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Anomalous Josephson effect in *d*-wave superconductor junctions on a topological insulator surface

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We study the Josephson effect of d-wave superconductor (DS)/ferromagnet insulator (FI)/DS junctions on a surface of topological insulator (TI). We calculate Josephson current $I(\varphi)$ for various orientations of the junctions where φ is the macroscopic phase difference between two DSs. In certain configurations, we find anomalous current-phase relation $I(\varphi) = -I(-\varphi + \pi)$ with 2π periodicity. In the case where the first-order Josephson coupling is absent without magnetization in FI, $I(\varphi)$ can be proportional to $\cos\varphi$. In symmetric junctions, the magnitude of the obtained Josephson current is enhanced due to the zero-energy states on the edge of DS on TI. Even if we introduce an s-wave component of pair potential in DS, we can still expect the anomalous current-phase relation in asymmetric DS junctions with $I(\varphi = 0) \neq 0$. This can be used to probe the induced d-wave component of pair potential on a TI surface in high-T_c cuprate/TI hybrid structures.

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I. INTRODUCTION

The Josephson effect has been a fundamental and central topic in superconductivity and contributed to determine the pairing symmetry in unconventional superconductors [1–3]. It is well known that standard current-phase relation (CPR) of Josephson current $I(\varphi)$ between two superconductors is $I(\varphi) \sim \sin\varphi$, where φ is the macroscopic phase difference. For d-wave superconductor (DS) junctions, due to the presence of Andreev bound state at the interface [4–6], exotic quantum interference effects exist. One is the nonmonotonic temperature dependence of maximum Josephson current [7–11] and second is the anomalous CPR [7,9,10,11]. Due to the presence of Andreev bound state (ABS), the $\sin 2\varphi$ component of $I(\varphi)$ is enhanced and the free energy of the junction can locate neither $\varphi = 0$ nor $\pm\pi$. Furthermore, a pure $\sin 2\varphi$ CPR is possible for $d_{x^2-y^2}$-wave/d_{xy}-wave superconductor junction [7,9]. Thus, d-wave junctions have really rich current phase relation and their functionalities are worthy of further research.

On the other hand, three-dimensional (3D) topological insulators (TIs) [12–19] are materials with a topologically protected surface state due to the strong spin-orbit coupling. The generation of superconductivity on the surface state of a TI via the proximity effect has been verified by the presence of supercurrent through the Josephson junctions on a TI [20–24]. The Josephson current in superconductor (S)/ferromagnetic insulator (FI)/S junction on a TI stimulates us since anomalous CPR $I(\varphi) \sim \sin(\varphi - \varphi_0)$ discussed in conventional S/ferromagnet (F)/S junction without a TI [25–31] can be realized easily. It is noted that φ_0 can be tunable by magnetization [32].

We can imagine that more dramatic features will be expected in d-wave superconductor (DS) /FI/DS Josephson junctions on a TI. Although there have been several works about DS/FI/DS junctions [33–35], CPR has not been clarified for general orientations of the junctions. Recent experiments have shown that induced gap function on the surface of a TI which is formed on high-T_c cuprate is almost isotropic. This shows that the induced pair potential has predominant s-wave symmetry [36]. The possibility of inducing d-wave pairing on a TI surface in the actual experiment is still in debate now [37–39]. Therefore, aside from the CPR in d-wave superconductor junctions, we must study the CPR in junctions with $(s + d)$-wave symmetry for comparison with actual experiments.

In this paper, in order to calculate dc Josephson current, we develop a formalism of Green's function of quasiparticles on the surface of a TI [40]. Both Josephson current and local density of states (LDOS) can be calculated for general orientations of junctions. In general, the obtained current-phase relation $I(\varphi)$ has a complex φ dependence. $I(\varphi) = -I(-\varphi)$ is easily to be broken by magnetization in FI and $I(\varphi)$ can not be simply expressed by $I_0 \sin(\varphi - \varphi_0)$ with nonzero φ_0. The extreme case is $d_{x^2-y^2}$/FI/d_{xy}-wave junctions, where CPR becomes $\sin 2\varphi$ without magnetization due to the absence of the first-order Josephson coupling. If we switch on magnetization, exotic CPR becomes possible depending on the direction of magnetization in FI: (i) mixture of $\cos\varphi$ and $\sin(2\varphi)$ terms with $I(\varphi) = -I(-\varphi + \pi)$ and (ii) $\sin(2\varphi - 2\varphi_0)$. The complex CPR $I(\varphi) = -I(-\varphi + \pi)$ with 2π periodicity in (i) is not realized in the preexisting high-T_c cuprate junctions without TI [1–3]. We also calculate Josephson current where s- and d-wave pair potentials mix. It is found that the anomalous CPR with $I(\varphi = 0) \neq 0$ exists for junctions of asymmetric orientations even if the s-wave component becomes dominant. This feature serves as a guide to detect the proximity-induced d-wave component of pair potential on the surface of a TI.

