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Diamagnetism in wire medium metamaterials: Theory and experiment

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A strong diamagnetic response of a wire medium with a finite wire radius is reported. Contrary to the previous works where it was assumed that the wire medium exhibits only an electric response, we show that the nonzero magnetic susceptibility has to be taken into account for a proper effective medium description of the wire medium. Analytical and numerical results are supported by experimental measurements.

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I. INTRODUCTION

Wire metamaterials are composed of arrays of optically thin metallic rods embedded in a dielectric matrix [1]. These structures have been extensively studied both theoretically and experimentally, due to their possible applications in subwavelength image transfer [2–7] and control over the spontaneous emission lifetime of quantum emitters [8,9]. While there are different geometries of the wire media, in our work we consider a special class, formed by a two-dimensional array of parallel wires of finite radius and length, shown schematically in Fig. 1. For practical applications and specifically in order to model the electromagnetic properties of wire metamaterial blocks of complex geometry, effective material parameters such as permittivity and permeability are required for proper homogenization. The effective dielectric permittivity of an array of perfectly conducting parallel wires has been obtained in a number of papers [10–12]. The permittivity and permeability tensors are given by [12]

$$\hat{\varepsilon} = \varepsilon_m \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \tilde{\varepsilon}_{zz} \end{pmatrix}, \quad \hat{\mu} = \mu_m \mathrm{I}, \tag{1}$$

where I is the unity matrix, ε_m, μ_m are the permittivity and permeability of the host material, and $\tilde{\varepsilon}_{zz}$ reads

$$\tilde{\varepsilon}_{zz} = \left(1 - \frac{2\left(r_0^2 \log \frac{d^2}{4r_0(d-r_0)}\right)^{-1}}{(\omega/c)^2 - k_z^2} \right), \tag{2}$$

where r_0 is the wire radius, and d is the period of the structure. We note that the dielectric permittivity tensor is nonlocal, i.e., it depends on the wave vector k_z. However, in the case of an infinite wire structure along the lateral direction, at normal incidence, we can consider only the tangential components of the dielectric permittivity which coincide with the dielectric permittivity of the matrix. Thus, at normal incidence, the light effectively interacts with a uniform dielectric with a permittivity ε_m and a magnetic permeability equal to unity. However, it is clear that the surface currents at the wire interfaces should lead to nonzero magnetic susceptibility. In this Rapid Communication, we present analytical, numerical, and experimental results to conclude that for a proper description of the effective parameters of wire media, it is required to account for the diamagnetic response of these structures.

II. THEORETICAL MODELING

First, let us consider the quasistatic approximation for a single unit cell. We assume that the uniform static magnetic field is aligned along the x direction. Then, the surface currents originating at the surface of the wire are aligned along the wires and can be expressed as $\mathbf{j}_{s,z} = -2\mu_m H \sin(\phi)\delta(r - r_0)$, where ϕ is the azimuthal angle, and r is the projection of the radius vector on the xy plane. Knowing the currents we can calculate the magnetic moment of the unit cell M as $\frac{1}{2}\int d^3\mathbf{r}[\mathbf{r} \times \mathbf{j}]$. Dividing it by the unit cell volume $V = d^2 L_z$ will give us the magnetization density and the effective permeability. The integral can be taken analytically, yielding

$$\mu_{\text{eff}} = \mu_m \left(1 - \frac{\pi r_0^2}{d^2} \right). \tag{3}$$

We note that when deriving expression (3) we assumed a freestanding wire. Thus, no interaction with the neighboring wires is accounted for. As will be seen later, this approximation works well for thin wires. To obtain the effective permittivity we use the results of Ref. [13], where it is shown that the wire medium belongs to a class of so-called *isorefractive* media, where the relation $\varepsilon_{\text{eff}}\mu_{\text{eff}} = n^2 = \varepsilon_{\text{m}}\mu_m$ holds. Thus, for the effective permittivity we have

$$\varepsilon_{\text{eff}} = \varepsilon_m \left(1 - \frac{4r_0^2}{d^2} \right)^{-1}. \tag{4}$$

Thus, we argue that for a correct description of the wire medium, ε_m and μ_m in Eq. (1) should be substituted with the right-hand sides of Eqs. (4) and (3), respectively. We note that there are many works dedicated to artificial diamagnetism in metamaterials [14–16]. At the same time, surprisingly, diamagnetism in wire media metamaterials is conventionally abandoned.

