Orbital Kondo effect in fractional quantum Hall systems

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We study the transport properties of a charge qubit coupling two chiral Luttinger liquids, realized by two antidots placed between the edges of an integer $\nu = 1$ or fractional $\nu = 1/3$ quantum Hall bar. We show that in the limit of a large capacitive coupling between the antidots, their quasiparticle occupancy behaves as a pseudospin corresponding to an orbital Kondo impurity coupled to a chiral Luttinger liquid, while the interantidot tunneling acts as an impurity magnetic field. The latter tends to destabilize the Kondo fixed point for the $\nu = 1/3$ fractional Hall state, producing an effective interedge tunneling. We relate the interedge conductance to the susceptibility of the Kondo impurity and calculate it analytically in various limits for both $\nu = 1$ and $\nu = 1/3$.

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I. INTRODUCTION

Fractional quantum Hall (FQH) systems [1] are strongly correlated topological states, realized in clean two-dimensional electron gases under a large perpendicular magnetic field, where the bulk contains an incompressible fluid and the low-energy dynamics is controlled by chiral Luttinger liquids at the edges [2]. There has recently been a renewed interest in these systems due to a promise of the celebrated topological quantum computation using non-Abelian anyons [3–6] and also in connection to impurities in helical liquids of the quantum spin Hall systems [7–11]. However, there is still a considerable gap between theoretical and experimental studies of Abelian anyons in the FQH edge states, which motivates a more thorough study of their properties. Here, we study the problem of elastic co-tunnelling of Laughlin quasiparticles through two antidots and show that in certain limits, it maps to a Kondo impurity [12,13] embedded between two chiral Luttinger liquids [14–19] and exhibits interesting transport signatures.

Transport through antidots in the FQH regime has been studied in the past, experimentally [20–25] and theoretically [26–29] in a regime where the transport was dominated by correlated but incoherent transfers of individual quasiparticles. In contrast, in this paper, we are interested in a regime where this sequential tunneling is blocked due to a large interantidot capacitive coupling.

Combining the pseudospin of a double dot with the intrinsic spin, Borda *et al.* [30] predicted an SU(4) Kondo effect which has been recently observed [31]. The use of double dots to realize a pseudospin SU(2) Kondo model and its generalizations at $\nu = 1$ integer quantum Hall regime was proposed in Ref. [32]. Here, we extend those ideas by studying the realization and transport properties of a Kondo impurity coupled to chiral Luttinger liquid edge states in the FQH regime. A similar model arises in the study of a double dot inserted in a spinless nonchiral Luttinger liquid, once the occupancy of the dots is limited so that they act as an effective spin. In contrast to most previous studies that focus on zero temperature, we provide analytical expressions for the conductance in all asymptotic temperature regimes. In this paper, we only deal with fully polarized (or spinless) systems and spin refers to the orbital pseudospin.

II. THE MODEL

We consider the system depicted in Fig. 1, in which each antidot is represented by a single fermionic quasiparticle level. This is valid for small enough antidot radius. In this limit, the system can be described by the following Hamiltonian:

$$H = H_0 - [t_R \psi^\dagger_{qp,R}(0) d_R + t_L \psi^\dagger_{qp,L}(0) d_L + \text{H.c.}] + U(d^\dagger_R d_R + d^\dagger_L d_L - 1)^2 - t_C(d^\dagger_R d_L + \text{H.c.}), \quad (1)$$

where t_L, t_R, and t_C are the tunneling amplitudes and U is the Coulomb energy. Here, $d_{L/R}$ annihilates quasiparticles on the upper/lower antidot and $\psi_{qp,R/L}(x)$ annihilates right/left-moving quasiparticles on the upper/lower edge of the Hall bar, with the corresponding Hamiltonian H_0. We are interested in a parameter regime $T, t_L, t_R, t_C \ll D$, where $D \sim \min(\delta\epsilon, U) \ll \Delta$. Here, $\delta\epsilon$ is the antidot level spacing and Δ is the bulk energy gap (we set $k_B = 1$ throughout the paper). Then U limits the antidots charge configuration to the (0,1) and (1,0) sectors. Sequential tunneling is blocked in this large U limit and different methods must be developed to study the system. Temporarily ignoring the interdot tunneling t_C, we see that to transfer one quasiparticle from the upper to the lower edge, we must start in a state with the lower dot occupied, pass through a high-energy intermediate state with both dots occupied or empty and end up with only the upper dot occupied. Thus it is convenient to identify L and R with pseudospin up and down, respectively. The Schrieffer-Wolff transformation [33] then yields a Kondo model with impurity pseudospin operators $\vec{S} \equiv d^\dagger_\alpha \vec{\sigma}_{\alpha\beta} d_\beta/2$ and quasiparticle pseudospin density $\vec{\mathcal{J}}(x) \equiv \psi^\dagger_{qp,\alpha}(x)\vec{\sigma}_{\alpha\beta}\psi_{qp,\beta}(x)/2$. The Kondo interaction, $J_\perp[S^x \mathcal{J}_x(0) + S^y \mathcal{J}_y(0)] + J_z S^z \mathcal{J}_z(0)$ contains Kondo couplings $J_\perp = 4t_L t_R/U$, $J_z = 2(t_L^2 + t_R^2)/U + \delta J_z$, where δJ_z is an additional positive contribution arising from the Coulomb interaction between quasiparticles on the antidots and edges [29,34]. Interantidot tunneling corresponds to a magnetic field term in the Kondo Hamiltonian, coupled to the impurity spin only, $-t_C S^x$.

While $\psi_{qp,L/R}(x)$ are simply free chiral fermion fields for the integer Hall state occurring at $\nu = 1$; for $\nu = 1/3$, it is very useful to bosonize $\psi_{qp,R/L}(x) \propto e^{\pm i\varphi_{R/L}(x)}$ in terms of chiral bosons $\varphi_{R/L}(x)$, obeying the basic commutation

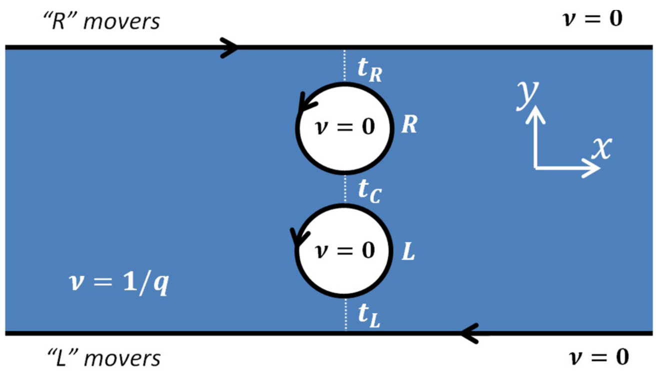

FIG. 1. (Color online) The system considered here. Two antidots enable tunneling of quasiparticles between the outer edge states in the $\nu = 1/q$ Laughlin FQH liquids. The gapped incompressible liquid (blue) plays the role of tunnel barrier for the quasiparticles. Only one state per antidot is considered. The capacitive coupling between the antidots is large enough to keep their total relative occupancy constant.

relation $[\varphi_{R/L}(x),\varphi_{R/L}(y)] = \pm i\pi\nu\,\mathrm{sign}(x-y)$. Then $H_0 = [v/(2\pi\nu)]\int_{-\infty}^{\infty} dx[(\partial_x\varphi_R)^2 + (\partial_x\varphi_L)^2]$, where v is the quasiparticle velocity. It is then convenient to define commuting right-moving spin and charge bosons, $\varphi_{s,c}(x) \equiv [\varphi_R(x) \pm \varphi_L(-x)]/\sqrt{2}$, since only the spin boson appears in the Kondo interaction. Then, we obtain $\mathcal{J}_- \equiv \mathcal{J}_x - i\mathcal{J}_y \propto e^{i\sqrt{2}\varphi_s}$ and $\mathcal{J}_z = \partial_x\varphi_s/(2\pi\nu\sqrt{2})$. The renormalized Kondo couplings grow larger as the energy scale is reduced [35], becoming large at the crossover scale T_K. For the $\nu = 1$ case, $T_K = De^{-1/\lambda}$, where $\lambda \equiv \varrho J$, ϱ is the density of states, and we have assumed $J_\perp = J_z$ (similar behavior occurs in the anisotropic case). For $\nu = 1/3$, $e^{i\sqrt{2}\varphi_s}$ has renormalization group (RG) scaling dimension 1/3. Thus $\lambda_\perp(E) = (D/E)^{2/3}\lambda_\perp$, so $T_K \propto D\lambda_\perp^{3/2}$.

