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Anomalous transport phenomena in $p_x + ip_y$ superconductors

Songci Li, A. V. Andreev, and B. Z. Spivak

Department of Physics, University of Washington, Seattle, Washington 98195, USA

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Spontaneous breaking of time-reversal symmetry in superconductors with the $p_x + ip_y$ symmetry of the order parameter allows for a class of effects which are analogous to the anomalous Hall effect in ferromagnets. These effects exist below the critical temperature, $T < T_c$. We develop a kinetic theory of such effects. In particular, we consider anomalous Hall thermal conductivity, the polar Kerr effect, the anomalous Hall effect, and the anomalous photo- and acousto-galvanic effects.

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Introduction. One of the leading candidates for p-wave pairing in electronic systems is Sr_2RuO_4. Numerous experiments indicate that the superconducting state of Sr_2RuO_4 has odd parity, breaks time-reversal symmetry, and is spin triplet [1–6]. The order parameter consistent with these experiments is given by the chiral p-wave state [7] which is an analog of the ^{3}He-A-phase. The Fourier transform of the real space order parameter $\Delta_{\alpha\beta}(\mathbf{r}-\mathbf{r}')$ has the form $\Delta_{\alpha\beta}(\mathbf{p}) \sim (p_x \pm ip_y)\mathbf{S}\cdot\boldsymbol{\sigma}_{\alpha\beta}$, where $\mathbf{S}$ is the spin of the Cooper pair, $\boldsymbol{\sigma}$ are the Pauli matrices, and α and β are spin indices. However, the observation of power law temperature dependence of specific heat [8] and NMR [9], the absence of electric currents along edges [10], and the absence of a split transition in the presence of an in-plane magnetic field [11] are inconsistent with the theoretically expected properties of a simple chiral superconductor. Consideration of additional experimental manifestations of spontaneous breaking of time-reversal symmetry in $p_x + ip_y$ superconductors may clarify the nature of the superconducting state in Sr_2RuO_4.

Due to spontaneous breaking of time-reversal symmetry, $p_x + ip_y$ superconductors must exhibit anomalous transport phenomena in the absence of external magnetic fields, similar to those in metallic ferromagnets (see Refs. [12,13] for a review). In this Rapid Communication we develop a theory of several such effects in $p_x + ip_y$ superconductors: the anomalous Hall effect, polar Kerr effect for microwave radiation, anomalous Hall thermal conductivity, and anomalous photo- and acousto-galvanic effects.

It should be noted that p-wave superconductivity exists only in the clean regime, $l > \xi$, where electron transport may be described semiclassically. Generally, in the semiclassical regime there are three contributions to anomalous transport phenomena: skew scattering, side jumps, and the intrinsic contribution. The side jump contribution arises from the shift of the center of mass of electron wave packets during the scattering events, while the intrinsic contribution is related to the anomalous velocity due to Berry curvature. The magnitude of these contributions is independent of the mean free path. In contrast, the magnitude of the skew scattering contribution is proportional to the quasiparticle mean free path l. As a result, the skew scattering contribution exceeds the intrinsic and side jump contributions by a large factor $Cp_F l$, where p_F is the Fermi momentum, and C is the prefactor that depends on the impurity strength and the details of the band structure. In $p_x + ip_y$ superconductors $l > \xi$, the semiclassical parameter is sufficiently large, $p_F l > 10^3\hbar$, so that the condition $Cp_F l \gg 1$ is satisfied for most types of impurities. Therefore in this Rapid Communication we will take into consideration only the skew scattering contribution. We focus on anomalous transport phenomena in the vicinity of the critical temperature, where quasiparticles play a major role.

