

Waveguiding by a locally resonant metasurface

A. A. Maznev^{1,*} and V. E. Gusev^{2,†}

¹*Department of Chemistry, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*

²*Laboratoire d'Acoustique de l'Université du Maine, UMR CNRS 6613, F-72085 Le Mans, France*

(Received 13 February 2015; revised manuscript received 19 July 2015; published 14 September 2015)

Dispersion relations for acoustic and electromagnetic waves guided by resonant inclusions located at the surface of an elastic solid or an interface between two media are analyzed theoretically within the effective medium approximation. Oscillators on the surface of an elastic half-space are shown to give rise to a Love-type surface acoustic wave only existing below the oscillator frequency. A simple dispersion relation governing this system is shown to also hold for electromagnetic waves guided by Lorentz oscillators at an interface between two media with equal dielectric constants. Different kinds of behavior of the dispersion of the resonantly guided mode are identified, depending on whether the bulk wave in the absence of oscillators can propagate along the surface or interface.

DOI: [10.1103/PhysRevB.92.115422](https://doi.org/10.1103/PhysRevB.92.115422)

PACS number(s): 78.67.Pt, 43.35.Pt, 78.68.+m, 73.20.Mf

I. INTRODUCTION

Studies of the interaction of propagating waves with resonant inclusions go back to the Lorentz oscillator model, which forms the basis of the classical dispersion theory [1]. In acoustics, this phenomenon has attracted renewed attention more recently [2–6] in the context of sound propagation in artificial media, referred to as locally resonant metamaterials [2]. It is well known that in a medium containing resonant inclusions [2,7–12], propagating waves hybridize with the local resonance, resulting in an avoided crossing band gap, as shown in Fig. 1(a).

The question we are aiming to address in this paper is what happens when a wave propagating in a three-dimensional (3D) medium interacts with a two-dimensional (2D) array of local oscillators, forming a locally resonant “metasurface” [13]. Will the hybridization with the local resonance yield a guided mode, and what will the dispersion of this guided mode look like? While transmission and reflection of electromagnetic and acoustic waves by metasurfaces are being actively investigated [11,13–16], general waveguiding properties of metasurfaces remain largely unexplored. We will show that a 2D oscillator array supports a guided mode at frequencies below the local resonance frequency, as shown in Fig. 1(b). The dispersion of this guided mode resembles the lower branch of the classic “avoided crossing” case, and the mode becomes increasingly localized on the oscillators on approach to the resonance. Below we consider two examples: (i) horizontal mechanical oscillators at the surface of an isotropic elastic half-space interacting with the shear horizontal (SH) acoustic wave, and (ii) the interaction of undamped Lorentz oscillators at an interface between two media having the same refractive index with a transverse electric (TE) electromagnetic wave. The analysis is performed within the effective medium approximation assuming that the wavelength is much greater than the distance between the oscillators. We will show that both systems yield similar behavior, with a virtually identical dispersion relation for the guided mode. A common feature

of these two cases is that in the absence of the oscillators, the bulk wave can propagate along the surface/interface (i.e., a bulk wave with the wave vector along the surface satisfies the boundary conditions). In this case, the guided mode dispersion does not have a low-frequency cutoff, as shown in Fig. 1(b). A different situation arises when, in the absence of the oscillators, the bulk wave with the wave vector along the interface does not satisfy the boundary conditions, as in the case of different values of the refractive index in the two media. We will see that, in the latter case, the guided mode exists in a band between the lower cutoff frequency and the local resonance frequency. We believe that the properties of locally resonant waveguiding discussed in this paper are generic and will be encountered in many different physical systems.

II. ACOUSTIC CASE: INTERACTION OF AN SH WAVE IN A SOLID HALF-SPACE WITH OSCILLATORS ON THE SURFACE

The mass-spring oscillator has often been used as the simplest model of a mechanical resonator on the surface of an elastic half-space [12,17,18] or a plate [19,20]. We consider the interaction of horizontal mass-spring oscillators of mass M and spring constant K on the surface $z = 0$ of an elastically isotropic solid half-space with a shear horizontal wave propagating in the x direction and polarized in the y direction. The equation of motion reads