II. MODEL AND FORMULAS

As depicted in Fig. 1(a), we consider a DS/FI/DS junction on a 3D TI surface. The effective Hamiltonian for the Bogoliubov–de Gennes (BdG) equations is given by

$$\mathcal{H} = \begin{bmatrix} h(k_x,k_y) + M & i\hat{\sigma}_y \Delta(\theta) \\ -i\hat{\sigma}_y \Delta^*(\theta) & -h^*(-k_x,-k_y) - M^* \end{bmatrix}, \quad (1)$$

where $h(k_x,k_y) = \hbar v_f(k_y\hat{\sigma}_x - k_x\hat{\sigma}_y) - \mu[\Theta(-x) + \Theta(x - L)]$. $\hat{\sigma}_{x,y,z}$ is the Pauli matrix in the spin space and

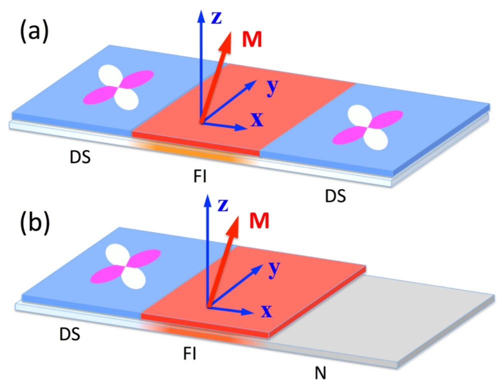

FIG. 1. (Color online) Schematics of the system: (a) d-wave superconductor (DS)/ferromagnetic insulator (FI)/DS Josephson junction and (b) DS/FI/normal metal (N) junction on the surface of a 3D topological insulator.

μ is the chemical potential in the superconducting region. The exchange field in FI region is $M = \sum_{i=x,y,z} m_i \hat{\sigma}_i \Theta(x)\Theta(L-x)$ [32]. The pair potential is given by $\Delta_0(T)\cos(2\theta - 2\chi_1)\Theta(-x) + \Delta_0(T)\cos(2\theta - 2\chi_2)e^{-i\varphi}\Theta(x-L)$, where φ and θ are the macroscopic superconducting phase and the propagating angle, respectively. The quantity χ is taken to be the angle between the x axis and the a axis of cuprate crystals which can be tuned independently in the two DSs by proper fabrication of junction [10,11,41]. In our model, we assume the d-wave pairing potential on TI surface without determining self-consistently. For most cuprates, the superconducting gap is weakly temperature dependent at low temperature and thus we adopt the BCS relation for the simplicity: $\Delta_0(T) = \Delta_0 \tanh(1.74\sqrt{T_c/T - 1})$ with $\Delta_0 = 1.76 k_B T_c$ and T_c is the critical temperature. It is noted that this ratio $\Delta_0/k_B T_c$ does not influence our main conclusion qualitatively at all.

To construct the retarded Green's function, we first seek the solutions for the four types of quasiparticle injection processes: left injection for electron (hole) $\psi_1(\psi_2)$ and right injection for electron (hole) $\psi_3(\psi_4)$. Because of the translational invariance along the y axis, the wave functions $\psi_{1(2)} = \psi_{1(2)}(x)e^{ik_y y}$ in the left superconducting region can be expressed as

$$\psi_1(x) = \hat{A}_1 e^{ik_x x} + a_1 \hat{A}_4 e^{ik_x x} + b_1 \hat{A}_3 e^{-ik_x x}, \qquad (2a)$$

$$\psi_2(x) = \hat{A}_2 e^{-ik_x x} + a_2 \hat{A}_3 e^{-ik_x x} + b_2 \hat{A}_4 e^{ik_x x}, \qquad (2b)$$