Kinetic scheme. Transport theory in conventional time-reversal invariant superconductors was developed long ago (see, for example, reviews in Refs. [14,15]). Below we generalize this approach to superconductors without time-reversal symmetry, which exhibit anomalous transport phenomena. In the clean regime, $l \gg \xi$, and at sufficiently low frequencies, $\omega \ll |\Delta|$, where $|\Delta|$ is the modulus of the order parameter, the quasiparticle dynamics can be described by the Boltzmann kinetic equation for the quasiparticle distribution function $n_{\mathbf{p}}(\mathbf{r},t)$,

$$\frac{\partial n_{\mathbf{p}}(\mathbf{r},t)}{\partial t} + \frac{\partial \tilde{\epsilon}_{\mathbf{p}}}{\partial \mathbf{p}}\frac{\partial n_{\mathbf{p}}}{\partial \mathbf{r}} - \frac{\partial \tilde{\epsilon}_{\mathbf{p}}}{\partial \mathbf{r}}\frac{\partial n_{\mathbf{p}}}{\partial \mathbf{p}} = I_{\mathrm{st}}, \tag{1}$$

where the collision integral $I_{\mathrm{st}} = I_{\mathrm{st}}^{(el)} + I_{\mathrm{st}}^{\epsilon}$ in Eq. (1) describes both elastic and inelastic scattering, and

$$\tilde{\epsilon}_{\mathbf{p}} = \epsilon_{\mathbf{p}} + \mathbf{v}\cdot\mathbf{p}_s, \quad \epsilon_{\mathbf{p}} = \sqrt{\tilde{\xi}_{\mathbf{p}}^2 + |\Delta|^2}, \tag{2a}$$

$$\tilde{\xi}_{\mathbf{p}} = \xi_{\mathbf{p}} + \Phi + \frac{\mathbf{p}_s^2}{2m}, \quad \xi_{\mathbf{p}} = \frac{p^2}{2m} - \epsilon_F. \tag{2b}$$

In Eq. (2) m is the electron mass, while $\mathbf{p}_s$ and Φ are given by

$$\mathbf{p}_s = \frac{\hbar}{2}\nabla\chi - \frac{e}{c}\mathbf{A}, \quad \Phi = \frac{\hbar}{2}\partial_t\chi + e\phi, \tag{3}$$

where χ is the order parameter phase, and ϕ and $\mathbf{A}$ are the scalar and vector potentials. From Eq. (3) one obtains the equation for the acceleration of the condensate,

$$\partial_t \mathbf{p}_s = e\mathbf{E} + \nabla\Phi. \tag{4}$$

Equations (1)–(3) should be supplemented by the expression for the current density,

$$\mathbf{j} = \frac{eN}{m}\mathbf{p}_s + e\int d\Gamma\, \mathbf{v} n_{\mathbf{p}}, \tag{5}$$

and by the charge neutrality condition,

$$\nu\Phi = \int d\Gamma\, \frac{\tilde{\xi}_{\mathbf{p}}}{\tilde{\epsilon}_{\mathbf{p}}} n_{\mathbf{p}}, \tag{6}$$

that relates the gauge-invariant scalar potential and the odd in ξ part of the quasiparticle distribution function, and by the self-consistency equation for the order parameter. Here ν is the density of states at the Fermi level, $d\Gamma = d^3p/(2\pi\hbar)^3$ and $\mathbf{v} = d\xi_{\mathbf{p}}/d\mathbf{p}$.

We work in linear response to external perturbations, and neglect corrections to equilibrium value of $|\Delta|$. We also assume that $\tau_\epsilon \gg \tau$, where τ_ϵ and τ are inelastic and elastic mean free time, respectively. Therefore the main contribution to the aforementioned anomalous effects comes from elastic scattering, which is described by the collision integral

$$I_{\mathrm{st}} = \int (W_{\mathbf{pp}'} n_{\mathbf{p}'} - W_{\mathbf{p}'\mathbf{p}} n_{\mathbf{p}})\delta(\tilde{\epsilon}_{\mathbf{p}} - \tilde{\epsilon}_{\mathbf{p}'}) d\Gamma'. \tag{7}$$

