

Collective modes, ac response, and magnetic properties of the three-dimensional Dirac semimetal in the triplet superconducting state

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It was recently shown that conventional phonon-electron interactions may induce a triplet pairing state in time-reversal invariant three-dimensional Dirac semimetals. Starting from the microscopic model of the isotropic Dirac semimetal, the Ginzburg-Landau equations for the vector order parameter is derived using the Gor'kov technique. The collective modes including gapless Goldstone modes and gapped Higgs modes of various polarizations are identified. They are somewhat analogous to the modes in the B phase of He³, although in the present case quantitatively there is a pronounced difference between longitudinal and transverse components. The difference is caused by the vector nature of the order parameter leading to two different coherence lengths or penetration depths. The system is predicted to be highly dissipative due to the Goldstone modes. The time-dependent Ginzburg-Landau model in the presence of external fields is used to investigate some optical and magnetic properties of such superconductors. The ac conductivity of a clean sample depends on the orientation of the order parameter. It is demonstrated that the difference between the penetration depths results in rotation of the polarization vector of microwave passing a slab made of this material. The upper critical magnetic field H_{c2} was found. It turns out that at fields close to H_{c2} the order parameter orients itself perpendicular to the field direction. In certain range of parameters the triplet superconducting phase persists at arbitrarily high magnetic field like in some p -wave superconductors.

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I. INTRODUCTION

Recently, three-dimensional (3D) Dirac semimetals (DSM) such as Na₃Bi and Cd₃As₂ with electronic states described by Bloch wave functions, obeying the pseudorelativistic Dirac equation (with the Fermi velocity v_F replacing the velocity of light), were observed [1] and attracted widespread attention. The discovery of the 3D Dirac materials makes it possible to study their physics including remarkable electronic properties. This is rich in new phenomena, such as giant diamagnetism that diverges logarithmically when the chemical potential approaches the 3D Dirac point, a linear-in-frequency ac conductivity that has an imaginary part [2], and quantum magnetoresistance showing linear field dependence in the bulk [3]. Most of the properties of these new materials were measured at relatively high temperatures. However, recent experiments at low temperature on topological insulators and suspected 3D Dirac semimetals exhibit superconductivity. Early attempts to either induce or discover superconductivity in Dirac materials were promising. The well-known topological insulator Bi₂Se₃ doped with Cu, becomes superconducting at $T_c = 3.8$ K [4]. At present its pairing symmetry is unknown. Some experimental evidence [5] point to a conventional phononic pairing mechanism. The spin-independent part of the effective electron-electron interaction due to phonons was studied theoretically [6]. For a conventional parabolic dispersion relation, typically independent of spin, the phonon mechanism leads to the s -wave superconductivity. The layered, noncentrosymmetric heavy element PbTaSe₂ was found to be superconducting [4]. Its electronic properties such as specific heat, electrical resistivity, and magnetic susceptibility indicate that PbTaSe₂ is a moderately coupled, type-II BCS superconductor with large electron-phonon coupling constant of $\lambda = 0.74$. It was shown theoretically to possess a very

asymmetric 3D Dirac point created by strong spin-orbit coupling. If the three dimensionality is confirmed, it might indicate that the superconductivity is conventional phonon mediated.

More recently when the Bi₂Se₃ was subjected to pressure [7], T_c increased to 7 K at 30 GPa. Quasilinear temperature dependence of the upper critical field H_{c2} , exceeding the orbital and Pauli limits for the singlet pairing, points to the triplet superconductivity. The band structure of the superconducting compounds is apparently not very different from its parent compound Bi₂Se₃, so that one can keep the two-band $\mathbf{k} \cdot \mathbf{p}$ description (Se p_z orbitals on the top and bottom layer of the unit cell mixed with its neighboring Bi p_z orbital). Electronic-structure calculations and experiments on the compounds under pressure [7] reveal a single bulk three-dimensional Dirac cone like in Bi with large spin-orbit coupling. Moreover very recently some pnictides were identified as exhibiting Dirac spectrum. This effort recently culminated in discovery of superconductivity in Cd₃As₂ [8]. It is claimed that the superconductivity is p wave at least on the surface.

The case of the Dirac semimetals is very special due to the strong spin dependence of the itinerant electrons' effective Hamiltonian. It was pointed out [9] that in this case the triplet possibility can arise although the triplet gap is smaller than that of the singlet, the difference sometimes is not large for spin independent electron-electron interactions. Very recently the spin-dependent part of the phonon-induced electron-electron interaction was considered [10] and it was shown that the singlet is still favored over the triplet pairing. Another essential spin-dependent effective electron-electron interaction is the Stoner exchange among itinerant electrons leading to ferromagnetism in transition metals. While in the best 3D Weyl semimetal candidates it is too small to form a ferromagnetic state, it might be important to determine the

nature of the superconducting condensate. It turns out that it favors the triplet pairing [11]. Also a modest concentration of magnetic impurities makes the triplet ground state stable.

In a multicomponent superconductor collective modes including gapless Goldstone modes (sound), and gapped Higgs modes of various polarizations play an important role in determining thermal and optical properties of the material [12]. In addition, as mentioned above, generally the applied magnetic field is an ultimate technique to probe the superconducting state. In a growing number of experiments, in addition to magnetotransport, magnetization curves, the magnetic penetration depth, and upper critical magnetic field were measured [13]. It is therefore of importance to construct a Ginzburg-Landau (GL) description [14] of these novel materials. This allows us to study inhomogeneous order parameter configurations (junctions, boundaries, etc.), the collective modes (somewhat analogous to the modes in the B phase of He³) and magnetic and optical response that typically involve inhomogeneous configurations (such as vortices) not amenable to a microscopic description.

In the present paper we derive such a GL-type theory for triplet superconductor from the microscopic isotropic DSM model with attractive local interaction. The order parameter in this case is a vector field and the GL theory of vector field already considered in literature [15–17] in connection with putative *p*-wave superconductors have several extraordinary features, both quantitative and qualitative.

The paper is organized as follows. The model of the (phonon mediated or unconventional) local interactions of 3D Dirac fermion is presented and the method of its solution (in the Gorkov equations form) including the symmetry analysis of possible pairing channels and the vectorial nature of the triplet order parameter is given in Sec. II. In Sec. III the Gorkov formalism, sufficiently general to derive the GL equations, is briefly presented. The most general form of the GL energy of the triplet superconductor in magnetic field consistent with the symmetries is given in Sec. IV. The coefficient of the relevant terms are calculated from the microscopic DSM model in Sec. V. Section VI is devoted to applications of the GL model. The ground-state degeneracy, the character of its excitations, and basic magnetic properties are discussed. The vector order parameter is akin to optical phonons with sharp distinction between transverse and longitudinal modes. Transverse and longitudinal coherence lengths and penetration depths are calculated and the upper critical magnetic field is discussed. Section VI includes generalizations to include Pauli paramagnetism, discussion of an experimental possibility of observation of the excitation and conclusion.

II. LOCAL PAIRING MODEL IN DIRAC SEMIMETAL

A. Pairing Hamiltonian in the Dirac semimetal

Electrons in the 3D Dirac semimetal are described by field operators $\psi_{fs}(\mathbf{r})$, where $f = L, R$ are the valley index (pseudospin) for the left/right chirality bands with spin projections taking the values $s = \uparrow, \downarrow$ with respect to, for example, the z axis. To use the Dirac (pseudorelativistic) notations, these are combined into a four-component bispinor creation operator, $\psi^\dagger = (\psi_{L\uparrow}^\dagger, \psi_{L\downarrow}^\dagger, \psi_{R\uparrow}^\dagger, \psi_{R\downarrow}^\dagger)$, whose index $\gamma = \{f, s\}$ takes four

values. The noninteracting massless Hamiltonian with Fermi velocity v_F and chemical potential μ reads [18]

$$K = \int_{\mathbf{r}} \psi^\dagger(\mathbf{r}) \widehat{K} \psi(\mathbf{r}); \quad (1)$$

$$\widehat{K}_{\gamma\delta} = -i\hbar v_F \nabla^i \alpha_{\gamma\delta}^i - \mu \delta_{\gamma\delta}, \quad (2)$$

where the three 4×4 matrices, $i = x, y, z$,

$$\alpha = \begin{pmatrix} \sigma & 0 \\ 0 & -\sigma \end{pmatrix}, \quad (3)$$

are presented in the block form via Pauli matrices σ . They are related to the Dirac γ matrices (in the chiral representation, sometimes termed spinor) by $\alpha = \beta\gamma$ with

$$\beta = \begin{pmatrix} 0 & \mathbf{1} \\ \mathbf{1} & 0 \end{pmatrix}. \quad (4)$$

Here $\mathbf{1}$ is the 2×2 identity matrix.

We consider a special case of 3D rotational symmetry that in particular has an isotropic Fermi velocity. Moreover we assume time-reversal and inversion symmetries although the pseudo-Lorentz symmetry will be explicitly broken by interactions. The spectrum of single-particle excitations is linear. The chemical potential μ is counted from the Dirac point.

As usual in certain cases the actual interaction can be approximated by a model local one:

$$V_{\text{eff}} = -\frac{g}{2} \int_{\mathbf{r}} \psi_\alpha^\dagger(\mathbf{r}) \psi_\beta^\dagger(\mathbf{r}) \psi_\beta(\mathbf{r}) \psi_\alpha(\mathbf{r}). \quad (5)$$

Unlike the free Hamiltonian K , Eq. (1), this interaction Hamiltonian does not mix different spin components.