where $k_x = \mu\cos\theta/\hbar v_F$. Here, the magnitudes of the momenta for electrons and holes are approximated to be equal since we have made the assumption of E, $\Delta \ll \mu$. The spinors are given by $\hat{A}_1 = (i, e^{i\theta}, -e^{i\theta}\gamma_1, i\gamma_1)^T$, $\hat{A}_2 = (ie^{i\theta}\gamma_2, -\gamma_2, 1, ie^{i\theta})^T$, $\hat{A}_3 = (ie^{i\theta}, -1, \gamma_2, ie^{i\theta}\gamma_2)^T$, and $\hat{A}_4 = (i\gamma_1, e^{i\theta}\gamma_1, -e^{i\theta}, i)^T$ with $\gamma_{1(2)} = \Delta_{1(2)}/(E + \sqrt{E^2 - \Delta_{1(2)}^2})$ and $\Delta_{1(2)} = \Delta_0\cos(2\theta \mp 2\chi_1)$. Other wave functions can be solved in a similar way. The coefficients $a_{1,2}$ and $b_{1,2}$ are determined by the continuity of wave function at each interface between DS and FI. The retarded Green's function $G^r(x,x',y,y') = \sum_{k_y} G^r_{k_y}(x,x')e^{ik_y(y-y')}$ can be obtained by combining all the injection processes [40]:

$$G^r_{k_y}(x,x') = \begin{cases} \alpha_1\psi_1(x)\tilde{\psi}_3^T(x') + \alpha_2\psi_1(x)\tilde{\psi}_4^T(x') \\ +\alpha_3\psi_2(x)\tilde{\psi}_3^T(x') + \alpha_4\psi_2(x)\tilde{\psi}_4^T(x'), \\ \qquad (x > x'), \\ \beta_1\psi_3(x)\tilde{\psi}_1^T(x') + \beta_2\psi_4(x)\tilde{\psi}_1^T(x') \\ +\beta_3\psi_3(x)\tilde{\psi}_2^T(x') + \beta_4\psi_4(x)\tilde{\psi}_2^T(x'), \\ \qquad (x < x'), \end{cases} \qquad (3)$$

where $\tilde{\psi}_{i=1\text{–}4}$ are the corresponding conjugated processes of $\psi_{i=1\text{–}4}$. The coefficients $\alpha_{i=1\text{–}4}$ and $\beta_{i=1\text{–}4}$ are determined by satisfying the boundary conditions for all x, x' across the regions:

$$G^r_{k_y}(x+0,x) - G^r_{k_y}(x-0,x) = \hbar^{-1}v_f^{-1}(i\hat{\tau}_z\hat{\sigma}_y), \qquad (4)$$

where $\hat{\tau}_{x,y,z}$ is the Pauli matrix in the particle-hole space. The dc Josephson current for DS/FI/DS junction is determined by electric charge conservation rule

$$\partial_t P + \partial_x J_x + S = 0, \qquad (5)$$

where $P = e(\Psi_\uparrow^\dagger\Psi_\uparrow + \Psi_\downarrow^\dagger\Psi_\downarrow)$, $J_x = iev_f(\Psi_\uparrow^\dagger\Psi_\downarrow - \Psi_\downarrow^\dagger\Psi_\uparrow)$, and $S = 2e\,\mathrm{Im}[\Delta^*\Psi_\downarrow\Psi_\uparrow - \Delta^*\Psi_\uparrow\Psi_\downarrow]$ are electric charge density, electric current, and source term, respectively. After straightforward derivation following Ref. [42], we find that the total Josephson current is given by

$$I_x = \frac{ek_BT}{2\hbar}\sum_{k_y,\omega_n}\mathrm{sgn}(\omega_n)\left[\frac{\Delta_1 a_1(i\omega_n)}{\sqrt{\omega_n^2+\Delta_1^2}} - \frac{\Delta_2 a_2(i\omega_n)}{\sqrt{\omega_n^2+\Delta_2^2}}\right]. \qquad (6)$$

$a_{1(2)}(i\omega_n)$ is obtained by analytical continuation E to $i\omega_n$, where ω_n is the Matsubara frequency $\omega_n = \pi k_B T(2n+1)$ $(n = 0, \pm1, \pm2, \ldots)$. Equation (6) looks similar to the extended Furusaki-Tsukada formula [42] for anisotropic d-wave pair potential [7,9]. In addition, Eq. (6) is also applicable to the Josephson current of $(s+d)$-wave pairing in which one substitutes $\Delta_{1(2)}$ by $\Delta_0 + \eta\Delta_0\cos(2\theta \mp 2\chi_1)$ of which $\eta \geqslant 0$ is the ratio between d-wave pairing and s-wave pairing.

