# Quenches and dynamical phase transitions in a nonintegrable quantum Ising model

Shraddha Sharma,[1] Sei Suzuki,[2] and Amit Dutta[1]
[1]*Department of Physics, Indian Institute of Technology, Kanpur 208 016, India*
[2]*Department of Liberal Arts, Saitama Medical University, Moroyama, Saitama 350-0495, Japan*
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We study quenching dynamics of a one-dimensional transverse Ising chain with nearest neighbor antiferromagnetic interactions in the presence of a longitudinal field which renders the model nonintegrable. The dynamics of the spin chain is studied following a slow (characterized by a rate) or sudden quenches of the longitudinal field. Analyzing the temporal evolution of the Loschmidt overlap, we find different possibilities of the presence (or absence) of dynamical phase transitions (DPTs) manifested in the nonanalyticities of the rate function of the return probability. Even though the model is nonintegrable, there are periodic occurrences of DPTs when the system is slowly ramped across the quantum critical point (QCP) as opposed to the ferromagnetic version of the model; this numerical finding is qualitatively explained by mapping the original model to an effective integrable spin model which is appropriate for describing such slow quenches. Furthermore, concerning the sudden quenches, our numerical results show that in some cases, DPTs can be present even when the spin chain is quenched within the same phase or even to the QCP, while in some other situations they completely disappear even after quenching across the QCP. These observations lead us to the conclusion that it is the change in the nature of the ground state that determines the presence of DPTs following a sudden quench.

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Following the remarkable advancement of the experimental studies of ultracold atoms trapped in optical lattices [1,2], there is an upsurge in the studies of nonequilibrium dynamics of closed quantum systems, in particular from the viewpoint of quantum quenches across a quantum critical point (QCP) [3,4]. The relaxation time of the quantum system diverges at the QCP resulting in a nonadiabatic dynamics and proliferation of topological defects in the final state reached after the quench. Particularly, the study of the dynamics of a quantum system ramped slowly across its QCP(s) has gained importance because of the possible *universal* Kibble-Zurek (KZ) scaling [5,6] of the defect density and the residual energy measured in the final state reached following the quench [7,8]; this scenario has been extensively studied in recent years [9–14]. Similar scaling relations for the residual energy and the defect density have also been derived using an adiabatic perturbation theory for a sudden quench of small magnitude [15] (for a review, see [16–18]). In this paper, we consider the real time evolution of a nonintegrable system following a quench (both slow as well as sudden) and probe the nonanalyticities, known as a *dynamical phase transition* (DPT), occurring at different instants of time. In particular, our emphasis is on the slow quenching across the QCP which, to the best of our knowledge, has not received much attention. Furthermore, we also present some remarkable results in the context of sudden quenches.

It is well established that the phase transition in a thermodynamic system is marked by the nonanalyticities in the free-energy density whose information can be obtained by analyzing the zeros of the partition function in a complex temperature plane as proposed by Fisher [19]. These zeros of the partition function coalesce into a line (or area [20]) in complex temperature plane, crossing the real axis in the thermodynamic limit; these crossings mark the nonanalyticities in the free-energy density. A similar observation was made earlier by Yang and Lee [21] for a complex magnetic plane. In a similar spirit, a recent work by Heyl *et al.* [22] introduced the notion of a DPT in connection to quantum quenches probing the nonanalyticities in the *dynamical free energy* in the complex time plane. The idea stems from the similarity between the canonical partition function

$$Z(\beta) = \mathrm{Tr}e^{-\beta H}, \tag{1}$$

of an equilibrium system (where $\beta$ is the inverse temperature) and that of the overlap amplitude [the Loschmidt overlap (LO)] defined at an instant of time $t$ as

$$G(t) = \langle\psi_0|e^{-iHt}|\psi_0\rangle, \tag{2}$$

where, in the above equation, $H$ is the final Hamiltonian of the system reached through a sudden quenching of parameters, while $|\psi_0\rangle$ is the ground state of the initial Hamiltonian. Generalizing to the complex time ($z$) plane, one can define the dynamical free energy, $f(z) = -\ln G(z)$; one then looks for the zeros of the $G(z)$, known as Fisher zeros, and can claim the occurrence of DPTs (at real times) when the lines of Fisher zeros cross the imaginary axis. These DPTs are manifested in sharp nonanalyticities in the rate function of the return probability $[I(t) = -\ln|G(t)|^2/N]$ at those instants of time. This usually happens when the system is quenched across the QCP [23]. The quantity $|G(t)|^2$ can also be viewed as the work ($W$) distribution function $P_t(W=0) = |\langle\Psi_0|e^{-i\hat{H}t}|\Psi_0\rangle|^2$; this represents the probability of doing no work in a time $t$ during a double-quench process, in which the system is quenched to the final Hamiltonian at $t=0$ and quenched back to the initial Hamiltonian at a time $t$ [22].

The initial observation by Heyl *et al.* [22] for a transverse Ising chain led to a series of works for both the integrable and the nonintegrable models [24–28] where DPTs were observed for sudden quenches across the QCP, although, a later work [29] showed that DPTs can occur even when the system is quenched within the same phase. These studies have also been generalized to two dimensions [30,31] where topology may play a nontrivial role [30]. Furthermore, a dynamical topological order parameter that changes its discrete values at a DPT has been introduced [32].