Spin density in Dirac semimetal has the form

$$\mathbf{S}(\mathbf{r}) = \frac{1}{2} \psi^\dagger(\mathbf{r}) \boldsymbol{\Sigma} \psi(\mathbf{r}), \quad (6)$$

where the matrices

$$\boldsymbol{\Sigma} = -\alpha\gamma_5 = \begin{pmatrix} \sigma & 0 \\ 0 & \sigma \end{pmatrix}, \quad (7)$$

$$\gamma_5 = \begin{pmatrix} -\mathbf{1} & 0 \\ 0 & \mathbf{1} \end{pmatrix},$$

are also the rotation generators.

B. Symmetry classification of possible pairing channels

Since we consider the local interactions as dominant, the superconducting condensate (the off-diagonal order parameter) will be local

$$O = \int_{\mathbf{r}} \psi_\alpha^\dagger(\mathbf{r}) M_{\alpha\beta} \psi_\beta^\dagger(\mathbf{r}), \quad (8)$$

where the constant matrix M should be a 4×4 antisymmetric matrix. Due to the rotation symmetry they transform covariantly under infinitesimal rotations generated by the spin S^i operator, Eq. (6):

$$\int_{\mathbf{r}, \mathbf{r}'} [\psi_\alpha^\dagger(\mathbf{r}) M_{\alpha\beta} \psi_\beta^\dagger(\mathbf{r}') + \psi_\gamma^\dagger(\mathbf{r}') \boldsymbol{\Sigma}_{\gamma\delta}^i \psi_\delta(\mathbf{r})]$$

$$= - \int_{\mathbf{r}} \psi_\gamma^\dagger(\mathbf{r}) (\boldsymbol{\Sigma}_{\gamma\delta}^i M_{\delta\kappa} + M_{\gamma\delta} \boldsymbol{\Sigma}_{\delta\gamma}^{ii}) \psi_\kappa^\dagger(\mathbf{r}). \quad (9)$$

Here and in what follows t denotes the transpose matrix. The representations of the rotation group therefore characterize various possible superconducting phases.

Out of 16 matrices of the four-dimensional Clifford algebra, six are antisymmetric and one finds one vector and three scalar multiplets of the rotation group. The multiplets contain the following.

(i) A triplet of order parameters:

$$\{M_x^T, M_y^T, M_z^T\} = \{-\beta\alpha_z, -i\beta\gamma_5, \beta\alpha_x\} = \{T_x, T_y, T_z\}. \quad (10)$$

The algebra is

$$\Sigma_i T_j + T_j \Sigma_i = 2i\epsilon_{ijk} T_k. \quad (11)$$

Note that the three matrices T_i are Hermitian.

(ii) Three singlets

$$M_1^S = i\alpha_y; \quad M_2^S = i\Sigma_y; \quad M_3^S = -i\beta\alpha_y\gamma_5. \quad (12)$$

Which one of the condensates is realized at zero temperature is determined by the parameters of the Hamiltonian and is addressed next within the Gaussian approximation. As was shown in our previous work [11,19], either exchange interactions or magnetic impurities make the triplet state a leading superconducting channel in these materials. Therefore we will consider in the next section only the vector channel.

III. GORKOV EQUATIONS AND THE TRIPLET PAIRING

A. Gorkov equations for Green's functions in matrix form

Using the standard BCS formalism, the Matsubara Green's functions (τ is the Matsubara time)

$$\begin{aligned} G_{\alpha\beta}(\mathbf{r}, \tau; \mathbf{r}', \tau') &= -\langle T_\tau \psi_\alpha(\mathbf{r}, \tau) \psi_\beta^\dagger(\mathbf{r}', \tau') \rangle; \\ F_{\alpha\beta}(\mathbf{r}, \tau; \mathbf{r}', \tau') &= \langle T_\tau \psi_\alpha(\mathbf{r}, \tau) \psi_\beta(\mathbf{r}', \tau') \rangle; \\ F_{\alpha\beta}^+(\mathbf{r}, \tau; \mathbf{r}', \tau') &= \langle T_\tau \psi_\alpha^\dagger(\mathbf{r}, \tau) \psi_\beta^\dagger(\mathbf{r}', \tau') \rangle, \end{aligned} \quad (13)$$

obey the Gor'kov equations [20]:

$$\begin{aligned} -\frac{\partial G_{\gamma\kappa}(\mathbf{r}, \tau; \mathbf{r}', \tau')}{\partial \tau} - \int_{\mathbf{r}''} \langle \mathbf{r} | \widehat{K}_{\gamma\beta} | \mathbf{r}'' \rangle G_{\beta\kappa}(\mathbf{r}'', \tau; \mathbf{r}', \tau') \\ - g F_{\beta\gamma}(\mathbf{r}, \tau; \mathbf{r}, \tau) F_{\beta\kappa}^+(\mathbf{r}, \tau; \mathbf{r}', \tau') &= \delta^{\gamma\kappa} \delta(\mathbf{r} - \mathbf{r}') \delta(\tau - \tau'); \\ \frac{\partial F_{\gamma\kappa}^+(\mathbf{r}, \tau; \mathbf{r}', \tau')}{\partial \tau} - \int_{\mathbf{r}''} \langle \mathbf{r} | \widehat{K}_{\gamma\beta}^t | \mathbf{r}'' \rangle F_{\beta\kappa}^+(\mathbf{r}'', \tau; \mathbf{r}', \tau') \\ - g F_{\gamma\beta}^+(\mathbf{r}, \tau; \mathbf{r}, \tau) G_{\beta\kappa}(\mathbf{r}, \tau; \mathbf{r}', \tau') &= 0. \end{aligned} \quad (14)$$

These equations are conveniently presented in matrix form (superscript t denotes transposed and I is the identity matrix):

$$\begin{aligned} \int_{X''} [D^{-1}(X, X'') G(X'', X') - \Delta(X) F^+(X, X')] &= I \delta(X - X'); \\ \int_{X''} (D^t)^{-1}(X, X'') F^+(X'', X') + \Delta^{t*}(X) G(X, X') &= 0. \end{aligned} \quad (15)$$

Here $X = (\mathbf{r}, \tau)$, $\Delta_{\alpha\beta}(X) = g F_{\beta\alpha}(X, X)$ and

$$D_{\alpha\beta}^{-1}(X, X') = -\delta_{\alpha\beta} \frac{\partial}{\partial \tau} \delta(X - X') - \delta(\tau - \tau') \langle \mathbf{r} | \widehat{K}_{\alpha\beta} | \mathbf{r}' \rangle. \quad (16)$$

In the homogeneous case the Gor'kov equations for Fourier components of the Green's functions simplify considerably:

$$\begin{aligned} D^{-1}(\omega, \mathbf{p}) G(\omega, \mathbf{p}) - \Delta F^+(\omega, \mathbf{p}) &= I; \\ \widetilde{D}^{-1}(\omega, \mathbf{p}) F^+(\omega, \mathbf{p}) + \Delta^{t*} G(\omega, \mathbf{p}) &= 0. \end{aligned} \quad (17)$$

The matrix gap function can be chosen as

$$\Delta_{\beta\gamma} = g F_{\gamma\beta}(0) = \Delta_z M_{\gamma\beta}, \quad (18)$$

with real constant Δ_z . Here $D^{-1}(\omega, \mathbf{p}) = i\omega + \mu - \alpha \cdot \mathbf{p}$, is the noninteracting inverse Dirac Green's function for the Hamiltonian Eq. (1) and $\widetilde{D}^{-1}(\omega, \mathbf{p}) = i\omega - \mu - \alpha' \cdot \mathbf{p}$, where $\omega = \pi T(2n + 1)$ is the fermionic Matsubara frequency.

Solving these equations one obtains (in matrix form)

$$\begin{aligned} G^{-1} &= D^{-1} + \Delta \widetilde{D} \Delta^{t*}; \\ F^+ &= -\widetilde{D} \Delta^{t*} G, \end{aligned} \quad (19)$$

with the gap function to be found from the consistency condition

$$\Delta^{t*} = -g \sum_{\omega p} \widetilde{D} \Delta^{t*} G. \quad (20)$$

Now we find solutions of this equation for each of the possible superconducting phases.

B. Homogeneous triplet solution of the gap equation

In this phase rotational symmetry is spontaneously broken simultaneously with the electric charge $U(1)$ (global gauge invariance) symmetry. Assuming z direction of the p -wave condensate the order parameter matrix takes a form:

$$\Delta = \Delta_z T_z = \Delta_z \beta \alpha_x, \quad (21)$$

where Δ_z is a constant. The energy scale will be set by the Debye cutoff T_D of the electron-phonon interactions, see below.

The spectrum of elementary excitations at zero temperature was discussed in Ref. [11]. There is a saddle point with energy gap $2\Delta_z$ on the circle $p_\perp^2 \equiv p_x^2 + p_y^2 = \mu^2/v_F^2$, $p_z = 0$. The gap Δ_z as a function of the dimensionless phonon-electron coupling $\lambda = gN$, where N being the density of states (all spins and valleys included), increases upon reduction in μ . At large $\mu \gg T_D$, as in BCS, the gap becomes independent of μ and one has the relation

$$\frac{1}{g} = \frac{N}{12} \sinh^{-1} \frac{T_D}{\Delta_z}; \quad N = \frac{2\mu^2}{\pi^2 v_F^3 \hbar^3}, \quad (22)$$

leading to an exponential gap dependence on λ when it is small: $\Delta_z = T_D / \sinh(12/\lambda) \simeq 2T_D e^{-12/\lambda}$.