III. JOSEPHSON EFFECT IN DS/FI/DS JUNCTION

In this section, we show the results of Josephson current I in DS/FI/DS junctions, which has been normalized to $eR_N I/\Delta_0$ where R_N is the interface resistance per unit area in the normal state. To analyze the CPR further, we decompose the Josephson current into a series of different orders of Josephson coupling

$$I(\varphi) = \sum_n I_n \sin(n\varphi) + J_n \cos(n\varphi), \qquad (7)$$

where $n \geqslant 1$ is an integer. Figure 2(a) shows CPR without magnetization. In this case, the CPR is expressed as $\sum_n I_n \sin(n\varphi)$ and J_n is zero. In the condition with $\chi_1 = 0$ and $\chi_2 = \pi/4$, the CPR $I(\varphi)$ becomes $\sum_n I_n \sin(n\varphi)$ $(n = 2, 4, \ldots)$. The feature of this CPR is the same as that in the standard d-wave junctions without TI with the pair potential considered here. However, as the magnetization switches on, the CPR dramatically changes. Figure 2(b) shows that m_y gives

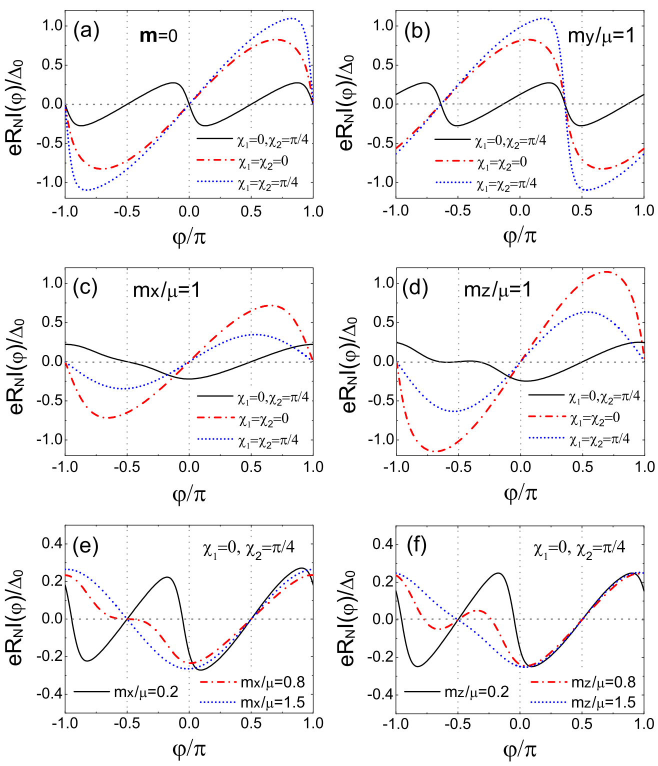

FIG. 2. (Color online) Josephson currents as a function of φ in the DS/FI/DS junctions. Magnetization in FI is (a) zero, (b) along the y axis, (c) along the x axis, and (d) along the z axis. Three geometries are considered in panels (a)–(d): $d_{x^2-y^2}$/FI/d_{xy}, $d_{x^2-y^2}$/FI/$d_{x^2-y^2}$, and d_{xy}/FI/d_{xy}. (e) Josephson currents in $d_{x^2-y^2}$/FI/d_{xy} junctions with $m_x/\mu = 0.2, 0.8$, and 1.5 and (f) those with $m_z/\mu = 0.2, 0.8$, and 1.5. Other parameters are set as $T = 0.05T_c$, $\mu = 1$, $\hbar v_f = 1$, $\Delta = 0.01$, and $L = 1$.

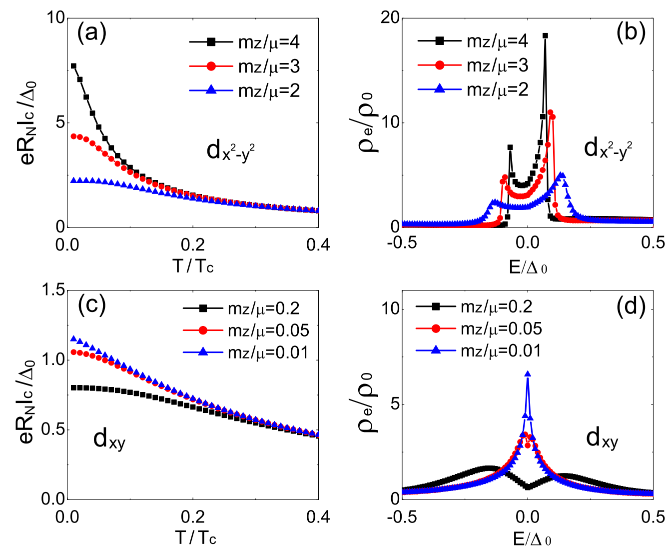

FIG. 3. (Color online) (a) The maximum Josephson currents in the $d_{x^2-y^2}$/FI/$d_{x^2-y^2}$ junctions. (b) The LDOS on the surface of DS in the $d_{x^2-y^2}$/FI/N junctions. ρ_0 is the electron density of states of the bulk N at Fermi energy. (c), (d) The maximum Josephson currents and LDOS in d_{xy}/FI/d_{xy} junctions, respectively. The magnetization is along the z axis in all panels. Other parameters are set as the same as Fig. 2.