 

A pertinent question at this point is, how does the DPT depend on the integrability of the model under consideration or the nature of driving (slow or sudden)? Is quenching across a QCP essential to observe this? In this paper, we shall address these issues in the context of a specific nonintegrable model. We note in passing that the quantity $|G(t)|^2$ denotes the Loschmidt echo which has been studied in recent years in the context of decoherence [33–41]. The LO is also connected to the work statistics [42] and the entropy generation following a quench [43].

The model we consider here is a one-dimensional Ising model with a nearest neighbor antiferromagnetic (AFM) interaction $J$ (scaled to unity in the subsequent discussion) subjected to a transverse field ($\Gamma$) as well as a longitudinal field ($h$). It is described by the Hamiltonian [44]

$$H = \sum_i \sigma_i^z \sigma_{i+1}^z - \Gamma \sum_i \sigma_i^x - h \sum_i \sigma_i^z, \tag{3}$$

where $\sigma_i$'s are Pauli matrices. For $h = 0$, the model is integrable with QCPs at $\Gamma = \Gamma_c = \pm 1$, while any nonzero value of $h$ renders the model nonintegrable. Furthermore, since the AFM interaction and the field $h$ compete with each other, there is a quantum phase transition (QPT) from the AFM ordered phase to the disordered phase at a particular value of $\Gamma_c(h)$ for a given value of $h$. As a result, one finds a phase diagram in the $\Gamma$-$h$ plane (separating the ordered from the disordered paramagnetic phase starting from the integrable QCP (at $\Gamma_c = 1$, $h = 0$) at one end and terminating at first order transition points at $\Gamma = 0$, $h = \pm 2$ on the $h$ axis. The phase diagram of the model and the quenching path are shown in Fig. 1.

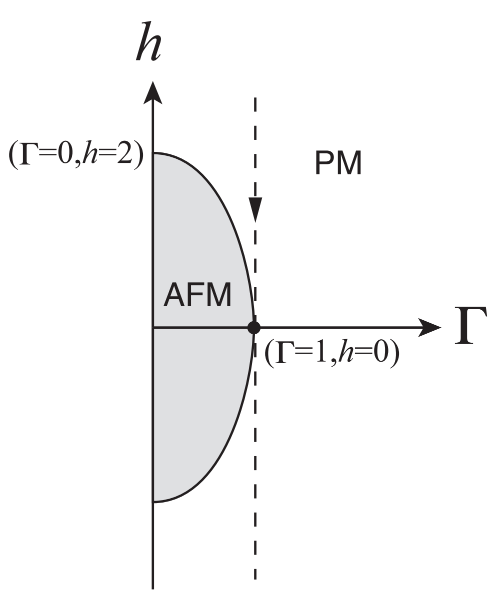


FIG. 1. The schematic phase diagram of the model given in Eq. (3); the solid line extending from ($\Gamma = 0$, $h = 2$) to ($\Gamma = 0$, $h = -2$) through ($\Gamma = 1$, $h = 0$) separates the antiferromagnetic (AFM) phase from the paramagnetic (PM) phase. The points ($\Gamma = 0$, $h = \pm 2$) denote the first order transition while ($\Gamma = 1$, $h = 0$) corresponds to the integrable quantum critical point. Throughout this paper, $\Gamma$ is set equal to 1 and $h$ is quenched along the dashed line shown with an arrow.

In this paper, we shall restrict our attention to the case when $\Gamma$ is fixed to $\Gamma_c = 1$ so that the system is at the QCP when $h = 0$ (see Fig. 1). Preparing the system in the ground state of the initial Hamiltonian, the longitudinal field $h$ is driven slowly (i.e., defined by a rate $\tau^{-1}$) or suddenly in the vicinity of the QCP. In the presence of a small $h$, a gap ($\Delta E$) opens up in the energy spectrum and a perturbation theoretic calculation, valid for small $h$, yields [44] $\Delta E \sim h^{\nu_h z} = h^2$, where $\nu_h$ is the correlation length exponent associated with the relevant perturbation $h$ and $z$ is the dynamical exponent associated with the QCP at $\Gamma = 1$. Noting that $z = 1$, one concludes that the exponent $\nu_h = 2$. The expected scaling relations of the residual energy for both slow and sudden quenches obtained using exponents $\nu_h = 2$ and $z = 1$ are indeed numerically established as shown in Appendix A. We note that a similar study was reported in Ref. [45] for a ferromagnetic (FM) Ising chain in a skewed field (having both $\Gamma$ and $h$); however, the universal behavior associated with the FM case is different.

Our results establish that for a sudden quench starting from the QCP as well as a slow quench up to the QCP, numerically obtained residual energies per spin exhibit scaling relations which perfectly match earlier predictions (see the discussion in Appendix A). On the contrary, there is a series of interesting and unexpected results concerning the scenario of DPTs following these quenches. Even though the model is nonintegrable, we find prominent existence of DPTs when the longitudinal field is slowly ramped across the QCP. This is remarkable, given the fact that in the FM case [45] sharp nonanalyticities are present in $I(t)$ in the integrable case for $h = 0$ when $\Gamma$ is ramped across the QCP; on the contrary, those get smoothened out when the skewed field is quenched through the QCP at $\Gamma = 1$ so that the system is always nonintegrable except at the QCP. This apparently leads to a conclusion that nonintegrability may wipe out DPTs. On the other hand, for sudden quenches the DPTs are found to occur whenever there is a difference in the nature of the ground states of the initial and the final Hamiltonians irrespective of the fact whether the system is quenched across a QCP or not.

