

Universal postquench coarsening and aging at a quantum critical point

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The nonequilibrium dynamics of a system that is located in the vicinity of a quantum critical point is affected by the critical slowing down of order-parameter correlations with the potential for novel out-of-equilibrium universality. After a quantum quench, i.e., a sudden change of a parameter in the Hamiltonian, such a system is expected to almost instantly fall out of equilibrium and undergo aging dynamics, i.e., dynamics that depends on the time passed since the quench. Investigating the quantum dynamics of an N -component φ^4 model coupled to an external bath, we determine this universal aging and demonstrate that the system undergoes a coarsening, governed by a critical exponent that is unrelated to the equilibrium exponents of the system. We analyze this behavior in the large- N limit, which is complementary to our earlier renormalization-group analysis, allowing in particular the direct investigation of the order-parameter dynamics in the symmetry-broken phase and at the upper critical dimension. By connecting the long-time limit of fluctuations and response, we introduce a distribution function that shows that the system remains nonthermal and exhibits quantum coherence even on long time scales.

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I. INTRODUCTION

Nonequilibrium behavior of interacting many-body systems is becoming a field of increasing importance in different areas of physics. It is largely driven by two main experimental advances: (i) the ability to bring a system into an out-of-equilibrium state in a controlled and reproducible manner and (ii) the ability to observe and follow the subsequent dynamics in real time, i.e., on microscopic time scales. Different tools have been developed for different experimental systems. Important examples are various cold-atom setups [1–6], ultrafast pump-probe spectroscopy of quantum materials [7–10], and heavy-ion collisions that explore the dynamics of the quark-gluon plasma [11]. These experiments clearly reveal that observations far away from equilibrium can yield new insights.

Nonstationary out-of-equilibrium dynamics is often characterized by a lack of time-translation invariance. As a result, the fluctuation-dissipation theorem does not typically hold and memory effects occur; i.e., the dynamics depends on the initial state of the system. Major questions in nonequilibrium are (i) what are the effects of interactions; do nonlinearities, aging, and memory effects occur, and how are they characterized [12–14]; (ii) what are the properties of transient metastable “prethermal” states [15–27]; and (iii) how does the system eventually reach a steady state and is it given by a (generalized) thermal distribution [2,28–30]? Aging describes the phenomenon that correlation and response functions $G^K(t, t')$ and $G^R(t, t')$ depend on both time arguments t, t' and not just on their time difference $t - t'$ as is the equilibrium case; i.e., they depend on the age of the system. These effects are well known to occur in kinetic Ising models, structural glasses, spin glasses, and disordered systems [31–37]. As these effects are related to spatial and temporal fluctuations it is often required to perform an analysis beyond the mean-field approximation.

It is in general difficult to make quantitative predictions for the nonequilibrium dynamics of interacting quantum many-

body systems beyond the mean-field approximation [38]—except in cases where exact solutions are available [39–49] or when controlled numerical approaches are established such as for impurity models [50–57] and for one-dimensional systems [22,58–66]. Here, we exploit the presence of a quantum critical point to analytically solve for the universal dynamics of an interacting quantum φ^4 model, which is an effective description of a number of experimental systems [67]. We obtain solutions both for short as well as for long times.

Universality close to (quantum) critical points is well established in equilibrium and follows from a divergence of the correlation length ξ and time ξ_τ near criticality [67–69]. Observables such as the order parameter or correlation functions can be expressed in terms of universal scaling functions with dimensionless arguments. As a function of, for example, the distance to the critical point they follow power laws with universal critical exponents. In nonequilibrium, the correlation length is itself a function of time. For example, if a parameter of a system is changed sufficiently slowly, the state and the correlation length of the system can adiabatically follow this change. Close to a critical point, however, adiabaticity demands that the parameter change must occur infinitesimally slowly due to the divergence of the correlation time. At any finite sweep rate, the system eventually falls out of equilibrium. In the Kibble-Zurek description of such a parameter sweep through a critical point, the correlation length is assumed to remain constant at this freeze-out length scale for the remainder of the sweep [70,71]. It then follows that the number of (topological) excitations depends on the rate via a universal scaling law that solely contains equilibrium critical exponents [72–76]. Similarly, it follows that the long-time approach to equilibrium of a system that is suddenly quenched close to a (quantum) critical point is governed by equilibrium exponents [77]. We confirm this behavior in our study as well. Kibble-Zurek scaling has been experimentally observed, in particular in the regime of slow sweeps [2,78–80]. In the

opposite regime of fast parameter quenches also different scaling behavior has been reported [81].

Recently, however, it was shown for the Kibble-Zurek regime of a gradual sweep that the dynamics of a system does not completely freeze-out once the system falls out of equilibrium [82]. Instead, the correlation length $\xi(t)$ remains a function of time. If the system is located close to a critical point, where ξ diverges in equilibrium, the correlation length in fact diverges in a light-cone-like fashion according to a power law

$$\xi(t) \propto t^{1/z_d} \quad (1)$$

with a characteristic dynamic coarsening exponent z_d [82]. While the coarsening exponent is in general different from the dynamic critical exponent z that characterizes the quantum dynamics in equilibrium, it turns out that the two are equal in our approach. This scaling behavior (1) is also one of the results of our calculations for the case of a quantum quench. A crucial implication of it is that correlation and response functions G^K, G^R obey a generalized scaling form, which incorporates the time dependence of $\xi(t)$. Using this scaling form, we are able to find an analytic solution of the universal dynamics of correlation and response functions, both at short and at long times.

In this article we study the postquench nonequilibrium dynamics of a dissipative N -component φ^4 model. Dissipation is introduced via coupling to an external bosonic bath with Ohmic, sub-Ohmic, or super-Ohmic spectrum. In equilibrium, the model exhibits a quantum critical point (QCP) that separates a disordered (symmetry-unbroken) from an ordered (symmetry-broken) phase. The equilibrium properties of this model are discussed, for example, in Refs. [67,83,84]. We consider a system that is initially prepared in an equilibrium state away from the quantum critical point, where the correlation length ξ is finite. We discuss both the situation of an initial state in the symmetry-broken and in the symmetry-unbroken phase. Then, at time $t = 0$ a parameter in the system is rapidly changed, i.e., quenched, to a value that corresponds to the quantum critical point in equilibrium. Experimentally, this parameter can, for example, be pressure, strain, magnetic field, or interaction strength.

We are interested in the postquench dynamics of the correlation and response functions as well as the time evolution of the order parameter itself. We obtain analytic expressions for the universal part of the time dependence, inspired by Refs. [12,85]. For a system at high and low temperature due to the rapid quench the correlation length first collapses to a nonuniversal value of the order of a microscopic length scale in the system. Since the system is located at the (quantum) critical point, however, it then recovers by critical coarsening in the light-cone fashion $\xi(t) \propto t^{1/z_d}$. An important implication of this power-law recovery of ξ , both for classical and quantum dynamics, is a universal time regime, that resembles the frequently discussed prethermal regime. There, the dynamics of the correlation function and the order parameter $\phi(t)$ is described by the universal critical exponent

$$\phi(t) \propto t^\theta. \quad (2)$$

For $\theta > 0$ it follows that the order parameter increases after an initial collapse due to the fast spreading of correlations

in the system, while for $\theta < 0$ the order decreases [86]. The exponent θ also characterizes the divergence in the classical and quantum response functions at short times. While the dynamics of the order parameter and the response function turns out to be similar in both the high and the low temperature regime, we find crucial differences in the behavior of the correlation function and the value of the exponent θ . For example, in the limit $t \gg t'$, the free quantum correlation function $G^K(q, t, t')$ takes the form

$$G^K(q, t, t') = G^R(q, t) \propto t^{-1+2/z} \quad (3)$$

for short times $t < \gamma^{z/2} q^{-z}$, where γ is the coupling to the bath, in contrast to the classical result $G_{\text{cl}}^K(q, t, t') \propto t^{2/z}$.

The duration of the prethermalization regime is determined by the quench amplitude and increases for smaller quench amplitudes, i.e., if the initial state of the system was close to criticality. We note that while universality is often associated with large length and time scales, it here arises from the critical light-cone recovery of the correlation length over time, similar to the coarsening in heterogeneous systems [87].

After the prethermalized regime at large times, we show that the system eventually relaxes to equilibrium quasiadiabatically with a power law that contains equilibrium critical exponents. Still, the prethermalization exponent θ enters the long-time expressions as a universal prefactor in the relaxation amplitudes of the order parameter and the correlation functions. We note that the dominant slow dynamics is determined by the interactions that are present within the model and the coupling to the bath mainly ensures equilibration in the long-time limit. At long times we can define a time-dependent distribution function $n(t, \omega)$ using the relation between response and correlation functions known as the fluctuation-dissipation theorem in equilibrium. We find that the deviation δn from the equilibrium Bose-Einstein distribution is given by

$$\delta n(t, \omega) = -\coth\left(\frac{\omega}{2T}\right) \frac{\gamma\theta\Gamma(2/z)}{\sin(\pi/z)t^{2/z}} \text{Re } G_{\text{eq}}^R(q, \omega), \quad (4)$$

where T is the bath temperature, G_{eq}^R denotes the equilibrium retarded Green's function at the QCP, and t is the time after the quench. As we show, this result implies that the distribution function is nonthermal even in the long-time regime: δn decays only algebraically at large frequencies, while a thermal distribution decays exponentially. The approach to equilibrium over time is slow and described by a power law $\propto t^{-2/z}$. Finally, the fact that δn changes sign and can become negative for $z < 2$ implies that the density matrix of the system is nondiagonal in the energy basis and quantum coherence is present.

The quench protocol to the critical point that we consider is inspired by the pioneering work on classical critical points by Janssen, Schaub, and Schmittmann in Ref. [12] who analyzed the postquench dynamics of a classical φ^4 theory in contact with an Ohmic bath. The analysis of the classical case was extended to colored noise in Ref. [85]. Previously, we have reported results on the dissipative quantum φ^4 model using a renormalization-group (RG) approach in nonequilibrium and calculated the new exponent θ in an expansion in $\epsilon = 4 - d - z$ close to the upper critical dimension [88]. For a closed φ^4 model without an external bath, θ was recently derived in Refs. [89,90] using a different nonequilibrium

renormalization-group method. In this paper, we complement our previous analysis by an expansion in $1/N$, where N is the number of components of the field. We also largely extend our previous study, which focused on a quench starting from the symmetry-unbroken phase, by a direct investigation of the order parameter dynamics $\phi(t)$ starting from an initial state with nonzero ϕ . We explicitly prove that the exponent θ is independent of the initial state and the particular quench protocol. All previous investigations were limited to space dimensions below the upper critical dimension. We also discuss the behavior at the upper critical dimension $d_{uc} = 4 - z$, where logarithmic corrections occur and

$$\phi_{uc}(t) \propto [\ln(t/t_\gamma)]^{\theta_{uc}} \quad (5)$$

with t_γ being a microscopic time scale that marks the beginning of the universal prethermalization regime.

The remainder of the paper is organized as follows: In Sec. II, we introduce the model and the Hamiltonian, and define the quench protocol and the coupling to the external bath. In Sec. III, we present a nonequilibrium formulation of the large- N expansion and derive the large- N saddle-point equations. To solve these equations self-consistently we employ a scaling form for the Green's functions that we discuss in Sec. IV. We present the large- N solution for a quench to the quantum critical point starting from the disordered phase in Sec. V and starting from the ordered phase in Sec. VI. In Sec. VA, we relate the short-time scaling exponent θ to the time dependence of the self-energy, which can be captured by a time-dependent effective mass, and determine the short-time behavior of the Green's functions. In Sec. VB, we elaborate on the long-time behavior of the Green's function, which enters into the central calculation in Sec. VC, where we show the self-consistency of the solution, which fixes the value of the exponent θ . In Sec. VD we show that the system is characterized by a nonthermal distribution function at long times. In Secs. VE and VIC we discuss the dynamics at the upper critical dimension. In Sec. VI, we consider a quench that starts from the ordered phase, and find in Sec. VIA the scaling of the order parameter and the crossover time that separates the regimes of prethermal scaling dynamics from the adiabatic long-time dynamics that is characterized by equilibrium critical exponents as discussed in Sec. VIB. We finally summarize our main results and conclude in Sec. VII. In the appendices, we provide further details on the analytical calculations, in particular on the different short time scales in the problem in Appendix A, on the derivation of the large- N equations in Appendix B, on the free postquench Keldysh function in Appendix C, and on the long-time limit of the full Green's functions in Appendix D.

II. THE MODEL AND THE QUENCH PROTOCOL

A. The φ^4 model

We consider the postquench nonequilibrium dynamics of a φ^4 model coupled to an external bath. The system is schematically depicted in Fig. 1(a). To be specific, we also show one particular experimental realization, the dimer antiferromagnet realized, for example, in TlCuCl_3 [91] in Fig. 1(b). Other realizations of our theory are discussed below.

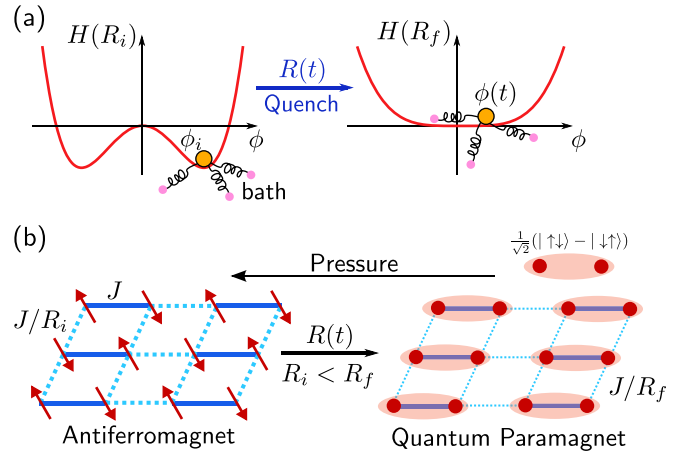


FIG. 1. (Color online) (a) Schematic description of the φ^4 model coupled to an external bath. The order parameter ϕ experiences a potential landscape that depends on a set of parameters R . If the initial parameters R_i are such that the system is prepared in the symmetry-broken state with finite order parameter ϕ_i , we consider a sudden change of the parameters from R_i to R_f that brings the system to the quantum critical point, where $\phi = 0$ in equilibrium. (b) One experimental realization of the φ^4 model for $N = 3$ is a quantum dimer model in contact with phonons. The quench can be performed by rapidly changing the pressure.

The φ^4 model is described by the Hamiltonian

$$H_s(t) = \frac{1}{2} \int_x \left[\pi^2 + (\nabla \varphi)^2 + r_0(t) \varphi^2 + \frac{u(t)(\varphi \cdot \varphi)^2}{2N} - \mathbf{h}(t) \cdot \varphi \right]. \quad (6)$$

Here, $\varphi(x) = (\varphi_1(x), \dots, \varphi_N(x))$ is an N -component vector field and $\pi(x)$ its canonically conjugated momentum with $[\varphi_l(x), \pi_{l'}(x')]_- = i \delta_{ll'} \delta(x - x')$. In addition, we used the shorthand notation $\int_x \equiv \int d^d x$. The nonequilibrium dynamics is a consequence of the explicit time dependence of either the bare mass $r_0(t)$, the interaction strength $u(t)$, or the externally applied field $\mathbf{h}(t)$, respectively. In the case of constant parameters the system is in equilibrium and undergoes at temperature $T = 0$ and for $\mathbf{h} = \mathbf{0}$ a quantum phase transition at a critical value $r_{0,c}(u)$ of the bare mass r_0 . This transition can be captured analytically by using an expansion for small $1/N$. In fact, the coefficient $1/N$ in front of the φ^4 term was introduced to allow for a well-defined $N \rightarrow \infty$ limit. The large- N approximation is complementary to our earlier nonequilibrium renormalization-group theory of Ref. [88]. In particular, it allows for a rather straightforward analysis of the dynamics of a system that was initially in the symmetry-broken phase.

B. The quench protocol

We consider the following protocols for the time dependence of

$$R(t) \equiv (r_0(t), u(t), \mathbf{h}(t)). \quad (7)$$

The system is initially prepared in the ground state with $R_i = (r_{0,i}, u_i, \mathbf{h}_i)$ somewhat away from the critical point located

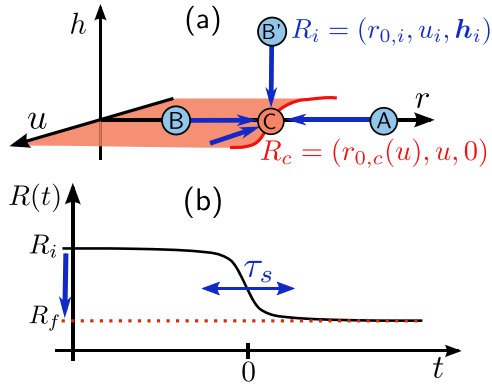


FIG. 2. (Color online) (a) Schematic equilibrium phase diagram of the φ^4 model at $T = 0$ including the different quench paths that we consider in this article. We either approach the quantum critical point C from the symmetric side (path $A \rightarrow C$) or from the symmetry-broken side (path $B \rightarrow C$). We also consider a quench in the presence of a magnetic field \mathbf{h}_i that induces a finite initial order parameter ϕ_i (path $B' \rightarrow C$). (b) Schematic plot of the quench protocol. We consider fast quenches where the switching time τ_s from the initial parameter set R_i to the final one R_f occurs on a microscopic time scale.

at $R_c = (r_{0,c}(u), u, 0)$. Then we switch $R(t)$ such that it approaches a final value $R(t \rightarrow \infty) = R_f$ that is closer to the critical point. The schematic phase diagram showing the parameter quenches that we consider in the following are shown in Fig. 2(a). The switching of $R(t)$ takes place on a time scale τ_s . We consider the limit where τ_s is of the order of the microscopic time scales of the system, i.e., the fastest time that is consistent with the validity of the φ^4 model as an effective low energy description. This situation is depicted in Fig. 2(b). Keeping this in mind, we can safely perform the limit $\tau_s \rightarrow 0$ in the calculation such that

$$R(t) = R_i + \theta(t)(R_f - R_i), \quad (8)$$

with step function $\theta(t)$. After the quench the system instantly falls out of equilibrium.