a shift of phase difference, which is similar to φ_0 junctions realized in conventional s-wave superconductor/ferromagnet hybrid systems [25–31]. As the magnetization along the x or z axis appears, the qualitative features of CPR of symmetric $d_{x^2-y^2}$/FI/$d_{x^2-y^2}$ and d_{xy}/FI/d_{xy} junctions do not change as compared to the case without magnetization as shown in dotted and dashed-dotted lines of Figs. 2(c) and 2(d). However, in the asymmetric $d_{x^2-y^2}$/FI/d_{xy} junction, the CPR is quite anomalous and the component proportional to $\sum_n J_n \cos(n\varphi)$ is generated. We find that $I(\varphi)$ can be expressed by $\sum_k [I_{2k} \sin(2k\varphi) + J_{2k-1} \cos(2k-1)\varphi]$ where $k \geqslant 1$ is an integer and, therefore, $I(\varphi)$ becomes zero at $\varphi = \pm\pi/2$ as shown in solid lines in Figs. 2(c) and 2(d). The present CPR is completely different from that of the standard $d_{x^2-y^2}$/FI/d_{xy} junction without TI. We can see that the term proportional to $J_1 \cos(\varphi)$ becomes dominant in the limit of large m_x or m_z in Figs. 2(e) and 2(f).

To explain the anomalous CPR for nonzero m_x or m_z in $d_{x^2-y^2}$/FI/d_{xy} junction on a TI surface, we focus on the symmetry of this Hamiltonian. We consider the mirror reflection symmetry with respect to the xz plane $M_{xz} = i\sigma_y\tau_0$ and the time-reversal symmetry $T = -i\sigma_y\mathcal{K}\tau_0$, where $\mathcal{K}$ is the complex-conjugation operator. In the present system, both symmetries are broken. However, since the pair potential of $d_{x^2-y^2}$ (d_{xy}) is mirror even (odd) with respect to the xz plane, the M_{xz} operation produces additional phase $I(\varphi) \to I(\varphi+\pi)$. It is also known that time-reversal operation transforms $I(\varphi)$ to $-I(-\varphi)$. Hence, the composition operator $\tilde{T} = M_{xz}T$ will give rise to $I(\varphi) \to -I(-\varphi+\pi)$. Taking into account the fact that $\tilde{T}$ makes the (k_x,k_y) state to the $(-k_x,k_y)$ one, we can arrive at

$$\tilde{T}\mathcal{H}(-i\partial_x,k_y,\varphi)\tilde{T}^{-1} \to \mathcal{H}(i\partial_x,k_y,-\varphi+\pi). \qquad (8)$$

It means that $-I(-\varphi+\pi) = I(\varphi)$ will be satisfied at any φ if we consider the junctions between a mirror-even and mirror-odd pair potential. In the $d_{x^2-y^2}$/FI/d_{xy} junction with m_x or m_z, we can find that relation (8) fulfills at any φ, which indicates $I(\varphi = \pm\pi/2) = 0$. The above analysis based on mirror reflection symmetry has been applied in the analysis of Josephson current [43,44]. Now, let us look at the Josephson current at $\varphi = 0$. In the standard $d_{x^2-y^2}$/FI/d_{xy} junctions without TI substrate, due to the spin SU(2) symmetry, the rotation or mirror reflection of the ferromagnetism does not change the CPR and one can always find $I(\varphi = 0) = 0$ [45]. However, this SU(2) symmetry is broken on TI surface due to its nature of spin-momentum locking and thus nonzero $I(\varphi = 0)$ becomes possible which generates exotic 2π-periodic CPR $-I(-\varphi+\pi) = I(\varphi)$.

Next, we plot the temperature dependence of the maximum Josephson current I_c of DS/FI/DS junctions in the left panels of Fig. 3. For simplicity, only the z component of magnetization m_z and symmetric junctions are considered. We concentrate on the low-temperature region $T/T_c \leqslant 0.4$ in which the behavior of I_c is highly influenced by the zero-energy states (ZESs). Therefore, we display the LDOS at the edge of DS. It is obtained by calculating the LDOS $\rho_e(x,E) = -\frac{1}{\pi}\sum_{k_y} \mathrm{Im}[G^r_{k_y,11}(x,x,E) + G^r_{k_y,22}(x,x,E)]$ at the DS/FI interface in the DS/FI/N junction as illustrated in Fig. 1(b). From

