

# Ferroelectricity in $d^0$ double perovskite fluoroscandates

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(Received 6 July 2015; published 27 August 2015)

Ferroelectricity in strain-free and strained double perovskite fluorides,  $\text{Na}_3\text{ScF}_6$  and  $\text{K}_2\text{NaScF}_6$ , is investigated using first-principles density functional theory. Although the experimental room temperature crystal structures of these fluoroscandates are centrosymmetric, i.e.,  $\text{Na}_3\text{ScF}_6$  ( $P2_1/n$ ) and  $\text{K}_2\text{NaScF}_6$  ( $Fm\bar{3}m$ ), lattice dynamical calculations reveal that soft polar instabilities exist in each prototypical cubic phase and that the modes harden as the tolerance factor approaches unity. Thus the double fluoroperovskites bear some similarities to  $\text{ABO}_3$  perovskite oxides; however, in contrast, these fluorides exhibit large acentric displacements of alkali metal cations (Na, K) rather than polar displacements of the transition metal cations. Biaxial strain investigations of the centrosymmetric and polar  $\text{Na}_3\text{ScF}_6$  and  $\text{K}_2\text{NaScF}_6$  phases reveal that the paraelectric structures are favored under compressive strain, whereas polar structures with in-plane electric polarizations ( $\sim 5\text{--}18 \mu\text{C cm}^{-2}$ ) are realized at sufficiently large tensile strains. The electric polarization and stability of the polar structures for both chemistries are found to be further enhanced and stabilized by a coexisting single octahedral tilt system. Our results suggest that polar double perovskite fluorides may be realized by suppression of octahedral rotations about more than one Cartesian axis; structures exhibiting in- or out-of-phase octahedral rotations about the  $c$  axis are more susceptible to polar symmetries.

DOI: [10.1103/PhysRevB.92.054111](https://doi.org/10.1103/PhysRevB.92.054111)

PACS number(s): 31.15.A–, 77.80.–e

## I. INTRODUCTION

Many 21st century technologies such as lasers, sensors, transistors, and nonvolatile recording media depend on materials without spatial inversion symmetry [1–4]. However, noncentrosymmetric compounds occur infrequently in nature, particularly in solid-state inorganic materials where they comprise less than  $\sim 18\%$  of all known materials [5–8]. One way to overcome the limited material availability is to elucidate the forces favoring noncentrosymmetric distortions, and then use that understanding to design, for example, new ferroelectric materials with polar atomic displacements.

Within this context, one of most studied crystal families for realizing new polar materials are oxide perovskites with chemical formula  $\text{ABO}_3$  owing to the perovskite structure's chemical and structural flexibility [5,9]. Although there remains an incomplete chemical recipe for the design of new polar perovskites, research in ternary transition metal oxides have successfully identified key atomic and electronic structure features that reliably describe the tendency for a given composition to be polar and ferroelectric: octahedral rotations and antipolar displacements of the 12-fold coordinated  $A$ -site cations may suppress ferroelectricity [10,11], and thus they should be deactivated. Prior work has established that for ordered perovskites, for example,  $(AA')\text{BO}_3$ , polar displacements can stabilize polar structures in  $A$ -site driven ferroelectric perovskites [12,13]; however, the magnitude of the polar displacements is often small in the absence of covalent interactions. Furthermore, larger polarizations are obtained in  $\text{ABO}_3$  perovskites by incorporating stereo-active ( $ns^2np^0$ ) cations on the  $A$  site or  $d^0$  transition metal cations on the  $B$  site, which stabilize polar distortions by increasing

covalent interaction between the metal-ligand framework through the second order Jahn-Teller effect (SOJT) [9,14–16].

Although these operating principles are useful for perovskite oxides, the guidelines are not universal across all anion chemistries in structures characterized by metal-anion octahedra with different covalency and polarizability [17]. Intriguingly, inversion symmetry lifting distortions are more scarce in the more ionic fluoroperovskites ( $\text{ABF}_3$ ) than in oxides [18], despite polar phases commonly appearing in fluorides belonging to the  $\text{ABF}_4$ ,  $\text{ABF}_5$ ,  $\text{A}_5\text{B}_3\text{F}_{19}$ , and  $\text{A}_3\text{B}_5\text{F}_{15}$  families [17,19,20]. Two specific centrosymmetric transition metal (M) double perovskite fluorides ( $\text{A}_2\text{BMF}_6$ ) include those belonging to the cryolite and elpasolite crystal families (Fig. 1): elpasolites with the chemical formula  $\text{A}_2\text{BMF}_6$  are usually cubic,  $Fm\bar{3}m$  symmetry without any octahedral rotations ( $a^0a^0a^0$ ). Cryolite fluorides,  $\text{A}_3\text{MF}_6$ , where  $B = A$ , are distorted and frequently crystallize in  $P2_1/n$  symmetry with the Glazer tilt system  $a^-a^-c^+$  [21]. These observations lead us to pose the following question: are octahedral rotations or increased ionicity in perovskite fluorides responsible for the suppression of ferroelectricity? Also, given the possibility for thin film growth of fluorides [22–25], we propose a related challenge: can polar phases be stabilized in fluoroperovskites with epitaxial strains as is achieved in oxides [26–28]?

In this work, we use first principles calculations to examine the lattice dynamics of the cryolite  $\text{Na}_3\text{ScF}_6$  ( $P2_1/n$ ) [31,32] and elpasolite  $\text{K}_2\text{NaScF}_6$  ( $Fm\bar{3}m$ ) [33,34]. We find that the low-frequency polar IR-active phonons in these double perovskite fluorides are dominated by the displacements of alkali metals (K and Na) with smaller accompanying displacements of the  $\text{Sc}^{3+}$  ion. Similar to  $\text{ABO}_3$  perovskites, the frequency of the polar modes harden with increasing tolerance factor,  $t$  [11]. Specifically, we find a ferroelectric instability at the equilibrium unit cell volume for the high symmetry cubic structure of  $\text{Na}_3\text{ScF}_6$  (with the smaller  $A$ -site cation), whereas

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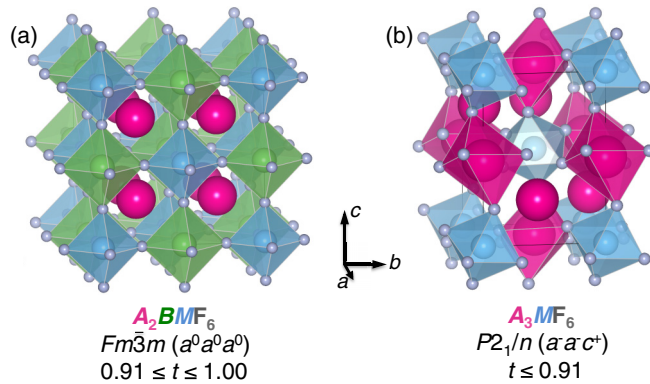