C. Coupling to an external bath

In the context of condensed matter systems, the generic situation is an open system that is coupled to external bath degrees of freedom. The bath, here conveniently described by a set of harmonic oscillators, is assumed to stay at a constant temperature, T . In particular, if $T = 0$ and $R_f = R_c$, the system does indeed reach the quantum critical point in the limit $t \rightarrow \infty$. This is to be contrasted with the behavior of a closed system, where energy conservation alone implies that the system will not be in the ground state of the postquench Hamiltonian [89,92]. Even if the system thermalizes, a quench towards the critical point will heat up a closed system to a finite temperature. If we include the coupling to the bath, the full Hamiltonian reads

$$H(t) = H_s(t) + H_b + H_{sb}. \quad (9)$$

Here, H_b describes the bath of harmonic oscillators and H_{sb} the linear coupling between system and bath:

$$H_b = \frac{1}{2} \sum_j \int d^d x (\mathbf{P}_j^2 + \Omega_j^2 \mathbf{X}_j^2), \quad (10)$$

$$H_{sb} = \sum_j c_j \int d^d x \mathbf{X}_j \cdot \boldsymbol{\varphi}.$$

In accordance with Eq. (8) the full Hamiltonian before and after the quench is then $H_i = H(t < 0)$ and $H_f = H(t > 0)$, respectively.

In Eq. (10) Ω_j are the frequencies of bath oscillators that couple with coupling constants c_j . As the bath stays in equilibrium throughout, it is fully characterized in terms of the retarded function

$$\eta(\omega) = - \sum_j \frac{c_j^2}{(\omega + i0^+)^2 - \Omega_j^2}. \quad (11)$$

The imaginary part of $\eta(\omega)$ is the spectral function of the bath. We consider a bath with spectral function

$$\text{Im}\eta(\omega) = \gamma \omega |\omega|^{\alpha-1} e^{-|\omega|/\omega_c}. \quad (12)$$

Here, the damping coefficient γ determines the overall strength of the coupling to the bath and ω_c is the ultraviolet cutoff of the bath spectrum. The exponent α characterizes the low-frequency behavior. $\alpha = 1$ corresponds to an Ohmic, $\alpha > 1$ to a super-Ohmic, and $\alpha < 1$ to a sub-Ohmic bath [93]. The frequency dependence of $\text{Im}\eta(\omega)$ for different α is depicted in Fig. 3(b). A Kramers-Kronig transformation yields the real part of $\eta(\omega)$. We obtain

$$\begin{aligned} \delta\eta(\omega) &\equiv \eta(\omega) - \eta(0) \\ &= \gamma \left(-\cot\left(\frac{\pi\alpha}{2}\right) + i \text{sign}(\omega) \right) |\omega|^\alpha \end{aligned} \quad (13)$$

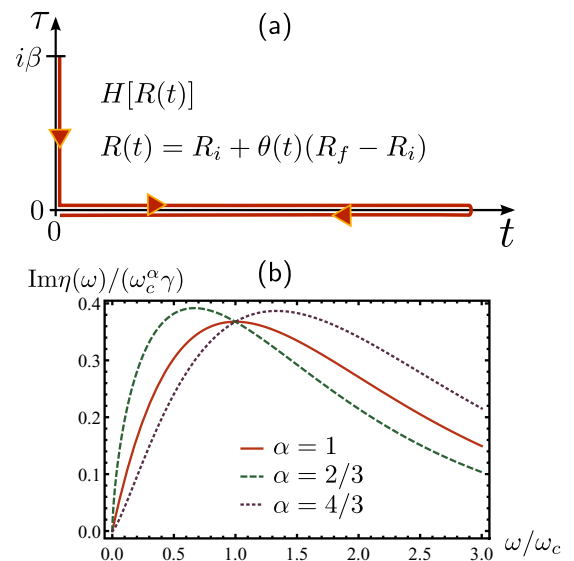


FIG. 3. (Color online) (a) Keldysh three-time contour. (b) Bath spectral function for different power-law exponents corresponding to the sub-Ohmic case $\alpha < 1$, the Ohmic case $\alpha = 1$, and the super-Ohmic case $\alpha > 1$.

for frequencies small in magnitude compared to the cutoff ω_c . The zero-frequency value $\eta(0) = \frac{\gamma\alpha}{2\pi}\omega_c^\alpha$ depends explicitly on the value of the bath cutoff. As we will see, $\eta(0)$ merely shifts the location of the quantum critical point of the system but does not affect the generic behavior near it. Below we will also need the analytic continuation $\eta^M(\omega_n) = \eta(0) + \delta\eta^M(\omega_n)$ of $\eta(\omega)$ to the Matsubara axis ($\omega + i0^+ \rightarrow i\omega_n = i2n\pi T$). It holds that

$$\delta\eta^M(\omega_n) = -\frac{\gamma}{\sin\frac{\pi\alpha}{2}}|\omega_n|^\alpha, \quad (14)$$

again valid for frequencies that are small compared to the bath cut off: $|\omega_n| \ll \omega_c$.

D. Experimental realizations

There are a number of experimental systems that can be effectively described by an N -component φ^4 theory that we are considering. Realizations for $N = 1$ are the magnetic insulators CoNb_2O_6 [94] and $\text{LiHo}_x\text{Y}_{1-x}\text{F}_4$ [95], which can be described by the transverse field quantum Ising model. Dissipation in these systems arises via coupling to phonons. Realizations for $N = 2$ are systems near the superconductor-insulator (or superfluid-insulator) transition, Josephson junction arrays, and quantum antiferromagnets in a magnetic field [67]. Superfluid-insulator transitions in the Bose-Hubbard model have been experimentally investigated using cold-atom quantum gases confined to optical lattice potentials [96,97]. Dissipation in those system has been engineered, for example, by coupling to other species [98]. Dissipative nanowires near a transition to a superconducting state [99] and an ensemble of qubits in a photon cavity [100,101] provide further realizations for $N = 2$. A realization for $N = 3$ is the quantum dimer antiferromagnet TlCuCl_3 shown in Fig. 1(b), which can be driven across the quantum phase transition by either changing pressure [91] or magnetic field [102].

III. NONEQUILIBRIUM FORMULATION OF THE LARGE- N EXPANSION

The natural technique to treat the postquench dynamics is the nonequilibrium many-body formalism due to Schwinger [103], Kadanoff-Baym [104], and Keldysh [105] (see also Ref. [106]), where the field theory is placed on the three-time Keldysh contour shown in Fig. 3(a). A subtlety of our problem is that the initial state at $t = 0$ is itself a many-body state where (i) bath and system are entangled and (ii) interactions cannot be neglected. Thus, we cannot assume that we evolve from a noninteracting initial state and switch on the interactions adiabatically afterwards. How to modify the approach to the scenario where we prepare system and bath in an entangled interacting equilibrium state governed by H_i and where the subsequent time evolution is then determined by H_f was discussed by Danielewicz [107] and Wagner [108]. In what follows we will first summarize and then use this approach.

A. Action within the three-time-contour formalism

Let us consider an initial state of system and bath in equilibrium at temperature T . The state right before the quench

at $t = 0$ is then characterized by the density matrix

$$\rho_i = \frac{1}{Z}e^{-\beta H_i}, \quad (15)$$

where $\beta = 1/T$ and $Z = \text{tr}e^{-\beta H_i}$ is the prequench equilibrium partition function. In the limit $T \rightarrow 0$ it holds $\rho_i = |\Psi_0\rangle\langle\Psi_0|$, where $|\Psi_0\rangle$ is the ground state of the coupled system and bath with Hamiltonian H_i . The subsequent time evolution is governed by H_f ; i.e., the density matrix is given as

$$\rho(t) = U(t,0)\rho_i U(0,t), \quad (16)$$

where $U(t,t') = \exp[-i(t-t')H_f]$. The expectation value of an arbitrary operator (for specificity we consider our vector field φ) is then

$$\langle\varphi(t)\rangle = \text{tr}[\rho(t)\varphi] = \frac{1}{Z}\text{tr}[e^{-\beta H_i}U(0,t)\varphi U(t,0)]. \quad (17)$$

For simplicity we suppress for the moment the component index l and spatial coordinate x of the field; i.e., $\varphi(t) = \varphi_l(x,t)$. The time evolution that enters the expectation value can be efficiently placed on the contour \mathcal{C} shown in Fig. 3(a) with evolution operator such that

$$\langle\varphi(t)\rangle = \frac{\text{tr}[T_{\mathcal{C}}e^{-i\int_{\mathcal{C}}dsH(s)}\varphi(t)]}{\text{tr}[T_{\mathcal{C}}e^{-i\int_{\mathcal{C}}dsH(s)}]}. \quad (18)$$

The time argument of φ denotes that the Schrödinger operator has to be inserted at time t . $T_{\mathcal{C}}$ is the time-ordering operator along the contour from $i\beta \rightarrow 0 \rightarrow \infty \rightarrow 0$. In the denominator we used the fact that $e^{-\beta H_i} = U(0,t)U(t,0)e^{-\beta H_i}$. In full analogy one can consider two-time correlation functions

$$\mathcal{G}(t,t') = -i\langle T_{\mathcal{C}}\varphi(t)\varphi(t')\rangle = -i\frac{1}{Z}\text{tr}[T_{\mathcal{C}}e^{-i\int_{\mathcal{C}}dsH(s)}\varphi(t)\varphi(t')]. \quad (19)$$

To determine these and other observables we consider the generating functional on the contour \mathcal{C} ($\int_{t\in\mathcal{C}} \equiv \int_{\mathcal{C}} dt$)

$$W[\mathbf{h}] = \int D\varphi D\mathbf{X} e^{iS[\varphi,\mathbf{X}] - i\int_{s,t\in\mathcal{C}} \mathbf{h}(t)\cdot\varphi(t)} \quad (20)$$

with action

$$S[\varphi,\mathbf{X}] = S_s[\varphi] + S_b[\mathbf{X}] + S_{sb}[\varphi,\mathbf{X}]. \quad (21)$$

$S[\varphi,\mathbf{X}]$ consists of the action of the system, the bath, and the coupling term. The individual terms are

$$S_s = \frac{1}{2} \int_{x,t\in\mathcal{C}} \left\{ [\partial_t\varphi(t)]^2 - r_0(t)\varphi(t)^2 - [\nabla\varphi(t)]^2 - \frac{u(t)}{2N}\varphi(t)^4 \right\} \quad (22)$$

for the action of the system and

$$S_b = \frac{1}{2} \sum_j \int_{x,t\in\mathcal{C}} ([\partial_t\mathbf{X}_j(t)]^2 - \Omega_j^2\mathbf{X}_j(t)^2), \quad (23)$$

$$S_{sb} = - \sum_j c_j \int_{x,t\in\mathcal{C}} \mathbf{X}_j(t) \cdot \varphi(t) \quad (24)$$

for the bath action and the system-bath coupling, respectively.

We integrate out the bath variables at the expense of a bare propagator that is highly nonlocal in time:

$$\mathcal{G}_0^{-1}(t, t') = -[\partial_t^2 + r_0(t) - \nabla^2]\delta(t - t') + \Delta(t - t'). \quad (25)$$

The effects of the bath enter via the nonlocal self-energy $\Delta(t - t')$ that is formally given as

$$\Delta(t - t') = -\sum_j c_j^2 (\partial_t^2 + \Omega_j^2)^{-1} \delta(t - t'). \quad (26)$$

The resulting action

$$S[\boldsymbol{\varphi}] = \frac{1}{2} \int_{x, t, t' \in \mathcal{C}} \boldsymbol{\varphi}(t) \mathcal{G}_0^{-1}(t, t') \boldsymbol{\varphi}(t') - \frac{1}{4N} \int_{x, t \in \mathcal{C}} u(t) [\boldsymbol{\varphi}(t) \cdot \boldsymbol{\varphi}(t)]^2 \quad (27)$$

depends only on the collective field $\boldsymbol{\varphi}$ and yields the generating functional via

$$W[\mathbf{h}] = \int D\boldsymbol{\varphi} e^{iS[\boldsymbol{\varphi}] - i \int_{x, t \in \mathcal{C}} \mathbf{h}(t) \cdot \boldsymbol{\varphi}(t)}. \quad (28)$$

In the usual Schwinger-Keldysh formalism it is convenient to place the four possible arrangements of the times t and t' on the forward and backward branch of the contour in a 2×2 matrix. In our case we have three branches of the contour which can be captured in terms of a 3×3 matrix structure of the Green's function [108]

$$\mathcal{G} = \begin{pmatrix} iG^M & \tilde{G}^< & \tilde{G}^< \\ \tilde{G}^> & G^T & G^< \\ \tilde{G}^> & G^> & G^{\tilde{T}} \end{pmatrix}. \quad (29)$$

The first matrix element is up to a factor i the imaginary-time Green's function in equilibrium prior to the quench:

$$G^M(\tau - \tau') = -\langle T_\tau \varphi_M(\tau) \varphi_M(\tau') \rangle, \quad (30)$$

where $\varphi_M(\tau) = e^{\tau H_i} \varphi e^{-\tau H_i}$. T_τ is the time-ordering operator along the vertical segment of the contour. The inverse of the bare Matsubara function is

$$(g_i^M)^{-1}(\tau, \tau') = (\partial_\tau^2 - r_{0,i} + \nabla^2) \delta(\tau - \tau') + \eta^M(\tau - \tau'), \quad (31)$$

where $\eta^M(\tau)$ is the Fourier transform of $\eta^M(\omega_n)$ in Eq. (14). $G^M(\tau, \tau') \equiv G^M(\tau - \tau')$ only depends on the difference between the two time variables. The Fourier transform of the bare Matsubara function yields

$$g_i^M(\omega_n) = \frac{1}{-\omega_n^2 - \bar{r}_{0,i} - q^2 + \delta\eta^M(\omega_n)}. \quad (32)$$

Here, we have introduced $\bar{r}_{0,i} = r_{0,i} - \eta^M(0)$ to account for a trivial shift of the bare mass due to the bath coupling. The functions

$$\begin{aligned} \tilde{G}^<(\tau, t') &= i \langle \varphi_M(\tau) \varphi_H(t') \rangle, \\ \tilde{G}^>(\tau, t') &= i \langle \varphi_H(t') \varphi_M(\tau) \rangle \end{aligned} \quad (33)$$

describe correlations across the quench, where $\varphi_H(t) = e^{iH_f t} \varphi e^{-iH_f t}$ is given in the Heisenberg representation after

the quench. The remaining 2×2 block with

$$\begin{aligned} G^T(t, t') &= -i \langle T_i \varphi_H(t) \varphi_H(t') \rangle, \\ G^{\tilde{T}}(t, t') &= -i \langle \tilde{T}_i \varphi_H(t) \varphi_H(t') \rangle, \\ G^>(t, t') &= -i \langle \varphi_H(t) \varphi_H(t') \rangle, \\ G^<(t, t') &= -i \langle \varphi_H(t') \varphi_H(t) \rangle \end{aligned} \quad (34)$$

makes up the usual Keldysh matrix where T_i and \tilde{T}_i refer to the time-ordering and anti-time-ordering operator along the horizontal real-time branch of the contour \mathcal{C} , respectively. Of those 9 Green's functions that occur in the 3×3 matrix structure, only a few contain truly independent information. By transforming \mathcal{G} with the rotation matrix

$$L = \begin{pmatrix} -1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}, \quad (35)$$

one can benefit from this redundancy among the Green's functions and obtain

$$L^{-1} \mathcal{G} L = \begin{pmatrix} G^M & -\sqrt{2} \tilde{G}^< & 0 \\ -\sqrt{2} \tilde{G}^> & G^K & G^R \\ 0 & G^A & 0 \end{pmatrix}. \quad (36)$$

The scalar field forms a three-component vector $(\varphi_M, \varphi_H^c, \varphi_H^q)$ with so called classical and quantum components

$$\varphi_H^{c,q}(t) = \frac{1}{\sqrt{2}} [\varphi_H^+(t) \pm \varphi_H^-(t)], \quad (37)$$

in addition to the Matsubara component. The two relevant Green's functions on the forward and backward branch of the contour are the retarded Green's function

$$G^R(t, t') = -i \theta(t - t') \langle [\varphi_H(t), \varphi_H(t')]_- \rangle \quad (38)$$

and the Keldysh function

$$G^K(t, t') = -i \langle [\varphi_H(t), \varphi_H(t')]_+ \rangle, \quad (39)$$

where $[A, B]_\pm = AB \pm BA$. The retarded function measures the response of the order parameter at time t caused by an external field that couples to it and that acted at time $t' < t$ after the quench, while the advanced function

$$G^A(t, t') = i \theta(t' - t) \langle [\varphi_H(t), \varphi_H(t')]_- \rangle \quad (40)$$

contains the same information as the retarded function: $G^A(t, t') = G^R(t', t)$. In distinction, the Keldysh function is a measure of the strength of correlations between order parameter configurations at distinct times after the quench. Couplings between the horizontal and vertical branches of the contour are determined by correlation functions $\tilde{G}^<$ and $\tilde{G}^>$. These functions take into account the coupling ‘‘across the quench.’’ Notice that the Matsubara field φ_M only couples to the quantum component φ_H^q . For the bare Green's functions this implies that only the Keldysh function contains prequench memory effects. Thus, the bare postquench Keldysh function $g_f^K(t, t')$ of a noninteracting system will depend on both time variables, i.e., display aging effects. How to determine $g_f^K(t, t')$ will be discussed in detail below. The bare retarded function only depends on the time differences $g_f^R(t, t') = g_f^R(t - t')$

(both before and after the quench) and can be Fourier transformed to yield

$$g_f^R(\omega) = \frac{1}{\omega^2 - \bar{r}_{0,f} - q^2 + \delta\eta(\omega)}, \quad (41)$$

where $\bar{r}_{0,f} = r_{0,f} - \eta(0)$. Notice that this form is only correct for the bare retarded function, i.e., for $u = 0$. As we will see, many-body interactions couple response and correlation functions and lead to aging effects in the retarded function as well.