$$M\ddot{Y} = K(u_y - Y), \quad (1)$$

where Y is the displacement of the oscillator, and u_y is the horizontal surface displacement at the point of contact of the oscillator with the surface. We assume the dependence $\exp(i\omega t - ikx)$ for both the acoustic wave in the half-space and the vibrations of the oscillators; in the following, this factor will be omitted, and the variables such as Y and u_y will be used to represent the complex amplitudes. From Eq. (1), we get the following expression relating the displacements of the oscillator and of the surface,

$$Y = \frac{\omega_0^2 u_y}{(\omega_0^2 - \omega^2)}, \quad (2)$$

*Corresponding author: alexei.maznev@gmail.com

†Corresponding author: vitali.goussev@univ-lemans.fr

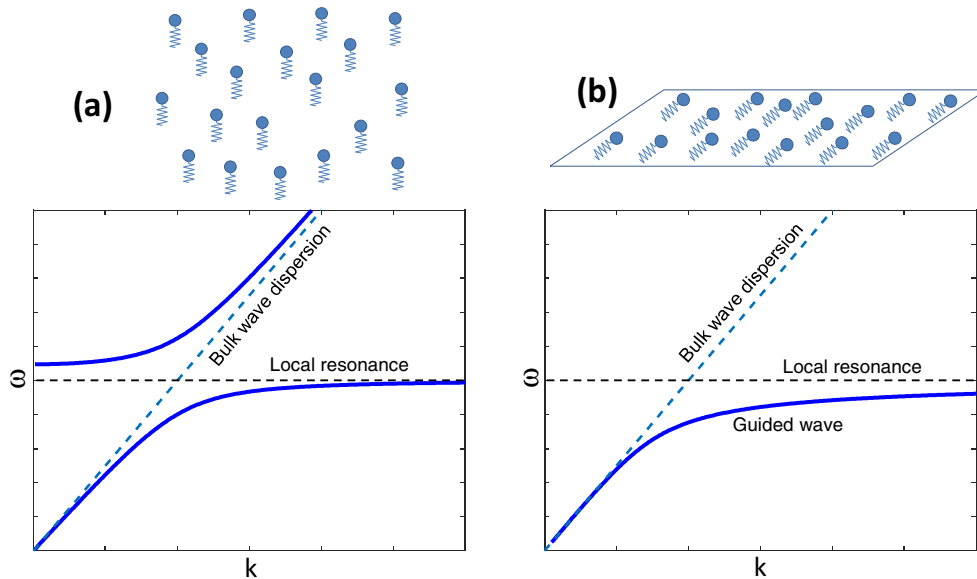


FIG. 1. (Color online) (a) Interaction of a bulk wave with resonant inclusions dispersed in 3D results in hybridization and an avoided crossing band gap; (b) interaction with resonant inclusions contained in 2D results in a guided mode below the local resonance frequency.

where $\omega_0 = (K/M)^{1/2}$ is the resonance frequency of the oscillator. We adopt the effective medium approach, assuming that the wavelength $\lambda = 2\pi/k$ is much greater than the distance between the oscillators. In this approximation, it is not essential to know whether the arrangement of the oscillators is regular or random; the only parameter we need is the average number of oscillators per unit area n . The force exerted by an oscillator on the substrate is $K(Y - u_y)$. In the effective medium approach [21], the force exerted by a distribution of oscillators is averaged over a large area compared to the distance between the oscillators. This average force yields the boundary condition for the shear stress component σ_{zy} at the surface $z = 0$,

$$\sigma_{zy} = -\frac{\omega^2 \omega_0^2 M n u_y}{(\omega_0^2 - \omega^2)}, \quad (3)$$

which replaces the boundary condition $\sigma_{zy} = 0$ for a mechanically free surface. In the half-space, the displacement in an SH wave polarized along y is described by the wave equation,

$$-\omega^2 u_y = c_t^2 \left(-k^2 u_y + \frac{\partial^2 u_y}{\partial z^2} \right), \quad (4)$$

where c_t is the transverse acoustic velocity, with the general solution given by

$$u_y = u_0 e^{-\gamma z} + u_1 e^{\gamma z}, \quad (5)$$

$$\gamma = \left(k^2 - \frac{\omega^2}{c_t^2} \right)^{1/2}. \quad (6)$$

At $\omega > c_t k$, γ is imaginary, yielding bulk waves propagating at an oblique angle to the surface; those solutions are of no interest to us. At $\omega < c_t k$, γ is real, in which case the second divergent term in Eq. (5) should be eliminated, yielding a surface wave decaying with increasing depth. Expressing the stress component σ_{zy} in terms of displacement,