The critical temperature is obtained from Eq. (20) with discrete ω by substituting $\Delta_z = 0$. To utilize the orthonormality of T_i , $\text{Tr}(T_i T_j^*) = 4\delta_{ij}$, one multiplies the gap equation by the matrix T_z/g and takes the trace:

$$\frac{1}{g} \text{Tr}(T_z T_z^*) = \frac{4}{g} = T_c \mathcal{B}_{zz}. \quad (23)$$

The bubble integral is

$$B_{ij} = \sum_{\omega \mathbf{p}} \text{Tr}(T_i \tilde{D} T_j^* D) = 4\delta_{ij} T_c \times \sum_{n \mathbf{p}} \frac{v_F^2(p_\perp^2 - p_z^2) + \mu^2 + \omega_n^2}{\omega_n^4 + (v_F^2 p^2 - \mu^2)^2 + 2\omega_n^2(v_F^2 p^2 + \mu^2)}. \quad (24)$$

Performing first the sum over Matsubara frequencies and then integrate over q one obtains, similarly to the singlet BCS, (see Appendix A for details):

$$T_c = \frac{2\gamma_E}{\pi} T_D e^{-12/\lambda}, \quad (25)$$

where $\log \gamma_E = 0.577$ is the Euler constant.

IV. GENERAL GL DESCRIPTION OF A TRIPLET SUPERCONDUCTOR IN A MAGNETIC FIELD

In this section the effective description of the superconducting condensate in terms of the varying (on the mesoscopic scale) order complex parameter vector field $\Delta_i(\mathbf{r})$ is presented.

A. GL description for a vector order parameter

The static phenomenological description is determined by the GL free-energy functional $F[\mathbf{A}(\mathbf{r}), \mathbf{A}(\mathbf{r})]$ expanded to second order in gradients and fourth order in Δ . In a magnetic field \mathbf{B} , as usual, space derivatives of the microscopic Hamiltonian become covariant derivatives $\nabla \rightarrow \mathcal{D} = \nabla + i \frac{e^*}{\hbar c} \mathbf{A}$, $e^* = 2e$ due to gauge invariance under $\Delta_i \rightarrow e^{i\chi(\mathbf{r})} \Delta_i$, $A_i \rightarrow A_i - \frac{\hbar c}{e^*} \nabla \chi$. Naively the only modification of the GL energy is in the gradient term, Eq. (26); the most general gradient term consistent with rotation symmetry and the $U(1)$ gauge symmetry is

$$F_{\text{grad}} = N \int_{\mathbf{r}} \{u_T \{(\mathcal{D}_j \Delta_i)^* (\mathcal{D}_j \Delta_i) - (\mathcal{D}_i \Delta_j)^* (\mathcal{D}_j \Delta_i)\} + u_L (\mathcal{D}_i \Delta_j)^* (\mathcal{D}_j \Delta_i)\}. \quad (26)$$

Here u_T, u_L are the phenomenological parameters that will be calculated in Sec. IV B. The factor N , the density of states, is customarily introduced into energy [11]. It was noted in Ref. [16] that, unlike in the usual scalar order parameter case, the longitudinal and transverse coefficients are in general different, leading to two distinct coherence lengths, see Sec. V. Possibilities for the local terms are [15]

$$F_{\text{loc}} = N \int_{\mathbf{r}} \left\{ \alpha(T - T_c) \Delta_i^* \Delta_i + \frac{\beta_1}{2} (\Delta_i^* \Delta_i)^2 + \frac{\beta_2}{2} |\Delta_i \Delta_i|^2 \right\}. \quad (27)$$

The magnetic part, $F_{\text{mag}} = B^2/8\pi$, completes the free energy.

B. Set of time-independent GL equations for triplet order parameter

The set of the GL equations corresponding to this energy are obtained by variation with respect to Ψ_j^* and A_i . The first is:

$$-\{u_T (\delta_{ij} \mathcal{D}^2 - \frac{1}{2} \{ \mathcal{D}_i, \mathcal{D}_j \}) + \frac{1}{2} u_L \{ \mathcal{D}_i, \mathcal{D}_j \} \} \Delta_j + \alpha(T - T_c) \Delta_i + \beta_1 \Delta_i \Delta_j^* \Delta_j + \beta_2 \Delta_i^* \Delta_j \Delta_j = 0. \quad (28)$$

The anticommutator appears due complex conjugate terms in Eq. (26) [21]. The Maxwell equation for the supercurrent density is:

$$J_i = \frac{ie^*}{\hbar} N (u_T \Delta_j^* \mathcal{D}_i \Delta_j + u \Delta_j^* \mathcal{D}_j \Delta_i) + \text{c.c.}, \quad (29)$$

where $u = u_L - u_T$.

Having defined coefficients $u_{T,L}$, $\beta_{1,2}$ and α , our aim in Sec. V is to deduce them from the microscopic Dirac semimetal model.

V. GL COEFFICIENTS FROM THE GOR'KOV EQUATIONS

For the calculation of coefficients of the local part, the homogeneous Gor'kov equation, Eq. (20) suffices, while for calculation of the gradient terms a general linearized equation, Eq. (15) is necessary.

A. Local (potential) terms in Gor'kov

Iterating once the equation Eq. (20) with help of Eq. (19) one obtains the local terms to third order in the gap function:

$$\frac{1}{g} \Delta^{*t} + \sum_{\omega \mathbf{p}} \{ \tilde{D}(\omega, \mathbf{p}) \Delta^{*t} D(\omega, \mathbf{p}) - \tilde{D}(\omega, \mathbf{p}) \Delta^{*t} D(\omega, \mathbf{p}) \Delta \tilde{D}(\omega, \mathbf{p}) \Delta^{*t} D(\omega, \mathbf{p}) \}. \quad (30)$$

Using $\Delta^{*t} = \Delta_i^* T_i$, multiplying by T_i^t and taking the trace, one gets the linear local terms

$$N \alpha (T - T_c) \Delta_i^* = \frac{4}{g} \Delta_i^*, \quad (31)$$

where the bubble integral was given in Eq. (24). Expressing g via T_c , see Eq. (25) allows us to write the coefficient a of Δ_i^* in the Gorkov equation Eq. (20) is

$$\alpha(T - T_c) = \frac{8\mu^2}{3\pi^2 v_F^3 \hbar^3 N} \log \frac{T}{T_c} \approx \frac{4}{3} \frac{T - T_c}{T_c}. \quad (32)$$

The cubic terms in Eq. (30), multiplied again by T_i^t and traced take the form

$$N (\beta_1 \Delta_j^* \Delta_j \Delta_i^* + \beta_2 \Delta_j^* \Delta_j^* \Delta_i) = -\Delta_j^* \Delta_k \Delta_i^* \sum_{\omega \mathbf{p}} \text{Tr} \{ T_i^t \tilde{D} T_j^{t*} D T_k \tilde{D} T_i^{t*} D \}. \quad (33)$$

The calculation is given in Appendix A and results in:

$$\beta_1 = \frac{7\zeta(3)}{20\pi^2} \frac{1}{T_c^2}; \quad \beta_2 = -\frac{1}{3} \beta_1. \quad (34)$$

The Riemann ζ function is $\zeta(3) = 1.2$.

B. Linear gradient terms

To calculate the gradient terms, one first linearizes the Gor'kov equations, Eq. (15)

$$\int_{X''} D^{-1}(X, X'') G(X'', X') = I \delta(X - X') \rightarrow G = D^{-1};$$

$$F^+(X, X') = - \int_{X''} D^t(X - X'') \Delta^{*t}(X'') D(X'' - X'). \quad (35)$$

In particular,

$$\begin{aligned} \frac{1}{g} \Delta^{*t}(X) &= F^+(X, X) \\ &= - \int_{X'} D^t(X - X') \Delta^{*t}(X') D(X' - X). \end{aligned} \quad (36)$$

The anomalous Green's functions are no longer space translation invariant, so that the following Fourier transform is required: The (time-independent) order parameter is also represented via Fourier components $\Delta^*(X) = \sum_{\mathbf{P}} e^{-i\mathbf{P}\cdot\mathbf{r}} \Delta^*(\mathbf{P})$. The linear part Gor'kov equation (this time including nonlocal parts) depending on the external momentum \mathbf{P} reads:

$$\frac{1}{g} \Delta^{*t}(\mathbf{P}) + \sum_{\omega\mathbf{p}} \tilde{D}(\omega, \mathbf{p}) \Delta^{t*}(\mathbf{P}) D(\omega, \mathbf{p} - \mathbf{P}). \quad (37)$$

To find the coefficients of the gradient terms, one should consider contributions quadratic in P from the expansion of both $\Delta^{t*}(\mathbf{P})$ and $D(\omega, \mathbf{p} - \mathbf{P})$. In view of the gap equation (23), (24), the expansions of $\Delta^{t*}(\mathbf{P})$ cancel each other up to small corrections of order $T - T_c$. So that multiplying by T_i^t and taking the trace

$$\frac{1}{2} P_k P_l \sum_{\omega\mathbf{p}} \text{Tr} \{ T_i^t \tilde{D}(\omega, \mathbf{p}) T_j^{t*} D''_{kl}(\omega, \mathbf{p}) \} \Delta_j^*, \quad (38)$$

where $D''_{kl}(\omega, \mathbf{p}) \equiv \frac{\partial^2 D(\omega, \mathbf{p})}{\partial p_k \partial p_l}$. Comparing this with the gradient terms in the GL equation, (28), see Appendix B for details, one deduces

$$u_T = \frac{28\zeta(3)}{15\pi^2} \frac{v_F^2 \hbar^2}{T_c^2}; \quad u_L = \frac{1}{32} u_T. \quad (39)$$

Note the very small longitudinal coefficient, $u_L \ll u_T$. As we shall see in the following section it has profound phenomenological consequences.