Having summarized our main observations, we now probe the scenario of possible DPTs by tracking the temporal evolution with the time-independent final Hamiltonian $H_f$, of the final wave function $|\psi_f\rangle$ reached following a quench. Notably, for a sudden quench $|\psi_f\rangle$ is the ground state of the initial Hamiltonian. The Loschmidt overlap at an instant $t$ (where the initial time $t = 0$ is set immediately after the quenching is complete) is given by $G(t) = \langle\psi_f|\exp(-iH_f t)|\psi_f\rangle$; subsequently, one can define the rate function of the return probability $I(t)$ and investigate its temporal evolution to probe the signature of possible DPTs [namely, the nonanalyticities in $I(t)$]. Results obtained for the slow and sudden quenches of the system described by Eq. (3) (of chain length $N$) obtained by using the time-dependent density matrix renormalization group (t-DMRG) method [46] are presented in Figs. 2 and 3; below we analyze the remarkable findings.

We first analyze the slow quenching of the model (3) with $h \sim -t/\tau$, where $h$ is varied from a large positive to a large negative value to arrive at the final wave function $|\psi_f\rangle$, and then the overlap $G(t)$ and the rate function $I(t)$, as defined above, are evaluated. Referring to Fig. 2, we find that $I(t)$ shows nonanalyticities which appear at regular (and periodic)

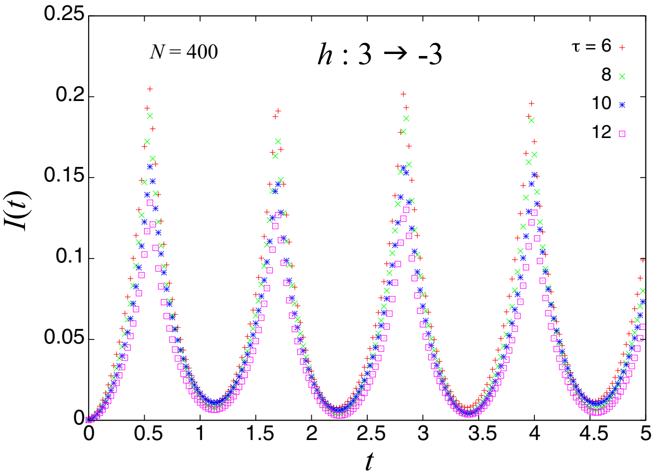


FIG. 2. (Color online) Our numerical results show prominent periodic occurrences of DPTs when the longitudinal field is slowly ramped from a large positive value to a large negative value as $h \sim -t/\tau$. This periodic pattern can be qualitatively explained by studying $I(t)$ of the equivalent integrable Hamiltonian (4) as demonstrated in Appendix B.

intervals when $\tau \gg 1$ in contrast to the FM case [45]. To analyze this, we recall that for a sufficiently slow driving the dynamics is always adiabatic except in the vicinity of the QCP ($h \ll 1$) where the relaxation time diverges. Remarkably, the nonintegrable Hamiltonian (3) can be mapped to an effective integrable model for $h \ll 1$, described by the Hamiltonian

$$H_{\rm eff} = (1 - bh^2) \sum_i \tau_i^z \tau_{i+1}^z - \sum_i \tau_i^x, \tag{4}$$

where $\tau_i$'s are Pauli spin matrices and $b$ is a constant which in our case can be chosen to be of the order of unity; this mapping to model (4) is shown to exactly describe the low-lying excitations of Hamiltonian (3) in the thermodynamic limit [44]. Consequently, so far as the slow quenching is concerned, when the dynamics is nonadiabatic only in the vicinity of a QCP, one can work with the effective Hamiltonian (4) which represents an AFM transverse Ising chain and is equivalent to a FM transverse Ising chain by simple gauge and duality transformations. Both the models are exactly solvable by Jordan-Wigner transformation. Focusing only at the QCP at $h = 0$ and considering a slow ramp of $h$ from a large positive value to a large negative value with the system initially in its ground state, one can derive the final wave function by numerically integrating the corresponding Schrödinger equation; the rate function thus obtained indeed shows occurrences of the DPTs thereby qualitatively explaining the phenomena we observe here (see Appendix B for details).

Interestingly, the mapping to the effective Hamiltonian (4) also enables us to explain the absence of DPTs following a sudden quench of small amplitude across the QCP of the original model as presented above in Fig. 3(a) because the interaction term in the equivalent Hamiltonian (4) does not change sign, which implies that this quenching does *not* take the system across the QCP of Hamiltonian (4). This explains the absence of DPT in this case though there is a crossing of the QCP in the original model. Though the mapping to the equivalent Hamiltonian is strictly valid for $h \ll 1$, in Fig. 3(a), we show this argument can be extended to explain the absence of DPTs when $h$ is quenched from $+0.7$ to $-0.7$ crossing the QCP at $h = 0$.