$\sigma_{zy} = \mu(\partial u_y / \partial z)$, where μ is the shear modulus, and plugging stress and displacement at the surface into the boundary condition given by Eq. (3), we get the following dispersion equation,

$$\left(k^2 - \frac{\omega^2}{c_t^2} \right)^{1/2} (\omega_0^2 - \omega^2) = \frac{\omega^2 \omega_0^2 n M}{\mu}, \quad (7)$$

We note that a similar equation can be obtained from Eq. (4) of Ref. [21], describing the interaction of an absorbed monolayer with an elastic half-space, if the interaction within the monolayer is disregarded. By introducing dimensionless frequency and wave vector,

$$\omega' = \omega / \omega_0, \quad k' = k c_t / \omega_0, \quad (8)$$

the dispersion relation can be written in the form

$$(k'^2 - \omega'^2)^{1/2} (1 - \omega'^2) = F \omega'^2, \quad (9)$$

containing a single dimensionless parameter

$$F = \frac{c_t \omega_0 n M}{\mu}, \quad (10)$$

which determines the strength of the interaction between the SH wave and the oscillators. If F vanishes, the dispersion equation separates into two equations describing bulk SH waves and the oscillator resonance. For a nonzero F , the dispersion equation has a single solution for any k , yielding a guided mode at frequencies below ω_0 . Figure 2 shows the dispersion curves calculated for $F = 0.01$ and $F = 1$. In the limit of small k , the dispersion curve approaches the line $\omega = c_t k$, and γ vanishes, which means that the guided mode becomes delocalized and turns into a bulk wave propagating along x . In the opposite limiting case of large k , the frequency approaches ω_0 , phase and group velocities approach zero, and γ approaches infinity, which means that the guided mode turns into localized vibrations of the oscillators.

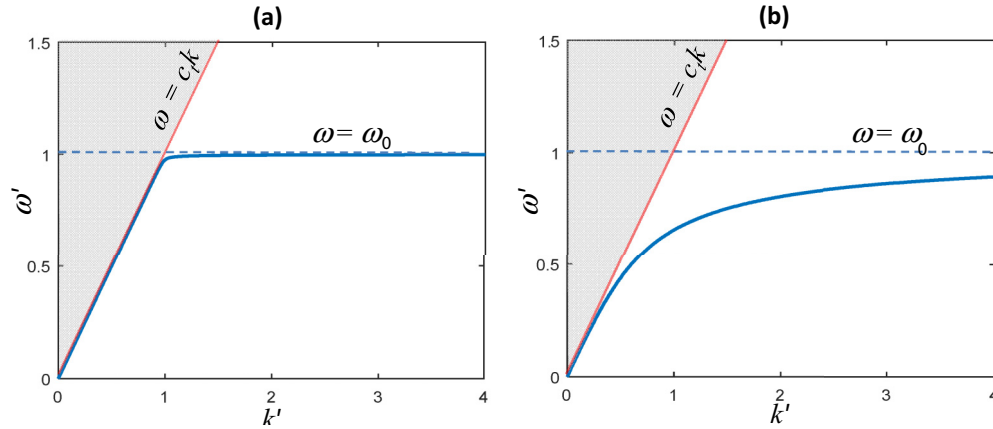


FIG. 2. (Color online) Dispersion of the guided mode (solid curve) as per Eq. (9) for (a) $F = 0.01$ and (b) $F = 1$. Gray-shaded area corresponds to the continuum of bulk modes.

If the interaction parameter is small, as in the case of Fig. 2(a), the guided mode exists only in a formal sense: In experiment, one would either see the bulk SH wave or the nonpropagating local resonance mode. One may ask why, in the weak coupling case, the local resonance mode does not exist at low wave vectors $k' < 1$. The answer is that in this case, the local resonance exists as a leaky mode radiating energy into bulk SH waves. Indeed, from Eq. (9), it is easy to see that at $k' < 1$ and $F \ll 1$, the local resonance frequency will acquire an imaginary part equal to $F/2\sqrt{1-k'^2}$. With increasing F , the leaky local resonance branch will become overdamped and disappear.