Skew scattering of quasiparticles corresponds to the part of scattering probability $W_{\mathbf{pp}'}$ in Eq. (7) that is associated with breaking of time-reversal symmetry, $\delta W_{\mathbf{pp}'} = W_{\mathbf{pp}'} - W_{-\mathbf{p}'-\mathbf{p}} \neq 0$. Thus, all the aforementioned effects are proportional to $\delta W_{\mathbf{pp}'}$. Skew scattering arises beyond the lowest Born approximation for the scattering amplitude. Below we consider pointlike impurities. In the normal state such impurities scatter electrons only in the s-wave channel and do not cause skew scattering. Therefore in the superconducting state skew scattering of quasiparticles is entirely due to the breaking of time-reversal symmetry by the $p_x + ip_y$ order parameter. The elastic scattering probability for quasiparticles with energy ϵ can be characterized by $\xi \equiv \xi_{\mathbf{p}}$, $\xi' \equiv \xi_{\mathbf{p}'} = \pm\xi$ and the azimuthal angles φ, φ', which define the direction of $\mathbf{p}$ and $\mathbf{p}'$ in the xy plane. For simplicity, we assume cylindrical Fermi surface (with energy independent of p_z) and obtain for the scattering probability [16]

$$W_{\mathbf{pp}'} = W_0 + W_1[1 - \cos(\varphi - \varphi' + 2\delta_\epsilon)]. \tag{8}$$

Here δ_ϵ is the energy-dependent scattering phase shift. It is related to the normal state s-wave phase shift δ_n by

$$\delta_\epsilon = \arctan \frac{\delta_n \epsilon}{\sqrt{\epsilon^2 - |\Delta|^2}}. \tag{9}$$

We assume weak impurities, for which $\delta_n \approx \tan\delta_n \equiv -\pi\nu V_0$, is small. Here V_0 is the impurity pseudopotential [17]. In this case W_0 and W_1 are given by

$$W_0(\xi,\xi') = \frac{\zeta(\epsilon)}{2\nu\tau}\frac{(\xi+\xi')^2}{2\epsilon^2}, \tag{10a}$$

$$W_1(\xi,\xi') = \frac{\zeta(\epsilon)}{2\nu\tau}\frac{|\Delta|^2}{\epsilon^2}. \tag{10b}$$

Here $\tau^{-1} = 2\pi n_i \nu V_0^2$, with n_i being the impurity density, is the elastic scattering rate in the normal state. The coefficient $\zeta(\epsilon) = (\epsilon^2 - |\Delta|^2)/[\epsilon^2(1+\delta_n^2) - |\Delta|^2]$ represents the enhancement factor of the quasiparticle scattering cross section over the normal state value. The first term in Eq. (8), W_0 given by Eq. (10a) has the same structure as in s-wave superconductors. It describes scattering only within the same (particlelike, $\xi > 0$, or holelike, $\xi < 0$) branch and does not lead to branch imbalance relaxation. The second term, W_1 in Eq. (8) is absent in s-wave superconductors. It leads to both skew scattering and scattering between branches of the quasiparticle spectrum with different signs of ξ. The skew scattering cross section, described by the $\sin(\varphi - \varphi')\sin 2\delta_\epsilon$ term in Eq. (8), is energy dependent. It follows from Eqs. (8), (9), and (10b) that it changes sign when impurity potential V_0 changes from repulsive to attractive.

Below we consider linear response to several external perturbations and look for the quasiparticle distribution function in the form $n_{\mathbf{p}} = n^{(0)} + n^{(1)}_{\mathbf{p}}$, where $n^{(0)}$ is a locally equilibrium Fermi distribution, and $n^{(1)}_{\mathbf{p}}$ describes the deviation from equilibrium. Noting that the collision integral (7) is nullified by an arbitrary function $n^{(0)}(\tilde{\epsilon}_{\mathbf{p}})$ we write the linearized Boltzmann equation in the form

$$\mathcal{S}(\mathbf{p}) = \int d\Gamma' W_{\mathbf{pp}'}\big(n^{(1)}_{\mathbf{p}} - n^{(1)}_{\mathbf{p}'}\big)\delta(\epsilon_{\mathbf{p}} - \epsilon_{\mathbf{p}'}), \tag{11}$$

where the source $\mathcal{S}(\mathbf{p})$ is obtained by linearizing the left-hand side of Eq. (1) about the equilibrium, and its specific form the depends on the type of perturbation.