III. CONDUCTANCE

We are interested in the interedge tunneling conductance, corresponding to backscattering, defined using the charge current operator $I = i\nu e t_C(d_L^\dagger d_R - \mathrm{H.c.}) = 2\nu e t_C S^y$. In the linear response regime, the Kubo formula gives [30]

$$G = -8\pi\nu t_C^2 \lim_{\omega\to 0}\frac{\chi''_{yy}(\omega)}{\omega}, \qquad (2)$$

in units of $\nu e^2/h$ where χ''_{yy} is the imaginary part of the dynamical impurity spin susceptibility of the Kondo model, $\chi_{yy}(\omega) \equiv -i\int_0^\infty dt e^{i\omega t}\langle[S^y(t),S^y(0)]\rangle$. Every transmitted quasiparticle contributing to the transport involves a spin-flip process at the impurity, relating the conductance to the spin relaxation.

A. High temperatures: $T_K, t_C \ll T$

In this limit, we may neglect t_C in the Hamiltonian and attempt to calculate the susceptibility using perturbation theory, but this gives a result which diverges at $\omega \to 0$:

$$\chi''_{yy}(\omega) = -\frac{\pi\nu}{2\omega}\left[\gamma_z^{(\nu)}\lambda_z^2 + \gamma_\perp^{(\nu)}\lambda_\perp^2(T/D)^{2\nu-2}\right]. \qquad (3)$$

Here, $\gamma_z^{(\nu)}$ and $\gamma_\perp^{(\nu)}$ are dimensionless coefficients of $\mathcal{O}(1)$. This surprising infrared divergence is not connected to the usual renormalization of the Kondo couplings since $T \gg T_K$. Nonetheless, it suggests that an infinite subset of diagrams must be resummed to get a finite conductance [36]. One way to solve this problem is to phenomenologically describe the impurity spin by the Bloch equations [37,38]

$$\partial_t\langle S_a\rangle = [\vec{h}(t)\times\langle\vec{S}\rangle]_a - \frac{\langle S_a\rangle - \tilde{\chi}_0 h_a(t)}{\tau_a}, \quad a = x,y,z. \qquad (4)$$

Here, $\vec{h}(t) = (-t_C, h_y(t), 0)$, where $h_y(t)$ is an infinitesimal time-dependent y component of the magnetic field, introduced to obtain χ_{yy}. We expect Eq. (4) to hold as the equation of motion for the averaged impurity spin in a theory where the quasiparticles are formally integrated out. Here, $\tilde{\chi}_0 \approx -1/4T$ is the static susceptibility $\langle\vec{S}\rangle_0 = \tilde{\chi}_0\vec{h}$, in the presence of the static field $-t_C$. $\langle S_a\rangle(t)$ rotates around the external magnetic field and relaxes towards it within the time scale τ_a because of its coupling to the quasiparticles. Therefore, using the definition $\chi_{yy}(\omega) \equiv \langle S_y\rangle_\omega/h_y(\omega)$ for the imaginary part of the susceptibility, we obtain

$$\chi''_{yy}(\omega\to 0) = \frac{\omega\tilde{\chi}_0\tau_y}{1 + t_C^2\tau_y\tau_z}. \qquad (5)$$

To obtain the conductance, we need $\tau_{y,z}$. The main "Bloch-equation" assumption, justifiable at $T \gg T_K, t_C$, is to neglect the frequency dependence of these rates, thus obtaining them from a large frequency limit of our perturbative result using $\tilde{\chi}_0/\tau_z = \lim_{\omega\to\infty}\omega\chi''_{zz}(\omega) \propto \lambda_\perp^2(T/D)^{2\nu-2}$ and $\tilde{\chi}_0/\tau_y = \lim_{\omega\to\infty}\omega\chi''_{yy}(\omega) = \gamma_z\lambda_z^2 + \gamma_\perp\lambda_\perp^2(T/D)^{2\nu-2}$. So at high temperatures,

$$G_{\nu=1} \propto \frac{t_C^2}{T^2(\lambda_\perp^2 + \lambda_z^2)}, \qquad G_{\nu=1/3} \propto \frac{t_C^2}{\lambda_\perp^2 T^{2/3} D^{4/3}}. \qquad (6)$$

(We show explicitly that this result can be obtained, at high T, from a resummation of Feynman diagrams in the special case $\lambda_y = \lambda_z = 0$, see Appendix F.) More correctly, $\lambda_\perp$, λ_z should be replaced by the renormalized quantities at the energy scale T, but this is an unimportant correction assuming $T \gg T_K$.

B. $T, T_K \ll t_C$

In this regime, the impurity spin becomes a classical field pointing in the direction of the instantaneous field $\vec{h}(t)$ (see Appendix E), so we may approximate:

$$H \approx H_0 + (1/2)\lambda_\perp[\mathcal{J}_x(0) + (h_y(t)/t_C)\mathcal{J}_y(0)]. \qquad (7)$$

This corresponds to a direct tunneling term between edges: $H_T = (1/4)\lambda_\perp[e^{ih_y(t)/t_C}\psi_{qp,L}^\dagger(0)\psi_{qp,R}(0) + \mathrm{H.c.}]$. For $\nu = 1$, this is a simple noninteracting tunneling model giving a conductance $G \propto \lambda_\perp^2$. [More accurately, $\lambda_\perp$ should be replaced by the renormalized coupling $\lambda_\perp(t_C)$ but this is again unimportant for $T_K \ll t_C$.] For the fractional quantum Hall case, the behavior is much different [39] since this direct tunneling interaction is relevant and $\lambda_\perp(T) = (t_C/T)^{2/3}\lambda_\perp(t_C)$. Therefore the conductance starts to grow as $G \propto T^{-4/3}$. It starts to level off at T of order T_K eventually saturating at ν, corresponding to perfect transmission through the double antidots. The nature of this zero temperature infrared fixed point can be straightforwardly understood from bosonization. The relevant tunneling term, $\propto -\lambda_\perp \cos[\sqrt{2}\varphi_s(0)]$, pins $\varphi_s(0)$ at 0. To understand the physical implications of this boundary condition, note that while the charge boson remains continuous at $x = 0$ at both high-T and low-T fixed points, implying $\varphi_R(0^+) - \varphi_L(0^-) = \varphi_R(0^-) - \varphi_L(0^+)$, the high-$T$ and low-T boundary conditions on the spin boson imply $\varphi_R(0^+) + \varphi_L(0^-) = \pm[\varphi_R(0^-) + \varphi_L(0^+)]$, respectively. Together, these boundary conditions merely imply continuity of $\varphi_{R/L}$ at the origin at high T but imply $\varphi_R(0^\pm) = -\varphi_L(0^\pm)$ at low T, corresponding to a breaking of the system into $x < 0$ and $x > 0$ parts, perfect transmission through the double dots and perfect backscattering. The leading low-T reduction of the conductance is conveniently calculated by considering the small horizontal current between the nearly disconnected $x < 0$ and $x > 0$ parts of the system [inset of Fig. 2(a)]. This involves *electrons* tunneling through vacuum (as opposed to quasiparticle tunneling through incompressible liquid) between the $x < 0$ and $x > 0$ sides and corresponds to a term in the effective Hamiltonian, $\propto \cos[(\varphi_R(0^+) - \varphi_R(0^-))/\nu]$, of RG scaling dimension $1/\nu$. Thus the horizontal conductance, *past* the double dots is $G_h \propto T^{2/\nu-2} = T^4$. By current conservation, we expect the vertical conductance *through* the antidots to behave as $G \to \nu - \alpha(T/T_K)^4$, for a dimensionless constant of $\mathcal{O}(1)$, α. The behavior of the conductance when $T_K \ll t_C$ for various temperature ranges and $\nu = 1$ and $\nu = 1/3$, is plotted in Fig. 2(a). The zero-temperature conductance, as well as the exponents for $T \ll t_C$ agree with previous numerical results [39].