III. ELECTROMAGNETIC CASE: INTERACTION OF A TE WAVE WITH A 2D ARRAY OF LORENTZ OSCILLATORS

We consider lossless Lorentz oscillators located at the interface $z = 0$ separating two semi-infinite media. A TE electromagnetic wave with electric field \mathbf{E} along y is propagating along x . The equation of motion of the oscillator reads

$$M\ddot{Y} = -KY + qE_y, \quad (11)$$

where Y is the position of charge q (of mass M), and K is the spring constant. Proceeding similarly to the acoustic case, we get the following expression relating the amplitudes of the oscillator and of the electric field,

$$Y = \frac{qE_y}{M(\omega^2 - \omega_0^2)}. \quad (12)$$

We adopt the effective medium approach, in which n oscillating charges per unit area yield an average current density

$$\mathbf{j} = \frac{i\omega n q^2 E_y}{M(\omega^2 - \omega_0^2)} \delta(z), \quad (13)$$

with $\delta(z)$ being Dirac's delta function. This current leads to the following boundary condition for the x component of the magnetic field at $z = 0$ obtained from Maxwell's equations in Gaussian units,

$$B_x(z = 0 - i\varepsilon) - B_x(z = 0 + i\varepsilon) = \frac{i4\pi\omega n q^2 E_y}{M c_0 (\omega^2 - \omega_0^2)}, \quad (14)$$

where c_0 is the speed of light in vacuum. From Maxwell's equations, the nonzero electric field component in the upper half-space $z > 0$ is given by

$$E_y = E_0 e^{-\gamma_1 z} + E_1 e^{\gamma_1 z}, \quad (15)$$

$$\gamma_1 = \left(k^2 - \frac{\omega^2}{c_1^2}\right)^{1/2}, \quad (16)$$

where c_1 is the speed of light in the upper half-space. At $\omega > c_1 k$, γ_1 is imaginary, yielding bulk waves propagating at an oblique angle to the interface $z = 0$. At $\omega < c_1 k$, γ_1 is real, in which case the second divergent term in Eq. (15) should be eliminated, yielding

$$E_y = E_0 e^{-\gamma_1 z}. \quad (17)$$

Analogously, the electric field in the lower half-space $z < 0$, with the speed of light c_2 , is given by

$$E_y = E_0 e^{\gamma_2 z}, \quad (18)$$

$$\gamma_2 = \left(k^2 - \frac{\omega^2}{c_2^2}\right)^{1/2}. \quad (19)$$

The amplitude factor E_0 is the same in Eqs. (17) and (18) because of the requirement of the continuity of the tangential component of the electric field across the interface. The x component of \mathbf{B} is now given by

$$B_x(z > 0) = \frac{i c_0 \gamma_1}{\omega} E_0 e^{-\gamma_1 z},$$

and

$$B_x(z < 0) = -\frac{i c_0 \gamma_2}{\omega} E_0 e^{\gamma_2 z}. \quad (20)$$

Let us first consider the case when the speeds of light on either side of the interface are equal, $c_1 = c_2 = c$. In this case, the boundary condition given by Eq. (14) yields the following dispersion relation

$$\left(k^2 - \frac{\omega^2}{c^2}\right)^{1/2} (\omega_0^2 - \omega^2) = \frac{2\pi\omega^2 n q^2}{M c_0^2}. \quad (21)$$

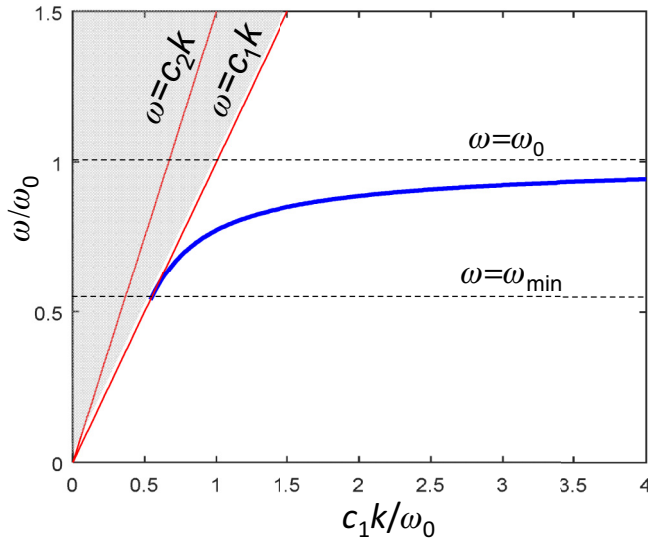


FIG. 3. (Color online) Dispersion of the guided mode (solid curve) in the case of different values of the refractive index on either side of the interface. Gray-shaded area corresponds to the continuum of bulk modes.

