{\alpha^{(p)}}{d}. \tag{12}$$

In the absence of spin pumping and bulk damping, the solutions of $s(q_x) = 0$ are standing waves where $q_x = N\pi/d$ and N is an integral number. When spin pumping is weak, the second term in Eq. (12) is small and the solutions of $s(q_x) = 0$ can be expanded around the solutions obtained in the absence of spin pumping. For the higher modes, when $N \neq 0$, we expand the wave vector q_x to the first order in the deviations from $N\pi/d$ and insert the resulting imaginary part of the wave vector into the dispersion relation of Eq. (11a) to find the spin-pumping lifetime $t_N^{(p)}$. For $N = 0$, we carry out a second-order expansion in terms of the small parameter $q_x d$ around 0 and insert this result into the dispersion of Eq. (11a) to find the lifetime $t_0^{(p)}$. We compute that

$$1/t_0^{(p)} = \frac{\alpha^{(p)}}{d}\omega_E, \tag{13a}$$

$$1/t_{N\neq 0}^{(p)} = 2\frac{\alpha^{(p)}}{d}\omega_E. \tag{13b}$$

In a striking contrast to ferromagnets, the spin-pumping-induced scattering rate $1/t^{(p)}$ (13) is proportional to the exchange energy. Similar expressions for the spin-pumping rates in ferromagnets scale with the ferromagnetic spin-wave energy, which is several orders of magnitude smaller than the exchange energy. We know that the spin-pumping-induced effective Gilbert damping coefficients $\alpha^{(p)}$ in insulating antiferromagnet–normal-metal systems are comparable to those of insulating ferromagnet–normal-metal systems (Ref. [29]). We will see that the short spin-pumping-induced AFI lifetimes of Eq. (13) imply a large heat conductance between AFIs and normal metals. Interestingly, we find that the spin-pumping-induced relaxation rate of the higher modes is twice as large as the uniform, but independent of the transverse (2D) wave vector $\mathbf{q}$. This ratio agrees with our previous result for the spin-pumping-induced ratio in thin-film ferromagnets and can be used to distinguish the spin-wave modes [29].

Next, we solve the linearized dynamic equation (8) with the fluctuating bulk forces and subject to the boundary condition (9b) where the fluctuating spin-pumping-induced forces appear. To compute the heat current, we represent the solution at $x = 0$ as $\tilde{m}_+ = \chi^{(p)}_{m+}\tilde{h}^{(p)}_{n+} + \int_0^d dx\, \chi^{(b)}_{m+}(x)\tilde{h}^{(b)}_{n+}(x)$ and $\omega_A\lambda_n^2\partial_x\tilde{n}_+/d = \chi^{(p)}_{n'+}\tilde{h}^{(p)}_{n+} + \int_0^d dx\, \chi^{(b)}_{n'+}(x)\tilde{h}^{(b)}_{n+}(x)$. We find that $\chi^{(p)}_{m+} = -1/[2d\omega_E s(q_x)]$, $\chi^{(b)}_{m+} = \chi^{(p)}_{m+} \cos q_x(d-x)/\cos q_x d$, $\chi^{(p)}_{n'+} = -q_x\lambda^2\omega_A \tan q_x d/d^2\omega s(q_x)$, and $\chi^{(b)}_{n'+} = -i\alpha^{(p)} \cos q_x(d-x)/d^2 s(q_x) \cos q_x d$.

We evaluate the variance of the fluctuating forces and find the heat current, $I_Q = -(2d\omega_E/\pi)\text{Im} \sum_{\mathbf{q}} \int_{-\infty}^{\infty} d\omega[\eta^{(p)}_Q + \eta^{(b)}_Q]$, where the spin-pumping and bulk contributions are

$$\eta^{(p)}_Q = \chi^{(p)}_{m+}(\chi^{(p)}_{n'+})^*\alpha^{(p)}\tilde{R}(\omega,T_1), \tag{14a}$$

$$\eta^{(b)}_Q = \int_0^d dx\, \chi^{(b)}_{m+}(x)[\chi^{(b)}_{n'+}(x)]^*\alpha^{(b)}\tilde{R}(\omega,T_A(x)). \tag{14b}$$

At equilibrium, $T_A(x) = T_1$, the heat current vanishes, $I_Q = 0$, as expected. In linear response, the temperature varies linearly in the AFI so that $T_A(x) = T_1 + (T_2 - T_1)x/d$. We then compute that the heat current is $I_Q = (T_2 - T_1)\kappa_Q$, where

$$\kappa_Q = \sum_{q_y,q_z} \int_{-\infty}^{\infty} d\omega \frac{\alpha^{(b)} \frac{\alpha^{(p)}}{d}}{\pi |s(q_x)|^2} \zeta(q_x) \frac{\partial \tilde{R}(\omega,T)}{\partial T} \qquad (15)$$

and $\zeta(q_x) = 2\int_0^d dx |\frac{\cos q_x(d-x)}{\cos q_x d}|^2 \frac{x}{d^2}$. By following the same methods, we also compute that the temperature-driven spin current, i.e., the spin Seebeck effect, vanishes, in agreement with Ref. [14]. However, we find that the temperature-driven staggered spin current is finite. Furthermore, the heat current is directly proportional to the staggered spin current.