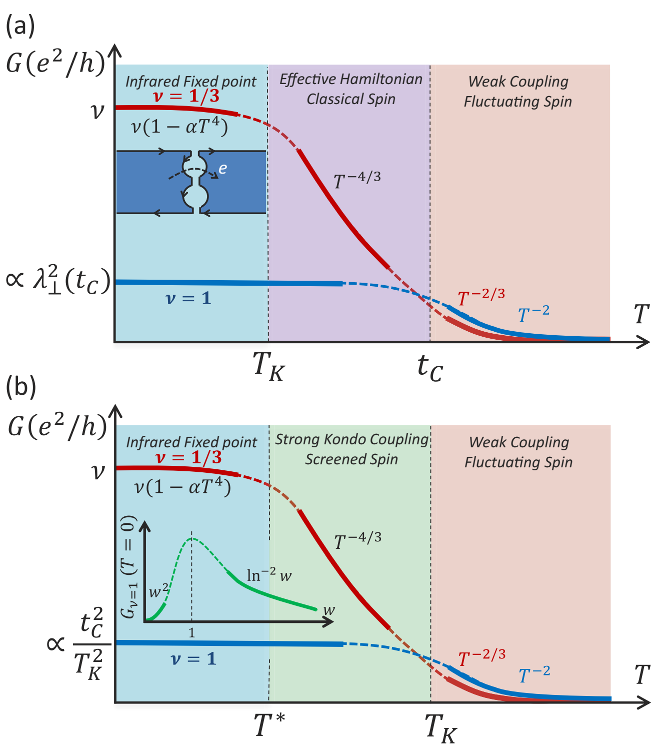

FIG. 2. (Color online) Interedge conductance vs temperature for $\nu = 1$ and $1/3$ quantum Hall states at (a) $t_C \gg T_K$ and (b) $t_C \ll T_K$. The dashed lines are interpolations, which we expect to be qualitatively correct for the crossover regimes. Whereas the $\nu = 1$ case exhibits a crossover only at larger crossover scale $\max(t_C, T_K)$, the $\nu = 1/3$ case has an additional crossover at T_K and T^* for the case (a) and (b), respectively. Insets: (a) schematic of the stable infrared fixed point in the case of $\nu = 1/3$. The leading irrelevant processes correspond to electron tunneling through vacuum. (b) The Kondo fixed point conductance in the case of $\nu = 1$ has a nonmonotonous dependence on $w \equiv t_C/T_K$, with a peak of $\mathcal{O}(1)$ at $w \sim 1$.

C. Strong Kondo coupling fixed point, $t_C \ll T \ll T_K$

In this parameter regime, the Kondo coupling constants $\lambda_\perp$ and λ_z renormalize to large values but the interdot tunneling t_C may be treated as a small perturbation. The impurity spin is then screened by the quasiparticles and, for $\nu = 1$, we may apply Fermi liquid theory. The impurity spin S^x appearing in the interdot tunneling Hamiltonian can then be represented by $(v/T_K)\psi_{qp}^\dagger(0)\sigma^x\psi_{qp}(0)$, the lowest dimension operator with the correct SU(2) spin transformation properties [40,41]. The factor of v/T_K can be inserted by dimensional analysis, recognizing that T_K is the characteristic energy scale, or reduced bandwidth at this fixed point. The corresponding Hamiltonian is noninteracting, with this tunneling term being marginal under the renormalization group. This leads to the familiar Shiba formula [42] giving $G \propto (t_C/T_K)^2$. Similar reasoning may be applied to the $\nu = 1/3$ case but now the effective interaction $\propto \psi_{qp}^\dagger(0)\sigma^x\psi_{qp}(0)$ is relevant, with dimension $1/3$. Thus calculating the conductance to lowest order in t_C gives $G \propto (t_C/T_K)^2(T_K/T)^{2(1-\nu)} \propto T^{-4/3}$. This diverges at low-$T$ signaling the breakdown of perturbation theory in t_C.

D. $T \ll t_C \ll T_K$ regime

For $\nu = 1$, there is no significant change in behavior as T is lowered to zero below t_C, with the conductance being approximately constant. On the other hand, for $\nu = 1/3$, the growth of the interdot tunneling term under renormalization signals the crossover to the same fixed point discussed above for $T, T_K \ll t_C$, corresponding to perfect transmission through the antidots. Renormalized interdot tunneling becomes strong at the scale $T^* \propto t_C^{3/2}/T_K^{1/2}$ and below this scale the conductance should again crossover to $\nu - \alpha(T/T^*)^4$ behavior. The behavior of the conductance when $t_C \ll T_K$ for various temperature ranges is plotted in Fig. 2(b).

Note that the conductance versus temperature looks rather similar in the two cases, $T_K \ll t_C$ and $t_C \ll T_K$. One essential difference is the crossover temperature scales. For $\nu = 1$, there is only one crossover that occurs at the larger of t_C and T_K. For $\nu = 1/3$, there are two crossover scales: t_C and T_K for $t_C \gg T_K$, but T_K and $t_C^{3/2}/T_K^{1/2}$ for $t_C \ll T_K$. It is also interesting to note that, for $\nu = 1$ and $\lambda_\perp = \lambda_z$, the $T = 0$ conductance is $\propto \lambda_\perp(t_C)^2 = 1/\ln^2(t_C/T_K)$ for $T_K \ll t_C$ but $\propto (t_C/T_K)^2$ for $t_C \ll T_K$. $G(0)$ decreases as t_C/T_K becomes large or small, going through a peak of $\mathcal{O}(1)$ at T_K of order t_C [see inset of Fig. 2(b)].

IV. CONCLUSION

We have mapped the conductance through two antidots in $\nu = 1$ integer and $\nu = 1/3$ fractional quantum Hall systems onto the susceptibility of a Kondo impurity in a Luttinger liquid, analyzed the fixed points and calculated the conductance in all asymptotic regimes. Calculations of noise, and extension to more exotic filling factors $\nu = 5/2$ and $12/5$ with non-Abelian statistics are left as future extensions of these results.