VI. BASIC PROPERTIES OF THE TRIPLET SUPERCONDUCTOR

A. Ground-state structure and degeneracy

A ground state is characterized by three independent parameters corresponding to three Goldstone bosons. The GL energy is invariant under both the vector $O(3)$ space rotations, $\Delta_i \rightarrow R_{ij} \Delta_j$, and the superconducting phase $U(1)$, $\Delta_i \rightarrow e^{i\chi} \Delta_i$. In the superconducting state characterized by the vector order parameter $\mathbf{\Delta}$ ($|\mathbf{\Delta}| = \Delta$, energy gap) the $U(1)$ is broken: $U(1) \rightarrow 1$, while the $O(3)$ is only partially broken down to its $O(2)$. There are therefore three Goldstone modes. Here we explicitly parametrize these degrees of freedom by phases following Ref. [15]. Generally a complex vector field can be written as

$$\mathbf{\Delta} = \Delta(\mathbf{n} \cos \chi + i\mathbf{m} \sin \chi), \quad (40)$$

where \mathbf{n} and \mathbf{m} are arbitrary unit vectors and $0 < \chi < \pi/2$, see Fig. 1.

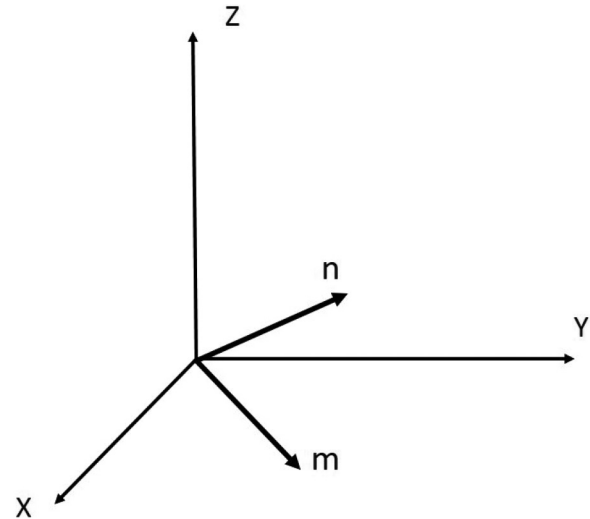


FIG. 1. A complex vector field can be written as $\mathbf{\Delta} = \Delta(\mathbf{n} \cos \chi + i\mathbf{m} \sin \chi)$, where \mathbf{n} and \mathbf{m} are arbitrary unit vectors and $0 < \chi < \pi/2$.

Using this parametrization the homogeneous part of the free-energy density, Eq. (27), takes the form

$$\frac{f_{loc}}{N} = \left\{ \alpha(T - T_c) \Delta^2 + \frac{1}{2} \beta_1 \Delta^4 + \frac{1}{2} \beta_2 [\cos^2(2\chi) + (\mathbf{n} \cdot \mathbf{m})^2 \sin^2(2\chi)] \Delta^4 \right\}. \quad (41)$$

This form allows us to make several interesting observations. The crucial sign is that of β_2 . In previous studies [15,22] only $\beta_2 > 0$ (so-called phase A) was considered. In our case however $\beta_2 < 0$ and different ground-state configurations should be considered. In phase B the minimization gives, $\mathbf{n} = \pm\mathbf{m}$. (The notation is coming from He₃. The phase A appears for $\beta_2 < 0$, while the phase B at $\beta_2 > 0$).

We do not have a general argument for negative value of this coefficient in possible generalizations of our microscopic Hamiltonian. Our point here was to try to calculate the coefficients in the vector case that, as far as we know, was not done even for such a well-known system like He₃. To have an explicit calculation of that sort is valuable for estimates of the values of the coefficients of real materials. Note two different solutions so that the vacuum manifold is

$$\mathbf{\Delta} = \Delta_0 \mathbf{n} e^{i\chi}. \quad (42)$$

Here the range of χ was enlarged, $-\pi/2 < \chi < \pi/2$, to incorporate $\mathbf{n} = \pm\mathbf{m}$. The ground-state energy density therefore is achieved at

$$\Delta_0^2 = \frac{\alpha(T_c - T)}{\beta_1 + \beta_2} = \frac{\alpha(T_c - T)}{\beta}. \quad (43)$$

Mathematically the vacuum manifold in phase B is isomorphic to $S_2 \otimes S_1/Z_2$. This determines the thermodynamics of the superconductor very much in analogy with the scalar superconductor with $\beta = \beta_1 + \beta_2$. However, the collective modes, the ac conductivity and the magnetic properties are markedly different [23].

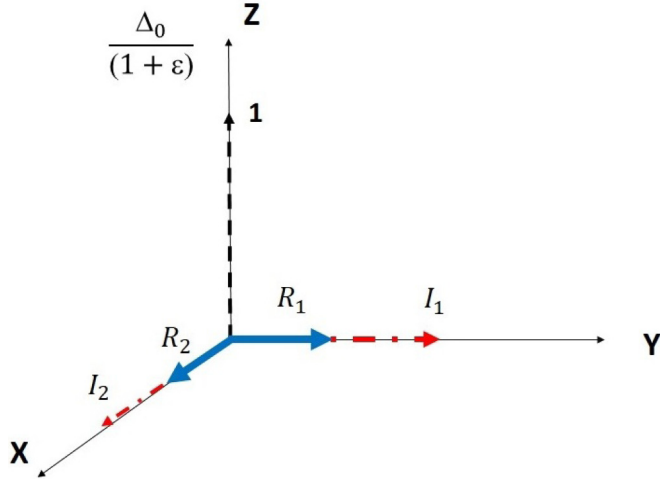


FIG. 2. (Color online) Fluctuations of the order parameter in the unitary gauge can be parametrized by the five real fields $\mathbf{\Delta} = \Delta_0(1 + \varepsilon)(R_1 + iI_1, R_2 + iI_2, 1)$.

B. Collective excitation modes

Here the response of the superconductor in phase B to an external perturbation, such as boundary or magnetic field, is considered. The basic excitation modes are uncovered by the linear stability analysis very similar to the so-called Anderson-Higgs mechanism in field theory applied to (scalar order parameter) superconductivity a long time ago [24]. Two basic scales, the coherence length (scale of variations of the order parameter) and magnetic penetration depth (scale of variations of the magnetic field), are obtained from the expansion of the GL energy to second order in fluctuations around superconducting ground state at zero field. In the superconducting state the most convenient gauge is the unitary gauge in which the $U(1)$ phase of the order parameter is set to zero, so that we are left with three massive vector potential fields A_1, A_2, A_3 . Choosing the ground state as $\Delta = \Delta_0(0, 0, 1)$, see Fig. 2, the order parameter in the unitary gauge can be parametrized by five real fields

$$\mathbf{\Delta} = \Delta_0(1 + \varepsilon)(R_1 + iI_1, R_2 + iI_2, 1). \quad (44)$$

The spontaneous breaking of the space rotation symmetry $O(3)$ into its $O(2)$ subgroup of rotations within the x - y plane according to the Goldstone theorem leads to two gapless modes R_α , $\alpha = 1, 2$. Due to the residual symmetry the Fourier components of the fluctuation fields can be generally written as combination of the radial and the tangential components:

$$R_\alpha = (R_r k_\alpha + R_t \varepsilon_{\alpha\beta} k_\beta) / k_\perp; \quad (45)$$

$$I_\alpha = (I_r k_\alpha + I_t \varepsilon_{\alpha\beta} k_\beta) / k_\perp \quad (46)$$

$$a_\alpha = \frac{e^* u_T^{1/2}}{\hbar c} (A_r k_\alpha + A_t \varepsilon_{\alpha\beta} k_\beta) / k_\perp, \quad (47)$$

$$a_3 = \frac{e^* u_T^{1/2}}{\hbar c} A_3$$

with $k_\perp = (k_x^2 + k_y^2)^{1/2}$. The GL energy to quadratic order in eight fluctuation fields $\eta = \{\varepsilon, I_r, I_t, R_r, R_t, a_r, a_t, a_3\}$,

$$f = \frac{N\Delta_0^2}{2} \sum_{\mathbf{k}} \eta_{\mathbf{k}}^* M \eta_{\mathbf{k}}, \quad (48)$$

TABLE I. Parameters of the various collective modes.

mode	mixed	Ω^2	C_\perp^2	C_\parallel^2
I_r	I_r, a_r	$2 \beta_2 \Delta_0^2$	u_L	$u_T + \frac{(u_T - u_L)^2}{u_T}$
a_r	I_r, a_r	1	0	λ_T^2
a_3	a_3	$\frac{u_L}{u_T}$	λ_T^2	0
I_t	I_t, a_t	$2 \beta_2 \Delta_0^2$	u_T	$u_T - \frac{(u_T - u_L)^2}{u_T}$
a_t	I_t, a_t	1	λ_T^2	λ_T^2
R_r	R_r	0	u_T	u_T
ε	ε	$\frac{1}{2}\beta\Delta_0^2$	u_L	u_T
R_t	R_r	0	u_T	u_L

decomposes into three independent sectors, where M is dimensionless fluctuation matrix.