B. Large- N equations

We consider interaction effects within the large- N approach. For a large number of field components N the generating functional $W[\mathbf{h}]$ can be evaluated in the saddle-point approximation controlled in small $1/N$. In Appendix B, we explicitly perform this approximation on the three-time contour and summarize the main steps of the large- N analysis for a system with quench. The resulting self-consistent large- N equations before the quench read

$$h_i = r_i \phi_i, \quad (42)$$

$$r_i = \bar{r}_{0,i} + \frac{u_i}{2} \phi_i^2 + u_i \int_{q,\omega_n} G_{r_i}^M(q,\omega_n), \quad (43)$$

and the equations after the quench read

$$h_f = - \int_0^\infty dt' (G_r^R)^{-1}(t,t') \phi(t') - \phi_i \int_{-\infty}^0 dt' \delta\eta(t-t'), \quad (44)$$

$$r(t) = \bar{r}_{0,f} + \frac{u_f}{2} \phi^2(t) + \frac{u_f}{2} \int_q i G_r^K(q,t,t). \quad (45)$$

We use the notation $\int_q \equiv \int \frac{d^d q}{(2\pi)^d}$ and $\int_{\omega_n} \equiv T \sum_n$ for the momentum integration and the summation over Matsubara frequencies. ϕ_i and r_i are the order parameter and renormalized mass prior to the quench. G_r^M is the renormalized Matsubara function, where $\bar{r}_{0,i}$ in Eq. (32) is replaced by r_i . In full analogy is the renormalized retarded function

$$(G_r^R)^{-1}(t,t') = -[\partial_t^2 + r(t) - \nabla^2] \delta(t-t') + \delta\eta(t-t') \quad (46)$$

governed by the renormalized, time-dependent mass $r(t)$. The nonequilibrium dynamics leads to a time dependence of the mass such that the response function is affected by aging behavior. The set of equations is closed by the Keldysh function that can be expressed in the form

$$G_r^K(t,t') = \int_{s,s'} G_r^R(t,s) M(s,s') G_r^A(s,t'), \quad (47)$$

where we introduced the memory function $M(s,s')$ that is explicitly determined in Appendix C. Once this memory function is known we have a closed set of equations for the nonequilibrium dynamics after a quantum quench.

For our subsequent analysis it is convenient to rewrite the last two equations in terms of an expansion in

$$\delta r(t) \equiv r(t) - r_f, \quad (48)$$

where $r_f \equiv r(t \rightarrow \infty)$ is the equilibrium value of the renormalized mass for the postquench parameters [see Eq. (54) below]. This includes a shift due to interactions. It implies in particular that $r_f = 0$ for a quench right to the critical point. The renormalized propagators can then be obtained from the Dyson equations

$$G_r^R(t,t') = g^R(t-t') + \int_{t'}^t ds g^R(t-s) \delta r(s) G_r^R(s,t') \quad (49)$$

and

$$G_r^K(t,t') = g^K(t,t') + \int_0^{t'} ds g^K(t,s) \delta r(s) G_r^A(s,t') + \int_0^t ds g^R(t-s) \delta r(s) G_r^K(s,t'). \quad (50)$$

Here, the function $g^R(t-t')$ is the solution of Eq. (46) with $r(t)$ replaced by r_f ; i.e., it reads in Fourier space as

$$g^R(\omega) = \frac{1}{\omega^2 - r_f - q^2 + \delta\eta(\omega)}. \quad (51)$$

The Keldysh function g^K is determined as in Eq. (47) via $g^K(t,t') = \int_{s,s'} g^R(t-s) M(s,s') g^A(s'-t')$.

In Secs. V and VI we present solutions of the large- N equations for several different quench protocols, both starting in the disordered and in the ordered phase. In the next sections, however, we first summarize the equilibrium limit without quench and motivate the scaling behavior out-of-equilibrium of various observables that will be confirmed by the subsequent explicit calculation.

C. The equilibrium limit of the large- N analysis

Without quench, the system remains in equilibrium and the large- N equations before and after the quench become identical. In equilibrium, the Keldysh function $G^K(q,t,t')$ only depends on the time difference $t-t'$, with Fourier transform determined by the fluctuation-dissipation theorem:

$$G_r^K(q,\omega) = 2i \coth\left(\frac{\omega}{2T}\right) \text{Im} G_r^R(q,\omega). \quad (52)$$

Using

$$T \sum_n G_r^M(q,\omega_n) = \frac{1}{2} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} i G_r^K(q,\omega) \quad (53)$$

it follows with $\int_{-\infty}^{\infty} dt \delta\eta(t) = 0$ that the equations before and after the quench give, as expected, the same solution. We can drop the subscripts i and f that distinguish between the pre- and postquench regimes. The renormalized mass fulfills

$$r = \bar{r}_0 + u \int_{q,\omega} G_r^M(q,\omega) + \frac{u}{2} \phi^2. \quad (54)$$

The equation of state for the order parameter reads

$$r \phi = h. \quad (55)$$

In the ordered phase it holds that $\phi \neq 0$ and using Eq. (55) it thus follows for $h = 0$ that $r(h = 0) = 0$. The excitation spectrum is massless due to the Goldstone theorem and the fact

that longitudinal excitations are of order $1/N$. The transition takes place when $r = 0$ and $\phi = 0$ and determines the critical value $\bar{r}_{0,c}(u)$. For $T = 0$ the latter is determined by

$$\bar{r}_{0,c} = -u \int_{q,\omega} \frac{1}{\omega^2 + q^2 + \delta\eta^M(\omega)}; \quad (56)$$

$\bar{r}_{0,c}$ depends on the momentum cutoff Λ and, via $\eta(0)$, on the frequency cutoff of the bath spectrum. However, universal behavior emerges as a function of the distance $\delta r = \bar{r}_0 - \bar{r}_{0,c}$ to the quantum critical point. For sufficiently small energies the dynamics is dominated by the bath. Comparing length and time scales in the propagator with $\delta\eta^M(\omega) \propto |\omega|^\alpha$ yields the dynamic scaling exponent

$$z = 2/\alpha. \quad (57)$$

Analyzing Eqs. (54) and (55) one obtains below the upper critical dimension $d < d_{uc} = 4 - z$ the well known results for the order parameter $|\phi| \propto (-\delta r)^\beta$ with $\beta = 1/2$, for the correlation length $\xi = r^{-1/2} \propto \delta r^{-\nu}$ with $\nu^{-1} = d + z - 2$, and for the order-parameter susceptibility at the critical point $G^R(q = 0, \omega = 0) \propto \delta r^{-\gamma}$ with $\gamma = 2\nu$. For $d > d_{uc}$ all exponents take their mean-field values.

IV. SCALING BEHAVIOR

In this section we discuss scaling arguments for the nonequilibrium dynamics after a quench towards the critical point. This analysis follows the logic developed in Refs. [109,110] for boundary critical phenomena. In addition to the dynamics of the order parameter $\phi(t)\hat{\phi} = \langle \phi(k,t) \rangle$ we are interested in the retarded and Keldysh Green's functions $G^R(k,t,t')$ and $G^K(k,t,t')$, respectively, where both time arguments are after the quench. We suppress the field index l here and in the following for simplicity. These scaling relations and the corresponding exponents will be determined from a self-consistent solution of the coupled large- N equation in the subsequent sections.

In equilibrium, the order parameter at $T = 0$ obeys

$$\phi_{\text{eq}}(\delta r, h) = b^{-\beta/\nu} \phi_{\text{eq}}(b^{1/\nu} \delta r, b^{\beta\delta/\nu} h), \quad (58)$$

with distance to the critical point $\delta r = \bar{r}_0 - \bar{r}_{0,c}$ and scaling parameter b . Note, δr can in fact be realized by either changing \bar{r}_0 or by changing $\bar{r}_{0,c}$ that depends on u . Thus, for the discussion of scaling behavior, it is sufficient to include only two scaling fields $R = (\delta r, h)$. The choice $b = \delta r^{-\nu}$ implies for $h = 0$ that $\phi_{\text{eq}}(\delta r) \propto \delta r^\beta$, where β is the order-parameter exponent. ν is the correlation length exponent with equilibrium correlation length

$$\xi \propto \delta r^{-\nu}. \quad (59)$$

In analogy, right at the critical point and for finite field follows with $b = h^{-\nu/(\beta\delta)}$ that $\phi_{\text{eq}}(h) \propto h^{1/\delta}$. Those are the well known scaling relations in equilibrium.

In a nonequilibrium setting the order parameter will, on the one hand, depend on time, which transforms under scaling as $t \rightarrow b^{-z}t$. Here, z is the dynamic scaling exponent that relates typical time and length scales. We consider a regime where the dynamics is dominated by the coupling to the bath. In this case, to leading order in $1/N$ we find that like in the equilibrium case discussed previously [see Eq. (57)] $z = 2/\alpha$ is determined

by the spectral function $\eta(\omega)$. On the other hand, our quench protocol implies that the order parameter depends on the initial distance δr_i and the final distance δr_f to the critical point as well as on the initial and final fields h_i and h_f . It just holds that

$$\phi(t, R_i(1), R_f(1)) = b^{-\beta/\nu} \phi(b^{-z}t, R_i(b), R_f(b)), \quad (60)$$

where $R_f(b) = (b^{1/\nu} \delta r_f, b^{\beta\delta/\nu} h_f)$ and $R_i(b) = (b^{\kappa/\nu} \delta r_i, b^{\kappa\beta\delta/\nu} h_i)$. The scaling dimensions of the order parameter ϕ and of the final values δr_f and h_f continue to take their equilibrium values β , ν^{-1} , and $\beta\delta/\nu$, respectively. This is a consequence of the fact that the system approaches equilibrium after very long time scales

$$\lim_{t \rightarrow \infty} \phi(t, R_i, R_f) = \phi_{\text{eq}}(R_f), \quad (61)$$

independently of the initial values δr_i and h_i . Thus, the corresponding scaling dimensions are the same as in equilibrium. In this model with a nonconserved order parameter there is, however, no reason why the scaling dimension of δr_i and h_i should also be equal to the equilibrium values. This is reflected in the new exponent κ that enters Eq. (60). At this point it is not obvious why the same exponent κ modifies the scaling dimension of δr_i and h_i . This will only become clear in our explicit analysis of the theory. To demonstrate this and to determine κ is one of the goals of this paper.

Let us further analyze implications of this scaling behavior. For simplicity, we first consider the case of a quench right to the critical point $\delta r_f = 0$ in vanishing field $h_i = h_f = 0$. In this case Eq. (60) simplifies to

$$\phi(t, \delta r_i) = b^{-\beta/\nu} \phi(b^{-z}t, b^{\kappa/\nu} \delta r_i), \quad (62)$$

with $b^{-z}t = t_\gamma$ and t_γ being a microscopic time scale. We can therefore express the order parameter in terms of a scaling function as

$$\phi(t, \delta r_i) = t^{-\frac{\beta}{\nu z}} \Psi(t/t^*), \quad (63)$$

where $t^* \propto \delta r_i^{-\frac{\nu z}{\kappa}}$ and t, t^* are in units of t_γ . For $t \gg t^*$ the scaling function is expected to approach a constant $\Psi(x \gg 1) \rightarrow \text{constant}$ and the order parameter decays adiabatically according to a power law characterized by equilibrium exponents

$$\phi(t \gg t^*) \propto t^{-\beta/(vz)}. \quad (64)$$

On the other hand, for short times $t \ll t^*$ one expects that $\phi \propto \delta r_i^\beta$ such that $\Psi(x \ll 1) \propto x^{\kappa\beta/(vz)}$. In this regime follows

$$\phi(t \ll t^*) \propto t^{(\kappa-1)\beta/(vz)}. \quad (65)$$

As shown below the exponent κ varies with z and is for some z values larger than unity, implying that the order parameter rises for intermediate time scales, while for other z values $\kappa < 1$ such that a slower decay of ϕ governs the regime up to the crossover scale t^* . Note that t^* diverges for shallow quenches where δr_i is small. An analogous behavior occurs in the regime where we perform a quench of the field h_i to $h_f = 0$ for a system at the critical point: $\delta r_i = \delta r_f = 0$. The characteristic time scale is now given as $t^* \propto h_i^{-\frac{\nu z}{\beta\delta\kappa}}$. The resulting time dependence for $t \ll t^*$ and $t \gg t^*$ is the same as before.

Let us now turn to the correlation and response functions. In equilibrium, the retarded and Keldysh Green's functions only depend on the difference $t - t'$ of the two time variables and obey at the critical point the established scaling behavior

$$G_{\text{eq}}^{R(K)}(q, t, t') = \frac{F_{\text{eq}}^{R(K)}(q^z(t - t')/\gamma^{z/2})}{q^{2-z-\eta}\gamma^{z/2}}, \quad (66)$$

with scaling functions $F_{\text{eq}}^R(x)$ and $F_{\text{eq}}^K(x)$. Note that in contrast to the classical limit at high temperatures, here G^K and G^R have the same scaling dimension. The factor $\gamma^{z/2}$ occurs to render the scaling functions and their arguments dimensionless. As discussed, this behavior is only valid for time scales sufficiently long to ensure that the dynamics is dominated by the bath and it is justified to neglect ω^2 compared to $|\omega|^\alpha$ in the bare propagators.

Considering now the out-of-equilibrium dynamics after the quench to the critical point, correlations and response depend in general on both time variables. This gives rise to the additional dimensionless ratio t/t' , compared to scaling in equilibrium. Thus, we expect from dimensional considerations the following behavior:

$$G^{R(K)}(k, t, t') = \left(\frac{t}{t'}\right)^{\theta(\theta')} \frac{F^{R(K)}(q^z t/\gamma^{z/2}, t/t')}{q^{2-z}\gamma^{z/2}}. \quad (67)$$

Here, the scaling functions F^R and F^K are chosen such that they depend only weakly on the ratio t'/t if $t \gg t'$. The possibility of a singular dependence on t/t' is captured by the exponents θ and θ' , respectively. Below we will determine scaling laws that relate the exponents θ , θ' , and κ in such a way that only one new independent exponent emerges. Thus, the postquench dynamics is governed by a single critical exponent that cannot be expressed in terms of equilibrium exponents.

V. QUENCH FROM THE DISORDERED PHASE

In what follows we present the solution of the coupled large- N equations [see Eqs. (42)–(47)] for the postquench nonequilibrium dynamics at the quantum critical point starting the quench from the disordered phase. The quench protocol is indicated by $A \rightarrow C$ in Fig. 2. The effective time dependent mass in Eq. (45) simplifies to

$$r(t) = \bar{r}_{0,f} + \frac{u}{2} \int \frac{d^d q}{(2\pi)^d} i G_r^K(q, t, t). \quad (68)$$

In the case of a quench to the critical point, $\bar{r}_{0,f}$ takes the value $\bar{r}_{0,c}$ in Eq. (56). It is convenient to express $\bar{r}_{0,c}$ in terms of the equilibrium Keldysh function such that

$$r(t) = \frac{u}{2} \int \frac{d^d q}{(2\pi)^d} [i G_r^K(q, t, t) - i G_{\text{eq}}^K(q)]. \quad (69)$$

For times larger than the microscopic time scales the Keldysh function obeys scaling. We refer to Appendix A for a discussion of the various short time scales in the problem. Based on dimensional arguments, we make the generalized *light-cone* ansatz

$$r(t) = \frac{\gamma a}{t^{2/z}}, \quad (70)$$

valid for t larger than the microscopic time scale. The prefactor γ , that determines the strength of the coupling to the bath, was chosen to ensure that the light-cone amplitude a is dimensionless. This ansatz corresponds to a time-dependent correlation length

$$\xi(t) = |r(t)|^{-1/2} = \frac{1}{(\gamma|a|)^{1/2}} t^{1/z}, \quad (71)$$

reflecting the fact that after time t perturbations have travelled a distance $\xi(t)$. For ballistic propagation with $z = 1$ this corresponds to the usual light-cone propagation during time t . In general, $\xi(t)$ is the length scale on which the signal of a perturbation can propagate. Among others, this implies that the system will eventually reach a state with diverging correlation length. This is expected as the coupling to the heat bath ensures that the system equilibrates at the quantum critical point since the bath temperature is zero.