We also support the analytical expressions with numerical modeling. For that we use a homogenization technique similar to the one described in Ref. [17]. We again assume the quasistatic approximation and consider the propagation of the transverse electric magnetic (TEM) mode along the wires. In this case, we can set that electric field is aligned along y and the magnetic field along x directions. We introduce static electric and magnetic potentials u_e, u_m defined as $\nabla u_{e(m)} = \mathbf{E}(\mathbf{H})$. Then, two Laplace equations for u_e and u_m can be solved. For the electric potential we use the following boundary conditions: $u_e|_{y=\pm d/2} = \pm u_{e0}$, $\partial u_e/\partial x|_{x=\pm d/2} = 0$, and $u_e|_{r=r_0} = 0$ at the wire interface. For the magnetic

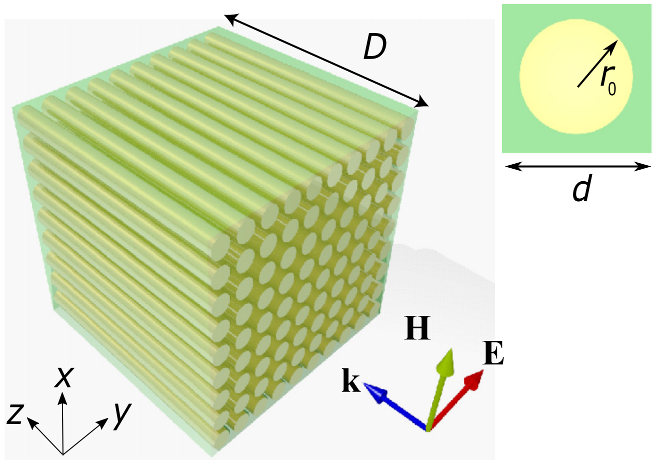

FIG. 1. (Color online) Geometry of the considered structure: An array of perfectly conducting wires of finite radius r_0 and length D is placed inside the dielectric matrix with permittivity ε_m.

potential, the boundary conditions are $u_m|_{x=\pm d/2} = \pm u_{m0}$, $\partial u_m/\partial y|_{y=\pm d/2} = 0$, and $\partial u_\mathrm{m}/\partial r|_{r=r_0} = 0$. We solve both Laplace equations numerically [18]. Knowing the electric and magnetic potentials and therefore the electric and magnetic fields, it is possible to express the effective permittivity and magnetic permeability. To obtain the effective permittivity, we assume the unit cell as a parallel plate capacitor with the capacitance C given by $C = Q_s/u_e = S\varepsilon_\mathrm{eff}/d$, where Q_s is the surface charge accumulated at the plate and $S = L_z d$ is the surface area. The surface charge Q_s can be found from the Gauss law, $\int \mathbf{D}d\mathbf{S} = Q_s$. Expressing the effective permittivity ε_eff gives

$$\varepsilon_\mathrm{eff} = \frac{\int_{-d/2}^{d/2} \varepsilon_m E_y|_{y=-d/2}dx}{u_{e0}}. \tag{5}$$

The analogous approach is applied to find the effective permeability μ_eff, yielding

$$\mu_\mathrm{eff} = \frac{\int_{-d/2}^{d/2} \mu_m H_x|_{x=-d/2}dy}{u_{m0}}. \tag{6}$$

The maps of the scalar electric and magnetic potentials as well as field lines are shown in Figs. 2(a) and 2(b).