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APPENDICES

In the Appendices, we provide details and proofs of some results presented in the paper. Appendix A discusses the weak coupling and strong Kondo coupling fixed points and the corresponding gluing conditions. Appendix B contains some discussion about the antidots and experimental considerations. Appendix C provides a calculation of the susceptibility using the semiclassical Bloch equation. Appendix D provides the result of perturbation theory to second order in Kondo coupling but exact in t_C. In Appendix E, we provide a detailed discussion of the effective Hamiltonian derivation in the case of $T, T_K \ll t_C$. Finally, Appendix F contains an exact solution of the case $\lambda_y = \lambda_z = 0$, using techniques developed to study the x-ray edge singularity. We show that it is possible to derive the Bloch equation as the high-temperature ($T \gg t_C$) formula for the susceptibility. The exact result in this special case also demonstrates the breakdown of perturbation theory at high temperatures.

APPENDIX A: GLUING CONDITIONS AT FIXED POINTS

1. Folding transformation

For a discussion of the fixed points, it is convenient to fold the chiral bosons, according to

$$\phi_{s,c} \equiv \frac{\varphi_{s,c}(x) + \varphi_{s,c}(-x)}{\sqrt{2}} \quad \theta_{s,c}(x) \equiv \frac{\varphi_{s,c}(x) - \varphi_{s,c}(-x)}{\sqrt{2}},$$

for $x > 0$, in terms of which the Hamiltonian becomes

$$H = H_0 + \frac{J_\perp}{2}(S^+ e^{i\phi_s(0)} + \text{H.c.}) + \frac{J_z}{4\pi\nu} S^z \partial_x \theta_s(0) - t_C S^x + h_z S^z, \tag{A1}$$

where we also allowed for a detuning $h_z \equiv \epsilon_R - \epsilon_L$ between the two antidots. Here, $H_0 = \int_0^\infty \frac{v dx}{4\pi\nu}[(\partial_x\phi_s)^2 + (\partial_x\theta_s)^2 + (\partial_x\phi_c)^2 + (\partial_x\theta_c)^2]$ and the nonchiral bosons obey $[\phi_s(x), \theta_s(y)] = 2\pi i \nu \Theta(x-y)$, where $\Theta(x)$ is the Heaviside step function.

2. Weak coupling

The only boundary condition at the weak coupling fixed point is that $\varphi_{s,c}(x)$ are continuous at $x = 0$, i.e., $\theta_{s,c}(0) = 0$. Using the definition $[\int dx \rho_{R/L}(x), \psi_{qp,R/L}(y)] = -\psi_{qp,R/L}(y)$, we obtain the density operator for both bosons to be $\rho_{R/L}(x) = \frac{1}{2\pi\nu}\partial_x\varphi_{R/L}$. From this, assuming a finite length L with periodic boundary conditions, the mode expansions are

$$\varphi_{R/L}(x) = \frac{2\pi\nu}{L} N_{R/L} x + \varphi_{R/L,0} + \sum_{n=1}^{\infty} \sqrt{\frac{2\pi\nu}{L k_n}} (\hat{a}_{R/L,n} e^{\pm i k_n x} + \hat{a}^\dagger_{R/L,n} e^{\mp i k_n x}) e^{-\frac{k_n a}{2}}. \tag{A2}$$

Here, $k_n = 2\pi n/L$, and the harmonic bosons and the zero mode obey the standard commutation relation $[a_{R/L,n}, a^\dagger_{R/L,m}] = \delta_{nm}$ and $[\varphi_{R/L,0}, N_{R/L}] = i$. This leads to the mode expansion of the charge/spin bosons

$$\varphi_{c/s}(x) = \frac{2\pi\nu}{\sqrt{2}L}(N_R \pm N_L)x + \frac{1}{\sqrt{2}}(\varphi_{R0} \mp \varphi_{L0}) + \cdots. \tag{A3}$$

Defining the total bulk charge $Q = N_R + N_L$ and spin $2s_z = N_R - N_L$, for Q even, s_z has to be integer, while for Q odd it must be half-integer. So, the gluing condition for (Q, s_z) at a weak-coupling fixed point is [41]

$$(Q, s_z) = (\text{even,integer}) \oplus (\text{odd,half-integer}). \tag{A4}$$

3. Kondo fixed point

By power counting, the J_z term is marginal but the $J_\perp$ terms is relevant. In order to account for this, we define dimensionless couplings $\lambda_z = 2\pi J_z/v$ and $\lambda_\perp = J_\perp D^{\nu-1}$ and will frequently switch between $J_{z,\perp}$ and $\lambda_{z,\perp}$ notations in the following. These couplings grow as the bandwidth is reduced [35] and the system flows to the Kondo fixed point. Although J_z is naively marginal, because of the coupling to S^z, it controls the scaling dimension of the $J_\perp$ term. This can be seen if we apply a unitary transformation [16,43] $H \to V^\dagger_\mu H V_\mu$ with $V_\mu = \exp[i\mu S^z \phi_s(0)]$, which gives

$$H \to H_0 + \frac{J_\perp}{2}(S^+ e^{i(1-\mu)\phi_s(0)} + \text{H.c.}) + \left(\frac{J_z}{4\pi\nu} - \mu v\right) S^z \partial_x \theta_s(0) + h_z S^z - t_C[S^x \cos\mu\phi_s(0) + S^y \sin\mu\phi_s(0)] \tag{A5}$$

and changes the dimension of $J_\perp$ from $1-\nu$ to $1-\nu(1-\mu)^2$. In order to understand the strong coupling Kondo fixed point, it is convenient to either tune this dimension to zero (the so-called Toulouse [44] point) so that it could be refermionized [16] or to 1 (the so-called decoupling point) so that it becomes a boundary magnetic field. We use the latter approach, which happens at $\mu = 1$ and the transverse Kondo coupling becomes $J_\perp S^x$. In the case of $t_C = h_z = 0$, and if $\lambda_z = 4\pi\nu$, the Kondo coupling reduces to a Zeeman field on the isolated (but dressed) impurity spin, which projects to the ground state of S^x at low energies. The Kondo temperature is $\sim J_\perp$ in this highly anisotropic Kondo model [45]. It is easy to check that $\varphi_s(x)$ develops a discontinuity at $x = 0$,

$$\tilde{\varphi}_s(x) = V^\dagger_\mu \varphi_s(x) V_\mu = \varphi_s(x) - \pi\nu\mu S^z \text{sign}(x), \tag{A6}$$

or equivalently, in the folded basis, the new boundary condition corresponds to $\tilde{\phi}_s(0) = \phi_s(0)$ and a new pinning of $\tilde{\theta}_s(0) = \mp\pi\nu$. The pinning is dynamically switching between these two values as in the instanton-gas representation of the Kondo problem [46]. The charge boson is unchanged by the unitary transformation followed by the projection and the new gluing condition at the strong Kondo coupling fixed point is [41]

$$(Q,s_z) = (\text{even,half-integer}) \oplus (\text{odd,integer}). \quad (A7)$$

This change in the gluing condition implies that a spin boson has decoupled from the edge states to screen the impurity spin. We have used the decoupling point to discuss the Kondo fixed point, and this requires tuning J_z to the large value of $4\pi v\nu$, which is not physical, as the bare Kondo coupling is usually assumed to be small. However, it is expected that other values of J_z would have a similar qualitative behavior.