FIG. 1. (Color online) Common structure types adopted by double perovskite fluorides. Elpasolites (a) have stoichiometry  $A_2BMF_6$  and are typically found in cubic  $Fm\bar{3}m$  symmetry. Cryolites (b) have stoichiometry  $A_3MF_6$  and often crystallize in a rotationally distorted monoclinic  $P2_1/n$  symmetry. In cryolites the 12-fold coordinated  $A$  cation is chemically identical to the octahedrally coordinated  $B$ -site cation ( $A = B$ ). The structure stability range for the each family within the given symmetry is specified by the tolerance factor,  $t$ , obtained using Shannon radii [29,30].

in  $K_2NaScF_6$  the polar mode is hard. Our analyses indicate that the driving force for the ferroelectric instability and octahedral rotations are identical, i.e., owing to underbonding of the  $A$ -site cation [13].

We also investigate the strain dependence of the polar phase stabilities compared to the centrosymmetric ground state structure in  $Na_3ScF_6$  and  $K_2NaScF_6$  and uncover that compressive strain favors centric geometries. However, we observe that tensile strain couples strongly to the polar instability, favoring displacements in the  $ab$  plane that stabilize lower symmetry polar structures over the bulk centric phases. For the polar structures, we compute the electric polarizations and find that they vary  $\sim 5$ – $18 \mu C cm^{-2}$  in the  $ab$  plane for  $Na_3ScF_6$  and  $K_2NaScF_6$  over the tensile strain range explored. The origin for the polar displacements is due to an increase in the metal-fluoride bond lengths from the strain, which destabilizes the  $A$  cation bond geometry. Thus stable polar geometries result from the acentric distortions that optimize the bond distances in the metal-ligand framework.

## II. COMPUTATIONAL DETAILS

We performed density functional theory (DFT) calculations with the general gradient approximation (GGA) of Perdew-Burke-Ernzerhof revised for solids [35] (PBEsol) as implemented in the Vienna *Ab initio* Simulation Package (VASP) [36,37] with the projector augmented wave (PAW) method [38] to treat the core and valence electrons using the following valence configurations:  $2p^63s^1$  for Na,  $3p^64s^13d^2$  for Sc, and  $2s^22p^5$  for F. We use a  $7 \times 7 \times 7$  Monkhorst-Pack  $k$ -point mesh [39] with Gaussian smearing (10 meV width) for Brillouin zone (BZ) integrations and a 600 eV plane wave cutoff. Structural relaxations are performed until the Hellmann-Feynman forces are less than  $1 \text{ meV } \text{\AA}^{-1}$  on each atom. We use density-functional perturbation theory to compute the lattice phonons of  $Na_3ScF_6$  and  $K_2NaScF_6$  in

the ideal cubic structure,  $Fm\bar{3}m$ . Owing to the large unit cell size (40 atoms), all phonon calculations are performed on  $\sqrt{2} \times \sqrt{2} \times \sqrt{2}$  supercells. The phonon mode symmetries are analyzed using the PHONOPY package [40].

## III. RESULTS

### A. $Na_3ScF_6$ lattice dynamical properties and low energy structures

$Na_3ScF_6$  crystallizes in the common cryolite symmetry, i.e.,  $P2_1/n$  and  $a^-a^-c^+$  tilt pattern, with a tolerance factor  $t = 0.87$  [31]. With this in mind, we first examine the tendency to ferroelectricity by exploring the phonon frequencies of the ideal cubic phase and considering how combinations of soft modes responsible for the monoclinic symmetry support or suppress polar displacements. We find a ferroelectric lattice instability transforming as the irreducible representation (irrep)  $\Gamma_4^- (I4mm)$  with a frequency of  $102i \text{ cm}^{-1}$ . Figure 2(a) shows the polar mode mainly consists of displacements of the 12-fold coordinate Na ions and fluoride anions, similar to what has been recently reported for  $ABF_3$  perovskites [23]. The character of the polar mode in  $Na_3ScF_6$  is distinct from that commonly observed in perovskite oxides, whereby the six-coordinate transition metal and oxide anions undergo polar displacements [11]. Given that the polar mode found in  $Na_3ScF_6$  is dominated by  $A$ -site displacements, the origin of the polar instability is unlikely to originate from a SOJT mechanism.

Next, we find the mode frequencies for the unstable  $ScF_6$  tilt modes are more unstable than the polar mode. The in-phase  $a^0a^0c^+$  rotation described by  $X_3^+$  has a mode frequency of  $\nu = 133i \text{ cm}^{-1}$  [Fig. 2(b)] and the out-of-phase  $a^-a^-c^0$  rotation [ $\Gamma_4^+$ , Fig. 2(c)] has a mode frequency of  $\nu = 135i \text{ cm}^{-1}$ . Condensing the in-phase and out-of-phase distortions into the  $Fm\bar{3}m$  aristotype produces the centrosymmetric  $P4/mnc$  and  $C2/m$  structures, respectively.

To investigate the relative stabilities of structures derived from the unstable lattice modes, we freeze-in each distortion mode into the ideal cubic phase and then perform full volume

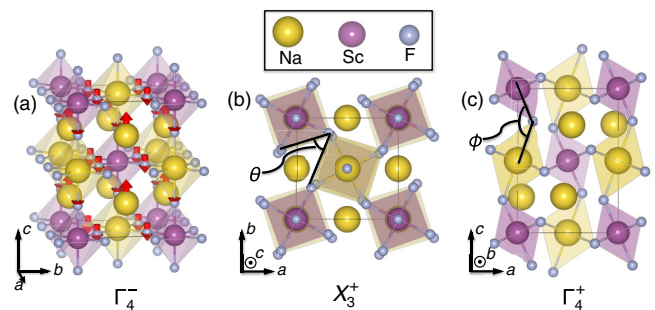


FIG. 2. (Color online) Unstable lattice distortions at  $\Gamma$  and  $X$  for  $Na_3ScF_6$ . (a)  $\Gamma_4^-$  is a polar displacement of  $A$ -site Na atoms and F atoms in the  $[001]$  direction indicated by the direction of the red arrows; the length indicates the relative magnitude of the displacement for the ions. (b)  $X_3^+$  is an in-phase octahedral rotation about the  $[001]$  axis. The rotation angle is measured as  $(90 - \theta)/2$ . (c)  $\Gamma_4^+$  is an out-of-phase rotation of octahedra about the  $[110]$  direction. The tilt angle is measured as  $(180 - \phi)/2$ . These modes may be visualized using the resources in the Supplemental Material [41].