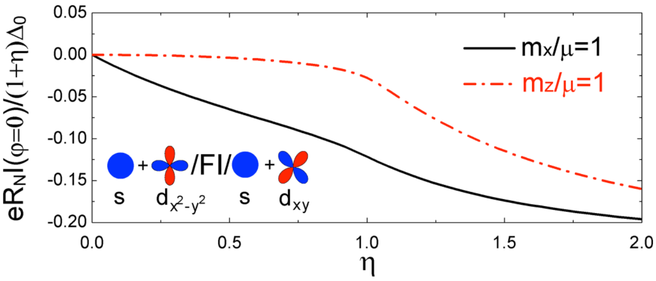

FIG. 4. (Color online) $I(\varphi = 0)$ as a function of η in $s + d_{x^2-y^2}$/FI/$s + d_{xy}$ junctions on a TI. Other parameters are set as the same as in Fig. 2.

Fig. 3(a), we can see that temperature dependence of I_c in $d_{x^2-y^2}$/FI/$d_{x^2-y^2}$ junctions changes from Kulik-Omelyanchuk (KO) [46] type to Ambegaokar-Baratoff (AB) [47] type with decreasing m_z. However, in d_{xy}/FI/d_{xy} junction, this tendency is reversed when we decrease m_z, as shown in Fig. 3(c). As seen from Figs. 3(b) and 3(d), we can see that as LDOS at zero energy is enhanced, temperature dependence of I_c is reduced to be the KO type. In the other case, it is in the AB type. Since the spin degeneracy is lifted on the surface states of a TI by spin-momentum locking, we obtain the highly asymmetric Yu-Shiba-Rusinov type of LDOS [48–50] in the $d_{x^2-y^2}$/FI interface. This finding is similar to that in the s-wave superconductor/FI/N junction [51].

IV. JOSEPHSON EFFECT WITH ($s + d$)-WAVE PAIRING

Recent experiments have shown that the induced energy gap by high-T_c cuprate on the surface of a TI is almost isotropic [36]. It is interesting to clarify the role of the induced s-wave pair potential on DS/FI/DS junctions. In this section, we calculate Josephson current in $(s + d)$-wave/FI/$(s + d)$-wave junctions on a TI. The pair potential is $\Delta = \Delta_0 + \eta\Delta_0 \cos(2\theta - 2\chi)$ with $\chi = 0(\pi/4)$ on the left (right) side. The ratio η is chosen to be 0.5 so that the system is s-wave dominant and fully gapped. The obtained $I(\varphi)$ has a typical sinusoidal shape of s-wave Josephson current where the first-order coupling $I_1 \sin\varphi$ plays the predominant role. Because of the s-wave component of pair potential, the symmetry $\tilde{T}$ at $\varphi = \pm\pi/2$ is broken and thus nonzero current $I(\varphi = \pm\pi/2)$ can be expected. Also, in the presence of m_x or m_z, we find a nonzero Josephson current at $\varphi = 0,\pi$ in such junctions. The obtained anomalous CPR in the $(s + d)$-wave Josephson junctions can be used to probe the d-wave component of the induced pair potential on a TI surface. For example, one can observe the supercurrent flow without macroscopic phase difference in $s + d_{x^2-y^2}$/FI/$s + d_{xy}$ junctions. As seen in Fig. 4, the existence of d-wave component generates a nonzero current $I(\varphi = 0)$ when one turns on either m_x or m_z.

In our calculations, the direct coupling between cuprate and FI without going through a TI region is not considered. To prevent this direct coupling, one can separate each segment with a distance as proposed in Ref. [51]. The feature of Josephson current will not be changed.

V. CONCLUSION

In summary, we have theoretically studied the Josephson effect in d-wave superconductor-ferromagnet insulator (FI) hybrids on the surface of a TI. Depending on the orientation of the magnetization in FI, the exotic current-phase relation which violates $I(\varphi) \neq -I(-\varphi)$ has been obtained in two different ways: (i) through a simple phase shift and (ii) mixture of $\cos\varphi$ term into the original CPR. The latter case can generate the exotic current-phase relation $I(\varphi) = -I(-\varphi + \pi)$ with 2π periodicity. We show that the Josephson current is enhanced due to the zero-energy states on the edge of d-wave superconductor. For comparison with actual experiments, we calculate the Josephson current when both s- and d-wave pair potentials exist. The anomalous current-phase relation is also found which provides a way to probe the fingerprint of d-wave pair potential in high-T_c cuprate/TI heterostructures. Our preliminary theoretical investigation has practical significance for controlling the Josephson current and designing new functional devices.