We compare the analytical expressions (3) and (4) with the more rigorous approach given by expressions (5) and (6) in Figs. 2(c) and 2(d), respectively. We use the following structure parameters: $\varepsilon_\mathrm{m} = 16, \mu_\mathrm{m} = 1$. It can be seen that for thin wires, the diamagnetism is negligible and both approaches match almost perfectly. At higher values of r_0/d the two methods start to give slightly different results but still match quite well. The discrepancies at large r_0/d appear, since the analytical approach considers a single wire and does not account for the structure periodicity and interaction between wires in different unit cells, which grow with the growth of r_0/d.

Moreover, in order to check if the quasistatic approximation is applicable in the case of realistic wire media structures, we have extracted the effective electric and magnetic susceptibility using the Nicolson-Ross-Weir (NRW) method [19–21]. We have calculated the reflection and transmission spectra of the wire medium slab with the same material parameters,

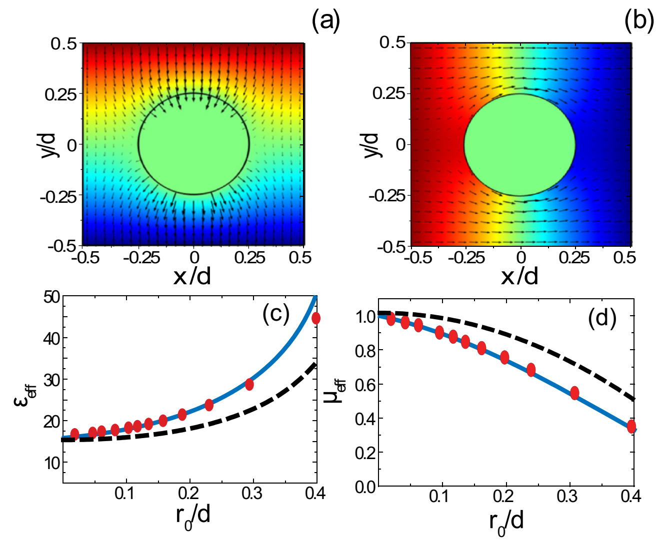

FIG. 2. (Color online) (a), (b) Maps of the scalar (a) electric and (b) magnetic (b) potentials and field lines for the (a) electric and (b) magnetic fields in a unit cell for the case of $r_0/d = 0.25$; the color scale is the same for both maps. (c) Effective dielectric permittivity vs r_0/d obtained via the analytical formula in Eq. (4) (dashed black line), the semianalytical expression (5) (solid blue lines), and from extracting the effective parameters with the NRW method (red circles). (d) Effective magnetic permeability vs r_0/d obtained via the analytical formula (3) (dashed black line), the semianalytical expression (6) (solid blue lines), and from extracting the effective parameters with the NRW method (red circles).

period $d = 25\ \mu$m, thickness $D = d$ in the full-wave electromagnetic simulation package [22]. The wire radius r_0 was a parameter which was changed in the region $[0.01d, 0.4d]$. The conventional NRW method essentially is an inversion of the Rayleigh expressions for the transmission and reflection from the uniform material slab with respect to the effective material refractive index $n = \sqrt{\varepsilon_\mathrm{eff}\mu_\mathrm{eff}}$ and impedance $Z = \sqrt{\varepsilon_\mathrm{eff}/\mu_\mathrm{eff}}$:

$$R = \frac{i(Z^2 - 1)}{2Z\cot(n\omega/cD) - i(Z^2 + 1)}, \tag{7}$$

$$T = \frac{2Z/\sin(n\omega/cD)}{2Z\cot(n\omega/cD) - i(Z^2 + 1)}. \tag{8}$$

The values of n, Z are then substituted to obtain the $\varepsilon_\mathrm{eff}, \mu_\mathrm{eff}$. The results are shown with the red dots in Figs. 2(c) and 2(d). As shown in Figs. 2(c) and 2(d), there is a good match between the full numerical simulation and the semianalytical approach.