TABLE I. Structural and energetic comparison for equilibrium  $\text{Na}_3\text{ScF}_6$  phases in space group symmetries obtained by condensing unstable lattice modes with frequencies,  $\nu$ . The interaxial angles for all DFT-PBEsol structures were constrained to  $\alpha = \beta = \gamma = 90^\circ$  except for the ground state  $P2_1/n$  phase, where  $\beta = 90.72^\circ$  is fixed to the experimentally reported monoclinic angle [31]. Relative energies differences per formula unit (f.u.),  $\Delta E$ , are reported with respect to the aristotype  $Fm\bar{3}m$  structure.

Symmetry	Glazer tilt	Mode	$\nu$ ( $\text{cm}^{-1}$ )	Lattice constants ( $\text{\AA}$ )	$\Delta E$ (meV/f.u.)	Rotation angle (deg)
$Fm\bar{3}m$	$a^0a^0a^0$			8.306 ( $a = b = c$ )	0	0.0
$I4mm$	$a^0a^0a^0$	$\Gamma_4^-$ [001]	102i	5.942 ( $a = b$ ), 8.404 ( $c$ )	-263	0.0
$P4/mnc$	$a^0a^0c^+$	$X_3^+$ [001]	133i	5.795 ( $a = b$ ), 8.196 ( $c$ )	-576	18.5
$C2/m$	$a^-a^-c^0$	$\Gamma_4^+$ [110]	135i	5.657 ( $a$ ), 5.865 ( $b$ ), 8.147 ( $c$ )	-926	21.9
$P2_1/n$	$a^-a^-c^+$	$X_3^+ \oplus \Gamma_4^+$		5.588 ( $a$ ), 5.794 ( $b$ ), 8.105 ( $c$ )	-1050	13.0, 21.8

and atomic relaxations. We find that although the polar  $I4mm$  structure is energetically more stable than the  $Fm\bar{3}m$  phase by 263 meV/f.u. (Table I), the phases containing octahedral rotations are always more energetically favorable. Indeed the monoclinic  $P2_1/n$  structure, which contains coexisting  $a^0a^0c^+$  and  $a^-a^-c^0$  rotations, is most stable ( $a^-a^-c^+$ ). Furthermore, we find that the sum of the relative energy gain  $\Delta E$  (and rotation angle amplitudes) obtained from the in-phase and out-of-phase rotations independent of any other distortions is greater than that computed for the ground state structure (Table I). This result indicates that the coexisting in-phase and out-of-phase rotations in  $P2_1/n$  compete with each other.

We now investigate whether the polar mode displacements are compatible with the single-tilt structures exhibiting the  $a^0a^0c^+$  and  $a^-a^-c^0$  octahedral rotations. We begin by computing the zone-center phonons for each relaxed intermediate  $P4/mnc$  and  $C2/m$  structure given in Table I. We find in the  $P4/mnc$  phase that the softest mode,  $\nu = 118i$   $\text{cm}^{-1}$ , describes out-of-phase  $\text{ScF}_6$  octahedral rotations and transforms like  $\Gamma_4^+$  [42]. Condensation of the mode would drive the  $P4/mnc \rightarrow P2_1/n$  symmetry reduction. Interestingly, we also find that a polar instability is present at  $\nu = 82i$   $\text{cm}^{-1}$  with  $\Gamma_4^-$  symmetry. Here, Na and F displacements along the [110] direction would produce a polar  $P4/mnc \rightarrow Pm$  transition. Hence in-phase rotations alone in  $\text{Na}_3\text{ScF}_6$  do not appear to suppress the polar instability, which is consistent with that reported for perovskite oxides [11], i.e., the frequencies of the soft ferroelectric modes decrease but remain unstable in the presence of the  $a^0a^0c^+$  tilt. On the other hand, the intermediate  $C2/m$  structure with tilt pattern  $a^-a^-c^0$  does not have a polar instability.  $C2/m$  has one unstable phonon,  $\nu = 66i$   $\text{cm}^{-1}$ , that produces the centrosymmetric  $C2/m \rightarrow P2_1/n$  transition.

### B. Strain effects on $\text{Na}_3\text{ScF}_6$ structure stability

Here we examine how biaxial strain affects the stability and local structure of the various  $\text{Na}_3\text{ScF}_6$  phases. For all structures, we assume epitaxy occurs on a cubic substrate that forces the in-plane lattice parameters to be equal ( $a = b$ ). At a value of 0% strain, we set the pseudocubic lattice constant of the substrate  $a_S = (a + b)/2$ , where  $a$  and  $b$  are the bulk equilibrium lattice constants of the phase being considered (see Table I) [43]. At each strain state, the  $c$  axis and atomic positions are relaxed until the forces and stresses are below the specified threshold.

Figure 3(a) shows the evolution in the total energy of  $\text{Na}_3\text{ScF}_6$  as a function of biaxial strain for the ground state

$P2_1/n$  ( $a^-a^-c^+$  tilt) and low-energy polar  $I4mm$  structure without octahedral rotations ( $a^0a^0c^0$  tilt). We observe that at 0% strain the monoclinic phase is  $\sim 0.7$  eV/f.u. more stable than the polar structure. We find that increasing compressive (negative) or tensile (positive) strain leads to an increase in the energy of both phases. Nonetheless, the energy difference between  $P2_1/n$  and  $I4mm$  is always greater than 0.5 eV [Fig. 3(b)], and the energy difference in the tensile range also becomes approximately strain independent. These results indicate that epitaxial strain is ineffective at stabilizing the polar, rotation-free, structure over the monoclinic phase with octahedral rotations.