Anomalous Hall thermal conductivity. We first consider the Hall component of the thermal conductivity κ_{xy} which describes the heat flux perpendicular to the direction (x axis) of the temperature gradient. In this case the source term in Eq. (11) has the form

$$\mathcal{S}(\mathbf{p}) = -\frac{\xi}{T}\,\mathbf{v}\cdot\nabla T\,\frac{\partial n^{(0)}}{\partial\epsilon}. \tag{12}$$

The expression for the heat flux is

$$\mathbf{j}^Q = \int d\Gamma\,\epsilon_{\mathbf{p}}\frac{\partial\epsilon_{\mathbf{p}}}{\partial\mathbf{p}} n^{(1)}_{\mathbf{p}}. \tag{13}$$

Note that $\partial\epsilon_{\mathbf{p}}/\partial\mathbf{p} = \mathbf{v}\xi/\epsilon$ is the group velocity of the quasiparticles, while $\mathbf{v}$ is the bare velocity as in a normal metal, $|\mathbf{v}| = v_F$. The solution of Eqs. (11) and (12) has the form

$$n^{(1)}_{\mathbf{p}} = -\frac{\xi}{T} v_F \nabla T \frac{\partial n^{(0)}}{\partial\epsilon}[\alpha_s(\epsilon)\sin\varphi + \alpha_c(\epsilon)\cos\varphi].$$

The Hall component of the thermal conductivity tensor, κ_{xy}, is determined by $\alpha_s(\epsilon)$ in the above expression, which is given by $\alpha_s(\epsilon) = a(\epsilon)/[b^2(\epsilon) + a^2(\epsilon)]$, with

$$a(\epsilon) = \frac{\zeta(\epsilon)}{2\tau}\frac{|\Delta|^2}{2\epsilon|\xi|}\sin 2\delta_\epsilon, \tag{14a}$$

$$b(\epsilon) = \frac{|\xi|}{\epsilon\tau} + \frac{\zeta(\epsilon)}{2\tau}\frac{|\Delta|^2}{2\epsilon|\xi|}(\cos 2\delta_\epsilon + 2). \tag{14b}$$

For weak impurities, $|\delta_n| \ll 1$, we obtain, close to T_c,

$$\kappa_{xy} = 3\kappa\left(\frac{|\Delta|}{\pi T}\right)^2\delta_n, \tag{15}$$

where $\kappa = \pi^2\nu T D/3$ (with $D = v_F^2\tau/2$ being the diffusion constant) is the normal state thermal conductivity.

Polar Kerr effect. Next we consider a linearly polarized electromagnetic wave at normal incidence to the xy surface of $p_x + ip_y$ superconductor. The reflected wave is elliptically polarized with the major axis rotated with respect to the incident one by the polar Kerr angle [18]

$$\theta_k = \frac{(1 - n^2 + \kappa^2)\Delta\kappa + 2n\kappa\Delta n}{(1 - n^2 + \kappa^2)^2 + (2n\kappa)^2}, \tag{16}$$

where n and κ are, respectively, the real and imaginary parts of the refraction index and

$$\Delta n + i\Delta\kappa = -\frac{4\pi}{\omega}\frac{(n - i\kappa)\sigma_{xy}}{n^2 + \kappa^2}, \tag{17}$$

where σ_{xy} is the complex ac conductivity.