This equation is entirely equivalent to Eq. (7) and can be represented in the dimensionless form of Eq. (9) with $F = 2\pi nq^2 c_1 / M\omega_0 c_0^2$.

Let us now consider the case when the media separated by the interface $z = 0$ have different dielectric constants. The dispersion equation now reads

$$\left[\left(k^2 - \frac{\omega^2}{c_1^2} \right)^{1/2} + \left(k^2 - \frac{\omega^2}{c_2^2} \right)^{1/2} \right] (\omega_0^2 - \omega^2) = \frac{4\pi\omega^2 nq^2}{Mc_0^2}. \quad (22)$$

Figure 3 shows an example of the dispersion curve described by Eq. (22) for $c_2/c_1 = 1.5$ and $2\pi nq^2 c_1 / M\omega_0 c_0^2 = 1$. In this case, the guided mode exists in a band between ω_{\min} and ω_0 , with the lower cutoff frequency ω_{\min} given by

$$\omega_{\min} = \sqrt{\omega_0^2 + \Delta^2} - \Delta, \quad \Delta = \frac{2\pi nq^2}{Mc_0(n_1^2 - n_2^2)^{1/2}}, \quad (23)$$

assuming $n_1 > n_2$, where $n_{1,2} = c_0/c_{1,2}$ is the refractive index in the upper/lower half-space, respectively. The smaller the difference between n_1 and n_2 and the larger the density and strength of the oscillators, the larger is the frequency band of the guided mode.

IV. DISCUSSION

We have seen that an array of Lorentz oscillators at an interface between two media with the same refractive index interacting with a TE wave and an array of mechanical oscillators on a solid surface interacting with an SH wave yield a guided mode described by the same dispersion relation. Incidentally, the same dispersion relation has been obtained for acoustic waves in air propagating above an array of Helmholtz resonators (soda cans) [22]. The common feature of these three cases is that in the absence of oscillators, the bulk wave with the wave vector along the surface/interface satisfies the boundary conditions. It is well known that such bulk wave can

easily be turned into a surface wave by a small perturbation. For example, an SH wave in a solid can be localized by a thin “slow” layer (Love waves [23]), by the piezoelectric effect (Gulyaev-Bleustein waves [24]), or by elastic nonlinearity [25,26]. The specificity of the “locally resonant waveguiding” is that the guided mode only exists below the local resonance frequency. In the case of SH acoustic waves, this can be understood by drawing an analogy with mass loading of the surface. In the case of mass loading, the boundary condition at the surface $z = 0$ reads

$$\sigma_{zy} = -\omega^2 \rho_s u_y, \quad (24)$$

where ρ_s is the mass loading per unit area of the surface. This equation becomes formally equivalent to Eq. (3) if we introduce frequency-dependent effective mass loading,

$$\rho_s^{\text{eff}} = \frac{\omega_0^2}{(\omega_0^2 - \omega^2)} Mn. \quad (25)$$

At low frequencies, the effective mass loading is equal to the mass of the oscillators per unit area; it increases and approaches infinity on the approach to the local resonance frequency (of course, the infinity results from disregarding the losses). It is well known that a layer of a soft dense material that can be modeled as mass loading gives rise to a surface Love wave [23]; because of the mass loading, the velocity of the surface wave is lower than that of the bulk SH wave; consequently, the surface wave can propagate without radiating energy into the bulk. On approaching the resonance frequency from below, the phase velocity approaches zero as the mass loading approaches infinity. Above the resonance frequency, however, the effective mass loading per Eq. (25) becomes negative, and hence no surface mode exists.

A more general explanation can be made based on the comparison with the classic avoided crossing case illustrated in Fig. 1(a). The phase velocity of the lower hybridized branch is below the velocity of the bulk wave in the absence of the oscillators, but the velocity of the upper branch is higher than that, which implies that resonant inclusions “soften” the system below ω_0 but “stiffen” it above ω_0 . A mode guided by a 2D array of resonant inclusions ought to be slower than the bulk wave (with the exception of rare occurrences of isolated supersonic surface waves [27]); otherwise, it will get attenuated by radiating energy into the bulk. Consequently, we have waveguiding below ω_0 , where the system “softens,” and no waveguiding above ω_0 , where it “stiffens.”