We next investigate the effect of in-plane epitaxial strain on the phase stability of the two intermediate,  $P4/mnc$  and  $Pm$ , phases [Fig. 4(a)]. We find that at 0% biaxial strain the  $Pm$  phase is more favorable than the centrosymmetric  $P4/mnc$  intermediate. The ground state  $P2_1/n$  ( $a^-a^-c^+$ ) structure is  $\sim 0.47$  eV/f.u. more stable than the polar  $Pm$  structure. We also observe that compressive strain increases the total energy of all  $\text{Na}_3\text{ScF}_6$  structures. For strains greater than  $-2\%$  strain, we find that the  $Pm$  phase with the polar displacements is destabilized relative to the nonpolar  $P4/mnc$  structure. In contrast, for increasing tensile strain this stability reversal among the two low-energy phases with only in-phase tilts does not occur: the  $Pm$  phase is always more stable than the

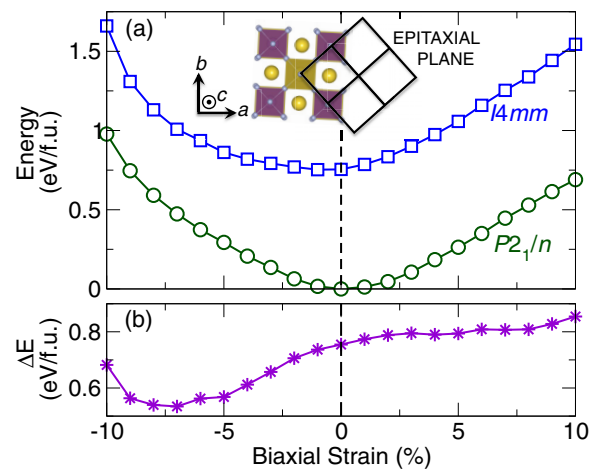


FIG. 3. (Color online) (a) Stability of the monoclinic  $P2_1/n$  ground state (circles) and polar  $I4mm$  (squares) structure with epitaxial strain. The inset shows a schematic of the epitaxial relationship between  $\text{Na}_3\text{ScF}_6$  and the cubic substrate. (b) The energy difference between the structures is obtained as  $\Delta E = E(I4mm) - E(P2_1/n)$ .



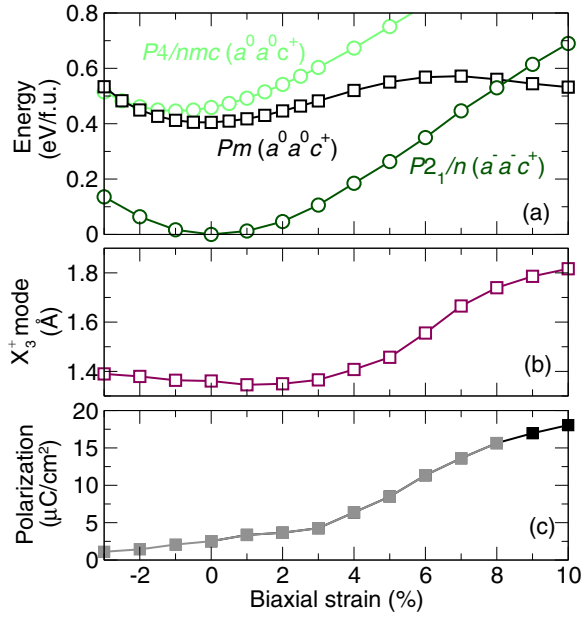


FIG. 4. (Color online) (a) Total energy evolution with biaxial strain for low-energy  $\text{Na}_3\text{ScF}_6$  polymorphs. The evolution in the (b) in-phase rotation mode amplitude and (c) electric polarization for the  $Pm$  phase with strain. The gray line in (c) indicates the region where the polarization is inaccessible at 0 K.

$P4/mnc$  structure with a single octahedral tilt and no polar distortion.

Remarkably, for sufficiently large tensile strains ( $>8\%$ ) we find it possible to stabilize the polar  $Pm$  structure relative to the bulk  $P2_1/n$  ground state. With increasing compressive or tensile strain the total energy for both centrosymmetric structures ( $P4/mnc$  and  $P2_1/n$ ) evolve in a similar fashion. In the tensile region, however, the total energy for the polar  $Pm$  phase initially increases for small strains between  $\sim 4\%$  and then the curvature above this value changes such that the energy difference between the  $P2_1/n$  and  $Pm$  phases decreases.

We next apply mode-crystallography analyses [44] to the  $Pm$  structures to understand this strain-dependent phase stability: the  $Pm$  phase is characterized by three main structural distortions, i.e., in-phase rotations about the [001] direction ( $X_3^+$ ), polar displacements ( $\Gamma_4^-$ ), and antipolar displacements ( $X_5^-$ ) [41]. We observe that the amplitude of the in-phase rotation remains fairly constant below  $+4\%$  strain [Fig. 4(b)]. However, the rate at which the amplitude of the polar mode changes with strain increases for strains  $\geq 4\%$ . By comparing the evolution of the electric polarization in the  $ab$  plane, shown in Fig. 4(c), with the amplitude of the rotation mode, we find similar strain-dependent behavior, including a marked increase in polarization near  $4\%$  strain. In contrast, the evolution of the antipolar mode amplitude shows no significant changes about  $4\%$  strain; it evolves in a nearly linear fashion [41].

To understand the nature of the strain interactions on the  $Pm$  phase, we next examine in detail the energetic dependence of the three coexisting  $\Gamma_4^-$ ,  $X_3^+$ , and  $X_5^-$  modes at  $4\%$  strain. Figures 5(a)–5(c) show that at this value of tensile strain, the  $\Gamma_4^-$  mode, which produces a  $Cm$  symmetry [see panel (d)], is an energy-raising distortion in the absence of

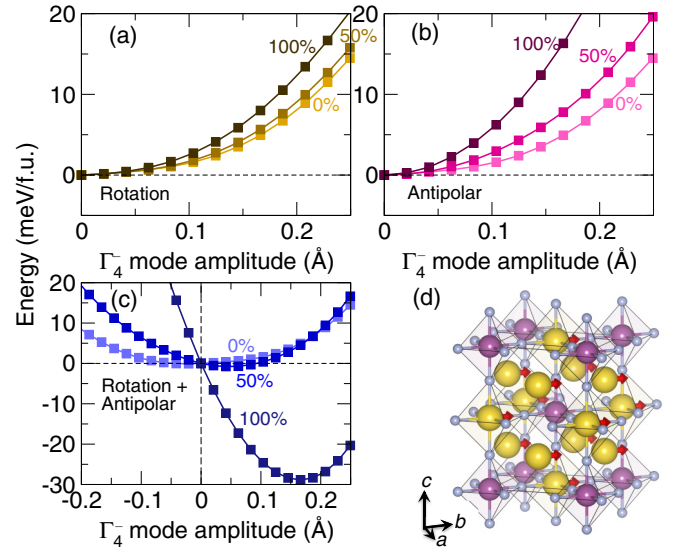


FIG. 5. (Color online) Energetic dependence of various mode distortions at  $4\%$  biaxial strain for  $\text{Na}_3\text{ScF}_6$ . Coupling between  $\Gamma_4^-$  and fixed amplitudes of (a) the in-phase rotation,  $X_3^+$ , (b) the antipolar distortion,  $X_5^-$ , and (c) the linear combination of the two modes,  $X_3^+ \oplus X_5^-$ . (d) The polar displacements of ions in the  $\Gamma_4^-$  mode. The normalized energy gains are obtained by increasing the amplitudes of the in-phase and antipolar distortions at fixed percentages found in the equilibrium structure at  $4\%$  tensile strain.