By comparing the equilibrium expectation value of the spin-wave internal energy with the quantum-mechanical result for a magnon gas or, alternatively, by using the fluctuation-dissipation theorem represented by Eq. (4.9) in Ref. [30], we identify that the correlation function $R(\omega,T)$ represents the mean energy at the temperature T of an oscillator at natural frequency ω, $\tilde{R}(\omega,T) = \frac{1}{2}\hbar|\omega| + \hbar\omega f(|\omega|,T)$, where $f(\omega,T)$ is the Bose-Einstein distribution function.

When damping is small, and the spin-pumping-induced damping is smaller than the bulk damping, we can expand the poles of the denominator of Eq. (15) around the spin-wave resonance $q_x d = n\pi$ in a similar way as in Ref. [22]. This results in an intuitive expression:

$$I_Q = \sum_{N=0}^{\infty} \frac{1}{t_N^{(p)}} \int_0^{\infty} d\omega\, D_N(\omega)\hbar\omega\{f(\omega,T_2)[1 - f(\omega,T_1)] - f(\omega,T_1)[1 - f(\omega,T_2)]\}. \qquad (16)$$

The heat current that flows between the normal metals via the antiferromagnet, at each frequency, is proportional to the spin-pumping-induced spin-wave relaxation rate $1/t_N^{(p)}$, the mode-dependent density of states, $D_N(\omega) = \sum_{q_y,q_z} 2\delta[\omega - (2\omega_A\omega_E\{1 + \lambda_n^2[(\frac{N\pi}{d}) + q_y^2 + q_z^2]\})^{1/2}]$. Furthermore, the heat current is determined by the Bose-Einstein occupation of the magnons and the electron-hole pairs in the normal metal. This expression (16) reveals that the thermal coupling between normal metals and AFIs is relatively strong. The heat current is proportional to the spin-pumping-induced spin-wave scattering rates that are proportional to the exchange energy and the Gilbert damping coefficient and therefore are orders of magnitude larger than in ferromagnets. At high temperature, we find $I_Q = \mathcal{A}\pi^2(k_B T_1)^3 k_B(T_2 - T_1)\alpha^{(p)}/(15\sqrt{2}A_{\rm ex}^{3/2}\sqrt{\omega_E}\hbar^{3/2})$, where $A_{\rm ex} = \hbar\omega_A\lambda_n^2$ is the exchange stiffness. For example, using material parameters from Refs. [31,32], we find $\kappa/\mathcal{A} \sim 10^7$ W/m^2 for $RbMnF_3$, whereas a calculation for F-N yields a value $\sim 10^5$W/m^2 for yttrium iron garnet, both at 30 K and assuming a spin mixing conductance $g = 5 \times 10^{18}$ m^{-2}.

Phonons also mediate heat currents between AFIs and normal metals. Experimentally, the magnon-induced heat current we predict here can be separated from the phonon heat current by the different material, temperature, and length dependence. For instance, at temperatures below the magnon gap, magnons do not contribute to the heat conductance. Also, different measurements in systems with normal metals that couple strongly or weakly to the antiferromagnets can be compared. Finally, one can use an external magnetic field to change the magnon dispersion and consequently the spin-wave density of states governing magnon-induced heat current of Eq. (16).

In conclusion, we demonstrated a strong thermal coupling between antiferromagnetic insulators and normal metals. The heat current is directly proportional to the staggered spin current.

We acknowledge support from the Research Council of Norway, Project No. 216700.