We now demonstrate that Eq. (70) indeed leads to a self-consistent solution of the coupled large- N equations and determine the light-cone amplitude a . Let us next explore some consequences of such a time-dependent mass $r(t)$.

A. Light-cone amplitude a and universal short-time exponent θ

We first establish a relation between the light-cone amplitude a of Eq. (70) and the exponents θ and θ' of the retarded and Keldysh Green's functions in Eq. (67). We will show that $\theta = \theta'$, which is in contrast to the regime near classical phase transitions [12,85] and to the dynamics in an isolated, quantum system [89]. The equality of both exponents in the quantum case follows from the fact that the dominant contributions to g^K arise from the high-frequency regime, in contrast to a classical system, where the low-frequency part is dominant. This relation can easily be determined from an analysis of the Dyson equation of G^R in the intermediate regime $t_\gamma \ll t \ll t_q \equiv (\sqrt{\gamma}/q)^z$ and $t \gg t'$. The leading corrections that follow from Eqs. (49) and (50) in this limit are given by

$$\delta G_r^R(k, t, t') \approx \int_{t_\gamma}^t ds g^R(k, t-s) \frac{\gamma a}{s^{2/z}} g^R(k, s-t'), \quad (72)$$

$$\delta G_r^K(k, t, t') \approx \int_{t_\gamma}^t ds g^R(k, t-s) \frac{\gamma a}{s^{2/z}} g^K(k, s, t'). \quad (73)$$

Here, the lower integration limit t_γ appears since the power-law decay of $r(t) \propto t^{-2/z}$ sets in only beyond this time scale.

We first consider the correction to the retarded function. For frequencies $\omega \gg (q/\sqrt{\gamma})^z$, which correspond to times $t \ll (\sqrt{\gamma}/q)^z$, the bare retarded propagator g^R at the critical point is local in space and entirely governed by the bath

$$g^R(q, \omega) \approx \frac{1}{\delta\eta(\omega)}. \quad (74)$$

Here, $\delta\eta(\omega)$ is the frequency-dependent contribution of the bath function $\eta(\omega)$ [see Eq. (13)]. Performing the inverse Laplace transformation we obtain

$$g^R(q, t) \approx -\frac{\sin(\pi/z)}{\gamma\Gamma(2/z)} t^{2/z-1}. \quad (75)$$

If we insert this into Eq. (73), it follows for $t' > t_\gamma$ that

$$\delta G_r^R(t, t') = \frac{a \sin^2(\pi/z)}{\gamma \Gamma(2/z)^2} \int_{t'}^t ds \frac{(t-s)^{2/z-1} (s-t')^{2/z-1}}{s^{2/z}}.$$

The integral can be performed analytically. Evaluating it in the limit $t \gg t'$ yields

$$\delta G_r^R(t, t') = \frac{a \sin^2(\pi/z)}{\gamma \Gamma(2/z)^2} t^{2/z-1} \ln \frac{t}{t'}. \quad (76)$$

To leading order it follows for the retarded propagator

$$G_r^R(t, t') = g^R(t-t') \left(1 + \theta \ln \frac{t}{t'} + \dots \right), \quad (77)$$

with

$$\theta = -\frac{a \sin(\pi/z)}{\Gamma(2/z)}. \quad (78)$$

This result demonstrates that a time-dependent mass $r(t)$ leads to aging behavior of the retarded Green's function; i.e., $G_r^R(t, t')$ depends on both time arguments t and t' separately. If we include higher order corrections, an analysis along the same lines yields higher powers of the logarithm with appropriate coefficients allowing us to exponentiate the logarithm if a is small. We can then put $G_r^R(t, t')$ in the scaling form of Eq. (67) and find that θ is indeed the exponent that appears in the scaling form. Thus, the dimensionless amplitude a of the time-dependent mass determines the exponent θ of the retarded Green's function. This analysis was performed under the assumption of negligible q dependence of the retarded propagator. Including this q dependence is technically slightly more involved, but leads to the same behavior since the $\ln(t/t')$ -time dependence is still the dominant one in the limit $t \gg t'$.

Let us now discuss the interaction correction to the short-time behavior of the Keldysh function using Eq. (73). We need to evaluate the short-time behavior of the noninteracting Keldysh function $g^K(t, t')$, for which it is useful to introduce a double Laplace transformation (see Ref. [85]):

$$g^K(q, \omega, \omega') = \int_0^\infty dt dt' e^{i(\omega+i0^+)t} e^{i(\omega'+i0^+)t'} g^K(q, t, t'). \quad (79)$$

As we show in Appendix C the Keldysh function can be expressed in terms of the memory function $M(q, \omega, \omega')$ as $g^K(q, \omega, \omega') = M(q, \omega, \omega') g^R(q, \omega) g^R(q, \omega')$. Using the deep quench result for the memory function in Eq. (C16) at $T = 0$, this takes the form

$$g^K(q, \omega, \omega') = i \frac{\text{sgn}(\omega) \delta\eta(\omega) + \text{sgn}(\omega') \delta\eta(\omega')}{\omega + \omega' + i0^+} \times g^R(q, \omega) g^R(q, \omega'). \quad (80)$$

The limit $t \gg t'$ corresponds to $\omega' \gg \omega$. In this limit $\delta\eta(\omega')$ is always large compared to $\delta\eta(\omega)$ and it follows

$$g^K(q, \omega, \omega' \gg \omega) \simeq i \frac{\delta\eta(\omega')}{|\omega'|} g^R(q, \omega) g^R(q, \omega'). \quad (81)$$

Using Eq. (74) for the retarded function in the short-time limit, it immediately follows

$$g^K(q, \omega, \omega' \gg \omega) \simeq \frac{i}{|\omega'|} g^R(q, \omega). \quad (82)$$

The back transformation is now straightforward and, using that $1/|\omega'|$ turns into a constant upon Laplace transformation, we find

$$g^K(q, t \gg t') \simeq g^R(q, t) = -\frac{\sin(\pi/z)}{\gamma \Gamma(2/z)} t^{2/z-1}. \quad (83)$$

In the short-time limit and for $t \gg t'$, the Dyson equations for the retarded and Keldysh functions thus only differ in the bare values of the two functions. Since we just demonstrated that $g^K(q, t \gg t')$ and $g^R(q, t)$ have the same t dependence in the relevant regime, both Dyson equations have the same solutions. This immediately yields $\theta' = \theta$ for the exponent θ' of the Keldysh function introduced in Eq. (67). Finally, let us remark on an important difference between quantum and classical dynamics: while in the quantum case the free Keldysh function $g^K(q, t, t')$ approaches a constant in the limit $t' \rightarrow 0$, even for small but finite t , the classical postquench Keldysh function vanishes for $t' \rightarrow 0$ according to the power law $t'^{2/z}$.

B. Long-time behavior of $G^K(q, t, t)$

We next determine the long-time decay of the Green's functions G^R and G^K of a system with time-dependent effective mass $r(t)$ as given in Eq. (70). To this end we analyze the equation of motion (44), in the long-time limit. From the equation of motion follows

$$[\partial_t^2 + r(t) + q^2] \varphi(q, t) = \int_{-\infty}^t ds \delta\eta(t-s) \varphi(q, s). \quad (84)$$

In order to analyze the impact of the time dependence of the correlation length, we perform an expansion for small $r(t)$. For the specific ansatz in Eq. (70), this amounts to an expansion in the dimensionless light-cone amplitude a :

$$\varphi(q, t) = \varphi_{\text{eq}}(q, t) + \varphi_1(q, t) + O(a^2), \quad (85)$$

where $\varphi_{\text{eq}}(q, t)$ obeys the equation of motion for $r(t) = 0$, i.e., the equation of motion in equilibrium at the critical point. We can split the integral on the right-hand side of Eq. (84) into

$$\Xi(t) = \int_{-\infty}^0 ds \delta\eta(t-s) \varphi(q, s) \quad (86)$$

and a contribution that only contains the postquench dynamics of $\varphi(q, t)$. As far as the postquench dynamics is concerned, $\Xi(t)$ acts as an inhomogeneity in Eq. (84). At long times we can neglect this inhomogeneity since $\delta\eta(t \rightarrow \infty)$ vanishes sufficiently rapidly. This determines the leading perturbative correction to

$$\varphi_1(q, t) = -\int_{t_0}^t ds G_{\text{eq}}^R(t-s) r(s) \varphi_{\text{eq}}(q, s), \quad (87)$$

where $G_{\text{eq}}^R(t-t') = -i\theta(t-t') \langle [\varphi_{\text{eq}}(t), \varphi_{\text{eq}}(t')] \rangle$ (suppressing the field index l again).

Using the definition of $G^{R/K}$ in Eqs. (38) and (39) we can now determine how the Green's functions relax towards their equilibrium expressions. Let us first consider the equal-time Keldysh function $G_r^K(q, t, t)$, which turns out to be of particular importance for the self-consistent solution of the large- N equation. Using Eq. (85) leads to

$$G_r^K(q, t, t) = G_{\text{eq}}^K(q) + \delta G_r^K(q, t, t), \quad (88)$$

where

$$\delta G_r^K(q, t, t) = -2 \int_{t_\gamma}^t ds G_{\text{eq}}^R(t-s) r(s) G_{\text{eq}}^K(t-s). \quad (89)$$

To evaluate this expression we express the retarded function in Laplace and the equilibrium Keldysh function in Fourier space and obtain

$$\begin{aligned} \delta G_r^K(q, t, t) &= -2i \int \frac{d\omega d\omega'}{2\pi^2} \text{Im}[G_{\text{eq}}^R(\omega)] G_{\text{eq}}^K(\omega') \\ &\times \int_{t_\gamma}^t ds r(s) e^{-i(\omega-\omega')(t-s)}. \end{aligned} \quad (90)$$

We want to evaluate this integral in the limit of large t . The typical frequency scale of the equilibrium Green's function is $\omega_q \approx t_q^{-1} = q^z/\gamma^{z/2}$. For times $t \gg \omega_q^{-1}$ the integrand is in general highly oscillatory, except for contributions that stem from the upper limit of the integration over time. This yields up to leading order in t

$$\int_{t_\gamma}^t ds e^{-i(\omega-\omega')(t-s)} r(s) \approx \frac{ir(t)}{\omega - \omega'}. \quad (91)$$

Performing the remaining frequency integration leads to

$$G_r^K(q, t, t) = G_{\text{eq}}^K(q) + \frac{2ir(t)}{c_K q^{4-z} \gamma^{z/2}} \quad (92)$$

with numerical coefficient

$$c_K = \frac{4 \sin(\pi z/2)}{z(2-z) \sin^{z/2}(\pi/z)}. \quad (93)$$

For later discussion it is useful to note that c_K is positive for $z < 4$ and for $z \rightarrow 2$ it holds $c_K \rightarrow \pi$. From Eq. (92) follows that the equal-time correlation function decays for large times according to a power law. Including higher order corrections in the Dyson equation leads to terms of order $r(t)^2$. Thus, the term $G_1^K(q, t, t) = 2ir(t)/(c_K q^{4-z} \gamma^{z/2})$ appearing in Eq. (92) is the slowest decaying correction at large times. This is an interaction effect, because the bare correlation function $g^K(q, t, t)$ decays for finite momenta always exponentially $\propto \exp(-q^z t/\gamma^{z/2})$ to its value in equilibrium. Critical fluctuations lead to a significant slowing down of the equilibration of the system.

C. Self-consistent determination of the light-cone amplitude

In this section, we demonstrate that the ansatz for the time-dependent mass $r(t)$ in Eq. (70) indeed leads to a self-consistent solution of the coupled large- N equations and we show how to determine a . Let us begin with some general remarks about Eq. (69) that we want to solve self-consistently:

$$\frac{a\gamma}{t^{2/z}} = \frac{uK_d}{2} \int_0^\Lambda dq q^{3-z-\epsilon} [iG_r^K(q, t, t) - iG_{\text{eq}}^K(q)]. \quad (94)$$

Here, $K_d = \Gamma(d/2)/[2\pi^{d/2}(2\pi)^d]$ takes into account the integration over angles and

$$\epsilon = 4 - d - z. \quad (95)$$

We want to determine the dimensionless light cone amplitude a self-consistently for small ϵ . Since u will turn out be also of order ϵ [see Eq. (100) below] it is tempting to speculate

that one only has to expand G_r^K in a . To leading order it would then suffice to insert the bare Keldysh function $g^K(q, t, t)$ in Eq. (94). Since g^K decays exponentially to G_{eq}^K the integral is convergent in the limit $\Lambda \rightarrow \infty$. However, as was shown in the previous paragraph, first-order corrections to the Keldysh function decay much more slowly. Using Eq. (92) such corrections $G_1^K(q, t, t) \propto r(t)/q^{4-z}$ are proportional to q^{-4+z} . Upon integration this generates terms that behave as $1/\epsilon$ for small ϵ :

$$\int_{q_0}^\Lambda dq \frac{1}{q^{1+\epsilon}} = -\frac{1}{\epsilon} (\Lambda^{-\epsilon} - q_0^{-\epsilon}), \quad (96)$$

where q_0 is some appropriate lower cutoff that we elaborate on below. Such a term, multiplied with $r(t) \propto a$ will be of same order $O(\epsilon^0)$ as the bare g^K . Thus, in the expansion of G_r^K in a we have to keep those slowly decaying terms proportional to q^{-4+z} . From G_1^K in Eq. (92) follows that large times t correspond to large momenta $q \gg q_0 \equiv \gamma^{1/2}/t^{1/z}$, which naturally introduces the lower cutoff q_0 in Eq. (96). As we have previously shown in Sec. V A, in the opposite limit of small momenta $q \ll q_0$ the Keldysh function becomes momentum independent [see Eq. (83)] and cannot generate contributions that behave as $1/\epsilon$.

With these considerations, we are able to solve the large- N equations self-consistently. We expand $G_r^K = g^K + G_1^K$ in small $r(t)$ to find

$$\begin{aligned} r(t) &= \frac{uK_d}{2} \int_0^\infty dq q^{3-z-\epsilon} [ig^K(q, t, t) - iG_{\text{eq}}^K(q)] \\ &+ \frac{uK_d r(t)}{c_K \epsilon \gamma^{z/2}} \left[\Lambda^{-\epsilon} - \left(\frac{\gamma^{1/2}}{t^{1/z}} \right)^{-\epsilon} \right]. \end{aligned} \quad (97)$$

In the first integral we already took the limit $\Lambda \rightarrow \infty$ because g^K approaches G_{eq}^K exponentially quickly and the integral is thus convergent at the upper limit. To proceed, we use the scaling form of Eq. (67) and introduce a dimensionless integration variable $x = q^z t/\gamma^{z/2}$ to obtain

$$\frac{a\gamma}{t^{2/z}} = \frac{uK_d}{2z\gamma^{z/2}} \frac{\gamma C_0 t^{\epsilon/z}}{\gamma^{\epsilon/z} t^{2/z}} + \frac{ua\gamma K_d}{c_K \epsilon \gamma^{z/2} t^{2/z}} \left(\Lambda^{-\epsilon} - \frac{t^{\epsilon/z}}{\gamma^{\epsilon/2}} \right), \quad (98)$$

where we have introduced the dimensionless integral

$$C_0 = \int_0^\infty dx x^{2/z-1} [if^K(x, 1) - iF_{\text{eq}}^K]. \quad (99)$$

Equation (98) must hold for all times $t \gg t_\gamma$, and therefore terms with and without $t^{\epsilon/z}$ must cancel separately. This allows us to extract the two conditions

$$u = u^* \equiv \frac{c_K \gamma^{z/2} \Lambda^\epsilon}{K_d} \epsilon, \quad (100)$$

$$a = \frac{c_K C_0}{2z} \epsilon. \quad (101)$$

The first equation determines the deep-quench fixed point value of u . It does not imply that u must be tuned precisely to this value. Instead, it should be understood as a consequence of using the deep-quench scaling form of G_r^K . Nonuniversal contributions in G_r^K will renormalize u , which is fully consistent with our renormalization-group-based reasoning in

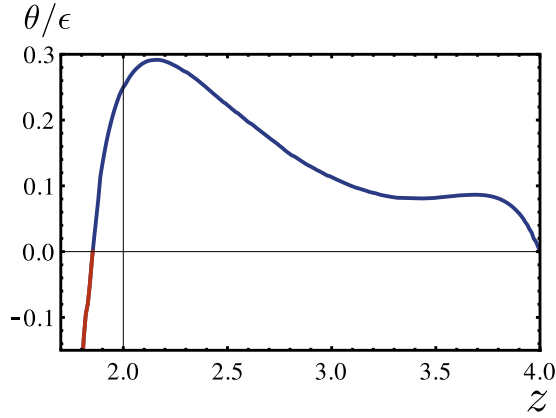


FIG. 4. (Color online) Prethermalization exponent θ as a function of dynamic critical exponents z . Here, $\epsilon = 4 - d - z$ and $z = 2/\alpha$ is determined by the form of the bath spectral function at low energies $\text{Im}\eta(\omega) \propto |\omega|^\alpha$. For $\theta < 0$ one observes an underdamped approach of the free Keldysh Green's function $G_0^K(t, t)$ towards equilibrium. The transition from overdamped to underdamped behavior occurs close the location of the sign change.