coupling to any other mode (see the  $0\%$  curves in all panels). Next we consider the coupling of the polar mode to another distortion at a finite fraction for which the latter is present in the equilibrium strained structure, indicated by the percentage labels. When the polar mode is coupled to finite amplitudes of the in-phase rotation or the antipolar distortion at  $4\%$  tensile strain [Figs. 5(a) and 5(b)], we observe that these polar  $Pm$  phases are further disfavored. Figure 5(c) shows that for fixed amplitudes of  $X_3^+ \oplus X_5^-$  coupled to  $\Gamma_4^-$ , however, there is a net energy gain that increases with the amplitude of the coexisting rotation and antipolar displacements. We conclude then for  $\text{Na}_3\text{ScF}_6$  that in-plane tensile strain favors the stabilization of a polar structure via cooperative coupling of the in-phase rotations and antipolar displacements to the polar mode.

We note that for  $\text{Na}_3\text{ScF}_6$  the  $9\%$  in-plane tensile strain (pseudocubic,  $a_S \approx 4.46 \text{ \AA}$ ) needed to stabilize the  $Pm$  phase over the  $P2_1/n$ , however, exceeds the pseudocubic lattice constants of available substrates [45]. Thus, although tensile strain favors polar distortions in the cryolite double perovskite, the energetic gain arising from the octahedral rotations is more favorable at most accessible strain values and would lead to centric fluorides. Therefore, we propose the use of biaxial strain in a fluoroperovskite that is free of octahedral rotations in its ground state. In this way, one would not be required to overcome the large stability provided by the octahedral rotations to the total energy. We explore such a scenario in the next section.

### C. Bulk and thin film $\text{K}_2\text{NaScF}_6$

Chemical substitution allows one to exploit steric effects in double perovskite fluorides. The replacement of Na cations on

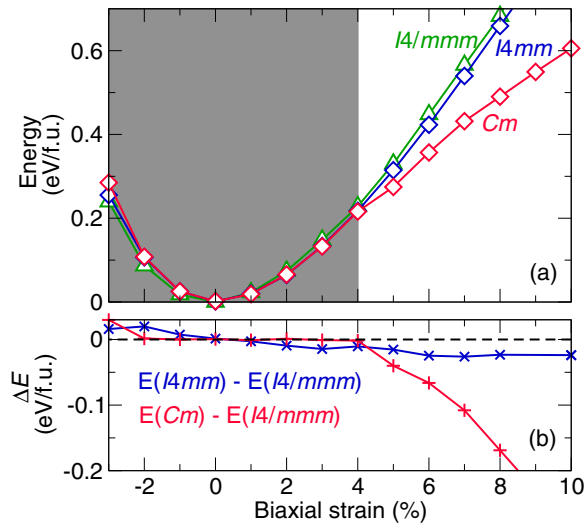


FIG. 6. (Color online) Evolution of the (a) relative total energy and (b) energy differences ( $\Delta E$ ) with biaxial strain for low-energy polar phases of  $K_2NaScF_6$  with respect to the equilibrium centrosymmetric ( $I4/mmm$ ) structure. In (a) the area shaded gray represents the biaxial strain region over which the centrosymmetric phase is stable.

the A site of  $Na_3ScF_6$  by a larger alkali metal would give an elpasolite ( $A_2NaScF_6$ ) with significantly less underbonding of the A cation. The larger A-site cation would therefore reduce the tendency towards octahedral rotations, which would be reflected in a larger tolerance factor. We conjecture this type of chemical substitution may provide a promising avenue for stabilizing polar phases at smaller pseudocubic lattice constants by eliminating competing (rotational) distortions.

To test our hypothesis, we investigate the chemistry  $K_2NaScF_6$  ( $t = 0.954$ ), which is experimentally reported to be a cubic double perovskite fluoride at room temperature ( $Fm\bar{3}m$ ,  $a = 8.47$  Å) [34]. We first examine the dynamical stability of  $K_2NaScF_6$  by performing phonon calculations on the PBEsol relaxed equilibrium cubic structure. For our calculations, we find the triply degenerate polar  $\Gamma_4^-$  modes are hard with  $\nu = 130$   $cm^{-1}$ , corresponding to displacements along either [001] or [110].

Given the sensitivity of ferroelectric modes to pressure [10,46] and the tendency of  $Na_3ScF_6$  to prefer polar displacements at large tensile strains, we next examine the pressure dependence of these polar modes in  $K_2NaScF_6$  at unit cell volumes greater than the equilibrium volume. This computational experiment corresponds to a negative pressure configuration. Upon increasing the equilibrium cubic lattice constant by 5%, we find unstable zone center modes ( $45i$   $cm^{-1}$ ) that are also threefold degenerate with  $\Gamma_4^-$  symmetry. Similar to  $Na_3ScF_6$ , the  $\Gamma_4^-$  [001] mode drives the  $Fm\bar{3}m \rightarrow I4mm$  symmetry reduction, whereas the  $\Gamma_4^-$  [110] mode governs the  $Fm\bar{3}m \rightarrow Cm$  transition. All atoms contribute to this mode with displacements along the [001] and [110] directions, respectively [41].