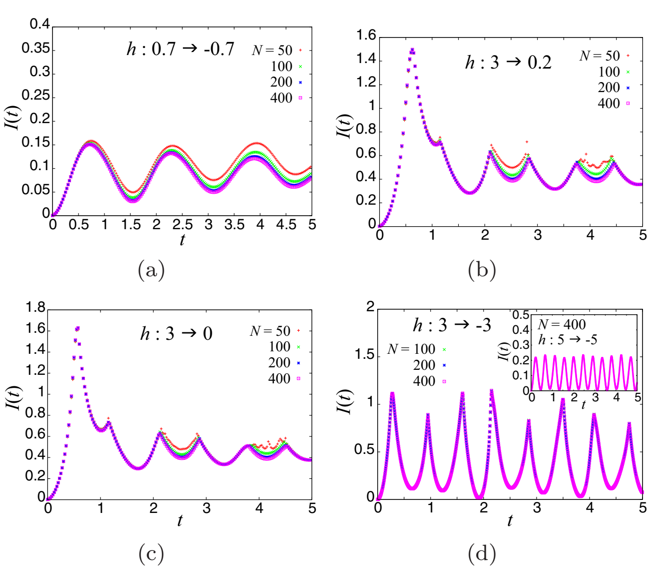

FIG. 3. (Color online) Numerically obtained $I(t)$ showing the absence and occurrence of the nonanalyticities (DPTs) in different situations when $h$ is suddenly quenched. (a) No DPTs are observed for a sudden quench of small amplitude of $h$ even if the system crosses the QCP in the process; (b) DPTs occur when $h$ is quenched within the same phase; (c) DPTs also appear when $h$ is quenched from a large positive value to the QCP at $h = 0$; (d) a regular (but not periodic) occurrence of DPTs is observed when $h$ is suddenly quenched from a large positive to a large negative value. The inset shows that the DPTs are rounded off when the quench amplitude is even larger, leading to Rabi oscillations.

Analyzing the original Hamiltonian (3), we note that the ground state is paramagnetic with all spins polarized in the direction of $h$, when $h \gg 1$; on the contrary, it is a quantum paramagnet with the majority of spins orienting in the direction of $\Gamma$ when $h \ll 1$. The change in the nature of the ground state is reflected in DPT, irrespective of the fact whether the system crosses the QCP in the process of quenching. In Fig. 3(b), we find a prominent presence of DPTs when $h$ is quenched from 3 to 0.2; here, even though the quenching does not take the original Hamiltonian across a QCP, the nature of the ground state changes. Similar DPTs are observed when quenched to the QCP also [Fig. 3(c)]. No such DPT is found to occur when the nature of the ground state is the same (e.g., when $h$ is changed from 3 to 2). Finally, when $h$ is suddenly quenched from $+3$ to $-3$ across the QCP, one finds a regular (but not periodic as shown in Fig. 2) occurrence of DPTs [see Fig. 3(d)]. This is a generic feature of a sudden quench across the QCP as also observed in the FM case [24] (while the periodic pattern is only a characteristic of the integrability of the underlying Hamiltonian). In this case, the initial and final ground states are nearly fully polarized states with their overlap being exponentially small with the system size; this difference of the ground states results in observed DPTs. When the quench amplitude is further increased [e.g., $h = +5$ to $-5$; see the inset of Fig. 3(d)], both the initial and final Hamiltonians essentially reduce to an assembly of noninteracting spins; in such situations DPTs are rounded off leading to Rabi oscillations between two fully polarized states.

Finally, we summarize the results: for the slow quenches, model (3) provides a unique example where one can work with an equivalent integrable model for $\tau \gg 1$. This mapping enables us to explain the KZ scaling and also a periodic occurrence of DPTs for a slow quenching across the QCP. This is remarkable in the sense that, to the best of our knowledge, the presence of DPTs following a slow quench of a nonintegrable model has not been reported earlier; in the FM situation, these nonanalyticities get smoothened out [45]. Concerning the sudden quench, we also present some remarkable observations: in some cases, DPTs do not occur even when the system is quenched across the QCP; but they may appear when the system is quenched within the same phase (even to the QCP). For very large amplitude quench of $h$ across $h = 0$, DPTs get rounded off. These observations lead us to the conclusion that concerning the sudden quenches, it is the change in the nature of the ground state that is responsible for DPTs. In short, our results establish that for slow quenches of a nonintegrable model across the QCP, DPTs can indeed occur periodically, while for sudden quenches they are not necessarily entangled with crossing the equilibrium QCP. These observations have not been reported in earlier studies specifically in the context of a nonintegrable model.

We would like to conclude with the note that the Hamiltonian (3) has been experimentally studied using Bose atoms in an optical lattice [47], with $\Gamma \ll h$. The field $\Gamma$ of the equivalent spin chain is determined by the hopping amplitude $t$ of the Bose atoms and is given by $2^{3/2}t$; $\Gamma$ is necessarily kept small to stabilize the Mott state necessary for the realization of a spin system. On the other hand, a quantum Monte Carlo study [48] shows that in one dimension it should be possible to achieve a field $\Gamma \approx 1$. Therefore it should be possible to verify some of the situations of the present study in experimental systems. It is noteworthy that the Loschmidt echo has already been studied close to the QCP of model (3) using NMR simulators [49].