In this case the electric field is uniform in the direction parallel to the surface of the sample, $\Phi = 0$, and the value of $\mathbf{p}_s(t)$ is determined by Eq. (4). The diagonal part of the conductivity is given by [14]

$$\sigma_{xx} \approx \sigma_D + \frac{iN_s(T)}{\omega}, \tag{18}$$

where $N_s(T)$ is the temperature-dependent superfluid density and $\sigma_D = e^2\nu D$ is the Drude conductivity. In contrast to the thermal conductivity consideration, in the present case $n^{(0)} = 1/(\exp[\epsilon_{\mathbf{p}} + \mathbf{v}\cdot\mathbf{p}_s(t)]/T + 1)$ gives a nonvanishing contribution to the current response [via $N_s(T)$] because the superfluid momentum depends on the electric field. The Kerr angle θ is determined by the value of the Hall component of conductivity σ_{xy}.

To find σ_{xy} we seek the solution of Eq. (1) in the form $n = n^{(0)}(\epsilon_{\mathbf{p}}/T) + n^{(1)}_{\mathbf{p}}$. The source in Eq. (11) becomes

$$\mathcal{S}(\mathbf{p}) = -i\omega n^{(1)}_{\mathbf{p}} - e\mathbf{v}\cdot\mathbf{E}\frac{\partial n^{(0)}}{\partial\epsilon}, \tag{19}$$

where the external electric field $\mathbf{E}$ is along the x direction. The nonequilibrium distribution $n^{(1)}_{\mathbf{p}}$ has the form

$$n^{(1)}_{\mathbf{p}} = -eEv_F\frac{\partial n^{(0)}}{\partial\epsilon}[\beta_s(\epsilon)\sin\varphi + \beta_c(\epsilon)\cos\varphi]. \tag{20}$$

The Hall conductivity depends only on the function $\beta_s(\epsilon)$, which is given by $\beta_s(\epsilon) = a(\epsilon)/\{[b(\epsilon) - i\omega]^2 + a^2(\epsilon)\}$, with $a(\epsilon)$ and $b(\epsilon)$ being defined in Eq. (14). Substituting Eq. (20) into Eq. (5) we obtain the Hall conductivity in the weak impurity limit, $|\pi\nu V_0| \ll 1$, in the form

$$\sigma_{xy}(\omega) = \sigma_D\frac{|\Delta|}{2T}\delta_n\int_0^\infty \frac{dx}{\cosh^2(\sqrt{x^2+1}|\Delta|/2T)} \times\frac{x^2+1}{(-i\omega\tau x\sqrt{x^2+1} + x^2 + 3/4)^2}, \tag{21}$$

where $x = |\xi|/|\Delta|$. At temperature close to T_c and at low frequencies, $\omega\tau \ll 1$, this expression yields

$$\sigma_{xy} = \frac{7\pi}{12\sqrt{3}}\,\delta_n\,\frac{|\Delta|}{T}\,\sigma_D. \tag{22}$$

This result was derived assuming $p_x + ip_y$ symmetry of the order parameter. In the $p_x - ip_y$ state the Hall conductivity σ_{xy} has opposite sign. It also changes sign if the impurity potential V_0 changes from repulsive, $\delta_n < 0$, to attractive, $\delta_n > 0$, in agreement with Ref. [19]. Note that our result for the low frequency Hall conductivity, Eq. (22), is proportional to the elastic mean free time τ and to the density of quasiparticles.

There is another contribution to σ_{xy} associated with the existence of the transverse component of the superfluid velocity $v_y \sim \dot{p}_x$, which is proportional to the condensate acceleration in the x direction. It may not be obtained within the present formalism that is based on the Boltzmann kinetic equation for the quasiparticles. At $T \sim T_c$ this contribution is smaller than the quasiparticle contribution, Eq. (22). However, at $T \ll T_c$ when the quasiparticle contribution becomes exponentially small, in Eq. (21) it becomes the dominant contribution. The requirement for this contribution to exist is violation of Galilean invariance in the system. Thus it should exist in any crystalline superconductors with $p_x + ip_y$ symmetry [20,21]. The conclusion that σ_{xy} remains finite even at $\omega \to 0$, and $T = 0$ is in agreement with the results presented in Fig. 2 of Ref. [20]. Galilean invariance can also be broken by impurities. The corresponding contribution to σ_{xy} is inversely proportional to the electron mean free time.