APPENDIX B: ANTIDOTS

We can find the spectrum of the antidot by inserting the mode expansion (A2) into the free Hamiltonian

$$H = \frac{v}{2\pi\nu}\int_0^L dx(\partial_x\varphi)^2 = \sum_{k>0} vka_k^\dagger a_k + E_C n^2, \quad (B1)$$

where $E_C = 2\pi v\nu/L$ acts like the "charging energy" of the antidot and L is its circumference. We see that the number of quasiparticles n is a good quantum number $\hat{n}|n\rangle_0 = n|n\rangle_0$. These two sectors are coupled to each other because $L = 2\pi R$ and $R = \sqrt{2N}\ell_B$ in terms of magnetic length $\ell_B = \sqrt{\hbar/eB}$, but we can assume that the antidots are large enough so that the radius of $|n\rangle_0$ and $|n+1\rangle_0$ are effectively the same [29] and assume that charge and neutral sectors decouple. This corresponds to the constant interaction model in quantum dots [47]. For the antidots, we are interested in a regime where the bosonic excitation energy $\delta\epsilon = 2\pi v/L$ is much larger than k_BT and we can assume that the harmonic part of the field is in its ground state $a_k|0\rangle_n = 0$. Note that $|n\rangle_0$ and $|\varphi\rangle_n$ are analogous to the number of charges and the excited states of normal quantum dots.

We also need to take into account the Aharonov-Bohm contribution of the magnetic flux going through the antidots. The number of quasiholes n_R in antidot R is such that it is equal to the number of flux quanta going through $\phi_R/\phi_0 = n_R$ where $\phi_0 = h/e$. This is another way of stating that $R_R = \sqrt{2n_R}\ell_B$, at the ground state. These numbers change as we change ϕ_R. This is done by replacing $E_C n_R^2$ in Eq. (B1) by $E_C(n_R - \phi_R/\phi_0)^2$ with $\varphi_0 \equiv h/e$. We are interested in a regime where two states in the same antidot with n_R and $n_R + 1$ quasiparticles become degenerate. This is possible for $\phi_R/\phi_0 = 2m_R + 1$, where m_R is an integer. Also we need a similar degeneracy to be valid for the second antidot $\phi_L/\phi_0 = 2m_L + 1$. To have both of these at the same magnetic field, we obviously need some fine tuning of the area of at least one of the antidots. We assume that this is possible by tuning the voltage applied to the gates that defined the antidots at the first place, or by a combination of voltages applied to the outer edges. To capture the deviation from this ideal case, one can add a term $h_z S^z$ to the Hamiltonian, but we assumed a perfect tuning in the paper.

If the temperature is low enough ($T \ll v_F/L \ll E_F$) so that the bosonic modes of the antidots are not excited, they effectively behave as hardcore fermions [29]. To see this, following Ref. [48], we assume that N and $N+1$ states of the antidot are degenerate and denoting them by $|0\rangle$ and $|1\rangle$, it can be seen that due to the commutation relation $[\varphi_0, N] = i$, the operators $s^\pm \propto e^{\pm i\varphi_0}$ are raising and lowering operators of the "spin" made of $|0\rangle$ and $|1\rangle$. From $[\varphi_0, N] = i$, it follows that

$$[N, e^{\pm i\varphi_0}] = \pm e^{\pm i\varphi_0} \quad \text{and} \quad N e^{\pm i\varphi_0} = e^{\pm i\varphi_0}(N \pm 1).$$

These can be combined with the bosonization Klein factors $\Gamma_{L,R}$ to represent the creation and annihilation operators for the additional fermion on the dot.

$$d_L^\dagger \equiv \Gamma_L e^{i\varphi_{L0}} \quad \text{and} \quad d_R^\dagger \equiv \Gamma_R e^{i\varphi_{R0}}. \quad (B2)$$

APPENDIX C: BLOCH EQUATION: NONZERO t_C

Considering that $\vec{h} = (-t_C, h_y, 0)$ and $\lambda_y \neq \lambda_z$, there is no spin symmetry present and we have to allow for different relaxation rates along each direction. Therefore we can write the Bloch equations [Eq. (4) of the paper] as

$$\partial_t\langle S_x\rangle = h_y\langle S_z\rangle - \frac{\langle S_x\rangle - \langle S_x^0\rangle}{\tau_x}, \quad (C1)$$

$$\partial_t\langle S_y\rangle = t_C\langle S_z\rangle - \frac{\langle S_y\rangle - \langle S_y^0\rangle}{\tau_y}, \quad (C2)$$

$$\partial_t\langle S_z\rangle = -t_C\langle S_y\rangle - h_y\langle S_x\rangle - \frac{\langle S_z\rangle - \langle S_z^0\rangle}{\tau_z}, \quad (C3)$$

where $\langle S_a^0\rangle$ are the components of the steady-state magnetization. To find the steady-state magnetizations, we do a rotation ($\tan\vartheta = -h_y/t_C$)

$$O(\vartheta) = \begin{pmatrix} \cos\vartheta & -\sin\vartheta \\ \sin\vartheta & \cos\vartheta \end{pmatrix} \quad (C4)$$

on

$$(\tilde{h}_x \;\; 0) = (-t_C \;\; h_y)O(\vartheta), \qquad \begin{pmatrix} S_x \\ S_y \end{pmatrix} = O(\vartheta)\begin{pmatrix} \tilde{S}_x \\ \tilde{S}_y \end{pmatrix}$$

to obtain $\tilde{h}_x = \cos\vartheta(-t_C + h_y^2/t_C) = -t_C + \mathcal{O}(h_y^2)$. The Hamiltonian is diagonal in this "tilde" basis and we find

$$\langle S_x^0\rangle = \cos\vartheta\langle\tilde{S}_x^0\rangle = -\frac{1}{2}\tanh\frac{\tilde{h}_x\beta}{2}\cos\vartheta, \quad (C5)$$

$$\langle S_y^0\rangle = -\frac{1}{2}\tanh\frac{\tilde{h}_x\beta}{2}\sin\vartheta, \qquad \langle S_z^0\rangle = 0. \quad (C6)$$

Since eventually we are interested in $\chi_{yy} = d\langle S\rangle_y/dh_y|_{h_y=0}$, we can drop $\mathcal{O}(h_y^2)$ and the above results simplify to

$$\langle S_x^0\rangle \approx \frac{1}{2}\tanh\frac{t_C\beta}{2}, \quad (C7)$$

$$\langle S_y^0\rangle \approx -\frac{1}{2}\frac{h_y}{t_C}\tanh\frac{t_C\beta}{2}, \qquad \langle S_z^0\rangle = 0. \quad (C8)$$

For $T \gg t_C$ where we expect the Bloch equation approach to be valid, $\tanh(t_C\beta/2) \approx t_C\beta/2$ and we get the linear response result $\langle S_y^0\rangle \approx \chi_0 h_y$, but for $T \ll t_C$, we have $\tanh(t_C\beta/2) \approx 1$. More generally, we can define an effective static susceptibility given by $\langle\vec{S}^0\rangle \approx \tilde{\chi}_0\vec{h}$, where

$$\tilde{\chi}_0(T) \equiv \frac{-1}{2t_C}\tanh\frac{t_C\beta}{2} = \chi_0\left(T \to \frac{t_C}{2\tanh\frac{t_C\beta}{2}}\right) \quad (C9)$$