(i) Three massive fields I_r, a_r, a_3 can mix due to kinetic terms:

$$M_1 = \begin{pmatrix} 2\Delta_0^2|\beta_2| + u_T k^2 + u k_\perp^2 & -i \frac{u}{u_T^{1/2}} k_3 & 0 \\ i \frac{u}{u_T^{1/2}} k_3 & 1 + \lambda_T^2 k_3^2 & -\lambda_T^2 k_3 k_\perp \\ 0 & -\lambda_T^2 k_3 k_\perp & u_L + \lambda_T^2 k_\perp^2 \end{pmatrix}, \quad (49)$$

where $\lambda_T^2 = \hbar^2 c^2 / 8\pi u_T e^{*2} N \Delta_0^2$.

To order k^2 the eigenvalues are,

$$E(k^2)/V = N\Delta_0^2(\Omega^2 + C_\perp^2 k_\perp^2 + C_\parallel^2 k_3^2). \quad (50)$$

The values of the gaps in the spectrum Ω^2 and corresponding velocities in directions perpendicular and parallel to the vector order parameter (taken to be z), C_\perp, C_\parallel are given in Table I.

(ii) The tangential massive, I_t, a_t , sector

$$M_2 = \begin{pmatrix} 2|\beta_2|\Delta_0^2 + u_T k^2 & i \frac{u}{u_T^{1/2}} k_3 \\ -i \frac{u}{u_T^{1/2}} k_3 & 1 + \lambda_T^2 k^2 \end{pmatrix}. \quad (51)$$

(iii) The Goldstone transverse mode R_t does not mix, $M_3 = u_T k^2$, so that $E_6/V = N\Delta_0^2 u_T k^2$.

(iv) Higgs and Goldstone radial collective modes ε, R_r form a 2×2 matrix:

$$M_4 = \begin{pmatrix} u_T k^2 + u k_\perp^2 & u k_\perp k_3 \\ u k_\perp k_3 & u_T k^2 + u k_3^2 + \Delta_0^2 \frac{\beta}{2} \end{pmatrix}. \quad (52)$$

C. Coherence length and penetration depth for massive collective modes

The six massive fields, ε, I_α and \mathbf{A} with different longitudinal and transversal characteristic lengths: $l_\perp = C_\perp/\Omega$; $l_\parallel = C_\parallel/\Omega$, and the same for ξ and λ , see Table I.

This is different compared with the one component (singlet) superconductor in two respects. First, the number of Higgs modes is larger since in addition to the superfluid density determined by ε , there are two additional components I_r and I_t . Second, as mentioned above, since the superconducting condensate is oriented, one has two different velocities. As far as (massive) photon modes are concerned, the number of modes remains the same but the anisotropy persists. Let us

TABLE II. Screening lengths λ in directions perpendicular and parallel to the order parameter direction (z).

mode	λ_{\perp}^2	λ_{\parallel}^2
A_r	0	λ_T^2
A_l	λ_T^2	λ_T^2
A_3	λ_L^2	0

start with magnetic excitations. The six screening lengths are given in Tables II and III (in the physical units).

Here definitions of the coherence lengths (of fluctuation of the superfluid density ε perpendicular and parallel to the direction of the order parameter \mathbf{n}), correlation lengths of the relative weights between different components of the vector order parameter (I_r, I_l) and screening lengths (vector potential \mathbf{A} fluctuations):

$$\begin{aligned} \xi_{\perp}^2 &= \frac{2u_L}{\beta\Delta_0^2}; & \xi_{\parallel}^2 &= \frac{u_T}{u_L}\xi_{\perp}^2; \\ l_{\perp}^2 &= \frac{u_T}{2|\beta_2|\Delta_0^2}; & l_{\parallel}^2 &= \frac{u_L}{u_T}l_{\perp}^2; \\ \lambda_T^2 &= \frac{\hbar^2 c^2}{8\pi u_T e^{*2} N \Delta_0^2}, & \lambda_L^2 &= \frac{u_T}{u_L}\lambda_T^2. \end{aligned} \quad (53)$$

In the isotropic superconductor one recovers the standard formulas since $u_T = u_L$.

Our calculation in the previous section for the Dirac semimetal, see Eq. (39), demonstrate that both are quite different since $u_L/u_T = 1/32 \ll 1$. This is obviously of great importance for large magnetic field properties of such superconductors and will be discussed below.

VII. MAGNETIC AND OPTICAL PROPERTIES

A. Strong magnetic fields: Is there an upper critical field H_{c2} ?

In strong homogeneous magnetic field H (assumed to be directed along the z axis) superconductivity typically (but not always, see an example of the p -wave superconductor that develops flux phases [23]) disappears at certain critical value H_{c2} . This bifurcation point is determined within the GL framework by the lowest eigenvalue of the linearized GL equations. This is an exact requirement of stability of the normal phase [14,25]. The linearized GL equation Eq. (28) reads:

$$\left[(\alpha(T_c - T) - u_T \mathcal{D}^2) \delta_{ij} - \frac{u}{2} \{ \mathcal{D}_i, \mathcal{D}_j \} \right] \Delta_j = 0, \quad (54)$$

TABLE III. Coherence lengths characterizing different collective modes ξ, l .

mode	perpendicular	parallel (z)
ε	ξ_{\perp}^2	ξ_{\parallel}^2
I_r	l_{\parallel}^2	$l_{\perp}^2(1 + u^2/u_T^2)$
I_l	l_{\perp}^2	$l_{\perp}^2(1 - u^2/u_T^2)$

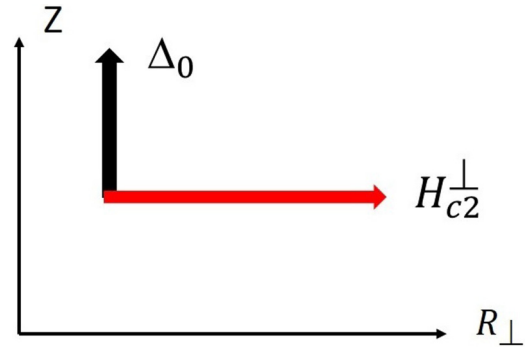


FIG. 3. (Color online) Superconductivity arises from the normal state when the order parameter is formed in direction perpendicular to the magnetic field.

where coefficients are in Eq. (39), and $u = u_L - u_T$. We use the Landau gauge, $A_x = H_{c2}y$; $A_y = A_z = 0$. Assuming translation symmetry along the field direction, $\partial_z \Delta_i = 0$, the operators of the eigenvalue problem depend on x and y only.

Since we have three components of the order parameter, there are three eigenvalues. It is easily seen from Eq. (54) that the z component of the order parameter Δ_z parallel to the external field direction is independent of the other two, Δ_x, Δ_y , leading to the ordinary Abrikosov value:

$$-u_T \mathcal{D}^2 \Delta_z = -\alpha(T_c - T) \Delta_z \rightarrow H_{c2}^{\parallel} = \frac{\Phi_0 \alpha(T_c - T)}{2\pi u_T}. \quad (55)$$

To avoid confusion with customary notations for layered materials (such as high- T_c cuprates), the material that is modeled here is isotropic and parallel, perpendicular and refer to the relative orientation of the magnetic field to the vector order parameter rather than to a layer. The orientation of the order parameter in isotropic material considered here, due to degeneracy of the ground state, is determined by the external magnetic field as we exemplify next.

The two remaining eigenvalues involving only the order parameter components Δ_x and Δ_y perpendicular to the field (see Fig. 3) are obtained from diagonalizing the Hamiltonian:

$$\begin{aligned} \mathcal{H} \begin{pmatrix} \Delta_x \\ \Delta_y \end{pmatrix} &= -\alpha(T_c - T) \begin{pmatrix} \Delta_x \\ \Delta_y \end{pmatrix}; \\ \mathcal{H} &= - \begin{pmatrix} u_T \mathcal{D}_y^2 + u_L \mathcal{D}_x^2 & \frac{u}{2} \{ \mathcal{D}_x, \mathcal{D}_y \} \\ \frac{u}{2} \{ \mathcal{D}_x, \mathcal{D}_y \} & u_T \mathcal{D}_x^2 + u_L \mathcal{D}_y^2 \end{pmatrix}. \end{aligned} \quad (56)$$

This nontrivial eigenvalue problem fortunately can be solved exactly, see Appendix C. The lowest eigenstate being a superposition of just two lowest even Landau levels, $|0\rangle$ and $|2\rangle$ are given. The lowest of these eigenvalues is

$$\frac{e^* H_{c2}^{\perp}}{\hbar c} \left(\frac{\frac{3}{2}(u_T + u_L)}{-\sqrt{3(u_T^2 + u_L^2) - 2u_T u_L}} \right) = \alpha(T_c - T). \quad (57)$$

The corresponding critical field H_{c2}^{\perp} (perpendicular refers to the order parameter direction with respect to that of the magnetic field, see Fig. 3) is

$$H_{c2}^{\perp} = \left(\frac{3}{2}(1+r) - \sqrt{3r^2 + 3 - 2r} \right)^{-1} H_{c2}^{\parallel}. \quad (58)$$

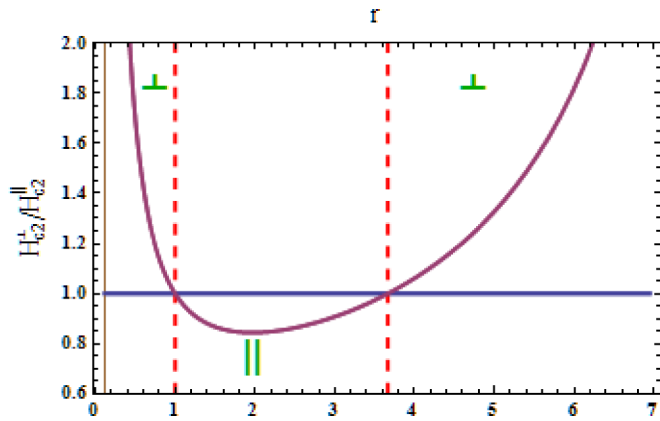


FIG. 4. (Color online) Magnetic phase diagram of the vector superconductor. Ratio of two upper critical fields, $H_{c2}^{\perp}/H_{c2}^{\parallel}$, is plotted as function of $r = u_T/u_L$. The parallel phase is separated from two perpendicular phases (dashed lines). At two values r_c and $1/r_c$ the parallel upper critical field diverges.