[1] A. S. Nunez, R. A. Duine, P. Haney, and A. H. MacDonald, Phys. Rev. B **73**, 214426 (2006).
[2] Z. Wei, A. Sharma, A. S. Nunez, P. M. Haney, R. A. Duine, J. Bass, A. H. MacDonald, and M. Tsoi, Phys. Rev. Lett. **98**, 116603 (2007).
[3] S. Urazhdin and N. Anthony, Phys. Rev. Lett. **99**, 046602 (2007).
[4] P. M. Haney and A. H. MacDonald, Phys. Rev. Lett. **100**, 196801 (2008).
[5] Y. Xu, S. Wang, and K. Xia, Phys. Rev. Lett. **100**, 226602 (2008).
[6] H. V. Gomonay and V. M. Loktev, Low Temp. Phys. **34**, 198 (2008).
[7] K. M. D. Hals, Y. Tserkovnyak, and A. Brataas, Phys. Rev. Lett. **106**, 107206 (2011).
[8] B. G. Park, J. Wunderlich, X. Marti, Y. Kurosaki, M. Yamada, H. Yamamoto, A. Nishide, J. Hayakawa, H. Takahashi, A. B. Shick, and T. Jungwirth, Nat. Mater. **10**, 347 (2011).
[9] X. Marti, B. G. Park, J. Wunderlich, H. Reichlova, Y. Kurosaki, M. Yamada, H. Yamamoto, A. Nishide, J. Hayakawa, H. Takahashi, and T. Jungwirth, Phys. Rev. Lett. **108**, 017201 (2012).
[10] E. G. Tveten, A. Qaiumzadeh, O. A. Tretiakov, and A. Brataas, Phys. Rev. Lett. **110**, 127208 (2013).
[11] X. Marti, I. Fina, C. Frontera, J. Liu, P. Wadley, Q. He, R. J. Paull, J. D. Clarkson, J. Kudrnovsky, I. Turek, J. Kunes, D. Yi, J-H. Chu, C. T. Nelson, L. You, E. Arenholz, S. Salahuddin, J. Foncuberta, T. Jungwirth, and R. Ramesh, Nat. Mater. **13**, 367 (2014).
[12] E. G. Tveten, A. Qaiumzadeh, and A. Brataas, Phys. Rev. Lett. **112**, 147204 (2014).
[13] R. Cheng, J. Xiao, Q. Niu, and A. Brataas, Phys. Rev. Lett. **113**, 057601 (2014).
[14] Y. Ohnuma, H. Adachi, E. Saitoh, and S. Maekawa, Phys. Rev. B **87**, 014423 (2013).
[15] G. E. W. Bauer, E. Saitoh, and B. J. van Wees, Nat. Mater. **11**, 391 (2012).
[16] K. Uchida, S. Takahashi, K. Harii, J. Ieda, W. Koshibae, K. Ando, S. Maekawa, and E. Saitoh, Nature (London) **455**, 778 (2008).
[17] J. Xiao, G. E. W. Bauer, K.-C. Uchida, E. Saitoh, and S. Maekawa, Phys. Rev. B **81**, 214418 (2010).

[18] H. Adachi, J.-I. Ohe, S. Takahashi, and S. Maekawa, Phys. Rev. B **83**, 094410 (2011).

[19] K. Uchida, J. Xiao, H. Adachi, J. Ohe, S. Takahashi, J. Ieda, T. Ota, Y. Kajiwara, H. Umezawa, H. Kawai, G. E. W. Bauer, S. Maekawa, and E. Saitoh, Nat. Mater. **9**, 894 (2010).

[20] J. Flipse, F. L. Bakker, A. Slachter, F. K. Dejene, and B. J. van Wees, Nat. Nanotechnol. **7**, 166 (2012).

[21] J. Flipse, F. K. Dejene, D. Wagenaar, G. E. W. Bauer, J. B. Youssef, and B. J. van Wees, Phys. Rev. Lett. **113**, 027601 (2014).

[22] S. Hoffman, K. Sato, and Y. Tserkovnyak, Phys. Rev. B **88**, 064408 (2013).

[23] Y. Tserkovnyak, A. Brataas, and G. E. W. Bauer, Phys. Rev. Lett. **88**, 117601 (2002).

[24] A. Brataas, Y. Tserkovnyak, G. E. W. Bauer, and B. I. Halperin, Phys. Rev. B **66**, 060404 (2002).

[25] Y. Tserkovnyak, A. Brataas, G. E. W. Bauer, and B. I. Halperin, Rev. Mod. Phys. **77**, 1375 (2005).

[26] S. Mizukami, Y. Ando, and T. Miyazaki, Jpn. J. Appl. Phys. **40**, 580 (2001).

[27] R. Urban, G. Woltersdorf, and B. Heinrich, Phys. Rev. Lett. **87**, 217204 (2001).

[28] J. Foros, A. Brataas, Y. Tserkovnyak, and G. E. W. Bauer, Phys. Rev. Lett. **95**, 016601 (2005).

[29] A. Kapelrud and A. Brataas, Phys. Rev. Lett. **111**, 097602 (2013).

[30] H. B. Callen and T. A. Welton, Phys. Rev. **83**, 34 (1951).

[31] C. G. Windsor and R. W. H. Stevenson, Proc. Phys. Soc., London **87**, 501 (1966).

[32] S. Klingler, A. V. Chumak, T. Mewes, B. Khodadadi, C. Mewes, C. Dubs, O. Surzhenko, B. Hillebrands, and A. Conca, J. Phys. D: Appl. Phys. **48**, 015001 (2015).