Now we investigate the stability of the polar  $I4mm$  and  $Cm$  phases relative to the cubic structure with biaxial strain (Fig. 6). Note that the centrosymmetric cubic symmetry of the bulk phase reduces to tetragonal  $I4/mmm$  owing to the mechanical constraint from epitaxy. We find that

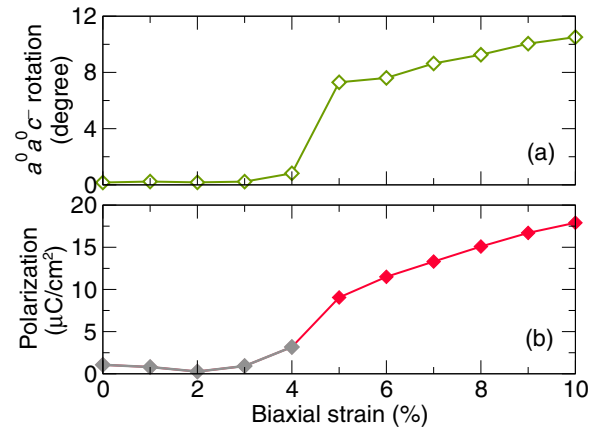


FIG. 7. (Color online) Evolution of the (a) out-of-phase rotation angle and (b) in-plane electric polarization in  $Cm$   $K_2NaScF_6$  as a function of tensile strain. The gray line represents the region where the polar state is inaccessible.

compressive strain favors the centrosymmetric phase over the polar structures ( $\sim 40$  meV/f.u.). Consistent with our negative pressure simulation, Fig. 6(a) shows that increasing tensile strain increases the stability of the polar structures over the centrosymmetric geometry. The energy of the  $I4mm$  phase is approximately 20 meV/f.u. or less lower in energy than the  $I4/mmm$  structure for tensile strains above 1% [Fig. 6(b)]. In contrast, the relative stability of the  $Cm$  phase over the  $I4/mmm$  structure increases steadily for strains above 4%. Below this value the polar phases are nearly degenerate with each other. These results show that tensile strain stabilizes polar ionic displacements in the epitaxial plane rather than orthogonal to it. Such coupling of the in-plane polar distortions to tensile strain is consistent with prior work in  $ABO_3$  perovskites [28,47,48].

A symmetry mode analysis of  $K_2NaScF_6$  above the 4% critical strain reveals that the  $Cm$  equilibrium structure is characterized by three main irreps. First, the polar mode  $\Gamma_4^-$  displaces the K, Na, and Sc atoms in the  $ab$  plane. The  $\Gamma_5^-$  mode contains polar displacements of the F ion along the  $ab$  plane. The third main distortion transforms as  $\Gamma_4^+$  and describes out-of-phase rotations about the  $c$ -axis, Glazer tilt system  $a^0a^0c^-$  [41].

The strain dependence of the out-of-phase rotation mode and  $\Gamma_4^-$  mode amplitudes, which we translate to a rotation angle and electric polarization, are shown in Fig. 7. The stabilization of the  $Cm$  phase above 5% strain as shown in Fig. 6 corresponds to the abrupt increase in the out-of-phase rotation angle and enhanced electric polarization (Fig. 7). This suggests that the octahedral rotations may be necessary to stabilize the polar phase. Interestingly, the latter result seemingly contradicts our initial hypothesis of eliminating competing rotational distortions to access polar geometries. However, we note that the polar phase of  $K_2NaScF_6$ , much like  $Na_3ScF_6$ , exhibits a single rotation about the [001] direction.

We next examine the coupling of the out-of-phase rotation modes to the polar modes at 5% strain. We observe that if the octahedral rotations coexist with either polar modes  $\Gamma_4^-$  [Fig. 8(a)] or  $\Gamma_5^-$  [Fig. 8(b)], the energy is always higher and the quadratic dependence of the energy landscape is greater

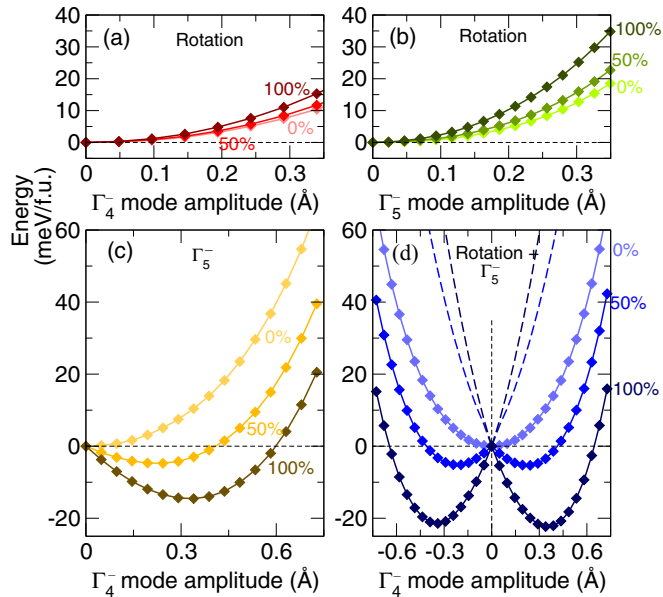


FIG. 8. (Color online) Evolution in the energy for  $K_2NaScF_6$  as a function of interacting lattice modes at 5% strain. (a) The coupling between the polar  $\Gamma_4^-$  and the out-of-phase rotation  $\Gamma_4^+$  modes, (b) the polar  $\Gamma_5^-$  and the out-of-phase rotation  $\Gamma_4^+$  modes, (c) the two polar modes, i.e.,  $\Gamma_4^- \oplus \Gamma_5^-$  (no rotations), and (d) the linear combination of all three modes,  $\Gamma_4^- \oplus \Gamma_5^- \oplus \Gamma_4^+$ . Here the symmetry equivalent minima are obtained by computing the energy for positive and negative amplitudes of the polar and rotational distortion modes. The normalized energy gains are obtained by increasing the amplitudes of the distortions at fixed percentages found in the equilibrium structure at 5% tensile strain.

than when the rotations are absent. However, Fig. 8(c) indicates that the  $\Gamma_4^- \oplus \Gamma_5^-$  coupling lowers the energy for increasing distortion amplitude, and together these modes give the  $Cm$  symmetry. This suggests that the two polar modes must coexist in order to drive the system to a stable polar configuration. Note the combination of any two ( $\Gamma_4^-$ ,  $\Gamma_5^-$ , or  $\Gamma_4^+$ ) modes will also produce the  $Cm$  symmetry. As such all three modes also couple to the lowest order via a trilinear term in the free energy,  $E_{\text{trilinear}} \propto \Gamma_4^- \Gamma_5^- \Gamma_4^+$ . When all three distortions are combined, i.e.,  $\Gamma_4^- \oplus \Gamma_5^- \oplus \Gamma_4^+$  [Fig. 8(d)], the polar ground state is further stabilized. However, our results suggest that the  $Cm$  phase of  $K_2NaScF_6$  can indeed exist without the out-of-phase rotation. The rotations act to enhance the stability of the ferroelectric phase provided by the biquadratic coupling between the two polar modes [Fig. 8(c)]. This result is distinct from  $Na_3ScF_6$  where the polar phase could not be stabilized in the absence of the in-phase rotation. Moreover, unlike many layered polar perovskites and complex antiferroelectrics [49], our results suggest that the trilinear coupling may be less important to the stability of the polar phase.