It is always larger than H_{c2}^{\parallel} , and therefore is physically realized. The upper field H_{c2}^{\perp} becomes infinite at $r \equiv u_L/u_T = r_c$ and $r = 1/r_c$, where $r_c = (13 - 4\sqrt{10})/3 \simeq 0.117$. The magnetic phase diagram, Fig. 4, therefore has two transition lines with reentrant perpendicular phase. This means that for superconductivity persists at any magnetic field like in some p -wave superconductors. The two critical fields as function of r are given in Fig. 4.

It was found in Sec. IV that for the simplest Dirac semimetal, $r = 1/32 < r_c$ see Eq. (39). Thus there is no upper critical field in this case. Of course, different microscopic models that belong to the same universality class, might have an arbitrary r . In any case the Abrikosov lattice is expected to be markedly different from the conventional one and even from the vector order parameter model studied in Ref. [15].

B. Dissipative dynamics

The set of the GL equations corresponding to this energy are obtained by variation with respect to Δ_j^* and A_i . The first is the time-dependent GL equation (the covariant derivative is replaced by partial since in a superconductor the scalar potential can be taken to be zero on the mesoscopic scale):

$$-\Gamma \partial_t \Delta_i = \frac{\delta F}{\delta \Delta_i^*}. \quad (59)$$

This should be supplemented by the Maxwell equation including the normal contribution to the current $\mathbf{J}^n = \mathbf{J} - \mathbf{J}^s$, $J_i^s = \frac{ie^*N}{\hbar} (u_T \Delta_j^* \mathcal{D}_i \Delta_j + u \Delta_j^* \mathcal{D}_j \Delta_i) + \text{c.c.}$ This determines the dynamics of the vector potential:

$$\frac{\sigma_n}{c} \partial_t A_i = J_i^s - \frac{c}{4\pi} (\partial^2 \delta_{ij} - \partial_i \partial_j) A_j = J_i. \quad (60)$$

In the small fluctuations approximation the dominant role is played by the two Goldstone modes, R_t, R_r , due to spontaneous breaking of the 3D rotation symmetry. The R_t mode is still isotropic, while R_r is not. Neglecting the massive excitations the dissipative dynamics of the diffusion type is

governed by

$$\begin{aligned} \partial_t R_t &= D_T k^2 R_t; \\ \partial_t R_r &= (D_T k_{\perp}^2 + D_L k_3^2) R_r, \end{aligned} \quad (61)$$

where $D_{T,L} = \frac{N}{T} u_{T,L}$. The diffusion coefficient of this equation is anisotropic and is discussed in Sec. VIII. This would lead to increase in thermal conductivity inside the superconducting state even at low temperature.

C. ac conductivity

In external ac field represented by (no spatial dispersion), $\mathbf{A} = \frac{ic}{\omega} \mathbf{E}(\omega) e^{i\omega t}$, one obtains in linear response

$$J_i = -\frac{2ie^{*2}N\Delta_0^2}{\omega\hbar^2} (u_T E_i + u E_3 \delta_{3i}) + \sigma_n E_i. \quad (62)$$

Therefore the conductivity tensor reads:

$$[\sigma(\omega)]_{ij} = \begin{pmatrix} \sigma_n - \sigma_T^s & 0 & 0 \\ 0 & \sigma_n - \sigma_T^s & 0 \\ 0 & 0 & \sigma_n - \sigma_L^s \end{pmatrix}, \quad (63)$$

where $\sigma_{T,L}^s(\omega) = \frac{2ie^{*2}N\Delta_0^2}{\omega\hbar^2} u_{T,L}$. The ac conductivity this is different for the order parameter and the perpendicular directions. One of the interesting consequences of this phenomenon is rotation of the polarization of microwave that passes the DSM film.

D. Rotation of the polarization of the microwave

The material becomes optically active, i.e., the polarization of the electromagnetic wave rotates. The dispersion relation is:

$$-\frac{ic^2k^2}{4\pi\omega} \sigma^{-1}(\omega) \mathbf{B} = \mathbf{B}. \quad (64)$$

The perpendicular to the order parameters are eigenvectors with eigenvalue $-\frac{ic^2k^2}{4\pi\omega(\sigma_n - \sigma_T^s)}$, while the third eigenvector (0,0,1) has the eigenvalue is $-\frac{ic^2k^2}{4\pi\omega(\sigma_n - \sigma_L^s)}$. Assume that the incident electromagnetic wave described on the surface of DSM by the magnetic field \mathbf{B}_0 is perpendicular to the order parameter direction taken as z (see Fig. 5).

Without loss of generality it can be taken as x [due to the residual $O(2)$ rotation symmetry]. The Fourier component of

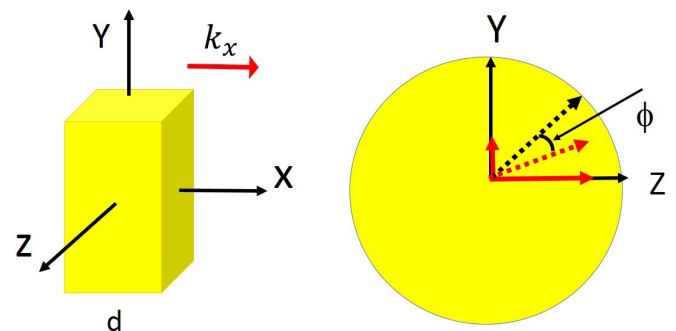


FIG. 5. (Color online) The polarization vector of the incident beam (x direction) rotates while passing a film of thickness d by angle ϕ $\tan \phi = |B_z(d)/B_y(d)| = \exp[(\lambda_T^{-1} - \lambda_L^{-1})d]$.

the magnetic field in the y - z plane are

$$\begin{aligned} B_y &= B_0 \exp(ik^{(2)}x) \\ B_z &= B_0 \exp(ik^{(3)}x). \end{aligned} \quad (65)$$

The complex wave vectors are

$$\begin{aligned} k^{(2)} &= \sqrt{\frac{1 + \sqrt{\lambda_T^4/\delta^4 + 1}}{2}} \left(-\frac{1}{\lambda_T} + i\frac{\lambda_T}{\delta^2} \right) \\ k^{(3)} &= \sqrt{\frac{1 + \sqrt{\lambda_L^4/\delta^4 + 1}}{2}} \left(-\frac{1}{\lambda_L} + i\frac{\lambda_L}{\delta^2} \right). \end{aligned} \quad (66)$$

Here the screening length is defined as usual $\delta = (c/4\pi\omega\sigma_n)^{1/2}$.

The penetration of the microwave radiation into the sample surface demonstrates differences for ac component parallel and perpendicular to the direction of the order parameter resulting in effective rotation of the incident wave vector like at Faraday effect.

VIII. DISCUSSION AND CONCLUSIONS

A. Vector nature of the order parameter

The physical properties of the triplet superconductivity appearing in 3D Dirac semimetals were considered. Starting from the microscopic model of the isotropic Dirac semimetal, the Ginzburg-Landau energy for this field is derived using the Gor'kov technique. The properties of the triplet superconductor phase of the Dirac semimetal has extremely unusual features that we would like to associate qualitatively with the characteristics of the Cooper pair. The superconducting state generally is a Bose-Einstein condensate of composite bosons—Cooper pairs, classically described by the Ginzburg-Landau energy as a functional of the order parameter. In the present case the Cooper boson is described by a vector field $\Delta_i(\mathbf{r})$. In this respect it is reminiscent to phonon and vector mesons in particle physics [24].

Vector fields generally have both the orbital and internal degrees of freedom often called polarization. The internal degree of freedom might be connected to the valley degree of freedom of constituents of the composite boson. We have provided evidence that the Cooper pair in DSM has finite orbital momentum, albeit, as will be shown shortly, the spin magnetic moment is zero. Microscopically the unusual nature is related to the presence of the valley degeneracy in Dirac semimetal. While in a single-band superconductor the Pauli principle requires a triplet Cooper pair to have both odd angular momentum and spin, it is no longer the case in the Dirac semimetal.

A massive bosonic vector field in isotropic situation (the case considered here) generally have distinct transversal and longitudinal polarizations (massless fields like photons in dielectric do not possess the longitudinal degree of freedom). The results for collective modes in triplet superconductor in DSM demonstrate pronounced disparity between dispersion of various polarizations, see Table I. In particular we have found sound velocity of the two Goldstone modes and, screening lengths of three gapped photon modes and three coherence

lengths of the other (Higgs) gapped modes. These all have an impact on transport, optical and magnetic properties of these superconductors.