Let us now turn our attention to the case when the bulk wave propagating along the surface/interface does not satisfy boundary conditions. Two principally different situations are possible here, depending on whether the surface/interface supports a surface wave in the absence of oscillators. The first case is exemplified by surface Rayleigh waves on a solid half-space. The interaction of vertical surface oscillators with a Rayleigh wave has been studied theoretically [17,18] and recently observed experimentally [5]. In this case, a classic avoided crossing takes place. However, in difference to the dispersion shown in Fig. 1(a), the upper dispersion branch of the hybridized mode terminates upon crossing the “transverse threshold” $\omega = c_t k$ [5].

The problem considered in Sec. III, i.e., electromagnetic waves at an interface between two media having different dielectric constants, exemplifies the situation when, in the absence of oscillators, a bulk wave cannot propagate along the interface, and no surface wave exists. In this case, an oscillator array supports a guided wave within a band between a cutoff frequency ω_{\min} given by Eq. (23) and ω_0 . A similar situation arises for SH waves at an interface between two solids having different values of c_t . If the oscillators are introduced at the interface without weakening the mechanical contact (e.g., heavy masses on springs can be placed inside small cavities located at the interface), one gets a dispersion relation entirely analogous to Eq. (22). Qualitatively, the origin of the lower cutoff frequency can be explained as follows: If the bulk wave propagating along the interface without oscillators does not satisfy the boundary conditions, it takes a finite perturbation of the boundary conditions to “localize” the bulk wave. However, at the low-frequency limit, the boundary condition perturbation tends to zero, as can be seen, e.g., from Eq. (3). Hence, there is a finite frequency threshold for the emergence of the guided mode.

Let us now consider an example of a realistic structure in which a guided mode described by Eq. (7) can be observed. Basically, the interaction parameter F should be large enough, i.e., on the order of unity or larger, and the oscillator frequency should be low enough for the effective medium approximation to be valid. Let us consider high-aspect-ratio pillars on an elastic half-space, with the fundamental bending mode of the pillars representing the oscillator. A pillar is not exactly a mass on a spring oscillator; however, since the motion in the bending mode is horizontal, the average force exerted by vibrating pillars on the substrate is also horizontal; hence, in the effective medium model, the pillars can be reasonably well represented by horizontal oscillators. The frequency of a fundamental bending mode of a high-aspect-ratio cylindrical pillar with one end fixed and another end free is given by [28]

$$\omega_0 = \frac{b_1^2 R}{2L^2} \left(\frac{E}{\rho} \right)^{1/2}, \quad (26)$$

where L and R are the height and radius of the pillar, respectively, E is Young’s modulus, ρ is the density, and b_1 is a constant equal to approximately 1.875 [28]. If the pillar is on an elastic substrate, its bottom end is not exactly fixed; however, a correction to Eq. (26) due to elasticity of the substrate is typically disregarded. Plugging Eq. (26) into Eq. (10) results in the following expression for the interaction parameter,

$$F = \frac{\pi b_1^2 n R^3 \sqrt{2(1+2\nu)}}{2L} \frac{Z_p}{Z_s}, \quad (27)$$

where ν is Poisson’s ratio of the pillars, and $Z = c_t \rho$ is the transverse acoustic impedance, with indexes p and s referring to the pillar and substrate materials, respectively. For tungsten pillars on an aluminum substrate arranged in a square lattice with a lattice constant of $3R$, we get $F \approx 7R/L$; hence for $L/R = 10$, we obtain a reasonably high value $F \approx 0.7$. For the same parameters, $k' = 1$ corresponds to a wavelength of $\sim 300R$, which means that the assumption of a large wavelength compared to the distance between the oscillators works quite well until at least $k' = 10$.