Here we successfully demonstrate that the critical strain state can be reduced to stabilize a polar polymorph by eliminating the competing out-of-phase  $a^-a^-c^0$  rotation mode present in the equilibrium  $Na_3ScF_6$  phase with chemical substitution. Importantly, we note that the large radius of K compared to Na results in larger equilibrium lattice constants for  $K_2NaScF_6$  over  $Na_3ScF_6$ . As a result, the 5% strain state

predicted to stabilize the  $Cm$  phase of  $K_2NaScF_6$  would require a pseudocubic lattice constant of 4.43 Å, which is also beyond that accessible in commercially available perovskite substrates [ $a_S(LaLuO_3) = 4.17$  Å] [50]. Our analysis suggests that the critical value could be reduced further with alternate chemistries by optimizing the tolerance factor.

#### IV. DISCUSSION

Ferroelectricity in fluoride perovskites has been speculated for some time. However, to date there have only been a few first principles studies attempting to elucidate routes towards the stabilization of polar phases in fluoride perovskites [23,51]. Our results show that the structural distortions present in bulk double perovskite fluorides, much like  $ABO_3$  perovskites, are mainly controlled by cation sizes [29,52]. Although fluoroperovskite structures are also prone to octahedral rotations, polar lattice instabilities may also be present and our lattice dynamical calculations for  $Na_3ScF_6$  ( $t = 0.87$ ) and  $K_2NaScF_6$  ( $t = 0.95$ ) reveal that the ferroelectric mode frequencies harden with increasing tolerance factor. The latter observation has previously been detailed in Ref. [11] for  $ABO_3$  perovskites. However, in contrast to  $ABO_3$  perovskites, the polar displacements in the cryolite and elpasolite fluorides are strongly dominated by displacements of the alkali metals (Na, K) and fluoride ions rather than the six-coordinate transition metal cations [23].

Our first principles calculations and structural analyses demonstrate that the double perovskites  $Na_3ScF_6$  and  $K_2NaScF_6$  show similarities to strain-activated ferroelectrics [23,27,28,53]. First, much like low tolerance factor incipient perovskites, the strain-stabilized polar  $Pm$  phase of  $Na_3ScF_6$  exhibits octahedral rotations. In simple perovskites without cation order, the acentric geometries have the  $a^-a^-c^+$  tilt pattern [23,28], but here, the  $a^0a^0c^+$  tilt supports the polar phase of  $Na_3ScF_6$ . Furthermore, the electric polarization in the  $ab$  plane of  $Na_3ScF_6$  is realized by coupling of the polar mode with the in-phase rotation about the  $c$  axis. The latter is in contrast to strained  $CaTiO_3$ , where the polarization that appears has little dependence on the amplitude of the rotation modes [28].

Second,  $K_2NaScF_6$  is stable in cubic symmetry at room temperature, similar to high tolerance factor incipient ferroelectric  $SrTiO_3$ . Under tensile strain both chemistries demonstrate strain-induced polarizations in the  $ab$  plane with coexisting out-of-phase octahedral rotations. For compressive strains  $K_2NaScF_6$  adopts the centrosymmetric phase.  $SrTiO_3$ , on the other hand, shows an enhancement of the polar mode along the [001] direction in the compressive strain region [27]. Indeed, experiments have confirmed that strain-induced polar phases of  $SrTiO_3$  can also coexist with antiferrodistortive octahedral rotations at low temperatures [53,54]. Here we find that the out-of-phase rotation in  $K_2NaScF_6$  may not be pivotal to the stability of the  $Cm$  phase, but there is a strong polarization-rotation coupling that augments the size of the electric polarization.

To better understand the mechanism responsible for the strain-induced polar phases of  $Na_3ScF_6$  and  $K_2NaScF_6$ , we compute the layer resolved polarization in both structures at 10% tensile strain. Figure 9 shows the layer polarizations decomposed along the  $a$  and  $b$  axes; all layers are found