B. Estimates of the characteristics of the collective modes and Faraday effect in a typical Dirac semimetal

Substituting the values of parameter of the vector GL equation found in Sec. IV into formulas for various collective modes, coherence lengths, described in Sec. VI, one obtains,

$$\begin{aligned} \xi_{0\perp}^2 &= \frac{7\zeta(3)}{80\pi^2} \frac{v_F^2 \hbar^2}{T_c^2}; & \xi_{0z}^2 &= 32\xi_{\perp}^2; \\ l_{0\perp}^2 &= 32\xi_{\perp}^2; & l_{0z}^2 &= \xi_{\perp}^2; \\ \lambda_{0T}^2 &= \frac{3\pi}{32} \frac{\hbar^3 c^2 v_F}{e^* \mu^2}, & \lambda_{0L}^2 &= 32\lambda_{0T}^2, \end{aligned} \quad (67)$$

where $\xi_{\perp}^2(T) = \xi_{0\perp}^2/(1 - T/T_c)$, etc.

For a typical DSM one estimates the Fermi velocity and chemical potential [1] $v_F = c/200$, $\mu = 0.2$ eV, and with the expected critical temperature [7,10] $T_c = 5$ K, one obtains for $\xi_{0\perp} = 230$ nm and $\lambda_{0T} = 220$ nm.

Let us estimate the characteristics of the two Goldstone modes, arising in the triplet superconducting state due to spontaneous breaking of the rotational $O(3)$ symmetry. The isotropic transverse mode R_i defined in Sec. III is isotropic and its dynamics is described by Eq. (61) with diffusion constant $D_T = Nu_T/\Gamma$, while the anisotropic mode R_r involves both the transverse and the longitudinal constant that is different, $D_L = Nu_L/\Gamma$. To estimate these, let us exploit the relation [14] the time constant as $\Gamma = \frac{\pi\hbar N}{8T_c}$. Thus

$$D_T = \frac{2^5 \times 7\zeta(3)}{15\pi^3} \frac{v_F^2 \hbar}{T_c} = 2 \times 10^4 \frac{\text{cm}^2}{\text{s}}, \quad (68)$$

$$D_L = \frac{7\zeta(3)}{15\pi^3} \frac{v_F^2 \hbar}{T_c} = 620 \frac{\text{cm}^2}{\text{s}}. \quad (69)$$

These considerations were made for superconductor without significant pinning—disorder on the mesoscopic scale. Recently the ac response of the disordered superconductor was utilized to probe Goldstone modes [26]. We have demonstrated that they are abundant in the triplet DSM superconductor. Therefore it would be interesting to look for effects of the Goldstone modes including damping resulting in strong sound absorption in these systems. For experimental verification and observation of the Higgs collective modes complex terahertz transmission and tunneling density of states might be a useful tools [27]. Heat capacity measurements demonstrating a significant increase is another method to fix the collective modes effect.

We have calculated the ac conductivity of DSM, see Eq. (64), and applied it to describe an intriguing effect of optical activity of DSM. According to Eq. (66), the polarization vector of the incident beam rotates while passing a film of thickness d by, see Fig. 5,

$$\tan \phi = |B_z(d)/B_y(d)| = \exp[(\lambda_T^{-1} - \lambda_L^{-1})d], \quad (70)$$

under the assumption the skin depth $\delta = (c/4\pi\omega\sigma_n)^{1/2}$ is much larger than both λ_T and λ_L . In particular for $\lambda_L = 32\lambda_T$, $\lambda_T = 220$ nm, and $d = 1$ mm, $\phi \rightarrow \pi/2$ significantly deviation from

the initial $\pi/4$ value. Let us stress that the effect is due to the difference between the two penetration depths.

C. DSM superconductor under constant magnetic field

Several new features appear when an external field is applied. The Ginzburg-Landau model was used to determine upper magnetic field H_{c2} . The magnetic phase diagram is presented in Fig. 4. For $r = u_L/u_T < 1$ and $r > 11/3$ it turns out that the order parameter of the texture orients itself perpendicular to the field direction. In between $1 < r < 11/3$ the reorientation phase transition takes place. We have shown that the upper field becomes infinite for $r = r_c$ or $1/r_c$, where $r_c = (13 - 4\sqrt{10})/3 \simeq 0.117$. In particular it is obeyed within our microscopic model. This means that in such material superconductivity persists at any magnetic field like in some p -wave superconductors. The expression for the H_{c2} for $u_L/u_T > 0.117$ is given by Eq. (58). Experimentally the phases can be identified by measuring attenuation of electromagnetic waves. The attenuation is different for linear polarization waves with respect to direction of the order parameter, Eq. (66).

The vortex physics of strongly type II triplet superconductors of this type is very rich and some of it has already been investigated in connection with heavy fermion and other superconductors suspected to possess p -wave pairing. In particular, their magnetic vortices appear as either vector vortices or so-called skyrmions [15]—coreless topologically nontrivial textures. The magnetic properties like the magnetization are very peculiar and even without a magnetic field the system forms a spontaneous flux state. The material therefore can be called a ferromagnetic superconductor. The superconducting state develops weak ferromagnetism and a system of alternating magnetic domains [22]. It was noted [15] that the phase is reminiscent to the phase B of superfluid He₃ [14] (with an obvious distinction that the order parameter in the later case is neutral rather than charged and tensorial rather than vectorial).

Since the prediction of the FFLO effect [14] in low- T_c superconductors it is well known that at very high magnetic fields the direct spin-magnetic field coupling on the microscopic level might not be negligible. The singlet channel Cooper pair is effectively broken by the splitting since the spins of the two electrons are opposite (Pauli paramagnetic limit). It is not clear what impact it has on Dirac semimetals. If the impact is large it could be incorporated as an additional paramagnetic term in the GL energy. In an isotropic Dirac superconductor one has only one possible term in the GL energy term linear in paramagnetic coupling and consistent with symmetries:

$$F_{\text{par}} = N\mu_p \int_{\mathbf{r}} i(\mathbf{\Delta}^* \times \mathbf{\Delta}) \cdot \mathbf{B}, \quad (71)$$

where μ_p is the effective spin of the Cooper pair sometimes called Zeeman coupling [15,25]. The single particle Hamiltonian in magnetic field has the Pauli term $\mu_B \mathbf{\Sigma} \cdot \mathbf{B}$, where the Bohr magneton, $\mu_B = e\hbar/2mc$, determines the strength of the coupling of the spin to magnetic field, with m being the free electron mass. The direct calculation, see Appendix A, shows that $\mu_p = 0$.

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APPENDIX A: LOCAL TERMS IN GL FREE ENERGY

1. Critical temperature calculation

Starting from equation Eq. (23) the angle integrations result in (for $\mu \gg T_D, T_c$)

$$\begin{aligned} \frac{1}{g} &= T \sum_{np} \frac{\mu^2 + \omega_n^2}{\omega_n^4 + (v_F^2 p^2 - \mu^2)^2 + 2\omega_n^2(v_F^2 p^2 + \mu^2)} \\ &= \frac{\mu^2}{12\pi^2 \hbar^3 v_F^3} \int_{\varepsilon=-T_D}^{T_D} \frac{\tanh(\varepsilon/2T)}{\varepsilon} \\ &\approx \frac{\mu^2}{6\pi^2 \hbar^3 v_F^3} \log \frac{2T_D \gamma_E}{\pi T_c}, \end{aligned} \quad (A1)$$

where $\varepsilon = v_F p - \mu$. See the last (BCS) integral in Ref. [20].

2. Cubic terms coefficients calculation

To fix the two coefficients, β_1 and β_2 in Eq. (28) we use only two components. The particular case $j = k = l = 1$ (the coefficient of $\psi_1^{*2} \psi_1$) gives after angle integration

$$N(\beta_1 + \beta_2) = \frac{2T}{15\pi^2 \hbar^3} \sum_n \int_{p=0}^{\infty} S(p, n), \quad (A2)$$

where

$$\begin{aligned} S(p, n) &= \frac{v_F^2 p^2 [v_F^4 p^4 + 10v_F^2 p^2 (\omega_n^2 - 5\mu^2) - 15(\mu^2 + \omega_n^2)^2]}{[v_F^4 p^4 + 2v_F^2 p^2 (\omega_n^2 - \mu^2) + (\mu^2 + \omega_n^2)^2]^2}. \end{aligned}$$

Performing finite integration (the upper bound on momentum, $\mu + T_D$, can be replaced by infinity), one obtains

$$N(\beta_1 + \beta_2) = \frac{8\mu^2}{15\pi^4 T^2 v_F^3 \hbar^3} s_3, \quad (A2)$$

where the sum is

$$s_3 = \sum_{n=0}^{\infty} \frac{1}{(2n+1)^3} = \frac{7\zeta(3)}{4}. \quad (A3)$$

Similarly taking $j = l = 2, k = 1$ (the coefficient of $\Delta_2^{*2} \Delta_1$) gives after the angle integration

$$\begin{aligned} N\beta_2 &= \frac{2T}{15\pi^2 \hbar^3} \sum_n \int_{p=0}^{\infty} \frac{v_F^2 p^2 [7v_F^4 p^4 + 10v_F^2 p^2 (\omega_n^2 + 3\mu^2) + 15(\mu^2 + \omega_n^2)^2]}{[v_F^4 p^4 + 2v_F^2 p^2 (\omega_n^2 - \mu^2) + (\mu^2 + \omega_n^2)^2]^2} \\ &= -\frac{4\mu^2}{15\pi^4 v_F^3 \hbar^3 T^2} s_3. \end{aligned} \quad (A4)$$

resulting in Eq. (34).