Ref. [88]. The second equation (101) determines the value of a and, using Eq. (78), the value of the new critical exponent

$$\theta = -\frac{\sin(\pi/z)a}{\Gamma(2/z)} = -\frac{c_K \sin(\pi/z)}{2z\Gamma(2/z)} C_0 \epsilon. \quad (102)$$

Note that exactly the same value was obtained in our previous renormalization-group analysis in the limit $N \rightarrow \infty$ [88], where we have shown that the integral C_0 can be performed analytically in the case of an Ohmic bath. For general dynamic critical exponents $1 < z < 4$ we evaluate it numerically and the result is presented in Fig. 4.

The relation between the light-cone amplitude a and the retarded short-time critical exponent θ is identical for a classical and a quantum system; however, the z dependence of θ is completely different due to the fact that quantum fluctuations are only present in the quantum system. Quantum fluctuations influence θ in two ways: (i) via the numerical constant c_K in Eq. (93) and (ii) via the result of the dimensionless integral C_0 over the Keldysh functions in Eq. (99).

The constant c_K determines the fixed point u^* as well as the long-time decay of G_r^K . For a quantum system, c_K is positive for $z \leq 4$ and linearly approaches zero for $z \rightarrow 4$. The constant C_0 is given by the difference of the postquench Keldysh scaling function $f^K(x)$ and the equilibrium Keldysh scaling function F_{eq}^K . For a classical system, the system is always overdamped and f^K is smaller than F_{eq}^K for all values of z . Hence, the exponent for a classical system is always positive $\theta_{cl} > 0$. For an Ohmic bath, for example, one finds $\theta_{cl} = \epsilon_{cl}/4$ with $\epsilon_{cl} = 4 - d$ [12]. For a quantum system, however, f^K can exhibit oscillations, in particular for super-Ohmic bath spectral functions where $z \lesssim 2$ the Keldysh function becomes underdamped. This leads to a sign change of θ at $z \approx 1.8$.

While the classical result of the exponent for an Ohmic bath is $\theta_{cl} = \epsilon_{cl}/4$, which is the same value as in the quantum case for $z = 2$, $\theta(z = 2) = \epsilon/4$, this is only a coincidence, since the constants c_K and C_0 take different values for $T = 0$ and $T \rightarrow \infty$. For an Ohmic bath, one can therefore still distinguish

the quantum postquench dynamics from the classical one by analyzing, for example, the long-time behavior of the Keldysh function. Moreover, since ϵ depends on z , we find the same short-time exponent for classical and quantum systems in different dimensions. For example, we predict $\theta_{cl} = 1/4$ for a classical system in contact with an Ohmic bath in $d = 3$ dimensions, while $\theta = 1/4$ for a quantum system in contact with an Ohmic bath in $d = 1$ dimension.

D. Distribution function $n(t, \omega)$

The analysis of the long-time behavior of the equal-time Keldysh function $G^K(q, t, t)$ in Sec. V B can be straightforwardly extended to different times t, t' as well as to the retarded function $G^R(q, t, t')$. We show in detail in Appendix D that in the limit where both time arguments are large compared to the typical mode time $t_q = q^{-z} \gamma^{z/2}$, but the relative time is small $t - t' \ll t_q$, this leads to

$$G_r^K(q, t, t') = G_{\text{eq}}^K(q, t - t') - 2r \left(\frac{t + t'}{2} \right) C^K(q, t - t'), \quad (103)$$

$$G_r^K(q, t, t') = G_{\text{eq}}^R(q, t - t') - 4i\theta(t - t')r \left(\frac{t + t'}{2} \right) C^K(q, t - t'), \quad (104)$$

where we have defined the functions

$$C^K(q, t) = \int \frac{d\omega}{2\pi} \text{Re}[G_{\text{eq}}^R(q, \omega)] G_{\text{eq}}^K(q, \omega) e^{-i\omega t}, \quad (105)$$

$$C^K(q, t) = \int \frac{d\omega}{2\pi} \text{Re}[G_{\text{eq}}^R(q, \omega)] \text{Im}[G_{\text{eq}}^R(q, \omega)] e^{-i\omega t}. \quad (106)$$

By performing a Wigner transformation of Eqs. (103) and (104) and by using the fluctuation-dissipation theorem for the equilibrium Keldysh function $G_{\text{eq}}^K = 2i \coth[\omega/(2T)] \text{Im}G_{\text{eq}}^R$, one can express G_r^K after a few steps as

$$G_r^K(q, t_a, \omega) = 2i \coth \left(\frac{\omega}{2T} \right) \left[1 + 2r(t_a) \text{Re}G_{\text{eq}}^R(q, \omega) \right] \times \text{Im}G_r^K(q, t_a, \omega), \quad (107)$$

where $t_a = (t + t')/2$. This result shows that we are in the limit of adiabatic relaxation, since G_r^K can be written as

$$G_r^K(q, t_a, \omega) = \coth \left(\frac{\omega}{2T} \right) \times \frac{2i \text{Im}\eta(\omega)}{[q^2 + r(t_a) + \text{Re}\eta(\omega)]^2 + [\text{Im}\eta(\omega)]^2}. \quad (108)$$

Further, the similarity between G_r^K and G_r^K suggests connecting them via the fluctuation-dissipation theorem and introducing a distribution function $n(t_a, \omega)$ via

$$G_r^K(q, t_a, \omega) = 2i[2n(t_a, \omega) + 1] \text{Im}G_r^K(q, t_a, \omega). \quad (109)$$

With $n(t_a, \omega) = n_B(\omega) + \delta n(t_a, \omega)$, where $n_B(\omega)$ is the Bose-distribution function, this yields for the correction

$$\delta n(t_a, \omega) = \coth \left(\frac{\omega}{2T} \right) r(t_a) \text{Re}G_{\text{eq}}^R(q, \omega). \quad (110)$$

This expression is only valid for large frequencies, which corresponds to small relative times $|t - t'| \ll t_q$. For $t_a \rightarrow \infty$ the system relaxes to the thermal equilibrium state. Aging effects, however, lead to a significant slowing down of thermalization. Explicitly, one finds

$$\delta n = \coth\left(\frac{\omega}{2T}\right) \frac{\theta \Gamma(2/z)}{(|\omega|t_a)^{2/z}} \left(\cos \frac{\pi}{z} + \frac{q^2}{\gamma |\omega|^{2/z}} \sin \frac{\pi}{z} \right), \quad (111)$$

where both $q^2|\omega|^{-2/z}/\gamma = (|\omega|t_q)^{-2/z} \ll 1$ as well as $(|\omega|t_a)^{-2/z} \ll 1$, since $t_a \gg t_q \gg |t - t'|$.

The correction to the Bose distribution is proportional to the short-time exponent θ and exhibits a (dominant) algebraic frequency dependence $\delta n \propto |\omega|^{-2/z}$ for $z \neq 2$ and $\delta n \propto |\omega|^{-2}$ in the Ohmic case. This slow decay at large frequencies clearly shows that the system is not thermal and cannot be characterized by a temperature as the decay would then be exponential in ω . The sign of δn is positive for a (sub-)Ohmic bath $z \geq 2$, showing that even long after the quench there are an increased number of excitations at large frequencies present in the system as compared to equilibrium. For super-Ohmic bath spectral densities, however, the sign of δn can change depending on the product of θ , which becomes negative for $z < 1.8$, and the last term in Eq. (111). This term stems from $\text{Re } G_{\text{eq}}^R$ and is positive (negative) for $z \geq 2$ ($z < 2$), except slightly below $z = 2$ where the sign depends on the interplay of the $\cos(\pi/z) < 0$ and $(|\omega|t_q)^{-2/z} \sin(\pi/z) > 0$ with $(|\omega|t_q)^{-2/z} \ll 1$. More importantly, a negative sign of δn implies that the density matrix of the system is not diagonal in the energy basis. The off-diagonal terms describe the presence of quantum coherence and thus make a straightforward interpretation of δn in terms of a distribution function impossible.

In Fig. 5 we illustrate this result for $2n(t_a, \omega) + 1$ in the Ohmic case, both for $\theta > 0$ as well as for $\theta < 0$ (although $\theta = \epsilon/4 > 0$ for $z = 2$). We include $\theta < 0$ in the plot to illustrate the behavior in cases where $\theta < 0$ which is qualitatively the same. Above the upper critical dimension, one finds $\theta = 0$ and thus $\delta n = 0$.

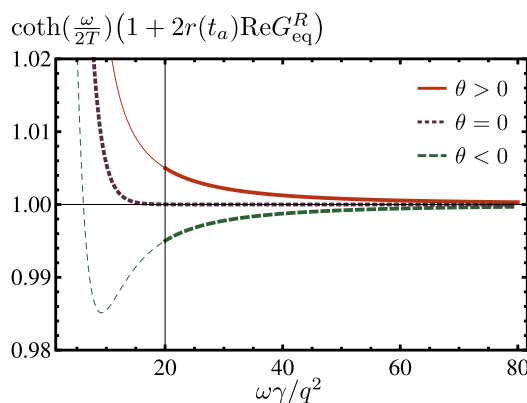


FIG. 5. (Color online) Visualization of the correction to the distribution function $\delta n(t_a, \omega)$ as a function of dimensionless variable $\omega\gamma/q^2$ [see Eq. (110)]. Plot shows $2n(t_a, \omega) + 1 = \coth(\frac{\omega}{2T})[1 + 2r(t_a)\text{Re}G_{\text{eq}}^R]$ [see Eq. (107)] as a function of the dimensionless variable $x = \omega\gamma/q^2$ for $z = 2$, $\gamma T/q^2 = 1.7$, and $|\theta|\gamma/(q^2 t_a^{2/z}) = 1$.

E. Upper critical dimension

From the bare Green's function g^R in Eq. (41) it follows immediately that the mean-field value of the exponent θ is zero. Therefore, θ vanishes above the upper critical dimension $d_{uc} = 4 - z$. In this section, we investigate the postquench behavior if the system is exactly at the upper critical dimension $d = d_{uc}$. We can straightforwardly extend our previous result to this case. The large- N equation for the mass reads

$$r_{uc}(t) = \frac{uK_d}{2} \int dq q^{3-z} [iG_{r_{uc}}^K(q, t, t) - iG_{\text{eq}}^K(q)], \quad (112)$$

where we used that $d = d_{uc}$. In a next step we expand $G_{r_{uc}}^K$ in $r_{uc}(t)$. The important slowly decaying correction comes from terms going with $G_1^K(q, t, t) \propto q^{-4+z}$, generating logarithmic divergences after the q integration. In Sec. VB we calculated this correction, without making any assumption about the dimensionality of the system or the concrete decay of the time-dependent mass $r(t)$. Therefore, one can directly use the result of Eq. (92). Again, the condition $t > t_q = \gamma^{z/2} q^{-z}$ translates into a lower cutoff in the q integral. For $q < q_0 = \gamma^{1/2}/t^{1/z}$ the Keldysh function $G_{r_{uc}}^K$ is again local in space, independent of the dimensionality, and cannot generate logarithmic terms. Focusing on the contribution of momenta $q > q_0$ it holds that

$$\begin{aligned} & \int_{q_0}^{\Lambda} dq q^{3-z} iG_1^K(q, t, t) \\ &= -\frac{2r_{uc}(t)}{c_K \gamma^{z/2}} \ln\left(\frac{\Lambda t^{1/z}}{\gamma^{1/2}}\right) \\ &= \frac{2r_{uc}(t)}{z c_K \gamma^{z/2}} \left[\ln\left(\frac{\gamma^{z/2}}{\Lambda^z t_\gamma}\right) - \ln\left(\frac{t}{t_\gamma}\right) \right]. \end{aligned} \quad (113)$$

Compared to the case $d < d_{uc}$ where $\epsilon > 0$, the slowly decaying correction is now logarithmically divergent both in the cutoff Λ and at the lower boundary q_0 . This is not surprising, because the same behavior occurs in equilibrium. The q integration of higher order terms in the expansion in $r(t)$ does not generate logarithmically divergent contributions and can therefore be neglected. Inserting the expansion $G_{r_{uc}}^K = G_{\text{eq}}^K + G_1^K$ into Eq. (112) and keeping only the logarithmically divergent corrections, we obtain

$$1 = \frac{u\gamma K_d C_0}{2z\gamma^{z/2} t^{2/z} r_{uc}(t)} + \frac{uK_d}{z c_K \gamma^{z/2}} \left[\ln\left(\frac{\gamma^{z/2}}{\Lambda^z t_\gamma}\right) - \ln\left(\frac{t}{t_\gamma}\right) \right]. \quad (114)$$

This equation holds for all times $t > t_\gamma$ if contributions with and without t dependence cancel separately. This gives rise to two conditions, one for the time-dependent mass $r_{uc}(t)$ and one for the deep-quench fixed point value of u :

$$r_{uc}(t) = \frac{\gamma c_K C_0}{2t^{2/z} \ln(t/t_\gamma)}, \quad (115)$$

$$u = u_{uc}^* = \frac{z c_K \gamma^{z/2}}{K_d \ln[\gamma^{z/2}/(\Lambda^z t_\gamma)]}. \quad (116)$$

Compared to the result below d_{uc} in Eqs. (100), (101), ϵ is replaced by $z/\ln(t/t_\gamma)$ in the effective mass. Since $t \gg t_\gamma$, $r_{uc}(t)$ is indeed small, justifying the earlier expansion of $G_{r_{uc}}^K$ in $r_{uc}(t)$. This additional logarithm in the self-energy correction

$\delta G_{ruc}^R(t, t')$ at intermediate times $t_\gamma \ll t' \ll t \ll t_q = \gamma^{z/2} q^{-z}$ implies that

$$\begin{aligned} \delta G_{ruc}^R(t, t') &= \int ds g^R(t, s) \frac{a\gamma}{s^{2/z} \ln(s/t_\gamma)} g^R(s, t') \\ &\approx \theta_{uc} \left\{ \ln \left[\ln \left(\frac{t}{t_\gamma} \right) \right] - \ln \left[\ln \left(\frac{t'}{t_\gamma} \right) \right] \right\}, \end{aligned} \quad (117)$$

with

$$\theta_{uc} = -\frac{c_K C_0 \sin(\pi/z)}{2\Gamma(2/z)}. \quad (118)$$

Since the logarithmic corrections are small, we can exponentiate θ_{uc} . This yields, instead of an algebraic divergence as in Eq. (67), a logarithmic divergence in the scaling function

$$G_{uc}^R(q, t, t') = \frac{1}{q^{2-z} \gamma^{z/2}} \left(\frac{\ln \frac{t}{t_\gamma}}{\ln \frac{t'}{t_\gamma}} \right)^{\theta_{uc}} F_{uc}^R \left(\frac{q^z t}{\gamma^{z/2}}, \frac{t}{t'} \right). \quad (119)$$

The same behavior can be extended for the Keldysh function and its scaling form at the upper critical dimension.

VI. QUENCH STARTING IN THE ORDERED PHASE

In this section we will consider a quench with a finite initial order parameter $\phi(t < 0) = \phi_i$. A finite order parameter can be achieved either by (i) a finite initial field h_i , which is switched off at $t = 0$ (path $B' \rightarrow C$ in Fig. 2) or by (ii) a quench in the mass parameter with $\bar{r}_{0,i} < \bar{r}_{0,c}$ (path $B \rightarrow C$ in Fig. 2). Both quench protocols yield the same exponent θ for the retarded and the Keldysh Green's functions. Furthermore, the exponent θ determines the short-time dynamics of $\phi(t)$ and the magnitude of the typical crossover time t^* from prethermalized dynamics $\phi \propto t^\theta$ to relaxation $\phi \propto t^{-\beta/(vz)}$.

Starting in the symmetry-broken phase, the postquench $1/N$ equations for a quench to the quantum critical point $r_f = 0$ read

$$\begin{aligned} r(t) &= \frac{u_f}{2} \phi^2(t) + \frac{u_f}{2} \int_q [iG_r^K(q, t, t) - iG_{eq}^K(q)], \quad (120) \\ \phi_i \int_{-\infty}^0 dt' \delta\eta(t-t') &= r(t)\phi(t) - \int_0^t ds \delta\eta(t-s)\phi(s). \end{aligned} \quad (121)$$

In the last line we dropped the $\partial_t^2 \phi$ term, since we concentrate on times $t \gg t_\gamma$, where the dynamics of the system is dominated by the bath. We also assume a spatially homogeneous order parameter $\phi(t)$.

In analogy to the quench starting in the disordered phase, we expand G^K in $G_{eq}^K + G_1^K$ for small $r(t)$ at long times, where G_1^K is given in Eq. (92). This yields

$$\begin{aligned} r(t) &= \frac{u_f}{2} \phi^2(t) + \frac{u_f K_d C_0}{2z \gamma^{2/z-1} t^{2/z}} \left(\frac{t}{\gamma^{z/2}} \right)^{\epsilon/z} \\ &\quad + \frac{u_f}{u^*} r(t) \left[1 - \left(\frac{\Lambda^z t}{\gamma^{z/2}} \right)^{\epsilon/z} \right]. \end{aligned} \quad (122)$$

This equation is only fulfilled for all times $t \gg t_\gamma$ if

$$u_f = u^*, \quad (123)$$

$$r(t) = \frac{\gamma a}{t^{2/z}} + \frac{c_K \epsilon \gamma^{z/2} \phi(t)^2}{2K_d} \left(\frac{\gamma^{z/2}}{t} \right)^{\epsilon/z}, \quad (124)$$

with u^* and a given by Eq. (100) and Eq. (101). The first equation determines the deep-quench fixed point of u . The value of u^* is not affected by the quench direction and is identical for $A, B, B' \rightarrow C$. The second equation determines the effective, time-dependent mass. The first term in $r(t)$ is exactly the same as for the quench starting in the symmetric phase. However, the mass is now modified by an additional term proportional to ϕ^2 . This time-dependent mass can be inserted into the equation of motion (121) of ϕ leading to a nonlinear, inhomogeneous differential equation.