## ACKNOWLEDGMENTS

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## APPENDIX A: THE KIBBLE-ZUREK SCALING

According to the KZ scaling relation [5,6], generalized to quantum critical systems [7,8], when a $d$-dimensional quantum system, initially prepared in its ground state, is driven across an isolated QCP, by changing a parameter of the Hamiltonian in a linear fashion as $t/\tau$, the density of defect satisfies the KZ scaling $\tau^{-d\nu/(z\nu+1)}$; here, $\nu$ and $z$ are the correlation-length exponent and the dynamical exponent associated with the QCP, respectively [9–11]. Subsequently several modifications of the scaling have been proposed [12–14]. Similarly when the system is quenched to the gapless QCP, the residual energy (the excess energy over the ground state of the final Hamiltonian) scales as $\tau^{-(d+z)\nu/(z\nu+1)}$; on the contrary, when quenched to the gapped phase, the residual energy follows a scaling relation identical to that of the defect density. Similar scaling relations for the residual energy and the defect density have also been derived using an adiabatic perturbation theory for a sudden quench of small magnitude [15] (see review articles [16–18]).

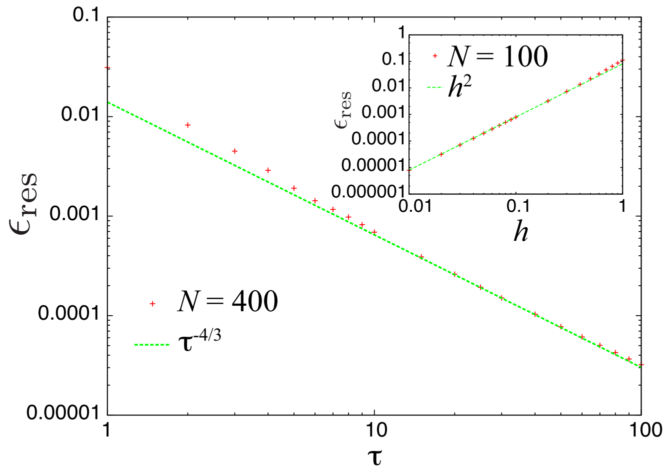


FIG. 4. (Color online) Scaling of residual energy for slow and sudden quenching by changing the longitudinal field $h$ with the transverse field $\Gamma = \Gamma_c = 1$, as obtained from DMRG studies. When $h$ is changed linearly as $-t/\tau$ to the QCP ($h = 0$), $\epsilon_{\rm res} \sim \tau^{-4/3}$ which is in perfect agreement with the KZ scaling. In the inset, we verify the scaling of $\epsilon_{\rm res} \sim h^2$ for a sudden quench starting from the QCP.

Let us first consider the situation when the field $h$ is ramped linearly to the QCP ($h = 0$) as $h = -t/\tau$ fixing $\Gamma = 1$ in the Hamiltonian given by Eq. (3). Denoting the final Hamiltonian $H_f$ with ground state energy $E_f^0$, and final wave function of the system (of length $N$) reached after the quench as $|\psi_f\rangle$, the residual energy per spin is defined by $\epsilon_{\rm res} = (\langle\psi_f|H_f|\psi_f\rangle - E_f^0)/N$. Using the t-DMRG calculations with an open boundary condition, we find $\epsilon_{\rm res} \sim \tau^{-4/3}$ (see Fig. 4). This is in perfect agreement with the KZ scaling prediction, $\epsilon_{\rm res} \sim \tau^{-\nu(d+z)/(\nu z+1)}$ with $\nu = \nu_h = 2$ and $z = 1$. We now turn our attention to the sudden quench, in which the system is initially at the QCP and suddenly a small longitudinal field $h$ is switched on; in this case, numerically we find $\epsilon_{\rm res} \sim h^2$ (inset, Fig. 4). According to the prediction of the adiabatic perturbation theory [15], for such a sudden quench of small magnitude starting from the QCP, $\epsilon_{\rm res}$ should scale as $h^{\nu_h(d+z)}$, as long as the exponent does not exceed 2; this is indeed true in the present case and as a result the exponent saturates to 2.

As emphasized in the main text, so far as the slow quenching of the longitudinal field $h$ is concerned (especially, for large $\tau$), one can equivalently work with the effective integrable Hamiltonian given by

$$H_{\rm eff} = (1 - bh^2)\sum_i \tau_i^z \tau_{i+1}^z - \sum_i \tau_i^x, \quad \text{(A1)}$$

where $b$ is a constant [44] which is inessential in the argument below, and hence set equal to unity hereafter. Using a gauge transformation (which flips the spins of alternate sites) and a duality transformation [50], the Hamiltonian in Eq. (A1) can be mapped to an equivalent dual Hamiltonian with a nearest neighbor FM interaction:

$$\begin{aligned}\tilde{H}_{\mathrm{eff}} &= -\sum_i \tilde{\tau}_i^z \tilde{\tau}_{i+1}^z - (1-h^2)\sum_i \tilde{\tau}_i^x \\ &= -\sum_i \tilde{\tau}_i^z \tilde{\tau}_{i+1}^z - \sum_i \tilde{\tau}_i^x + h^2 \sum_i \tilde{\tau}_i^x, \end{aligned} \qquad \text{(A2)}$$

which is a FM transverse Ising Hamiltonian in an effective transverse field $\Gamma_{\mathrm{eff}} = 1 - h^2$. We note that $\tilde{H}_{\mathrm{eff}}$ with $h = 0$ represents a critical Hamiltonian. Using the Jordan-Wigner transformation followed by the Fourier transformation, the model can be reduced to a two-level problem in the basis $|0\rangle$ (no fermion state) and $|k,-k\rangle$ (a state with a pair of fermions with quasimomenta $k$ and $-k$, respectively) [3,4]; the reduced $2 \times 2$ Hamiltonian is then given by

$$H_k(h) = 2\begin{pmatrix} (1-h^2) - \cos k & -i\sin k \\ i \sin k & -(1-h^2) + \cos k \end{pmatrix}. \qquad \text{(A3)}$$