Fourier transforming, we obtain

$$(1 - i\omega\tau_x)\langle S_x\rangle_\omega = -2\pi\delta(\omega)\chi_0^{\mathrm{eff}} t_C + \tau_x h_y(\omega) * \langle S_z\rangle_\omega,$$
$$(1 - i\omega\tau_y)\langle S_y\rangle_\omega = \chi_0^{\mathrm{eff}} h_y(\omega) + t_C\tau_y\langle S_z\rangle_\omega,$$
$$(1 - i\omega\tau_z)\langle S_z\rangle_\omega = -t_C T_2^z\langle S_y\rangle_\omega - \tau_z h_y(\omega) * \langle S_x\rangle_\omega.$$

These are easily generalized to the more general memory-full case, by allowing a frequency-dependence for $\tau_a(\omega)$. These set of equations are difficult to solve. One approximation that greatly simplifies this, is to ignore the fluctuations of the spin along the external field, x direction. This amounts to dropping the second (convolution) term on the right-hand side of the first equation and makes sense because we expect the second term to be $\mathcal{O}(h_y^2)$. Then everything simplifies: we get $\langle S_x\rangle_\omega \approx -2\pi\delta(\omega)\tilde{\chi}_0 t_C$, i.e., constant in time. Thus the convolution in the last line also simplifies and we obtain

$$(1 - i\omega\tau_z)\langle S_z\rangle_\omega \approx -t_C\tau_z[\langle S_y\rangle_\omega - \tilde{\chi}_0 h_y(\omega)], \tag{C10}$$

from which we get

$$\chi_{yy}(\omega) \equiv \lim_{h_y\to 0}\frac{\langle S_y\rangle_\omega}{h_y(\omega)} = \frac{\tilde{\chi}_0\left(1 - i\omega\tau_z + t_C^2\tau_z\tau_y\right)}{(1 - i\omega\tau_z)(1 - i\omega\tau_y) + t_C^2\tau_z\tau_y} \tag{C11}$$

with the imaginary part

$$\chi''_{yy}(\omega) = \frac{\omega\tilde{\chi}_0\tau_y\left[1 + t_C^2\tau_z\tau_y + \omega^2\tau_z^2\right]}{\left[1 + \left(t_C^2 - \omega^2\right)\tau_z\tau_y\right]^2 + \omega^2(\tau_y + \tau_z)^2}. \tag{C12}$$

Let us look at this formula, in various limits. Without Kondo coupling $\tau_{x,y,z} \to \infty$, and we get

$$\chi_{yy}(\omega) \to -\frac{t_C^2\tilde{\chi}_0}{\omega^2 - t_C^2} = \frac{t_C}{2}\frac{\tanh(\beta t_C/2)}{\omega^2 - t_C^2}. \tag{C13}$$

For $t_C\sqrt{\tau_z\tau_y} \ll 1$, we basically get the simple result $\chi_{yy}(\omega) = \tilde{\chi}_0/(1 - i\omega\tau_y)$ that we would get if we had neglected t_C from the beginning. Generally, we see that

$$\chi''_{yy}(\omega \to 0) = \frac{\omega\tilde{\chi}_0\tau_y}{1 + t_C^2\tau_y\tau_z}, \qquad G = \frac{-8\pi t_C^2\tilde{\chi}_0\tau_y}{1 + t_C^2\tau_y\tau_z}. \tag{C14}$$

However, for $\omega \to \infty$,

$$\chi''_{yy}(\omega \to \infty) = \frac{\tilde{\chi}_0}{\omega\tau_y}, \quad \to \quad \frac{\tilde{\chi}_0}{\tau_y} = \lim_{\omega\to\infty}\omega\chi''_{yy}(\omega). \tag{C15}$$

To obtain the conductance, we also need τ_z, which can be obtained using

$$\frac{\tilde{\chi}_0}{\tau_z} = \lim_{\omega\to\infty}\omega\chi''_{zz}(\omega) \tag{C16}$$

or from τ_y with a rotation along S_x, i.e., by interchanging $\lambda_y \leftrightarrow \lambda_z$.

APPENDIX D: SUSCEPTIBILITY TO ORDER $\mathcal{O}(\lambda^2)$ BUT EXACT IN t_C

In this section, we provide the result of perturbative calculations of the imaginary part of the susceptibility to second order in Kondo coupling but exact in interantidot tunneling t_C. The goal of this section is to demonstrate that once a finite t_C is included, the infrared divergence of the perturbation theory is cut off. Using equation of motion techniques, it can be shown that the correlation functions to second order in Kondo coupling are

$$\chi''^{(zz)}_{yy} = \frac{\lambda_z^2}{16}\frac{\omega^2}{\left(\omega^2 - t_C^2\right)^2}\mathrm{Im}\left[\Pi^R_{zz}(\omega)\right], \tag{D1}$$
$$\chi''^{(yy)}_{yy} = \frac{\lambda_y^2}{16}\frac{t_C^2}{\left(\omega^2 - t_C^2\right)^2}\mathrm{Im}\left[\Pi^R_{yy}(\omega)\right]. \tag{D2}$$

Here, $\Pi^R_{aa}(\omega) \sim \langle\mathcal{J}_a\mathcal{J}_a\rangle_\omega$ are retarded correlation functions of the current operators,

$$\Pi^R_{yy}(\omega,\nu = 1) = \Pi^R_{zz}(\omega,\nu = 1), \qquad \Pi^R_{zz}(\omega,\nu) = \frac{-i\omega}{8\pi\nu v^2},$$
$$\Pi^R_{yy}(\omega,\nu < 1/2) = -\left(\frac{2\pi}{\beta}\right)^{2\nu-1}\sin(\pi\nu)B\left(\nu - \frac{i\omega\beta}{2\pi},1-2\nu\right),$$

where $B(x,y)$ is the beta function. The λ_x^2 contribution has a more complicated form:

$$\chi''^{(xx)}_{yy}(\omega < t_C) = -g\left\{\frac{\pi}{2}\frac{\omega}{\omega^2 - t_C^2} + \frac{1}{2}\tanh(\beta t_C/2)\left[\frac{2\pi}{\beta}\frac{t_C}{\omega^2 - t_C^2} + H_\nu(\omega)\right]\right\}, \tag{D3}$$

where $g = \lambda_x^2/4$ and the function $H_\nu(\omega)$ for $\nu = 1$ is

$$H_{\nu=1}(\omega) \equiv \mathrm{Im}\left[\frac{\psi\left(1 - \frac{i\beta(\omega+t_C)}{2\pi}\right)}{\omega + t_C} - \frac{\psi\left(1 + \frac{i\beta(t_C-\omega)}{2\pi}\right)}{\omega - t_C}\right]$$

in terms of digamma function $\psi(z)$. Note that in the limit of $t_C \to 0$, these results reduce to Eq. (3) of the paper.