3. Effect of the Pauli interaction term

The single-particle Hamiltonian in magnetic field is

$$\widehat{K} = -iv_F \hbar \mathcal{D} \cdot \alpha - \mu + \mu_B \boldsymbol{\Sigma} \cdot \mathbf{B}, \quad (\text{A5})$$

In order to fix the coefficient of the paramagnetic term linear in both the order parameter and Pauli coupling it is enough to expand the linearized Gorkov equations Eq. (31) to the first order in the spin density. Normal Green's functions have the following corrections:

$$\begin{aligned} D_Z &\approx D - \mu_B D(\boldsymbol{\Sigma} \cdot \mathbf{B})D; \\ \widetilde{D}_Z &\approx \widetilde{D} + \mu_B \widetilde{D}(\boldsymbol{\Sigma}^t \cdot \mathbf{B})\widetilde{D}. \end{aligned} \quad (\text{A6})$$

The Pauli term in Gor'kov equation (after multiplying by T_i^t and taking the trace as usual), Eq. (28), therefore is obtained from expansion of Eq. (31),

$$\begin{aligned} \sum_{\omega \mathbf{p}} \text{Tr}\{T_i \widetilde{D} \Delta^* D\} &= -i\mu_Z \varepsilon_{ijk} \Delta_j^* B_k N = \mu_B \mathcal{B}_{ijk}^Z \Delta_j^* B_k, \\ \mathcal{B}_{ijk}^Z &= \sum_{\omega \mathbf{p}} \text{Tr}\{T_i \widetilde{D}(\Sigma_k^t \widetilde{D} T_j^* - T_j^* D \Sigma_k) D\}, \end{aligned} \quad (\text{A7})$$

The bubble sum is directly evaluated and vanishes $B_{ijk}^Z = 0$.

APPENDIX B: CALCULATION OF GRADIENT TERMS IN THE GL

Rotational invariance allows to represent the sum in Eq. (38) terms of coefficients u_T and u_L :

$$\begin{aligned} &-N[u_T(P^2 \delta_{mj} - P_m P_j) + u_L P_m P_j] \\ &= P_k P_l \sum_{\omega \mathbf{q}} \text{Tr}\{T_m^t \widetilde{D}(\omega, \mathbf{q}) T_j^{t*} D_{kl}''(\omega, \mathbf{q})\}, \end{aligned} \quad (\text{B1})$$

where

$$\begin{aligned} D_{ij}'' &= \frac{2}{(q^2 - (i\omega + \mu)^2)^2} \{q_j \alpha_i + q_i \alpha_j \\ &+ \delta_{ij}(i\omega + \mu + \alpha \cdot \mathbf{q}) + 2q_i q_j D\}. \end{aligned} \quad (\text{B2})$$

In particular

$$\begin{aligned} Nu_L &= - \sum_{\omega \mathbf{p}} \text{Tr}\{T_z \widetilde{D}(\omega, \mathbf{p}) T_z^* D_{zz}''(\omega, \mathbf{p})\} \\ &= \frac{\mu^2}{15\pi^4 T^2 v_F \hbar} s_3 \\ &= \frac{7\zeta(3)}{60\pi^4} \frac{\mu^2}{T^2 v_F \hbar} = \frac{7\zeta(3)(v_F \hbar)^2 N}{120\pi^2 T^2} \end{aligned} \quad (\text{B3})$$

and

$$Nu_T = - \sum_{\omega \mathbf{p}} \text{Tr}\{T_x \widetilde{D}(\omega, \mathbf{p}) T_x^* D_{zz}''(\omega, \mathbf{p})\} = 32u_L. \quad (\text{B4})$$

APPENDIX C: COLLECTIVE MODES

Here details of the calculation of free energy in harmonic approximation are given. The expansion of order parameter

Eq. (44) to quadratic order is

$$\Delta / \Delta_0 = (1 + \varepsilon)(\delta_1, \delta_2, 1) = (0, 0, 1) + (\delta_1, \delta_2, \varepsilon) + (\delta_1 \varepsilon, \delta_2 \varepsilon, 0). \quad (\text{C1})$$

The gradient terms that do not involve the vector potential \mathbf{A} are:

$$\begin{aligned} \frac{F_{\text{grad}}^{(1)}}{\Delta_0^2} &= u_T [\partial_j R_\alpha \partial_j R_\alpha + \partial_j I_\alpha \partial_j I_\alpha + (\partial_j \varepsilon)^2] \\ &+ u \{ \partial_1 R_1 \partial_1 R_1 + \partial_1 I_1 \partial_1 I_1 + \partial_2 R_2 \partial_2 R_2 + \partial_2 I_2 \partial_2 I_2 \\ &+ (\partial_3 \varepsilon)^2 + 2((\partial_2 R_1)(\partial_1 R_2) + (\partial_2 I_1)(\partial_1 I_2) \\ &+ (\partial_\alpha \varepsilon)(\partial_3 R_\alpha)) \}. \end{aligned} \quad (\text{C2})$$

The terms involving \mathbf{A} read

$$\frac{F_{\text{grad}}^{(2)}}{\Delta_0^2} = 2 \frac{e^* u}{\hbar c} (\partial_3 I_\alpha) A_\alpha + \left(\frac{e^*}{\hbar c} \right)^2 (u_T A^2 + u A_3^2). \quad (\text{C3})$$

Potential terms result in

$$\frac{F_{\text{pot}}}{\Delta_0^2} = \Delta_0^2 \left\{ \frac{\beta_1 + \beta_2}{2} \varepsilon^2 - 2\beta_2 (I_1^2 + I_2^2) \right\}, \quad (\text{C4})$$

while the magnetic energy is:

$$\frac{F_{\text{mag}}}{\Delta_0^2} = \frac{\hbar^2 c^2}{8\pi e^{*2} N \Delta_0^2} A_i^* (k^2 \delta_{ij} - k_i k_j) A_j. \quad (\text{C5})$$

The fluctuation matrix M in Eq. (48) thus was constructed.

APPENDIX D: EXACT SOLUTION FOR UPPER CRITICAL MAGNETIC FIELD

In this Appendix the matrix \mathcal{H} defined in Eq. (56) determining the perpendicular upper critical field is diagonalized variationally.

1. Creation and annihilation operators

Using Landau creation and annihilation operators in units of magnetic length $\frac{e^* B}{c} = l^{-2}$ for the state with $k_x = 0$ (independent of x), so that covariant derivatives are

$$\begin{aligned} D_x &= \partial_x + iy = iy = \frac{i}{\sqrt{2}}(a + a^+); \\ D_y &= \partial_y = \frac{1}{\sqrt{2}}(a - a^+). \end{aligned} \quad (\text{D1})$$

In terms of these operators the matrix operator \mathcal{H} takes a form:

$$\begin{aligned} \mathcal{H} &= u_T + \frac{u}{2} + \mathcal{V}; \\ \mathcal{V}_{11} &= 2u_T a^+ a + \frac{u}{2}(a^2 + a^{+2} + 2a^+ a); \\ \mathcal{V}_{12} &= \mathcal{V}_{21} = \frac{i u}{2}(a^{+2} - a^2); \\ \mathcal{V}_{22} &= 2u_T a^+ a - \frac{u}{2}(a^2 + a^{+2} - 2a^+ a). \end{aligned} \quad (\text{D2})$$

The exact lowest eigenvalue is a combination of two lowest Landau levels. Indeed applying the operator \mathcal{V} on a general

vector on the subspace gives

$$\begin{aligned} \nu \begin{pmatrix} \alpha|0\rangle + \beta|2\rangle \\ \gamma|0\rangle + \delta|2\rangle \end{pmatrix} = & \left(\frac{u}{\sqrt{2}}(i\delta - \beta)|0\rangle - \left(u \left(\frac{\alpha}{\sqrt{2}} + \frac{i\gamma}{\sqrt{2}} + 2\beta \right) + 4u_T\beta \right) |2\rangle + \frac{u}{2}(-\beta - i\delta)|4\rangle + \frac{u}{\sqrt{2}}(i\beta + \delta)|0\rangle \right. \\ & \left. + \left(u \left(-\frac{i\alpha}{\sqrt{2}} + \frac{\gamma}{\sqrt{2}} - 2\delta \right) - 4u_T\delta \right) |2\rangle + \frac{u}{2}(-i\beta + \delta)|4\rangle \right). \end{aligned} \quad (D3)$$

For $\delta = i\beta$, higher Landau levels decouple and one gets eigenvalue equations

$$\begin{vmatrix} -v & -\sqrt{2}u & 0 \\ -\frac{u}{\sqrt{2}} & -4u_T - 2u - v & -\frac{iu}{\sqrt{2}} \\ 0 & ui\sqrt{2} & -v \end{vmatrix} = 0, \quad (D4)$$

resulting in three eigenvalues of \mathcal{H}

$$h^{(1)} = u_T + u/2, \quad h^{(\pm)} = 3u_T + \frac{3}{2}u \pm \sqrt{4u_T^2 + 4u_Tu + 3u^2}.$$

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