Analyzing the spectrum,

$$\epsilon_k = 2\sqrt{\{(1-h^2) - \cos k\}^2 + \sin^2 k},$$

it is straightforward to show that model (A2) has three QCPs; the energy gap ($2\epsilon_k$) vanishes at critical points at $h = 0$ and $h = \pm\sqrt{2}$, with the corresponding critical wave vector (for which the energy gap vanishes) $k_c = 0$ and $\pi$, respectively. We are, however, interested in the transition at $h = 0$, which is the only relevant QCP to the context of the original Hamiltonian (3). To focus on the critical point at $h = 0$, we expand Hamiltonian (A3) in the vicinity of $k = 0$ to arrive at Hamiltonian

$$H_k(h) = 2\begin{pmatrix} -h^2 + \frac{k^2}{2} & -ik \\ ik & h^2 - \frac{k^2}{2} \end{pmatrix}, \qquad \text{(A4)}$$

which shows only one quantum critical point at $h = 0$. Analyzing the simplified form of the spectrum $\epsilon_k = \sqrt{(h^2 - k^2/2)^2 + k^2}$, one immediately finds for $h = 0$, the gap $(\Delta E_k = 2\epsilon_k) \sim k$, yielding $z = 1$ and for $k = 0$, gap scales as $h^2$ yielding $\nu z = 2$, and hence $\nu = 2$ (referred to as $\nu_h$ in the main text).

Let us now point out that the quenching $h = -t/\tau$, with $t$ going from $-\infty$ to 0, in the original Hamiltonian is equivalent to driving the reduced Hamiltonian (A4) from $h \to \infty$ to the QCP at $h = 0$ by a nonlinear protocol $(t/\tau)^2$; in both cases the system is initially prepared in its ground state. Even though the nonadiabatic transition probability for the mode $k$ ($p_k$) cannot be calculated directly using the Landau-Zener formula for such a nonlinear protocol, one can make appropriate rescaling in the corresponding Schrödinger equations [18,51] to argue that it would be a function of the dimensional combination of $k^2\tau^{4/3}$, i.e., $p_k = \mathcal{F}(k^2\tau^{4/3})$ where $\mathcal{F}$ is an unknown scaling function. Since the gapless QCP is characterized by gapless excitations $k$, the scaling of the residual energy can be obtained as $\epsilon_{\mathrm{res}} \sim \int dk\, k\mathcal{F}(k^2\tau^{4/3}) \sim \tau^{-4/3}$; this matches perfectly with the KZ prediction with $d = z = 1$ and $\nu = 2$ and the numerical result presented in Fig. 4. This establishes that the nonlinear reverse quenching of Hamiltonian (A4) indeed leads to the expected KZ scaling of the $\epsilon_{\mathrm{res}}$ following a linear quench of the longitudinal field of the original nonintegrable model (3).

## APPENDIX B: SLOW QUENCHING AND NONANALYTICITIES IN THE RATE FUNCTION

We shall now calculate the nature of the Fisher zeros of the effective partition function [22] obtained from the LO when the parameter $h$ of the Hamiltonian (A2) is quenched from a large positive to a large negative value following the protocol $h = -t/\tau$; this is equivalent to the slow quenching of the longitudinal field $h$ in the original Hamiltonian (3). But there is a subtle difference that needs to be emphasized: the field $h$ contributes a quadratic $h^2$ term to the transverse field of the equivalent model (A2), thus as $h$ is linearly changed from a large positive value to the negative value in model (3), the parameter $h^2$ changes from a positive initial value to zero (i.e., the QCP) and returns to the original initial value at the final time; this in a sense is a reverse quenching of the transverse field of Hamiltonian (A2) as studied in [52] in a nonlinear fashion.

To calculate the Loschmidt overlap of a system of length $N$ defined by $f(z) = -\ln\langle\psi_f|\exp(-H_f z)|\psi_f\rangle/N$, where $z$ is the complex time, $H_f$ is the final Hamiltonian, and $|\psi_f\rangle$ the state reached following the quantum quench, we focus on the reduced $2 \times 2$ Hamiltonian (A4). Summing over the contributions from all the momenta modes, a few lines of algebra leads us to the expression [53]

$$f(z) = -\int_0^\pi \frac{dk}{2\pi} \ln\left[(1-p_k) + p_k \exp\left(-2\epsilon_k^f z\right)\right], \qquad \text{(B1)}$$

where $p_k$ is the nonadiabatic transition probability for mode $k$. The zeros of the "effective" partition function [where $f(z)$ is nonanalytic] are given by

$$z_n(k) = \frac{1}{2\epsilon_k^f}\left[\ln\left(\frac{p_k}{1-p_k}\right) + i\pi(2n+1)\right], \qquad \text{(B2)}$$

where $n = 0, \pm 1, \pm 2, \cdots\cdots$ . For a nonlinear reverse quenching protocol, the expression for $p_k$ cannot be exactly