It should be noted that the propagation of surface acoustic waves in structures comprising periodic or aperiodic arrays of pillars has been investigated in a number of experimental and theoretical studies [29–31]. While the main emphasis of these studies was on sagittally polarized surface waves, the existence of guided SH waves associated with the resonances of the pillars has been predicted, and their dispersion has been calculated numerically using finite elements analysis [29]. The aspect ratio of pillars in Ref. [29] was smaller than unity, which makes a direct comparison with our model of high-aspect-ratio pillars difficult; moreover, resonance frequencies of short-aspect-ratio pillars are too high for the effective medium model to be valid. However, the lowest SH mode (mode 2) of Ref. [29] associated with the fundamental bending resonance of the pillars is essentially equivalent to the guided SH mode considered in this paper. In Ref. [29], this mode appears to have a low-frequency cutoff; we believe that this fact may be related to the finite depth of the numerical simulation domain [29]: As we have seen, at low frequencies, this mode becomes increasingly delocalized and resembles the bulk SH wave.

Experimental observations of SH waves guided by a one-dimensional array of high-aspect-ratio ridges were reported in Refs. [32,33]. The lowest SH mode observed in these studies, is, again, analogous to the SH guided wave considered in this paper, and its dispersion as calculated by the finite elements analysis [33] indeed looks similar to Fig. 2(b). Since ridges (just as real pillars) possess multiple resonances, multiple guided SH modes associated with higher-order resonances are observed above the fundamental mode.

A mention should be made of studies of acoustic modes of stubbed plates [34–36]. The situation here is more akin to that shown in Fig. 1(a), i.e., plate modes propagating in two dimensions interact with a 2D array of oscillators, yielding hybridization and avoided crossing behavior.

In electromagnetism, waveguiding by a slab comprising an array of conducting wires was considered in Refs. [37,38]. The guided modes in that case were TM (transverse magnetic) rather than TE, and since a wire yields multiple resonances, multiple guided modes are seen [37]. However, the fundamental guided mode associated with the lowest resonance is very much analogous to the mode guided by an array of Lorentz oscillators.

V. CONCLUSION

We have found that a locally resonant metasurface consisting of horizontal oscillators on the surface on an elastic half-space gives rise to a Love-type surface acoustic wave that exists at frequencies below the oscillator frequency. Furthermore, we found that the dispersion relation describing this wave appears in other situations, for example, for TE electromagnetic waves guided by Lorentz oscillators. We identified different kinds of behavior of the dispersion of the resonantly guided mode depending on whether the bulk wave can propagate along the surface or interface in the absence of the oscillators. We believe that a simple model of oscillators with a single degree of freedom yielding a simple dispersion relation is instructive for understanding the general behavior of waveguiding by resonant inclusions.

ACKNOWLEDGMENTS

The authors greatly appreciate stimulating discussions with Tian Gan, who independently derived Eq. (7), as well as with Keith Nelson, Jun Xu, and Nicholas Fang. The contribution

by A.A.M. was supported by the National Science Foundation Grant No. CHE-1111557. The contribution by V.E.G. was supported by the Région Pays-de-la-Loire through Grant “PROPASYM” (Pari Scientifique 2013).