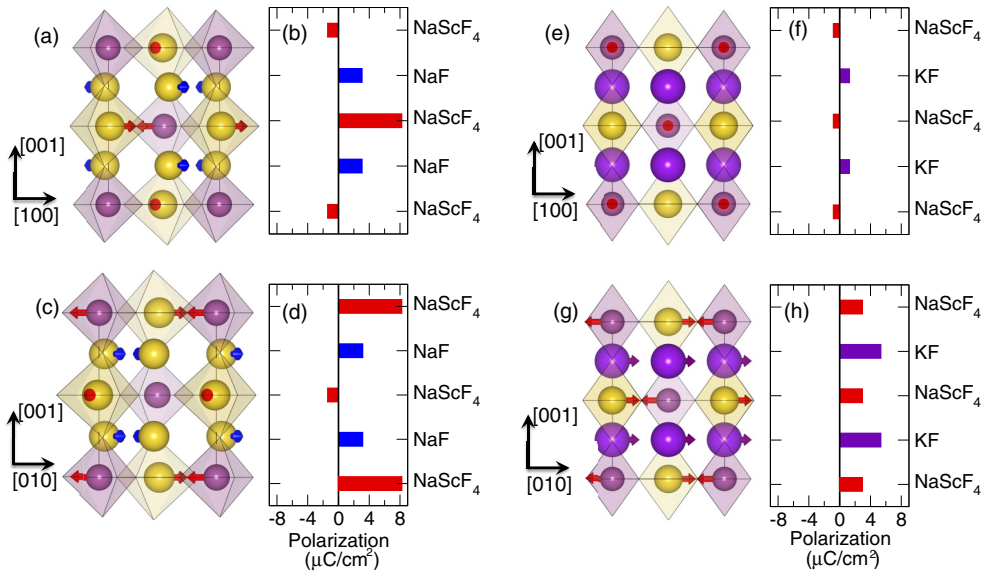


FIG. 9. (Color online) Layer-resolved polarizations for polar  $\text{Na}_3\text{ScF}_6$  ( $Pm$ ) and  $\text{K}_2\text{NaScF}_6$  ( $Cm$ ) at 10% tensile strain. (a) The cation displacements and (b) the layer-resolved polarization for  $\text{Na}_3\text{ScF}_6$  along the  $[100]$  direction for  $\text{Na}_3\text{ScF}_6$ . (c) The cation displacements and (d) the layer-resolved polarization for  $\text{Na}_3\text{ScF}_6$  along the  $[010]$  direction. Similarly, panels (e),(f) and (g),(h) depict the same information for  $\text{K}_2\text{NaScF}_6$  along the  $[100]$  and  $[010]$  directions, respectively. Berry phase calculations result in the total electric polarization for  $\text{Na}_3\text{ScF}_6$  at 10% to be  $P = 12.7\hat{x} + 12.8\hat{y}$ , which is close to that obtained by summing the layer dipoles, i.e.,  $\Sigma p = 13.0\hat{x} + 13.1\hat{y}$ . Similarly, for  $\text{K}_2\text{NaScF}_6$  at 10%,  $P = 1.4\hat{x} + 17.9\hat{y}$  and  $\Sigma p = 0.9\hat{x} + 16.6\hat{y}$ .

to contribute to the total polarization. These values are in excellent agreement with those obtained with our Berry phase calculations. Interestingly, for  $\text{Na}_3\text{ScF}_6$  we observe that the polarization from the  $\text{NaScF}_4$  layers is about twice that of the  $\text{NaF}$  layers. This behavior can be understood with the help from Fig. 9(a) and our mode coupling investigation (Fig. 5). Note that in the  $\text{NaScF}_4$  layers, the sixfold coordinated Na and Sc ions displace in an antipolar fashion with respect to each other. We conjecture that when the antipolar  $X_5^-$  and polar  $\Gamma_4^-$  mode couple they act to amplify the off-centering in the  $\text{NaF}_6$  octahedra over the more rigid  $\text{ScF}_6$  octahedra along the  $ab$  direction. The result is a net polarization in the direction of displacement of the Na ion [Fig. 9(b)]. Similar behavior is seen in the  $\text{NaF}$  layers; despite antipolar displacements of the Na ions, the polar mode leads to aligned layer polarizations in the  $\text{NaF}$  layers, thus resulting in a stable polar  $\text{Na}_3\text{ScF}_6$  structure that has a small  $A$ -site contribution to the total polarization [Figs. 9(b) and 9(d)].

Panels (e) through (h) in Fig. 9 reveal that the polarization in  $\text{K}_2\text{NaScF}_6$  is dominated by acentric displacements in the  $\text{KF}$  layers rather than the  $\text{NaScF}_4$  layers in the  $ab$  plane. From our coupling investigations (Fig. 8), we note that the  $Cm$   $\text{K}_2\text{NaScF}_6$  phase is stabilized by combining two polar modes. We assume that, in contrast to  $\text{Na}_3\text{ScF}_6$ , the absence of the canceling antipolar distortion in  $\text{K}_2\text{NaScF}_6$  leads to the larger contribution from the  $A$ -site cations (K ions) to the total polarization.

The rationale for the stabilization of the polar  $\text{Na}_3\text{ScF}_6$  and  $\text{K}_2\text{NaScF}_6$  phases at large tensile strain can be understood from bond valence arguments [55]. Increasing tensile strain leads to lattice deformations that stretch the metal-ligand bond network. Cations that sit at a center of a cavity that is too large will have a low bond valence sum. To recover the

preferred valence, the metal-ligand framework is optimized by compensating lattice distortions [15,55]. In these materials, the off-centering of the metal ions and the octahedral rotations are the lattice responses that improve the undercoordinated bond environments.

## V. CONCLUSION

We performed a first principles study based on density functional theory to understand the competition between polar and nonpolar lattice instabilities in double perovskite fluorides. We explored the feasibility of using strain engineering to stabilize polar polymorphs over centrosymmetric phases in  $\text{Na}_3\text{ScF}_6$  and  $\text{K}_2\text{NaScF}_6$ .

From our analysis of the polar displacements in  $\text{Na}_3\text{ScF}_6$  and  $\text{K}_2\text{NaScF}_6$ , we find that the primary contributors are the alkali metals (K, Na) and F ions. The ferroelectric modes computed at the equilibrium cubic volumes result in unstable modes in  $\text{Na}_3\text{ScF}_6$  but stable modes for  $\text{K}_2\text{NaScF}_6$ . The softening of the ferroelectric modes with negative hydrostatic pressure in  $\text{K}_2\text{NaScF}_6$  thus suggests that the polar instability is driven by increasing the underbonding between the  $A$ -site cation and the ligand [13]. This indicates that the same structural mechanism controls the tendency towards rotational instabilities and  $A$ -site dominant polar instabilities in  $\text{Na}_3\text{ScF}_6$  and  $\text{K}_2\text{NaScF}_6$ .

In both chemistries, large tensile strains stabilize polar geometries that couple strongly to a single rotational distortion about the  $c$  axis ( $a^0a^0c^+$  or  $a^0a^0c^-$ ). Our results imply that in the double perovskite fluorides, one may only need to suppress the two-tilt out-of-phase rotation,  $a^-a^-c^0$ , to tip the balance towards polar structures. Although thin film growth of fluorides has been successful using molecular beam



epitaxy [22,25], our predicted polar phases for  $\text{Na}_3\text{ScF}_6$  and  $\text{K}_2\text{NaScF}_6$ , however, would require critical strains which are greater than those that could be imposed by commercially available perovskite substrates. Nonetheless, we posit that strain stabilization of ferroelectric double perovskites may be experimentally realized with alternate chemistries. For instance, similar studies for double perovskites fluorides that substitute  $\text{Sc}^{3+}$  for the smaller  $\text{Al}^{3+}$  or  $\text{Ga}^{3+}$  cations may prove successful in reducing the critical strain value. We hope that this work motivates a resurgence of interest in experimental growth of fluoride films for use in present and future technologies.

## ACKNOWLEDGMENTS

This material is based upon work supported by the National Science Foundation under Grant No. DMR-1454688. DFT calculations were performed on the high-performance computing facilities available at the Center for Nanoscale Materials (CARBON Cluster) at Argonne National Laboratory, supported by the US DOE, Office of Basic Energy Sciences (BES), Grant No. DE-AC02-06CH11357, and at the Extreme Science and Engineering Discovery Environment (XSEDE), which is supported by the National Science Foundation Grant No. ACI-1053575.

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