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[1] C. C. Tsuei and J. R. Kirtley, Rev. Mod. Phys. **72**, 969 (2000).
[2] S. Kashiwaya and Y. Tanaka, Rep. Prog. Phys. **63**, 1641 (2000).
[3] A. A. Golubov, M. Yu. Kupriyanov, and E. Il'ichev, Rev. Mod. Phys. **76**, 411 (2004).
[4] C. R. Hu, Phys. Rev. Lett. **72**, 1526 (1994).
[5] S. Kashiwaya, Y. Tanaka, M. Koyanagi, H. Takashima, and K. Kajimura, Phys. Rev. B **51**, 1350 (1995).
[6] Y. Tanaka and S. Kashiwaya, Phys. Rev. Lett. **74**, 3451 (1995).
[7] Y. Tanaka and S. Kashiwaya, Phys. Rev. B **53**, R11957 (1996).
[8] Y. S. Barash, H. Burkhardt, and D. Rainer, Phys. Rev. Lett. **77**, 4070 (1996).
[9] Y. Tanaka and S. Kashiwaya, Phys. Rev. B **56**, 892 (1997).
[10] E. Il'ichev, M. Grajcar, R. Hlubina, R. P. J. IJsselsteijn, H. E. Hoenig, H.-G. Meyer, A. Golubov, M. H. S. Amin, A. M. Zagoskin, A. N. Omelyanchouk, and M. Yu. Kupriyanov, Phys. Rev. Lett. **86**, 5369 (2001).
[11] G. Testa, E. Sarnelli, A. Monaco, E. Esposito, M. Ejrnaes, D.-J. Kang, S. H. Mennema, E. J. Tarte, and M. G. Blamire, Phys. Rev. B **71**, 134520 (2005).
[12] L. Fu and C. L. Kane, Phys. Rev. B **76**, 045302 (2007).
[13] H. Zhang, C. X. Liu, X. L. Qi, X. Dai, Z. Fang, and S. C. Zhang, Nat. Phys. **5**, 438 (2009).