In the following two subsection we present a solution for $\phi(t)$ in the prethermalized regime and in the long-time limit. The special case where the system is at the upper critical dimension $d = d_{uc}$ will be treated in Sec. VIC.

A. Order parameter coarsening in the prethermalized regime and crossover time scale t^*

At intermediate times, one can assume that the order parameter ϕ is small due to the collapse of the correlations after the quench. This collapse follows *a posteriori* from our solutions which also predict a collapse of the correlation length to a microscopic length scale right after the quench. For a fast quench, the collapse occurs on microscopic time scales. In this regime, according to Eq. (124) the dominant contribution to $r(t)$ is given by

$$r(t \ll t^*) \approx \frac{\gamma a}{t^{2/z}}, \quad (125)$$

where t^* denotes the boundary of the prethermalization regime and will be defined below. This yields in the equation of motion (121)

$$\frac{\gamma a}{t^{2/z}} \phi(t) - \int_0^t ds \delta\eta(t-s)\phi(s) = \phi_i \int_{-\infty}^0 ds \delta\eta(t-s). \quad (126)$$

One can solve this equation with the retarded Green's function G^R introduced above or via Laplace transformation. Let us solve it here via Laplace transformation. We expand $\phi(t) = \phi_0 + \phi_1$ in small a and obtain after a few steps

$$\phi_0(t) = \phi_0(t_\gamma), \quad (127)$$

$$\phi_1(t) = \int_{t_\gamma}^t ds g^R(t, s) \frac{\gamma a}{s^{2/z}} \phi_0(s - t_\gamma). \quad (128)$$

The dominant contribution to the integral in the limit $t \gg t_\gamma$ is logarithmically divergent, yielding

$$\phi(t) \approx \phi_0(t_\gamma) \left(1 + \theta \ln \frac{t}{t_\gamma} \right), \quad (129)$$

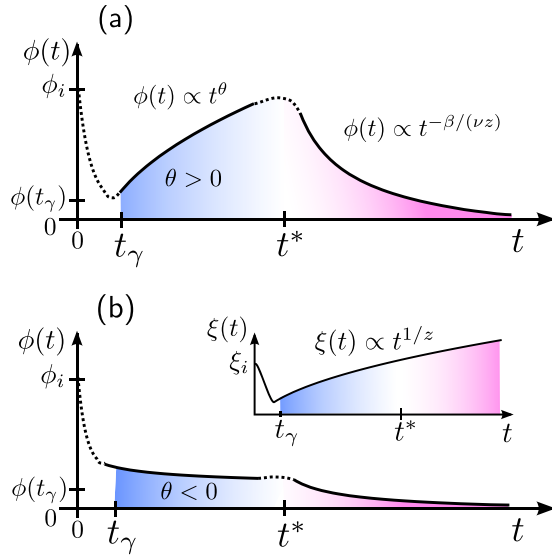


FIG. 6. (Color online) Time evolution of the order parameter $\phi(t)$ following a quench from an initial state $\phi_i \neq 0$. The two panels show the cases of (a) $\theta > 0$, and (b) $\theta < 0$. First, the order parameter collapses due to the quench. After a microscopic time scale t_γ (see also Appendix A), the order parameter recovers for a while $t_\gamma < t < t^*$ according to $\phi(t) \propto t^\theta$ for $\theta > 0$, before it eventually relaxes to zero for $t > t^*$ via a power law described by equilibrium exponents. For $\theta < 0$ the order parameter always decays in a power-law fashion but exhibits two different exponents for $t < t^*$ and $t > t^*$. The inset in (b) shows the time dependence of the correlation length which diverges after an initial collapse in a light-cone fashion $\xi(t) \propto t^{1/z}$.

with the same θ as in Eq. (78). For small θ the logarithm can again be exponentiated such that

$$\phi(t) = \phi_0(t_\gamma) \left(\frac{t}{t_\gamma} \right)^\theta. \quad (130)$$

For positive θ the order increases after a quench, while for negative θ it decays. Note that in both cases the order parameter first collapses right after the quench on microscopic time scales t_γ to a nonuniversal value $\phi(t_\gamma) < \phi_i$. The two cases are schematically shown in Fig. 6.

In both cases the order parameter varies slowly in time compared to the algebraic decay of $\gamma a/t^{2/z}$. Therefore, independent of the sign of θ there exists a time t^* where the underlying assumption of Eq. (125) that the second term in Eq. (124) may be neglected compared to the first is no longer valid. This yields the crossover time t^* between prethermalization and quasiadiabatic relaxation

$$t^* = \left(\frac{2|a|K_d t_\gamma^{2\theta}}{c_K \gamma^{2/z-1+\epsilon/2}} \right)^{z/(2+2z\theta-\epsilon)} \phi_0(t_\gamma)^{-\nu z/(\beta+\theta\nu z)}. \quad (131)$$

By performing a quench with a small amplitude such that the system was initially located close to the quantum critical point and $\phi_0(t_\gamma)$ is small, the duration of the prethermalization regime can be tuned to large time scales. Note that for $z \geq 2$ a large damping γ increases t^* , while for $z < 2$ a large damping shortens the prethermalized regime.

B. Quasiadiabatic relaxation to equilibrium at long times $t \gg t^*$

At long times the system thermalizes due to the external bath. For a quench right to the critical point, we expect the order parameter to eventually relax to zero. Thus, for $t \gg t^*$, where t^* is the crossover time defined in Eq. (131), the order parameter $\phi(t)$ varies slowly in time such that one can neglect the time derivatives of $\phi(t)$ as well as the convolution term:

$$\int dt' \delta\eta(t-t')\phi(t') \approx \phi(t) \int_0^\infty dt' \delta\eta(t-t') = 0. \quad (132)$$

The inhomogeneous term in Eq. (84) quickly decays to zero at long times, such that the equation of motion (121) reduces to

$$r(t \gg t^*)\phi(t) = 0. \quad (133)$$

A nontrivial solution of this equation requires that $r(t \gg t^*) = 0$. Inserting this solution into Eq. (124) yields for the dynamics of the order parameter

$$\begin{aligned} \phi(t \gg t^*) &= \left(\frac{2aK_d}{\epsilon\gamma^{2/z-1+\epsilon/2}c_K} \right)^{\frac{1}{2}} t^{-(2-\epsilon)/(2z)} \\ &= \left(\frac{2\gamma a\Lambda^\epsilon}{u^*\gamma^{\epsilon/2}} \right)^\beta t^{-\beta/(\nu z)}, \end{aligned} \quad (134)$$

where we used the large- N values of the exponents ν and β . The time dependence of ϕ in the long-time limit is the usual adiabatic decay and is fully determined by equilibrium exponents. This result confirms the scaling analysis in Sec. IV. Like in the long-time behavior of the Keldysh function, the light-cone amplitude a enters as a universal prefactor.

C. Dynamics of ϕ at $d = d_{uc}$

We can extend the previous discussion to the case of a system at the upper critical dimension $d = d_{uc}$. For $\epsilon = 0$ the self-consistent solution of the large- N equations is given by [cf. Eqs. (115) and (116)]

$$u = u_{uc}^*, \quad (135)$$

$$r_{uc}(t) = \frac{z c_K}{2 \ln(t/t_\gamma)} \left(\frac{\gamma C_0}{z t_\gamma} + \frac{\gamma^{2/z} \phi^2(t)}{K_d} \right). \quad (136)$$

In the adiabatic regime, the dynamics of ϕ is determined by the condition in Eq. (133), i.e., $r(t \gg t^*) = 0$, which yields

$$\phi(t) = \left(\frac{K_d C_0 \gamma^{1-z/2}}{z} \right)^\beta t^{\beta/(\nu z)}, \quad (137)$$

where ν and β now take their mean-field values $\beta_{MF} = \nu_{MF} = 1/2$. As expected, at $d = d_{uc}$ the long-time quasiadiabatic dynamics is given by the mean-field values of the equilibrium exponents.

To obtain the crossover time scale t^* , we again compare the size of the two terms in Eq. (136). In the prethermalized regime $t < t^*$, the ϕ^2 contribution in r_{uc} is much smaller than $(\gamma c_K C_0)/[2t^{2/z} \ln(t/t_\gamma)]$ and can thus be neglected. In this limit, the effective mass $r_{uc}(t)$ takes the form of Eq. (115) and it follows in analogy to the calculation in Sec. VIA for the

dynamics of the order parameter:

$$\begin{aligned}\phi_{uc}(t) &\approx \phi_0 \left(1 + \theta_{uc} \ln \left(\ln \frac{t}{t_\gamma} \right) \right) \\ &\approx \phi_0 \left(\ln \frac{t}{t_\gamma} \right)^{\theta_{uc}}.\end{aligned}\quad (138)$$

Like in the Green's function, the additional logarithm in the effective mass leads to a even slower increase (decrease) of order parameter after the quench. The typical crossover time t^* is now given by

$$t^* = \left(\frac{|C_0|K_d}{\gamma^{2/z-1}} \right)^{z/2} \phi_0(t_\gamma)^{-\nu z/\beta}.\quad (139)$$

Here, we have neglected the logarithmic contribution $\ln(t^*/t_\gamma)^{2\theta_{uc}}$. The typical crossover time for $\epsilon = 0$ is now given by the mean-field values of $\nu_{MF} = \beta_{MF} = 1/2$ and $\theta_{MF} = 0$ in the exponent of $\phi_0(t_\gamma)$.

VII. CONCLUSIONS

In this article we have provided a detailed analysis of the postquench dynamics of a φ^4 model coupled to an environment, that is suddenly moved from an equilibrium state near a quantum critical point towards the quantum critical point. We have employed nonequilibrium quantum field theory in the limit of large N , where N is the number of field components of φ . We considered different values of the dynamic exponent z , which at large N is determined by the bath spectral function that we assume to be of Ohmic, sub-Ohmic, or super-Ohmic form. We investigate quenches to the quantum critical point starting from both the symmetric and the symmetry-broken phases.

After the quench, the system is characterized by an effective, time-dependent mass $r(t) = \gamma a/t^{2/z}$ with universal amplitude a . The retarded and Keldysh Green's functions obey scaling $G^{R(K)} = (t/t')^\theta F^{R(K)}(q^z t/\gamma^{z/2}, t/t')/(q^{2-z}\gamma^{z/2})$. Away from equilibrium, G^R and G^K depend on both time arguments t, t' separately, implying interaction-induced aging behavior. The singular part of the time dependence in the short-time limit is characterized by a new, universal critical exponent θ , which is independent of the equilibrium exponents. This can be understood as a consequence of the fact that the scaling dimension of the initial magnetization reads $\kappa\beta/\nu$ which is different from the equilibrium scaling dimension of the magnetization β/ν (we note that this can be different in other models such as for a conserved order parameter). We have explicitly shown that one obtains the same value for θ , independent of the quench direction and the initial parameters. In the long-time limit the algebraic decay of $r(t)$ leads to a significant slowing down of thermalization. Here the value of θ enters a universal prefactor, while the aging behavior of G^R and G^K can be expressed by equilibrium exponents.

For a quench with a finite initial order parameter ϕ_i , we have identified three different time regimes in the dynamics of the order parameter: (i) a nonuniversal regime at short times $t < t_\gamma$, (ii) a prethermalized universal regime at intermediate times $t_\gamma < t < t^*$, and (iii) a regime at long times $t > t^*$ that is characterized by quasiadiabatic relaxation to equilibrium. In the prethermalized regime the order parameter fulfills

$\phi(t) \propto t^\theta$, where θ can be positive or negative, depending on the dynamic critical exponent z , which is determined by the bath spectral function. In the long-time limit the dynamics of ϕ is given by equilibrium exponents. The typical time scale t^* separating the prethermalization from the quasiadiabatic regime depends explicitly of the initial value of the order parameter ϕ_i . It diverges in the limit of a weak quench $\phi_i \rightarrow 0$. This permits tuning the prethermalization regime to extend to long time scales, however, at the cost of a small value of the order parameter.

We also analyzed the dynamics of the order parameter $\phi(t)$ and of the retarded and correlation function $G^{R/K}$ at the upper critical dimension $d = d_{uc} = 4 - z$. Compared to the dynamics below d_{uc} the power-law growth (or decay) of $\phi(t) \propto t^\theta$ in the prethermalization regime is now slowed down by the occurrence of an additional logarithm $\phi_{uc}(t) \propto [\ln(t/t_\gamma)]^{\theta_{uc}}$. In the long-time limit, the dynamics of ϕ is given by the mean-field values of equilibrium exponents. The special $[\ln(t/t_\gamma)]^{\theta_{uc}}$ behavior arises because at the mean-field level the exponent θ vanishes. Above the upper critical dimension $d > d_{uc}$, one cannot use a scaling ansatz for G^K . We expect that the order parameter remains constant during the prethermalization regime, which is followed by an adiabatic decay with mean-field exponents at long times.

The unique universal dynamical behavior we report is ultimately a consequence of the collapse of the correlation function ξ immediately after the quench and its recovery by critical coarsening according to $\xi(t) \propto t^{1/z}$. Due to the fast quench the system quickly falls out of equilibrium and is left in a highly excited state after the quench. Relaxation occurs due to interaction and via coupling to the external bath, which is we assume to remain at zero temperature. We observe substantial influences of quantum fluctuations and quantum coarsening on the postquench dynamics and relaxation. Quantum fluctuations are especially important for memory effects that the system exhibits which lead to a completely different z dependence of the exponent θ and of the aging prefactor of G^K compared to the classical result. Quantum fluctuations can even result in negative exponents $\theta < 0$ for super-Ohmic baths.

In the long-time limit, we show that it is possible to connect the response function to the correlation function, like in the equilibrium fluctuation-dissipation theorem. This allows us to introduce a nonthermal and time-dependent distribution function $n(t_a, \omega)$ which exhibits aging effects; i.e., it depends on the total time t_a passed since the quench. At a given time, we find $n(t_a, \omega) = n_B(\omega) + \delta n(t_a, \omega)$ with $\delta n \propto \theta(|\omega|t_a)^{-2/z} [\cos(\pi/z) + |\omega|^{-2/z} q^2 \gamma^{-1} \sin(\pi/z)]$. The correction δn exhibits an algebraic frequency dependence at large frequencies which clearly distinguishes it from a thermal distribution, which shows an exponential behavior for $T > \omega$. The amplitude of the correction δn is proportional to the short-time exponent θ . The sign of δn can either be positive or negative: $\delta n > 0$ can be interpreted as an increased number of excitations being present compared to equilibrium. The fact that δn can become negative implies that the density matrix of the system is not diagonal in the energy basis and quantum coherence is present even on long time scales.

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APPENDIX A: SHORT TIME SCALES

Our problem is governed by several short time scales determined by the momentum cut off Λ of the ϕ^4 theory, the strength of the damping coefficient γ , and the upper cutoff of the bath spectrum ω_c . Here we briefly discuss the hierarchy of these scales.

The large- N analysis clearly shows that we will only have a solution $r(t) \propto t^{-2/z}$ if $t \gg t_0$ with

$$t_0 = \gamma^{z/2} / \Lambda^z. \quad (\text{A1})$$

Otherwise the long-time expansion of the upper cutoff of the momentum integration is not allowed. t_0 obeys $\Lambda^2 = \gamma t_0^{-2/z}$; i.e., it is the scale where the damping $\gamma t_0^{-2/z}$ is comparable to the largest possible value of k^2 with momentum k . Clearly, this comparison is only sensible if in fact $t_0 > t_c$ with

$$t_c = 1/\omega_c \quad (\text{A2})$$

determined by the bath cutoff. For time scales smaller than t_c , the damping is very small and it cannot be estimated any longer via $\gamma t^{-2/z}$. The condition $t_0 > t_c$ translates into

$$\gamma \omega_c^{2/z} > \Lambda^2; \quad (\text{A3})$$

i.e., even the largest possible q^2 value is still smaller than the largest possible damping term in the propagator.

Another assumption of our analysis is that we ignore the ω^2 term relative to the damping term. This is only correct for sufficiently low energies or, equivalently, long time scales $t \gg t_\gamma$ with

$$t_\gamma = \gamma^{-z/[2(z-1)]}. \quad (\text{A4})$$

This result follows from $t_\gamma^{-2} = \gamma t_\gamma^{-2/z}$. For $z > 1$ discussed here, t_γ can be made small if the damping coefficient takes large values. However, comparing the ballistic and damping term makes sense only if $t_\gamma > t_c$, which corresponds to

$$\gamma \omega_c^{2/z} < \omega_c^2. \quad (\text{A5})$$

The conditions Eqs. (A3) and (A5) imply that it must hold

$$\omega_c > \Lambda. \quad (\text{A6})$$

One can also introduce the time scale

$$t_\Lambda = 1/\Lambda, \quad (\text{A7})$$

which is always larger than t_c . Thus, the shortest scale is always t_c .

Our prethermalized regime can only start after $\max(t_0, t_\gamma, t_\Lambda)$. One finds that no matter what the ratio of t_0 and t_γ , t_Λ is always between these two time scales and can therefore never be the largest. As for the ratio between t_0 and t_γ , it seems plausible to request $t_\gamma \gg t_\Lambda$ since t_γ is the only scale that does not depend on a cutoff. In this case we have the

hierarchy of scales

$$\omega_c \gg \Lambda^z / \gamma^{z/2} \gg \Lambda \gg t_\gamma^{-1} \quad (\text{A8})$$

and universal prethermalized behavior will set in after t_γ . Thus, an important condition for our analysis is that the damping due to the bath is sufficiently large such that there is a wide regime $t \gg t_\gamma$, where damping dominates the dynamics of the system.