APPENDIX E: EFFECTIVE HAMILTONIAN FOR $T,T_K \ll t_C$

Temporarily ignoring the infinitesimal time-dependent part of the field, $h_y(t)$, introduced to calculate the dynamical susceptibility, χ''_{yy}, it is clear that in this regime we may replace S^x by 1/2 (and $S^{y,z}$ by zero) since the impurity spin is polarized by the strong field. Now consider the effect of $h_y(t)$. We again wish to integrate out the impurity spin to obtain an effective Hamiltonian for the quasiparticles. It is now not appropriate to consider any relaxation term in the Bloch equations, since such terms arise from the integrating out the quasiparticles instead. So, lets consider the solutions of the simple spin torque equation

$$\partial_t\vec{S} = \vec{h}(t) \times \vec{S}(t) \tag{E1}$$

with $\vec{h}(t) \equiv (-t_C,h_y(t),0)$, $h_y(t) = \epsilon t_C\cos\omega t$, taking the limit where both $\epsilon \to 0$ and $\omega \to 0$. We also assume that the oscillating component of the field is turned on slowly in the infinite past. Thus we write

$$\vec{S}(t) = (1/2)(1,0,0) - \vec{S}'. \tag{E2}$$

We will see that $\vec{S}'$ is $O(\epsilon)$. Working to first order in ϵ,

$$\vec{h} \times \vec{S}/t_C \approx -(1/2)(0,0,\epsilon\cos\omega t) + (0,-S^{z\prime},S^{y\prime}). \tag{E3}$$

Thus

$$\frac{1}{t_C}\frac{dS^{z\prime}}{dt} = S^{y\prime} - \frac{\epsilon}{2}\cos\omega t, \tag{E4}$$

$$\frac{1}{t_C}\frac{dS^{y\prime}}{dt} = -S^{z\prime}. \tag{E5}$$

Thus

$$\frac{1}{t_C}\frac{d^2S^{z\prime}}{dt^2} = -t_C S^{z\prime} + \frac{1}{2}\epsilon\omega\sin\omega t \tag{E6}$$

with solution

$$S^{z\prime} \approx \frac{\epsilon t_C\omega}{2(t_C^2-\omega^2)}\sin\omega t \approx \frac{\epsilon\omega}{2t_C}\sin\omega t. \tag{E7}$$

Thus

$$\frac{dS^{y\prime}}{dt} \approx -\frac{\epsilon\omega}{2}\sin\omega t \tag{E8}$$

with solution

$$S^{y\prime} = \frac{\epsilon}{2}\cos\omega t. \tag{E9}$$

In summary,

$$\vec{S} \approx (1/2)(1, -\epsilon\cos\omega t, -\epsilon(\omega/t_C)\sin\omega t). \tag{E10}$$

At small ω we may drop the last component, giving

$$\vec{S} \approx -(1/2t_C)\vec{h}(t). \tag{E11}$$

Thus we see that, even with no relaxation term in the Bloch equations, purely a precession term, the spin tracks the instantaneous time-dependent field, in the limit where the time-dependent term in the field is small and slowly varying.

APPENDIX F: EXACT SPIN SUSCEPTIBILITY WHEN $\lambda_y = \lambda_z = 0$

1. Derivation of Bloch equation result for $t_C \approx 0$

The fact that Eqs. (3) and (6) of the paper contain a simple summation of $\lambda_\perp^2$ and λ_z^2 contributions to lowest order, suggests that the same IR divergence and the necessity to use Bloch equation occurs even if only one Kondo coupling, say $\lambda_z \neq 0$, is nonzero. In this limit and assuming we could neglect $t_C \approx 0$ at $T \gg t_C$, it is possible to map our Kondo problem to the x-ray absorption problem and find a nonperturbative formula for the susceptibility.

With only λ_z nonzero, the Hamiltonian is $H = H_0 + \lambda_z S^z \mathcal{J}_z(0)$ and the Kondo interaction is just a boundary magnetic field, depending on the spin state of the impurity. We are interested in the dynamic susceptibility $\langle S^y S^y\rangle_\omega$, defined after Eq. (2) of the paper. However, S^y itself, is not present in the Hamiltonian and its influence is just suddenly switching the sign of the boundary magnetic field, via $S^y|\uparrow\rangle = i|\downarrow\rangle$ and $S^y|\downarrow\rangle = -i|\uparrow\rangle$. Before this switching, the spin-up fermions see a phase shift and spin-down fermions another and these two phase shifts are suddenly switched. The ground states before and after switching are orthogonal to each other in the thermodynamic limit [49] and the transition creates lots of electron-hole pairs, the so-called orthogonality catastrophe. It is more convenient to discuss this in terms of the spin-up/down bosons. Setting the velocity to 1, dropping constants and following Ref. [50] by introducing $\varphi_{\uparrow/\downarrow} \equiv \varphi_{R/L}$, the Hamiltonian density at $\nu = 1$ is

$$\mathcal{H} = \mathcal{H}_0 + \delta(x)\frac{J_z}{4\pi}S^z(\partial_x\varphi_\uparrow - \partial_x\varphi_\downarrow), \tag{F1}$$

where $\mathcal{H}_0 = (\partial_x\varphi_\uparrow)^2 + (\partial_x\varphi_\downarrow)^2$. Depending on the state of the impurity spin $|\uparrow\rangle$ or $|\downarrow\rangle$, the Hamiltonian breaks into two sectors: $\mathcal{H} = \mathcal{H}_+|\uparrow\rangle\langle\uparrow| + \mathcal{H}_-|\downarrow\rangle\langle\downarrow|$, where using $\lambda_z = \varrho J_z = J_z/2\pi v$, we get

$$\mathcal{H}_\pm = \left[\partial_x\varphi_\uparrow \pm \frac{\pi\lambda_z}{2}\delta(x)\right]^2 + \left[\partial_x\varphi_\downarrow \mp \frac{\pi\lambda_z}{2}\delta(x)\right]^2, \tag{F2}$$

up to a constant. Defining $\lambda' \equiv \lambda_z/4$, these two Hamiltonians are related to $\mathcal{H}_0$ by the Schotte-Schotte unitary transformation [50]

$$U = e^{-i\lambda'\varphi_\uparrow(0)}e^{+i\lambda'\varphi_\downarrow(0)}, \tag{F3}$$

so that

$$\mathcal{H}_+ = U^\dagger\mathcal{H}_0 U, \qquad \mathcal{H}_- = U\mathcal{H}_0 U^\dagger. \tag{F4}$$

Writing S^y as $2iS^y = S^+ - S^-$ and applying the unitary evolution operator,

$$2iS^y(t) = e^{itH_+}S^+e^{-itH_-} - e^{itH_-}S^-e^{-itH_+}. \tag{F5}$$

Here, we used that before/after applying S^+, the system has to be in the $-/+$ sectors, respectively. Inserting this into the dynamic susceptibility and dropping $S^\pm S^\pm$ terms, we obtain

$$\chi_{yy}^R(t) \propto \Theta(t)[\langle e^{itH_+}e^{-itH_-}S^+S^-\rangle + \langle e^{itH_-}e^{-itH_+}S^-S^+\rangle - \langle e^{itH_+}e^{-itH_-}S^-S^+\rangle - \langle e^{itH_-}e^{-itH_+}S^+S^-\rangle].$$

Using Eq. (F4), we can write $e^{\pm itH_+} = U^\dagger e^{\pm itH_0}U$ and $e^{\pm itH_-} = Ue^{\pm itH_0}U^\dagger$. We have to apply the same procedure as in Eq. (F5) to the Boltzman factors. If the correlation function contains S^+S^-, the Boltzman factor becomes $e^{-\beta H}/Z \to e^{-\beta H_+}/Z = U^\dagger e^{-\beta H_0}U/2Z_0$ and a similar version for the S^-S^+ terms. After these substitutions, the spins have done their job and can be simply dropped from the correlation functions and we arrive at

$$\begin{aligned}\chi_{yy}^R(t) &\propto \Theta(t)[\langle e^{iH_0t}U^2e^{-iH_0t}U^{\dagger 2}\rangle + \langle e^{iH_0t}U^{\dagger 2}e^{-iH_0t}U^2\rangle \\ &\quad - \langle U^{\dagger 2}e^{iH_0t}U^2e^{-iH_0t}\rangle - \langle U^2e^{iH_0t}U^{\dagger 2}e^{-iH_0t}\rangle] \\ &\propto \Theta(t)[\langle U^2(t)U^{\dagger 2}\rangle + \langle U^{\dagger 2}(t)U^2\rangle \\ &\quad - \langle U^{\dagger 2}U^2(t)\rangle - \langle U^2U^{\dagger 2}(t)\rangle].\end{aligned} \tag{F6}$$