-
- [1] M. Born and E. Wolf, *Principles of Optics* (Pergamon Press, Oxford, 1980).
- [2] Z. Y. Liu, X. X. Zhang, Y. W. Mao, Y. Y. Zhu, Z. Y. Yang, C. T. Chan, and P. Sheng, *Science* **289**, 1734 (2000).
- [3] N. Fang, D. Xi, J. Xu, M. Ambati, W. Srituravanich, C. Sun, and X. Zhang, *Nat. Mater.* **5**, 452 (2006).
- [4] T. Still, W. Cheng, M. Retsch, R. Sainidou, J. Wang, U. Jonas, N. Stefanou, and G. Fytas, *Phys. Rev. Lett.* **100**, 194301 (2008).
- [5] N. Boechler, J. K. Eliason, A. Kumar, A. A. Maznev, K. A. Nelson, and N. Fang, *Phys. Rev. Lett.* **111**, 036103 (2013).
- [6] F. Lemoult, N. Kaina, M. Fink, and G. Lerosey, *Nature Phys.* **9**, 55 (2013).
- [7] J. H. Page, P. Sheng, H. P. Schriemer, I. Jones, X. Jing, and D. A. Weitz, *Science* **271**, 634 (1996).
- [8] Z. Liu, C. T. Chan, P. Sheng, A. L. Goertzen, and J. H. Page, *Phys. Rev. B* **62**, 2446 (2000).
- [9] R. Sainidou, N. Stefanou, and A. Modinos, *Phys. Rev. B* **66**, 212301 (2002).
- [10] C. Goffaux and J. Sánchez-Dehesa, *Phys. Rev. B* **67**, 144301 (2003).
- [11] Y. Pennec, B. Djafari Rouhani, H. Larabi, A. Akjouj, and G. Lèveque, *New J. Phys.* **14**, 073039 (2012).
- [12] V. E. Gusev and O. B. Wright, *New J. Phys.* **16**, 123053 (2014).
- [13] A. V. Kidishev, A. Boltasseva, and V. M. Shalaev, *Science* **339**, 1232009 (2013).
- [14] N. Yu and F. Capasso, *Nat. Mater.* **13**, 139 (2014).
- [15] N. K. Grady, J. E. Heyes, D. R. Chowdhury, Y. Zeng, M. T. Reiten, A. K. Azad, A. J. Taylor, D. A. R. Dalvit, and H.-T. Chen, *Science* **340**, 1304 (2013).
- [16] V. E. Gusev, *J. Appl. Phys.* **117**, 054903 (2015).
- [17] A. R. Baghai-Wadji, V. P. Plessky, and A. V. Simonian, *Akust. Zh.* **38**, 806 (1992) [*Sov. Phys. Acoust.* **38**, 442 (1992)].
- [18] E. A. Garova, A. A. Maradudin, and A. P. Mayer, *Phys. Rev. B* **59**, 13291 (1999).
- [19] Y. Xiao, J. Wen, and X. Wen, *J. Phys. D: Appl. Phys.* **45**, 195401 (2012).
- [20] D. Torrent, D. Mayou, and J. Sanchez-Dehesa, *Phys. Rev. B* **87**, 115143 (2013).
- [21] Yu. A. Kosevich and E. S. Syrkin, *Phys. Lett. A* **135**, 298 (1989).
- [22] A. A. Maznev, G. Gu, S. Sun, J. Xu, Y. Shen, N. Fang, and S. Zhang, *New J. Phys.* **17**, 042001 (2015).
- [23] B. A. Auld, *Acoustic Fields and Waves in Solids* (Krieger, Malabar, 1990).
- [24] S. V. Biryukov, Y. V. Gulyaev, V. V. Krylov, and V. P. Plessky, *Surface Acoustic Waves in Inhomogeneous Media* (Springer-Verlag, Berlin, 1995).
- [25] V. G. Mozhaev, *Phys. Lett. A* **139**, 333 (1989).
- [26] V. Gusev, W. Lauriks, C. Glorieux, and J. Thoen, *Phys. Lett. A* **232**, 77 (1997).
- [27] V. I. Alshits, A. N. Darinskii, and A. L. Shuvalov, *Phys. Ser. T* **44**, 85 (1992).
- [28] K. F. Graff, *Wave Motion in Elastic Solids* (Dover, New York, 1975).
- [29] A. Khelif, Y. Achaoui, S. Benchabane, V. Laude, and B. Aoubiza, *Phys. Rev. B* **81**, 214303 (2010).
- [30] Y. Achaoui, A. Khelif, S. Benchabane, L. Robert, and V. Laude, *Phys. Rev. B* **83**, 104201 (2011).
- [31] Y. Achaoui, V. Laude, S. Benchabane, and A. Khelif, *J. Appl. Phys.* **114**, 104503 (2013).
- [32] V. Laude, L. Robert, W. Daniau, A. Khelif, and S. Ballandras, *Appl. Phys. Lett.* **89**, 083515 (2006).
- [33] V. Laude, M. B. Dühring, H. Moubchir, N. Khelifaoui, and A. Khelif, in Proc. 2007 IEEE Ultrasonics Symposium (IEEE, Piscataway, 2007), p. 175.
- [34] T.-T. Wu, Z.-G. Huang, T.-C. Tsai, and T.-C. Wu, *Appl. Phys. Lett.* **93**, 111902 (2008).
- [35] Y. Pennec, B. Djafari-Rouhani, H. Larabi, J. O. Vasseur, and A. C. Hladky-Hennion, *Phys. Rev. B* **78**, 104105 (2008).
- [36] M. Oudich, Y. Li, B. M. Assouar, and Z. Hou, *New J. Phys.* **12**, 083049 (2010).
- [37] P. A. Belov and M. G. Silveirinha, *Phys. Rev. E* **73**, 056607 (2006).
- [38] F. Lemoult, G. Lerosey, and M. Fink, *Waves Random Complex Media* **21**, 591 (2011).