APPENDIX B: LARGE- N EXPANSION AFTER A QUANTUM QUENCH

In this Appendix we summarize the derivation of the large- N equations for the out-of-equilibrium dynamics after a sudden quench. After having integrated out the bath degrees of freedom the action of the collective order parameter field in Eq. (27) is given in terms of the Matsubara, quantum, and classical field variables as

$$S[\boldsymbol{\varphi}] = S_M[\boldsymbol{\varphi}_M] + S_K[\boldsymbol{\varphi}_c, \boldsymbol{\varphi}_q] + S_C[\boldsymbol{\varphi}_M, \boldsymbol{\varphi}_q]. \quad (\text{B1})$$

The first term is up to a trivial factor i the usual Matsubara action in equilibrium

$$S_M[\boldsymbol{\varphi}_M] = \frac{i}{2} \int_{x, \tau \tau'} \boldsymbol{\varphi}_M(\tau) (g_i^M)^{-1} (\tau - \tau') \boldsymbol{\varphi}_M(\tau') + i \frac{u_i}{4N} \int_{x, \tau} [\boldsymbol{\varphi}_M(\tau) \cdot \boldsymbol{\varphi}_M(\tau)]^2, \quad (\text{B2})$$

where $g_i^M(\tau - \tau')$ follows from Eq. (32) after Fourier transformation. We use the notation $\int_\tau \equiv \int_0^\beta d\tau$ with the usual imaginary time τ defined via $t = i\tau$. For the time integrals along the real part we use $\int_t \equiv \int_0^\infty dt$. The two segments on the real axis are given as

$$S_K[\boldsymbol{\varphi}_c, \boldsymbol{\varphi}_q] = \frac{1}{2} \int_{x, t, t'} \begin{pmatrix} \boldsymbol{\varphi}_c(t) \\ \boldsymbol{\varphi}_q(t) \end{pmatrix}^T \mathcal{G}_0^{-1}(t, t') \begin{pmatrix} \boldsymbol{\varphi}_c(t') \\ \boldsymbol{\varphi}_q(t') \end{pmatrix} - \frac{u_f}{2N} \int_{x, t} [\boldsymbol{\varphi}_c^2(t) + \boldsymbol{\varphi}_q^2(t)] \boldsymbol{\varphi}_c(t) \cdot \boldsymbol{\varphi}_q(t) \quad (\text{B3})$$

with matrix propagator

$$\mathcal{G}_0 = \begin{pmatrix} g_f^K & g_f^R \\ g_f^A & 0 \end{pmatrix}. \quad (\text{B4})$$

The inverse is given as

$$\mathcal{G}_0^{-1} = \begin{pmatrix} 0 & (g_f^A)^{-1} \\ (g_f^R)^{-1} & M_0 \end{pmatrix}, \quad (\text{B5})$$

where $M_0 \equiv (\mathcal{G}_0^{-1})^K = -(g_f^R)^{-1} g_f^K (g_f^A)^{-1}$. Since we integrated out the bath degrees of freedom the retarded Green's function is determined by Eq. (41) while

$$M_0(t, t') = -\nu(t - t'). \quad (\text{B6})$$

The fluctuation-dissipation theorem of the bath can be expressed by the Fourier transforms

$$\nu(\omega) = \coth\left(\frac{\omega}{2T}\right) [\eta(\omega) - \eta^*(\omega)]. \quad (\text{B7})$$

The coupling between the two parts of the contour is

$$S_C[\boldsymbol{\varphi}_M, \boldsymbol{\varphi}_q] = i \int_{t, \tau} \boldsymbol{\varphi}_M(\tau) \tilde{\eta}(i\tau, t) \boldsymbol{\varphi}_q(t), \quad (\text{B8})$$

where $\tilde{\eta}(t) = \sqrt{2}[\eta^<(t) - \eta^<(-t)]$ and $\eta^{\geq} = \frac{1}{2}(v \pm \eta \mp \eta^*)$.

Since $\eta_0 \equiv \eta(\omega = 0)$ is finite, we have $\eta(t - t') = \eta_0 \delta(t - t') + \delta\eta(t - t')$, where $\delta\eta(t - t')$ is purely nonlocal. This yields with $\bar{r}_{0,f} = r_{0,f} - \eta_0$ that

$$(g_f^R)^{-1}(t, t') = -(\partial_t^2 + \bar{r}_{0,f} - \nabla^2) \delta(t - t') + \delta\eta(t - t'). \quad (\text{B9})$$

Since $v(0) = 0$, $v(t - t')$ is purely nonlocal in time. Using $\eta(\omega = 0) = \eta_M(i\omega_n = 0)$ we can absorb the same zero-frequency part on the imaginary axis: $\bar{r}_{0,i} = r_{0,i} - \eta_0$, such that

$$(g_f^M)^{-1}(\tau, \tau') = (\partial_\tau^2 - \bar{r}_{0,i} + \nabla^2) \delta(\tau - \tau') + \delta\eta_M(\tau - \tau'). \quad (\text{B10})$$

Next we perform the large- N expansion and follow closely the procedure in chapter 30 of Ref. [111]. Important degrees of freedom are obviously $\rho_{\pm} = \frac{1}{N} \boldsymbol{\varphi}_{\pm}^2$ and $\rho_M = \frac{1}{N} \boldsymbol{\varphi}_M^2$ which yields in the case of the round trip segment

$$\begin{aligned} \rho_c &= \frac{1}{\sqrt{2}}(\rho_+ + \rho_-) = \frac{1}{\sqrt{2}N}(\boldsymbol{\varphi}_c^2 + \boldsymbol{\varphi}_q^2), \\ \rho_q &= \frac{1}{\sqrt{2}}(\rho_+ - \rho_-) = \frac{\sqrt{2}}{N} \boldsymbol{\varphi}_c \cdot \boldsymbol{\varphi}_q. \end{aligned} \quad (\text{B11})$$

The interaction term of the action along the real-axis part of the contour is then given by

$$\begin{aligned} S_{K,\text{int}} &= -\frac{u_f}{2N} \int_{x,t} [\boldsymbol{\varphi}_c^2(t) + \boldsymbol{\varphi}_q^2(t)] \boldsymbol{\varphi}_c(t) \cdot \boldsymbol{\varphi}_q(t) \\ &= -\frac{u_f N}{2} \int_{x,t} \rho_c(t) \rho_q(t). \end{aligned} \quad (\text{B12})$$

In order to enforce the above constraints, we introduce

$$\begin{aligned} 1 &= \int D\rho \prod_{x,t} \delta\left(\frac{1}{\sqrt{2}N}(\boldsymbol{\varphi}_c^2 + \boldsymbol{\varphi}_q^2) - \rho_c\right) \\ &\times \prod_{x,t} \delta\left(\frac{\sqrt{2}}{N} \boldsymbol{\varphi}_c \cdot \boldsymbol{\varphi}_q - \rho_q\right) \prod_{x,\tau} \delta\left(\frac{1}{N} \boldsymbol{\varphi}_M^2 - \rho_M\right) \\ &\propto \int D\rho D\mathbf{r} e^{-\frac{i}{2} \int_{x,t} r_c(t) (2\boldsymbol{\varphi}_c \cdot \boldsymbol{\varphi}_q - \sqrt{2}N\rho_q)} \\ &\times e^{-\frac{i}{2} \int_{x,t} r_q(t) (\boldsymbol{\varphi}_c^2 + \boldsymbol{\varphi}_q^2 - \sqrt{2}N\rho_c)} \\ &\times e^{-\frac{i}{2} \int_{x,\tau} r_M(t) (\boldsymbol{\varphi}_M^2 - N\rho_M)}, \end{aligned} \quad (\text{B13})$$

where $\int D\rho$ is the shorthand for $\int D\rho_c \int D\rho_q \int D\rho_M$ and similar for other degrees of freedom. The integration contour for the $r_{c,q}$ integrations is along the real axis, while the r_M integral is performed along the imaginary axis. It follows for the generating functional

$$Z = \int D\boldsymbol{\varphi} D\rho D\mathbf{r} e^{i(S_M + S_K + S_C)}$$

with unchanged coupling term S_C and

$$\begin{aligned} S_M &= \frac{i}{2} \int_{x,\tau,\tau'} \boldsymbol{\varphi}_M(\tau) (G_r^M)^{-1}(\tau, \tau') \boldsymbol{\varphi}_M(\tau') \\ &+ i \frac{N}{2} \int_{x,\tau} \left(\frac{u_i}{2} \rho_M^2(\tau) + [\bar{r}_{0,i} - r_M(\tau)] \rho_M(\tau) \right) \end{aligned}$$

as well as

$$\begin{aligned} S_K &= \frac{1}{2} \int_{x,t,t'} \begin{pmatrix} \boldsymbol{\varphi}_c(t) \\ \boldsymbol{\varphi}_q(t) \end{pmatrix}^T \mathcal{G}_r^{-1}(t, t') \begin{pmatrix} \boldsymbol{\varphi}_c(t') \\ \boldsymbol{\varphi}_q(t') \end{pmatrix} \\ &- \frac{N}{2} \int_{x,t} (\sqrt{2}[\bar{r}_{0,f} - r_c(t)] \rho_q(t) + u_f \rho_q(t) \rho_c(t)) \\ &- \frac{N}{2} \int_{x,t} \sqrt{2} r_q(t) \rho_c(t). \end{aligned} \quad (\text{B14})$$

The modified propagators are given as

$$\begin{aligned} (G_r^M)^{-1}(\tau, \tau') &= [-\partial_\tau^2 + r_M(\tau) - \nabla^2] \delta(\tau - \tau') \\ &+ \delta\eta_M(\tau - \tau') \end{aligned} \quad (\text{B15})$$

on the imaginary axis and

$$\mathcal{G}_r^{-1}(t, t') = \begin{pmatrix} 0 & (G_r^A)^{-1} \\ (G_r^R)^{-1} & M_0 \end{pmatrix}_{t,t'} - r_q(t) \delta(t - t') \mathbb{1} \quad (\text{B16})$$

with modified retarded function

$$(G_r^R)^{-1}(t, t') = -[\partial_t^2 + r_c(t) - \nabla^2] \delta(t - t') + \delta\eta(t - t') \quad (\text{B17})$$

on the real axis and 2×2 unit matrix $\mathbb{1}$. Thus, the bare masses have been replaced by in general time-dependent terms that correspond to become the usual self-energy corrections of the large- N theory.

We express the vectors $\boldsymbol{\varphi}_{c,q,M}$ in terms of the component $\boldsymbol{\varphi}_{c,q,M}$ along the direction of the field $\mathbf{h}_{i,m} = (h_{i,m}, 0, \dots, 0)$ and the $N - 1$ components $\boldsymbol{\pi}_{c,q,M}$ orthogonal to it:

$$\boldsymbol{\varphi}_{c,q,M} = (\sqrt{N} \boldsymbol{\varphi}_{c,q,M}, \boldsymbol{\pi}_{c,q,M}). \quad (\text{B18})$$

We first integrate over the transverse fields $\boldsymbol{\pi}_M$ along the imaginary contour and obtain

$$\begin{aligned} S &= S_K + \frac{iN}{2} \int_{x,\tau,\tau'} \boldsymbol{\varphi}_M(\tau) (G_r^M)^{-1}(\tau, \tau') \boldsymbol{\varphi}_M(\tau') \\ &+ iN \int_{x,\tau,t} \boldsymbol{\varphi}_M(\tau) \tilde{\eta}(i\tau, t) \boldsymbol{\varphi}_q(t) \\ &+ \frac{i}{2} (N - 1) \text{tr} \ln (G_r^M)^{-1} - \frac{i}{2} \int_{x,t,t'} \boldsymbol{\pi}_q(t) \mu(t, t') \boldsymbol{\pi}_q(t') \\ &+ i \frac{N}{2} \int_{x,\tau} \left(\frac{u_i}{2} \rho_M^2(\tau) + \bar{r}_{0,i} \rho_M(\tau) - r_M(\tau) \rho_M(\tau) \right), \end{aligned}$$

where

$$\mu(t, t') = \int_{\tau\tau'} \tilde{\eta}(t, i\tau) G_r^M(\tau, \tau') \tilde{\eta}(i\tau', t'). \quad (\text{B19})$$

The term $\int \boldsymbol{\pi}_q \mu \boldsymbol{\pi}_q$ can be absorbed into a redefinition of the bare inverse Keldysh function

$$M_0(t, t') \rightarrow M(t, t') = -v(t - t') - \mu(t, t'). \quad (\text{B20})$$

This is a crucial aspect of the theory demonstrating that the bare postquench correlation function is affected by the prequench behavior due to memory effects of the bath.

Next, we integrate over the transverse fields $\pi_{c,q}$ and obtain

$$\begin{aligned}
 S = & \frac{N}{2} \int_{x,t} \left(\begin{matrix} \phi_c(t) \\ \phi_q(t) \end{matrix} \right)^T \mathcal{G}_r^{-1}(t,t') \left(\begin{matrix} \phi_c(t') \\ \phi_q(t') \end{matrix} \right) \\
 & + \frac{iN}{2} \int_{x,\tau\tau'} \phi_M(\tau) (G_r^M)^{-1}(\tau,\tau') \phi_M(\tau') \\
 & + iN \int_{x,\tau t} \phi_M(\tau) \tilde{\eta}(i\tau,t) \phi_q(t) \\
 & - \frac{N}{2} \int_{x,t} (\sqrt{2}(\bar{r}_{0,f} - r_c(t)) \rho_q(t) + u_f \rho_q(t) \rho_c(t)) \\
 & + \frac{N}{2} \int_{x,t} \sqrt{2} r_q(t) \rho_c(t) \\
 & + i \frac{N}{2} \int_{x,\tau} \left(\bar{r}_{0,i} \rho_M(\tau) + \frac{u_i}{2} \rho_M^2(\tau) - r_M(\tau) \rho_M(\tau) \right) \\
 & + \frac{i}{2} (N-1) \text{tr} \ln \mathcal{G}_r^{-1} + \frac{i}{2} (N-1) \text{tr} \ln (G_r^M)^{-1}.
 \end{aligned} \tag{B21}$$

The action is of order N , allowing us to perform a saddle-point analysis. Minimizing the action with respect to ϕ , ρ , and r yields the nine saddle-point equations. Minimizing with respect to the fields on the Matsubara segment of the contour yields

$$\begin{aligned}
 h_M(\tau) &= \int_{\tau'} (G_r^M)^{-1}(\tau,\tau') \phi_M(\tau') + \int_t \tilde{\eta}(i\tau,t) \phi_q(t), \\
 \rho_M(\tau) &= \phi_M^2(\tau) + \frac{i}{2} \frac{\delta \text{tr} \ln \mathcal{G}_r^{-1}}{\delta r_M(\tau)} + \frac{\delta \text{tr} \ln (G_r^M)^{-1}}{\delta r_M(\tau)}, \tag{B22} \\
 r_M(\tau) &= \bar{r}_{0,i} + u_i \rho_M(\tau).
 \end{aligned}$$

The minimization with respect to the quantum field on the Keldysh contour yields

$$\begin{aligned}
 h_c(t) &= -r_q(t) \phi_q(t) + \int dt' (G_r^R)^{-1}(t,t') \phi_c(t') \\
 & \quad + \int dt' (G_r^{-1})^K(t,t') \phi_q(t') + \int_\tau \phi_M(\tau) \tilde{\eta}(i\tau,t), \\
 \rho_c(t) &= \frac{1}{\sqrt{2}} [\phi_c^2(t) + \phi_q^2(t)] - \frac{i}{\sqrt{2}} \frac{\delta \text{tr} \ln \mathcal{G}_r^{-1}}{\delta r_c(t)}, \tag{B23} \\
 r_c(t) &= \bar{r}_{0,f} + \frac{u_f}{\sqrt{2}} \rho_c(t).
 \end{aligned}$$

Finally, from the minimization with respect to the classical fields on the Keldysh contour follows that

$$\begin{aligned}
 h_q(t) &= -r_q(t) \phi_c(t) + \int_{t'} (G_r^A)^{-1}(t,t') \phi_q(t'), \\
 \rho_q(t) &= \frac{1}{\sqrt{2}} \phi_c(t) \phi_q(t) - \frac{i}{\sqrt{2}} \frac{\delta \text{tr} \ln \mathcal{G}_r^{-1}}{\delta r_q(t)}, \tag{B24} \\
 r_q(t) &= \frac{u}{\sqrt{2}} \rho_q(t).
 \end{aligned}$$

The natural saddle-point solution for the quantum components respecting causality is $r_q = \rho_q = \phi_q = 0$ for $h_q = 0$. We checked that this trivial solution is the only solution by an explicit analysis of the Heisenberg equation of motion of the order parameter and the propagators. Performing the functional derivatives and evaluating them at vanishing quantum fields yields

$$\begin{aligned}
 \frac{\delta \text{tr} \ln \mathcal{G}_r^{-1}}{\delta r_c(t)} &= \text{tr} \left(\mathcal{G}_r \frac{\delta \mathcal{G}_r^{-1}}{\delta r_c(t)} \right) \\
 &= -G_r^R(t,t) - G_r^A(t,t) = 0, \\
 \frac{\delta \text{tr} \ln \mathcal{G}_r^{-1}}{\delta r_q(t)} &= \text{tr} \left(\mathcal{G}_r \frac{\delta \mathcal{G}_r^{-1}}{\delta r_q(t)} \right) = -G_r^K(t,t), \tag{B25} \\
 \frac{\delta \text{tr} \ln (G_r^M)^{-1}}{\delta r_M(\tau)} &= (G_r^M)(\tau,\tau), \quad \frac{\delta \text{tr} \ln \mathcal{G}_r^{-1}}{\delta r_M(\tau)} = 0.
 \end{aligned}$$

We consider time-independent fields $h_M(\tau) = h_i$ and $h_c(t) = h_f$ before and after the quench, respectively. Since the imaginary-time evolution prior to the quench is in equilibrium we obtain time-independent solutions for $r_M = r_i$ and $\phi_M = \phi_i$, yielding the usual equilibrium version of the large- N equations. Eliminating ρ_M yields

$$\begin{aligned}
 h_M &= r_M \phi_M, \\
 r_M &= \bar{r}_{0,i} + \frac{u_i}{2} \phi_M^2 + u_i G_r^M(x,\tau;x,\tau). \tag{B26}
 \end{aligned}$$

In the last step we reintroduced spatial coordinates. The nonequilibrium large- N equations of the classical components r_c , ρ_c , and ϕ_c are nontrivial and time dependent. We obtain, after the elimination of ρ ,

$$\begin{aligned}
 h_c &= \int dt' (G_r^R)^{-1}(t,t') \phi_c(t') + \phi_M \int_\tau \tilde{\eta}(i\tau,t), \\
 r(t) &= \bar{r}_{0,f} + \frac{u_f}{2} \phi_c^2(t) + \frac{u}{2} i G_r^K(x,t;x,t). \tag{B27}
 \end{aligned}$$

These are the self-consistent large- N equations given in the main part of the paper. There we only used the more physically motivated notation $h_i = h_M$, $\phi_i = \phi_M$, and $r_i = r_M$ to indicate that the variables on the Matsubara branch refer to the initial field, order parameter, and renormalized mass, respectively. Similarly we have $h_f = h_c$, $\phi(t) = \phi_c(t)$, and $r(t) = r_c(t)$ for the corresponding variables after the quench.