[14] D. Hsieh, D. Qian, L. Wray, Y. Xia, Y. S. Hor, R. J. Cava, and M. Z. Hasan, Nature (London) **452**, 970 (2008).
[15] D. Hsieh, Y. Xia, L. Wray, D. Qian, A. Pal, J. H. Dil, J. Osterwalder, F. Meier, G. Bihlmayer, C. L. Kane, Y. S. Hor, R. J. Cava, and M. Z. Hasan, Science **323**, 919 (2009).
[16] Y. Xia, D. Qian, D. Hsieh, L. Wray, A. Pal, H. Lin, A. Bansil, D. Grauer, Y. S. Hor, R. J. Cava, and M. Z. Hasan, Nat. Phys. **5**, 398 (2009).
[17] Y. L. Chen, J. G. Analytis, J.-H. Chu, Z. K. Liu, S.-K. Mo, X. L. Qi, H. J. Zhang, D. H. Lu, X. Dai, Z. Fang, S. C. Zhang, I. R. Fisher, Z. Hussain, and Z.-X. Shen, Science **325**, 178 (2009).
[18] D. Hsieh, Y. Xia, D. Qian, L. Wray, F. Meier, J. H. Dil, J. Osterwalder, L. Patthey, A. V. Fedorov, H. Lin, A. Bansil, D. Grauer, Y. S. Hor, R. J. Cava, and M. Z. Hasan, Phys. Rev. Lett. **103**, 146401 (2009).
[19] Z. Ren, A. A. Taskin, S. Sasaki, K. Segawa, and Y. Ando, Phys. Rev. B **82**, 241306 (2010).
[20] B. Sacepe, J. B. Oostinga, J. Li, A. Ubaldini, N. J. G. Couto, E. Giannini, and A. F. Morpurgo, Nat. Commun. **2**, 575 (2011).
[21] D. M. Zhang, J. Wang, A. M. DaSilva, J. S. Lee, H. R. Gutierrez, M. H. W. Chan, J. Jain, and N. Samarth, Phys. Rev. B **84**, 165120 (2011).
[22] M. Veldhorst, M. Snelder, M. Hoek, T. Gang, V. K. Guduru, X. L. Wang, U. Zeitler, W. G. van der Wiel, A. A. Golubov, H. Hilgenkamp, and A. Brinkman, Nat. Mater. **11**, 417 (2012).
[23] J. R. Williams, A. J. Bestwick, P. Gallagher, S. S. Hong, Y. Cui, A. S. Bleich, J. G. Analytis, I. R. Fisher, and D. Goldhaber-Gordon, Phys. Rev. Lett. **109**, 056803 (2012).
[24] M. Snelder, C. G. Molenaar, Y. Pan, D. Wu, Y. K. Huang, A. de Visser, A. A. Golubov, W. G. van der Wiel, H. Hilgenkamp, M. S. Golden, and A. Brinkman, Supercond. Sci. Technol. **27**, 104001 (2014).
[25] R. Grein, M. Eschrig, G. Metalidis, and G. Schon, Phys. Rev. Lett. **102**, 227005 (2009).
[26] M. Eschrig, T. Lofwander, T. Champel, J. C. Cuevas, J. Kopu, and G. Schon, J. Low Temp. Phys. **147**, 457 (2007).
[27] V. Braude and Y. V. Nazarov, Phys. Rev. Lett. **98**, 077003 (2007).
[28] Y. Asano, Y. Sawa, Y. Tanaka, and A. A. Golubov, Phys. Rev. B **76**, 224525 (2007).
[29] M. Eschrig and T. Lofwander, Nat. Phys. **4**, 138 (2008).
[30] A. Buzdin, Phys. Rev. Lett. **101**, 107005 (2008).
[31] F. Konschelle and A. Buzdin, Phys. Rev. Lett. **102**, 017001 (2009).
[32] Y. Tanaka, T. Yokoyama, and N. Nagaosa, Phys. Rev. Lett. **103**, 107002 (2009). In this paper, the momentum-spin locking term in Hamiltonian is chosen to be $h(k_x,k_y) = v_f(k_x\sigma_x + k_y\sigma_y)$ and magnetization along the x axis only shifts the phase difference.
[33] J. Linder, Y. Tanaka, T. Yokoyama, A. Sudbø, and N. Nagaosa, Phys. Rev. Lett. **104**, 067001 (2010).
[34] J. Linder, Y. Tanaka, T. Yokoyama, A. Sudbø, and N. Nagaosa, Phys. Rev. B **81**, 184525 (2010).
[35] P. Lucignano, A. Mezzacapo, F. Tafuri, and A. Tagliacozzo, Phys. Rev. B **86**, 144513 (2012).
[36] E. Wang, H. Ding, A. V. Fedorov, W. Yao, Z. Li, Y.-F. Lv, K. Zhao, L.-G. Zhang, Z. Xu, J. Schneeloch, R. Zhong, S.-H. Ji, L. Wang, K. He, X. Ma, G. Gu, H. Yao, Q.-K. Xue, X. Chen, and S. Zhou, Nat. Phys. **9**, 621 (2013).
[37] Z. X. Li, C. Chan, and H. Yao, Phys. Rev. B **91**, 235143 (2015).
[38] S. Y. Xu, C. Liu, A. Richardella, I. Belopolski, N. Alidoust, M. Neupane, G. Bian, N. Samarth, and M. Z. Hasan, Phys. Rev. B **90**, 085128 (2014).
[39] K. Lee, A. Vaezi, M. H. Fischer, and E.-A. Kim, Phys. Rev. B **90**, 214510 (2014).
[40] W. L. McMillan, Phys. Rev. **175**, 559 (1968).
[41] E. Ilichev, V. Zakosarenko, R. P. J. IJsselsteijn, H. E. Hoenig, V. Schultze, H.-G. Meyer, M. Grajcar, and R. Hlubina, Phys. Rev. B **60**, 3096 (1999)
[42] A. Furusaki and M. Tsukada, Solid State Commun. **78**, 299 (1991).
[43] C. K. Lu and S. K. Yip, Phys. Rev. B **80**, 024504 (2009).
[44] G. Campagnano, P. Lucignano, D. Giuliano, and A. Tagliacozzo, J. Phys.: Condens. Matter **27**, 205301 (2015).
[45] In cuprate junctions where the geometry is the same as Fig. 1(a), the orientation of ferromagnetism plays no role in CPR due to the spin SU(2) symmetry. Therefore, in the case $\varphi = 0$, the system and its time-reversal copy have the same Josephson current, which implies $I(\varphi = 0) = 0$.
[46] I. O. Kulik and A. N. Omelyanchuk, Fiz. Nizk. Temp. **4**, 296 (1978) [Sov. J. Low Temp. Phys. **4**, 142 (1978)].
[47] V. Ambegaokar and A. Baratoff, Phys. Rev. Lett. **10**, 486 (1963);**11**, 104 (1963).
[48] L. Yu, Acta Phys. Sin. **21**, 75 (1965).
[49] H. Shiba, Prog. Theor. Phys. **40**, 435 (1968).
[50] A. I. Rusinov, Zh. Eksp. Teor. Fiz. **56**, 2047 (1969) [Sov. Phys. JETP **29**, 1101 (1969)].
[51] B. Lu, P. Burset, K. Yada, and Y. Tanaka, Superconductor Science and Technology **28**, 105001 (2015).