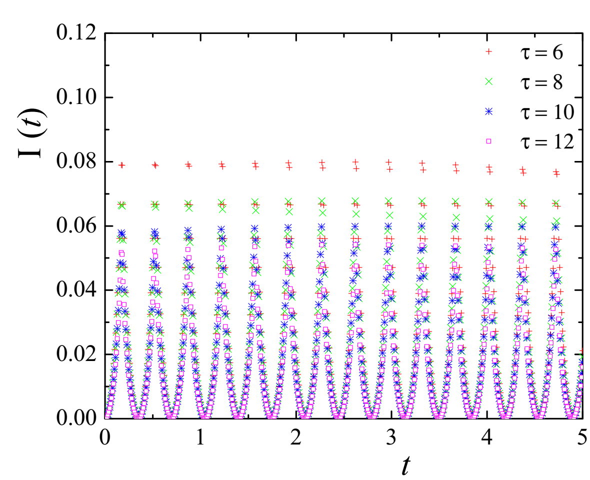


FIG. 5. (Color online) The rate function for quenching $h = 3$ to $h = -3$ shows sharp nonanalyticities at periodic intervals in time with system size $N = 400$ and several $\tau$'s.

determined using the LZ formula (though an exact form can be obtained for the linear case [52]). However, it can be argued $p_k = \mathcal{G}[(k-k_0)^2\tau^{4/3}]$, where $k_0$ is the wave vector for which $p_k$ is maximum which shifts to $k=0$ for large $\tau$ and $\mathcal{G}$ is an unknown function. We find from Eq. (B2) Fisher zeros cross the imaginary axis for a particular value of $k_*$ for which $p_{k_*} = 1/2$ [45,53] and the rate function shows sharp nonanalyticities at $t_n^* = \pi(n+\frac{1}{2})/\epsilon_{k_*}^f$. For the present case, to calculate the Fisher zeros and especially the rate functions $I(t)$ we shall use the form of the Hamiltonian near $k=0$ given in Eq. (A4) (to avoid the influence of the QCP at $h=\sqrt{2}$), when $h$ is quenched from $+3$ to $-3$. Numerically integrating the Schrödinger equation describing the dynamics of Hamiltonian (A4) with the initial condition that the system is in the ground state of the initial Hamiltonian, we obtain the value of $p_k$ which is then substituted in the expression of the rate function:

$$I(t) = -\int_0^{\pi} \frac{dk}{2\pi} \log\left[1 + 4p_k(p_k-1)\sin^2 \epsilon_k^f t\right]. \quad \text{(B3)}$$

As shown in Fig. 5, this qualitatively explains the periodic occurrence of DPTs presented in Fig. 2 in the main text.