Hall effect for normal current injection. Let us now consider a normal metal/($p_x + ip_y$)-superconductor junction, through which a steady current is flowing. At $T \ll |\Delta|$ this situation was considered in Ref. [22]. In this regime conversion of normal current to supercurrent is mediated by multiple Andreev reflections. Here we work near the critical temperature and consider a setup in which the normal current is injected into the superconductor in the x direction, as shown in the inset in Fig. 1. In this case the conversion of quasiparticle current to the supercurrent occurs in the superconductor. Just as in the case of the s-wave superconductor, near T_c, the electric field penetrates into the superconductor to a large distance $L_Q \gg l$, which is determined by the relaxation of imbalance between the populations of quasiparticles in electronlike, $\xi > 0$, and holelike, $\xi < 0$, branches of the spectrum (see, for example, Ref. [14], and references therein). The new feature of normal current injection that appears in $p_x + ip_y$ superconductors is that skew scattering of quasiparticles generates nonequilibrium current that is perpendicular to the electric field. Another aspect is that, in contrast to s-wave superconductors, impurity scattering leads to branch imbalance relaxation even if the magnitude of the order parameter $|\Delta|$ is isotropic in the Fermi surface. Below we assume that the inelastic scattering rate is smaller than $1/\tau$ and thus impurity scattering gives the dominant contribution to branch imbalance relaxation.

In linear response we write the quasiparticle distribution function in the superconductor in the form $n^{(0)}(\epsilon_{\mathbf{p}}/T) + n^{(1)}_{\mathbf{p}}$.

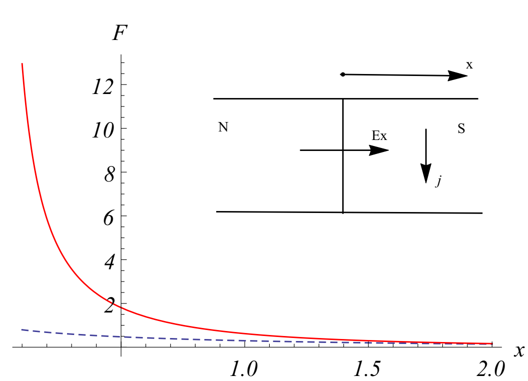

FIG. 1. (Color online) Plot of the functions $F_{-2}(x)$ (solid line) and $F_0(x)$ (dashed line) in Eq. (30). The inset shows a schematic setup of the normal current injection experiment. Electric current is injected into the superconductor S from the normal metal N along the x axis. Skew scattering of quasiparticles generates an anomalous Hall current in the y direction.

This yields the source term in Eq. (11),

$$\mathcal{S}(\mathbf{p}) = \frac{\xi}{\epsilon}\,\mathbf{v}\cdot\frac{\partial n^{(1)}_{\mathbf{p}}}{\partial \mathbf{r}}. \tag{23}$$

At length scales in excess of the mean free path we may employ the diffusive approximation. With the aid of Eq. (5) the Hall current j_y can be expressed in the form

$$j_y(x) = -4evD\,\delta_n \int d\xi \frac{|\Delta|^2}{\xi^2}\partial_x \bar{n}_a(\xi,x), \tag{24}$$

where $\bar{n}_a(\xi,x)$ is the antisymmetric in ξ part of the distribution function averaged over the momentum directions. The latter satisfies the diffusion equation with relaxation

$$D\frac{\partial^2}{\partial x^2}\bar{n}_a(\xi,x) = \frac{1}{\tau_Q(\xi)}\bar{n}_a(\xi,x), \tag{25}$$

with energy-dependent relaxation rate $\tau_Q^{-1}(\xi) = \tau^{-1}|\Delta|^2(\xi^2+2|\Delta|^2)/\xi^4$. The solution of Eq. (25) is

$$\bar{n}_a(\xi,x) = \bar{n}_a(\xi,0)\exp[-x/L_Q(\xi)], \tag{26}$$

where $L_Q(\xi) = \sqrt{D\tau_Q(\xi)}$ is the energy-dependent branch imbalance relaxation length.