The first term $\langle U^2(t)U^{\dagger 2}\rangle$ at $T = 0$ reduces to

$$\langle e^{-2i\lambda'\varphi_\uparrow(t)}e^{2i\lambda'\varphi_\uparrow(0)}\rangle\langle e^{2i\lambda'\varphi_\downarrow(t)}e^{-2i\lambda'\varphi_\downarrow(0)}\rangle$$

$$= e^{4i\pi\lambda'^2}\langle e^{-2i\lambda'[\varphi_\uparrow(t)-\varphi_\uparrow(0)]}\rangle\langle e^{2i\lambda'[\varphi_\downarrow(t)-\varphi_\downarrow(0)]}\rangle = \frac{e^{4i\pi\nu\lambda'^2}}{t^{8\nu\lambda'^2}}. \tag{F7}$$

Doing a similar procedure for the other terms and summing up all the terms, at zero temperature, we obtain

$$\chi_{yy}^R(t) = -\frac{\Theta(t)}{2t^{2g}}\sin\pi g, \qquad g \equiv 4\lambda'^2 = \frac{\lambda_z^2}{4}. \tag{F8}$$

This correlation function is a power law in absence of any bulk interaction, because of the physics of orthogonality catastrophe [49,50]. Using conformal mapping to a finite-radii cylinder

$t \to \frac{\beta}{\pi} \sinh \frac{\pi t}{\beta}$, we can bring this retarded function to the finite temperature,

$$\chi^R_{yy}(t) = -\left(\frac{\pi}{\beta}\right)^{2g} \frac{\Theta(t) \sin \pi g}{2\left|\sinh \frac{\pi t}{\beta}\right|^{2g}}. \quad \text{(F9)}$$

The Fourier transform gives [51]

$$\chi^R_{yy}(\omega) = -\left(\frac{2\pi}{\beta}\right)^{2g-1} B(g - i\omega\beta/2\pi, 1-2g) \frac{\sin \pi g}{2}.$$

where $B(x,y)$ is the beta function. We are interested in the limit of small Kondo coupling, $g \to 0$. Therefore, using properties of the beta function,

$$\chi^R_{yy}(\omega) \approx -\left(\frac{2\pi}{\beta}\right)^{-1} B(g - i\omega\beta/2\pi, 1) \frac{\pi g}{2} = \frac{\chi_0}{1 - i\omega\tau_K},$$

where $\chi_0 = -\beta/4$ and $\beta/\tau_K = 2\pi g = 2\pi\lambda_z^2/4$. We have dropped g in the second argument of the beta function, but kept it in the first argument. This is what one would obtain assuming $t_C \approx 0$ in Eq. (C11), which relies on the Bloch equation approach. This form was proved using some analytical assumption [37] or by using the phenomenological Bloch equation [38], but we provided an exact derivation here, when $\lambda_{x,y} = 0$. We expect a similar result for $\lambda_x \neq 0$ but $\lambda_{z,y} = 0$ by a spin rotation along the y direction.

2. Nonzero t_C

When t_C is nonzero, for $\lambda_y \neq 0$ or $\lambda_z \neq 0$, the Hamiltonian contains noncommuting spin terms and the problem is complicated. However, the special case of only $\lambda_x \neq 0$ (but $\lambda_{y,z} = 0$) can be still solved exactly using techniques similar to those described above. Note that having $\lambda_x \neq \lambda_y$ is unphysical as a Schrieffer-Wolff transformation would always produce equal transverse Kondo couplings. Nevertheless, this unphysical case can be used as a check on our Bloch equation result. Following the same technique as in the previous section, it can be easily shown that

$$\chi^R_{yy}(t) = \frac{i\Theta(t)}{4\left[\frac{\beta}{\pi}\sinh(\pi t/\beta)\right]^{2g}} (e^{-it_C t} A - e^{it_C t} A^*), \quad \text{(F10)}$$

where

$$A = -\cos(\pi g)\tanh(\pi b) + i\sin(\pi g). \quad \text{(F11)}$$

and $b = \beta t_C / 2\pi$. The Fourier transform of (F10) gives

$$-4i\chi^R_{yy}(\omega) = \left(\frac{2\pi}{\beta}\right)^{2g-1}\left[AB\left(g - \frac{i\beta(\omega - t_C)}{2\pi}, 1-2g\right) - A^* B\left(g - \frac{i\beta(\omega + t_C)}{2\pi}, 1-2g\right)\right]. \quad \text{(F12)}$$

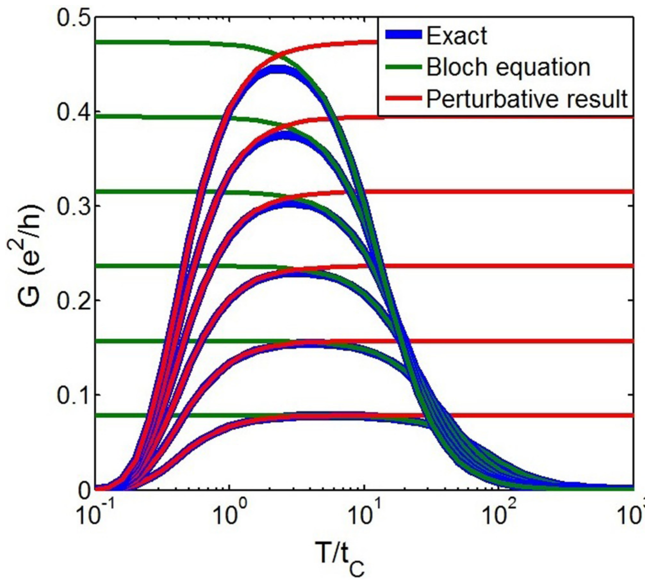

FIG. 3. (Color online) The special unphysical case of $\lambda_y = \lambda_z = 0$, but $\lambda_x \neq 0$ and $t_C \neq 0$. Conductance $G(b,g)$ as a function of T/t_C for various values of $g = \lambda_x^2/4$ on a logarithmic scale. These values are $g = 0.002$, 0.004, 0.006, 0.008, 0.010, and 0.012, from the lowest to highest conductance, respectively. The exact result is compared with perturbation theory to second order in Kondo coupling and exact in t_C (red color) and the Bloch equation result (green).

Using Eq. (2) of the paper, the conductance can be written as a closed formula

$$G = -4\pi b^2 \mathrm{Im}\{A(b,g)B(g+ib, 1-2g) \times [\psi(g+ib) - \psi(1-g+ib)]\}, \quad \text{(F13)}$$

where again $\psi(z)$ is the digamma function. Note that the conductance is a function of $b = \beta t_C/2\pi$ and $g = \lambda_x^2/4$ only. Higher values of Kondo coupling squared g correspond to larger conductance. This function is plotted in Fig. 3, as a function of $(2\pi b)^{-1} = T/t_C$ for various values of g, and it is compared with the perturbation theory result [Eq. (D3)] and the Bloch equation result [Eq. (C14)]. Although the second-order perturbation theory (but exact in t_C) is sufficient at $T \ll t_C$, it fails in the opposite regime of $T \gg t_C$ as pointed out in the paper. On the other hand, the Bloch equation result provides an accurate estimation of the conductance in this high-temperature regime of $T \gg t_C$.

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