[1] I. Bloch, J. Dalibard, and W. Zwerger, Rev. Mod. Phys. **80**, 885 (2008).
[2] M. Lewenstein, A. Sanpera, and V. Ahufinger, *Ultracold Atoms in Optical Lattices Simulating Quantum Many-Body Systems*, (Oxford University Press, Oxford, 2012).
[3] S. Sachdev, *Quantum Phase Transitions* (Cambridge University Press, Cambridge, UK, 1999).
[4] S. Suzuki, J-i Inoue, and Bikas K. Chkarabarti, *Quantum Ising Phases and Transitions in Transverse Ising Models*, Lecture Notes in Physics Vol. 862 (Springer, New York, 2013).
[5] T. W. B. Kibble, J. Phys. A **9**, 1387 (1976).
[6] W. H. Zurek, Nature (London) **317**, 505 (1985).
[7] W. H. Zurek, U. Dorner, and P. Zoller, Phys. Rev. Lett. **95**, 105701 (2005).
[8] A. Polkovnikov, Phys. Rev. B **72**, 161201(R) (2005).
[9] J. Dziarmaga, Phys. Rev. Lett. **95**, 245701 (2005).
[10] B. Damski, Phys. Rev. Lett. **95**, 035701 (2005).
[11] R. W. Cherng and L. S. Levitov, Phys. Rev. A **73**, 043614 (2006).
[12] V. Mukherjee, U. Divakaran, A. Dutta, and D. Sen, Phys. Rev. B **76**, 174303 (2007); U. Divakaran, A. Dutta, and D. Sen, *ibid.* **78**, 144301 (2008); U. Divakaran, V. Mukherjee, A. Dutta, and D. Sen, J. Stat. Mech. (2009) P02007; V. Mukherjee and A. Dutta, Europhys. Lett. **92**, 37004 (2010); F. Pellegrini, S. Montangero, G. E. Santoro, and R. Fazio, Phys. Rev. B **77**, 140404(R) (2008); S. Deng, G. Ortiz, and L. Viola, Europhys. Lett. **84**, 67008 (2008); Phys. Rev. B **80**, 241109(R) (2009).
[13] K. Sengupta, D. Sen, and S. Mondal, Phys. Rev. Lett. **100**, 077204 (2008); D. Sen, K. Sengupta, and S. Mondal, *ibid.* **101**, 016806 (2008); R. Barankov and A. Polkovnikov, *ibid.* **101**, 076801 (2008).
[14] A. Dutta, R. R. P. Singh, and U. Divakaran, Europhys. Lett. **89**, 67001 (2010); T. Hikichi, S. Suzuki, and K. Sengupta, Phys. Rev. B **82**, 174305 (2010).
[15] C. De Grandi, V. Gritsev, and A. Polkovnikov, Phys. Rev. B **81**, 012303 (2010).
[16] A. Polkovnikov, K. Sengupta, A. Silva, and M. Vengalattore, Rev. Mod. Phys. **83**, 863 (2011).
[17] J. Dziarmaga, Adv. Phys. **59**, 1063 (2010).
[18] A. Dutta, G. Aeppli, B. K. Chakrabarti, U. Divakaran, T. Rosenbaum, and D. Sen, *Quantum Phase Transitions in Transverse Field Spin Models: From Statistical Physics to Quantum Information* (Cambridge University Press, Cambridge, 2015).
[19] M. E. Fisher, *Boulder Lectures in Theoretical Physics* (University of Colorado, Boulder, 1965), Vol. 7.
[20] W. van Saarloos and D. Kurtze, J. Phys. A **17**, 1301 (1984).
[21] C. Yang and T. Lee, Phys. Rev. **87**, 404 (1952).
[22] M. Heyl, A. Polkovnikov, and S. Kehrein, Phys. Rev. Lett. **110**, 135704 (2013).
[23] These nonanalyticities for an integrable model was first reported in [45] in the context of a slow quenching.
[24] C. Karrasch and D. Schuricht, Phys. Rev. B **87**, 195104 (2013).
[25] J. N. Kriel, C. Karrasch, and S. Kehrein, Phys. Rev. B **90**, 125106 (2014).
[26] F. Andraschko and J. Sirker, Phys. Rev. B **89**, 125120 (2014).
[27] E. Canovi, P. Werner, and M. Eckstein, Phys. Rev. Lett. **113**, 265702 (2014).
[28] M. Heyl, arXiv:1505.02352.
[29] S. Vajna and B. Dora, Phys. Rev. B **89**, 161105(R) (2014).
[30] S. Vajna and B. Dora, Phys. Rev. B **91**, 155127 (2015).
[31] M. Schmidt and S. Kehrein, Phys. Rev. B **92**, 075114 (2015).
[32] J. C. Budich and M. Heyl, arXiv:1504.05599.
[33] H. T. Quan, Z. Song, X. F. Liu, P. Zanardi, and C. P. Sun, Phys. Rev. Lett. **96**, 140604 (2006).
[34] D. Rossini, T. Calarco, V. Giovannetti, S. Montangero, and R. Fazio, Phys. Rev. A **75**, 032333 (2007).
[35] F. M. Cucchietti, S. Fernandez-Vidal, and J. P. Paz, Phys. Rev. A **75**, 032337 (2007); C. Cormick and J. P. Paz, *ibid.* **77**, 022317 (2008).
[36] S. Sharma, V. Mukherjee, and A. Dutta, Eur. Phys. J. B **85**, 143 (2012).
[37] L. Campos Venuti and P. Zanardi, Phys. Rev. A **81**, 022113 (2010); L. Campos Venuti, N. T. Jacobson, S. Santra, and P. Zanardi, Phys. Rev. Lett. **107**, 010403 (2011).
[38] V. Mukherjee, S. Sharma, and A. Dutta, Phys. Rev. B **86**, 020301(R) (2012).
[39] T. Nag, U. Divakaran, and A. Dutta, Phys. Rev. B **86**, 020401(R) (2012).
[40] B. Dora, F. Pollmann, J. Fortagh, and G. Zarand, Phys. Rev. Lett. **111**, 046402 (2013); R. Sachdeva, T. Nag, A. Agarwal, and A. Dutta, Phys. Rev. B **90**, 045421 (2014).
[41] P. Zanardi, H. T. Quan, X. Wang, and C. P. Sun, Phys. Rev. A **75**, 032109 (2007).
[42] A. Gambassi and A. Silva, arXiv:1106.2671; P. Smacchia and A. Silva, Phys. Rev. E **88**, 042109 (2013).
[43] R. Dorner, J. Goold, C. Cormick, M. Paternostro, and V. Vedral, Phys. Rev. Lett. **109**, 160601 (2012); S. Sharma and A. Dutta, Phys. Rev. E **92**, 022108 (2015).

[44] A. A. Ovchinnikov, D. V. Dmitriev, V. Ya Krivnov, and V. O. Cheranovskii, Phys. Rev. B **68**, 214406 (2003).

[45] F. Pollmann, S. Mukerjee, A. G. Green, and J. E. Moore, Phys. Rev. E **81**, 020101(R) (2010).

[46] S. R. White and A. E. Feiguin, Phys. Rev. Lett. **93**, 076401 (2004).

[47] J. Simon, W. S. Bakr, R. Ma, M. E. Tai, P. M. Preiss, and M. Greiner, Nature (London) **472**, 307 (2011).

[48] V. G. Rousseau, D. P. Arovas, M. Rigol, F. Hébert, G. G. Batrouni, and R. T. Scalettar, Phys. Rev. B **73**, 174516 (2006).

[49] Jingfu Zhang, Fernando M. Cucchietti, C. M. Chandrashekar, Martin Laforest, Colm A. Ryan, Michael Ditty, Adam Hubbard, John K. Gamble, and Raymond Laflamme, Phys. Rev. A **79**, 012305 (2009).

[50] J. B. Kogut, Rev. Mod. Phys. **51**, 659 (1979).

[51] S. Mondal, K. Sengupta, and D. Sen, Phys. Rev. B **79**, 045128 (2009).

[52] U. Divakaran and A. Dutta, Phys. Rev. B **79**, 224408 (2009).

[53] S. Sharma, U. Divakaran, and A. Dutta (unpublished).