We work in the vicinity of the critical temperature T_c, where for typical thermal quasiparticles $\xi \sim T$, $(|\xi| \gg |\Delta|)$, the relaxation lengths are long, $L_Q(\xi) = l|\xi|/|\Delta| \gg l$. These quasiparticles diffuse into the bulk of the superconductor and contribute to the gauge-invariant potential Φ given by Eq. (6). The boundary value of the nonequilibrium quasiparticle population, $\bar{n}_a(\xi,0)$ in Eq. (26), is obtained by matching the solution of Eq. (25) with the solution of diffusion equation with energy relaxation for the electrons in the normal metal. The result depends on both the inelastic mean free path in the normal metal, l_e, and the branch imbalance relaxation length L_Q in the superconductor. For $l_e \gg L_Q$ the boundary condition is

$$\bar{n}_a(\xi,0) = \mathrm{sgn}(\xi)\frac{eE_x(0)L_Q(\xi)}{4T\cosh^2(\xi/2T)}, \tag{27}$$

where $E_x(0)$ is the electric field in the normal metal generating the steady current. Here we used the fact that in the stationary case $\mathbf{E} = -\nabla\Phi/e$, which follows from Eq. (4). Using this relation and substituting Eqs. (27) and (26) into Eqs. (6) and (24) we obtain the spatial distributions of the electric field $E_x(x)$ and the Hall current $j_y(x)$ in the superconductor,

$$E_x(x) = E_x(0)F_0\left(\frac{x}{\langle L_Q\rangle}\right), \tag{28}$$

$$j_y(x) = \sigma_D E_x(0)\delta_n\left(\frac{|\Delta|}{T}\right)^2 F_{-2}\left(\frac{x}{\langle L_Q\rangle}\right), \tag{29}$$

where $\langle L_Q\rangle = 2\ln 2(Tl/|\Delta|)$ and the functions F_n are defined as

$$F_n(x) = \int_0^\infty dy\frac{y^n}{\cosh^2(y)}\exp\left(-\ln 2\,\frac{x}{y}\right), \tag{30}$$

and are plotted in Fig. 1. The spatial distributions of the Hall current $j_y(x)$ and the electric field $E_x(x)$ are drastically different, and cannot be related by a local Hall conductivity σ_{xy}. At relatively short distances, $l \ll x \ll \langle L_Q\rangle$, we see from Eq. (29) that $j_y(x) \propto \sigma_D E_x(0)\delta_n(|\Delta|/T)^2\langle L_Q\rangle/x$, so that the Hall current is

$$I_y = \int dx\, j_y(x) \approx \sigma_D E_x(0)\delta_n\left(\frac{|\Delta|}{T}\right)^2\langle L_Q\rangle \ln\frac{\langle L_Q\rangle}{l}.$$

Anomalous photo- and acousto-galvanic effects. When an electromagnetic or an acoustic wave propagates through a conductor it generates an anisotropic in momentum $\mathbf{p}$ distribution function. The induced current density is proportional to the rate of the the momentum transfer from the wave to the electron system, [15]

$$J_x = I\alpha_{xx}.$$

Here I is the rate of momentum density transfer due to the wave adsorption, and x is the direction of the wave propagation. In $p_x + ip_y$ superconductors an anomalous current in the y direction is generated. Considerations similar to those leading to Eq. (22) near T_c yield

$$\alpha_{xy} \sim \alpha_{xx}\frac{|\Delta|}{T}\,\delta_n.$$

Finally, we note that all anomalous transport phenomena discussed above are driven by the underlying symmetry of the superconducting state. Therefore they should exist in any superconductor whose order parameter breaks time-reversal symmetry (see, for example, Refs. [23–27]). Although our consideration focused on $p_x + ip_y$ materials we believe our approach is applicable to other superconductors with broken time-reversal invariance.

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