APPENDIX C: THE FIXED-MASS POSTQUENCH BARE GREEN'S FUNCTIONS

In this subsection we summarize the main steps in the derivation of the postquench retarded and Keldysh function of the noninteracting system with constant masses $r_{0,i}$ and $r_{0,f}$. The knowledge of this propagator is essential for any development of a perturbative approach to include interactions.

In order to determine the bare propagators, we start from the Heisenberg equation of motion of the field operator after the quench:

$$\begin{aligned}
 (\partial_t^2 + r_{0,f} + q^2) \boldsymbol{\varphi}(q,t) &= \int_0^\infty ds \eta(t-s) \boldsymbol{\varphi}(q,s) \\
 & \quad + \boldsymbol{\Xi}(q,t) + \mathbf{h}(q,t) \tag{C1}
 \end{aligned}$$

with external field $\mathbf{h}(q,t)$ that is coupled to the order parameter. The source operator $\Xi(q,t)$ is given by

$$\Xi(q,t) = - \int_{-\infty}^0 ds \eta(t-s) \boldsymbol{\varphi}(q,s). \quad (\text{C2})$$

It is useful to express $\Xi(t)$ in terms of the initial bath operators $\mathbf{X}_j^0 = \mathbf{X}_j(q,t=0)$ and $\mathbf{P}_j^0 = \mathbf{P}_j(q,t=0)$:

$$\Xi(q,t) = - \sum_j c_j \left(\mathbf{X}_j^0(q) \cos(\Omega_j t) + \frac{1}{\Omega_j} \mathbf{P}_j^0(q) \sin(\Omega_j t) \right).$$

We solve the Heisenberg equation of motion via Laplace transform

$$\boldsymbol{\varphi}(q,\omega) = \int_0^\infty dt e^{i(\omega+i0^+)t} \boldsymbol{\varphi}(q,t) \quad (\text{C3})$$

with boundary condition $\boldsymbol{\varphi}(q,t=0) = \boldsymbol{\varphi}_0(q)$ and $\partial_t \boldsymbol{\varphi}(q,t=0) = \boldsymbol{\pi}_0(q)$. It follows that

$$\boldsymbol{\varphi}(q,\omega) = \mathbf{F}(q,\omega) g_f^R(q,\omega) \quad (\text{C4})$$

with force-operator

$$\mathbf{F}(q,\omega) = \boldsymbol{\pi}_0(q) - i\omega \boldsymbol{\varphi}_0(q) + \Xi(q,\omega) + \mathbf{h}(q,\omega) \quad (\text{C5})$$

and bare retarded postquench Green's function

$$g_f^R(q,\omega) = \frac{1}{\omega^2 - r_{0,f} - q^2 + \eta(\omega)}. \quad (\text{C6})$$

To check that this is indeed the correct result for the bare retarded Green's function one expresses the back transform $\boldsymbol{\varphi}(q,t)$ of $\boldsymbol{\varphi}(q,\omega)$ in terms of $g_f^R(k,\omega)$:

$$\begin{aligned} \boldsymbol{\varphi}(q,t) &= [\boldsymbol{\pi}_0(q) + \boldsymbol{\varphi}_0(q) \partial_t] g_f^R(q,t) \\ &+ \int_0^\infty [\Xi(q,s) + \mathbf{h}(q,s)] g_f^R(k,t-s) ds. \end{aligned} \quad (\text{C7})$$

Inserting this result into the definition Eq. (38) of the retarded function does indeed yield Eq. (C6) for zero external field $\mathbf{h}(q,t)$. Thus, for the bare retarded postquench Green's function follows the same result as in equilibrium, which only depends on the difference between the two time arguments. As we have shown in Eq. (77), this behavior of the retarded function will not carry over to the case where we include interactions.

Next we consider the bare postquench Keldysh function $g_f^K(q,t,t')$. This function will depend on both time scales already for a noninteracting system. We determine $g_f^K(q,t,t')$ using the same approach as for the retarded function, i.e., insert the solution Eq. (C7) into the definition Eq. (39) of the Keldysh function. Here, it is convenient to consider the double Laplace transform:

$$g_f^K(q,\omega,\omega') = \int_0^\infty dt dt' g_f^K(q,t,t') e^{i(\omega_+ t + \omega'_+ t')}, \quad (\text{C8})$$

with $\omega_+ = \omega + i0^+$. From our solution Eq. (C4) follows

$$g_f^K(q,\omega,\omega') = M(q,\omega,\omega') g_f^R(q,\omega) g_f^R(q,\omega'), \quad (\text{C9})$$

with memory function

$$M(q,\omega,\omega') = -i \langle [F_a(q,\omega), F_a(-q,\omega')]_+ \rangle, \quad (\text{C10})$$

given by the force-force correlation function. $M(q,\omega,\omega')$ can be obtained from the expectation values of $\mathbf{F}(q,\omega)$ as given in

Eq. (C5). To proceed we note that the operators that enter the memory function are $\boldsymbol{\varphi}_0(q)$, $\mathbf{X}_j^0(q)$, etc.; i.e., they are for system and bath variables at $t=0$. Thus, all relevant expectation values can be evaluated in equilibrium prior to the quench. The direct evaluation of these expectation values is straightforward but somewhat tedious. In particular one finds that numerous expectation values diverge in the limit of an infinite cutoff ω_c , divergences that cancel if one combines all terms that contribute to the memory function. To avoid these complications we present a significantly easier approach to this problem. We explicitly checked that both approaches lead to the same result. We stress again that all expectation values that enter the memory function can equally be determined prior to the quench when the system is still in equilibrium. We therefore consider a system without quench and with initial Hamiltonian H_i for all times. The Keldysh function of this equilibrium system should of course have the same formal structure as Eq. (C9); i.e.,

$$g_i^K(q,\omega,\omega') = M(q,\omega,\omega') g_i^R(q,\omega) g_i^R(q,\omega'). \quad (\text{C11})$$

The subscript i indicates that we are considering a system in equilibrium that is governed by the Hamiltonian H_i with bare mass $r_{0,i}$. The key insight is that the memory function $M(q,\omega,\omega')$ must be the same function as in Eq. (C9) as we have to determine the expectation value of the same operators $\mathbf{F}(q,\omega)$ with respect to the same state.

In equilibrium, the Keldysh function only depends on the difference of the time arguments. For the double Laplace transform this implies

$$g_i^K(q,\omega,\omega') = i \frac{g_i^K(q,\omega) + g_i^K(q,\omega')}{\omega + \omega' + i0^+}, \quad (\text{C12})$$

where $g_i^K(q,\omega)$ is the Laplace transform of the bare Keldysh function of the initial state in equilibrium:

$$g_i^K(q,\omega) = \int_0^\infty dt e^{i(\omega+i0^+)t} g_i^K(q,t). \quad (\text{C13})$$

In equilibrium we can then use the fluctuation-dissipation relation

$$g_i^K(q,t) = i \int_{-\infty}^\infty \frac{d\epsilon}{\pi} e^{-i\epsilon t} \coth\left(\frac{\epsilon}{2T}\right) \text{Im} g_i^R(q,\epsilon) \quad (\text{C14})$$

to determine this function. Thus, we obtain for the memory function

$$M(q,\omega,\omega') = i \frac{g_i^K(q,\omega) + g_i^K(q,\omega')}{\omega + \omega' + i0^+} g_i^R(q,\omega)^{-1} g_i^R(q,\omega')^{-1}. \quad (\text{C15})$$

As mentioned, we obtained the same result using a straightforward evaluation of the definition of the Keldysh function, inserting the solution Eq. (C7), and evaluating all expectation values explicitly. As limiting cases, Eq. (C15) includes the solution of Ref. [12] for a classical system with Ohmic bath and Ref. [85] for a classical system with colored noise. In addition it reproduces the findings of Ref. [92] for a noninteracting quantum system without bath.

As discussed in the context of the scaling behavior, the limit where we consider large distances $\delta r_{0,i}$ from the critical point prior to the quench is of particular interest. In this deep-quench

limit follows

$$M(q, \omega, \omega') = i \frac{\coth\left(\frac{\omega}{2T}\right)\delta\eta(\omega) + \coth\left(\frac{\omega'}{2T}\right)\delta\eta(\omega')}{\omega + \omega' + i0^+}. \quad (\text{C16})$$

For times $t, t' > t_i = (\gamma/\delta r_i)^{z/2}$ the memory function is always governed by this deep-quench limit, even if δr_i is nominally not large. This enables us to introduce a scaling form for G^K and to expect universal behavior after the quench. The justification to use the deep-quench limit in this paper is however not that we concentrate on time scales larger than t_i , but rather that even for a small quench the spatial correlation length collapses and the bare Green's function become completely local in space, corresponding to a large effective mass at time $t = 0^+$, i.e., right after the quench on universal time scales ($t \approx t_\gamma$), independent of the precise quench protocol.

APPENDIX D: LONG-TIME LIMIT OF THE GREEN'S FUNCTIONS

We can extend the analysis for the equal-time Keldysh function to different time arguments t, t' , but still in the limit

$$q^z t / \gamma^{z/2} \gtrsim q^z t' / \gamma^{z/2} \gg 1 \text{ and } q^z (t - t') / \gamma^{z/2} \ll 1. \quad (\text{D1})$$

We use the long-time expansion for $\phi \simeq \phi_{\text{eq}} - \int_{t_\gamma}^t ds G_{\text{eq}}^R(t-s)r(s)\phi_{\text{eq}}(s)$ to obtain

$$\begin{aligned} G_r^K(q, t, t') &= -i \langle [\phi(t), \phi(t')]_+ \rangle \\ &= G_{\text{eq}}^K(q, t - t') - \int_{t_\gamma}^t ds G_{\text{eq}}^R(t-s)r(s)G_{\text{eq}}^K(s, t') \\ &\quad - \int_{t_\gamma}^{t'} ds G_{\text{eq}}^R(t'-s)r(s)G_{\text{eq}}^K(s-t). \end{aligned} \quad (\text{D2})$$

To evaluate the s integration we express G^R by its Laplace transformation and G^K by its Fourier transformation:

$$\begin{aligned} & - \int_{t_\gamma}^t ds G_{\text{eq}}^R(t-s)r(s)G_{\text{eq}}^K(s, t') \\ &= -i \int \frac{d\omega d\omega'}{2\pi^2} \text{Im}G_{\text{eq}}^R(\omega)G_{\text{eq}}^K(\omega') \\ &\quad \times \int_{t_\gamma}^t ds r(s)e^{-i\omega(t-s)-i\omega'(s-t')}. \end{aligned} \quad (\text{D3})$$

Here, ω and ω' are of order of the typical mode frequency $q^{-z}\gamma^{z/2}$. In the limit where the time difference $t - t'$ is small, but t, t' are both large compared to the mode-frequency, the s integral can be approximated by

$$\int_{t_\gamma}^t ds r(s)e^{-i\omega(t-s)-i\omega'(s-t')} \simeq r(t) \frac{e^{-i\omega'(t-t')}}{i(\omega - \omega')}. \quad (\text{D4})$$

In a next step, we use the Kramers-Kronig relation to perform one frequency integration. This finally yields

$$\begin{aligned} & - \int_{t_\gamma}^t ds G_{\text{eq}}^R(t-s)r(s)G_{\text{eq}}^K(s, t') \\ &= -r(t) \int \frac{d\omega'}{2\pi} \text{Re}G_{\text{eq}}^R(\omega')G_{\text{eq}}^K(\omega')e^{-i\omega'(t-t')}. \end{aligned} \quad (\text{D5})$$

The same procedure leads for the second integral in Eq. (D2) to

$$\begin{aligned} & - \int_{t_\gamma}^{t'} ds G_{\text{eq}}^R(t'-s)r(s)G_{\text{eq}}^K(s-t) \\ &\simeq -r(t') \int \frac{d\omega'}{2\pi} \text{Re}G_{\text{eq}}^R(\omega')G_{\text{eq}}^K(\omega')e^{-i\omega'(t-t')}. \end{aligned} \quad (\text{D6})$$

For $r(t) \approx r(t') \approx r(t_a)$ with $t_a = (t + t')/2$ this yields the result given in Eq. (103) in the main text. In analogy to the long-time limit of $G_r^K(q, t, t')$ we can derive a similar behavior for the retarded Green's function $G_r^R(q, t, t')$. It follows with the long-time expansion for ϕ and the definition of G_r^R in Eq. (38) that

$$\begin{aligned} G_r^R(t, t') &= -i\theta(t-t') \langle [\phi(t), \phi(t')]_- \rangle \\ &\simeq G_{\text{eq}}^R(t-t') \\ &\quad + i\theta(t-t') \left(\int_0^t ds G_{\text{eq}}^R(t-s)r(s) \langle [\phi(s), \phi(t')]_- \rangle \right. \\ &\quad \left. + \int_0^{t'} ds \langle [\phi(t), \phi(s)]_- \rangle r(s)G_{\text{eq}}^R(t'-s) \right) \\ &= G_{\text{eq}}^R(t-t') - \int_{t_\gamma}^t ds G_{\text{eq}}^R(t-s)r(s)G_{\text{eq}}^R(s-t'). \end{aligned} \quad (\text{D7})$$

We express the equilibrium G_r^R by its Laplace transformation and obtain

$$\begin{aligned} \delta G_r^R(t, t') &= - \int_{t_\gamma}^t ds G_{\text{eq}}^R(t-s)r(s)G_{\text{eq}}^R(s-t') \\ &= \int \frac{d\omega d\omega'}{\pi^2} \text{Im}G_{\text{eq}}^R(\omega)\text{Im}G_{\text{eq}}^R(\omega') \\ &\quad \times \int_{t_\gamma}^t ds \frac{a\gamma}{s^{2/z}} e^{-i\omega(t-s)-i\omega'(s-t')}. \end{aligned} \quad (\text{D8})$$

ω and ω' are of order of the typical mode frequency $q^{-z}\gamma^{z/2}$. In the limit where the time difference $t - t'$ is small, but t, t' are both large compared to the mode frequency, the s integral can be approximated by

$$\begin{aligned} & \int_{t_\gamma}^t ds r(s)e^{-i\omega(t-s)-i\omega'(s-t')} \\ &\simeq r\left(\frac{t+t'}{2}\right) \frac{1}{i(\omega - \omega')} (e^{-i\omega'(t-t')} - e^{-i\omega(t-t')}). \end{aligned} \quad (\text{D9})$$

This yields the result given in Eq. (104):

$$\begin{aligned} \delta G_r^R(t, t') &\simeq -ir\left(\frac{t+t'}{2}\right) \int \frac{d\omega d\omega'}{\pi^2} \frac{\text{Im}G_{\text{eq}}^R(\omega)\text{Im}G_{\text{eq}}^R(\omega')}{\omega - \omega'} \\ &\quad \times (e^{-i\omega'(t-t')} - e^{-i\omega(t-t')}) \\ &= -2ir\left(\frac{t+t'}{2}\right) \int \frac{d\omega'}{\pi} \text{Re}G_{\text{eq}}^R(\omega')\text{Im}G_{\text{eq}}^R(\omega') \\ &\quad \times e^{-i\omega'(t-t')} \\ &= -4ir\left(\frac{t+t'}{2}\right) C^R(t-t'). \end{aligned} \quad (\text{D10})$$

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