We stress that omitting the effective permeability of the structure leads to incorrect values of the reflection and transmission coefficients if calculated with Rayleigh formulas, as shown in Fig. 3(a), where the reflection and transmission spectra for the cases when the permeability is accounted for or omitted are compared to the numerical modeling. The value of r_0/d is set to 0.25. We can see that omitting the magnetic response gives incorrect results for the reflection and transmission spectra. Moreover, as can be seen in Fig. 3(b), the extracted parameters correctly describe the electromagnetic properties of the wire medium slab in the case of oblique incidence, when diamagnetism is taken into account.

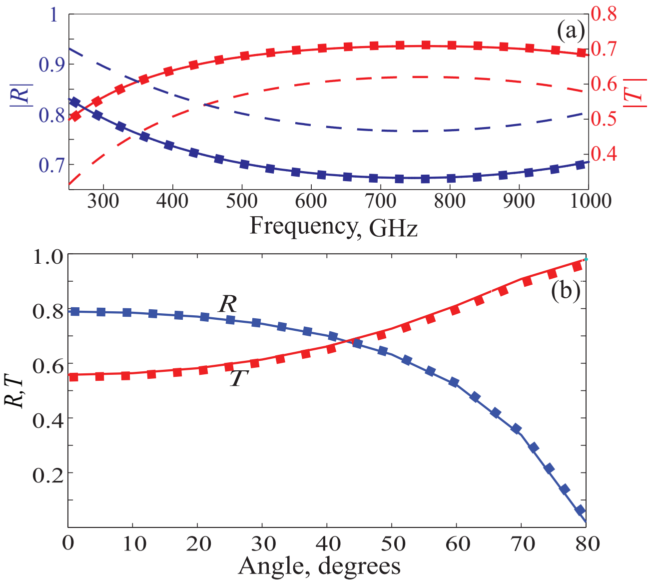

FIG. 3. (Color online) (a) Reflection and transmission spectra for the case of normal incidence. The solid and dashed lines correspond to the reflection and transmission calculated with Eqs. (7) and (8) with $\varepsilon = 16$, $\mu = 1$ (dashed lines), and with $\varepsilon = 23.19$, $\mu = 0.69$ (solid lines). The dots correspond to the CST Microwave Studio modeling results. (b) The dependence of reflection and transmission on the incidence angle at an incident wave frequency 250 GHz; solid lines correspond to reflection and transmission calculated with Eqs. (7) and (8) with $\varepsilon = 23.19$, $\mu = 0.69$, and the squares correspond to the CST modeling. Parameters of the wire medium structure are presented in the text.

III. EXPERIMENTAL MEASUREMENTS

In order to validate our results experimentally, we have constructed a wire metamaterial block with the following parameters: period of the structure $d = 11$ mm, wire radius $r_0 = 5$ mm, and wire length $D = 150$ mm. The block consists of 20×20 wires with an air dielectric matrix, i.e., $\varepsilon_m = 1$. The photograph of the structure is shown in the inset of Fig. 4. The experimental measurements were conducted in the following manner: A rectangular horn antenna (TRIM [23] 0.75–18 GHz; DR) connected to a transmitting port of the vector network analyzer Agilent E8362C was used to approximate a plane-wave excitation. The metamaterial block was placed at the far-field region of the antenna and a similar horn antenna (TRIM 0.75–18 GHz) was employed as a receiver. The effective scattering cross section was obtained from the imaginary part of the forward scattering amplitude (due to the optical theorem). Due to the fact that the horn antenna substantially modifies the wave front, we have normalized the scattering cross section to unity. After completing the experimental measurements, we performed a numerical simulation of the scattering at a metamaterial block of the same dimensions for three different models. In the first model, the metamaterial block was modeled as an array of perfectly conducting wires, reconstructing the geometry of the experimental setup. In the second model, the block consisted of a uniform anisotropic medium with the following parameters: $\varepsilon_\perp = 5.762$, $\varepsilon_\parallel = -10\,000.0 + 1.0i$, $\mu_\perp = 0.1735$, $\mu_\parallel = 1.0$,

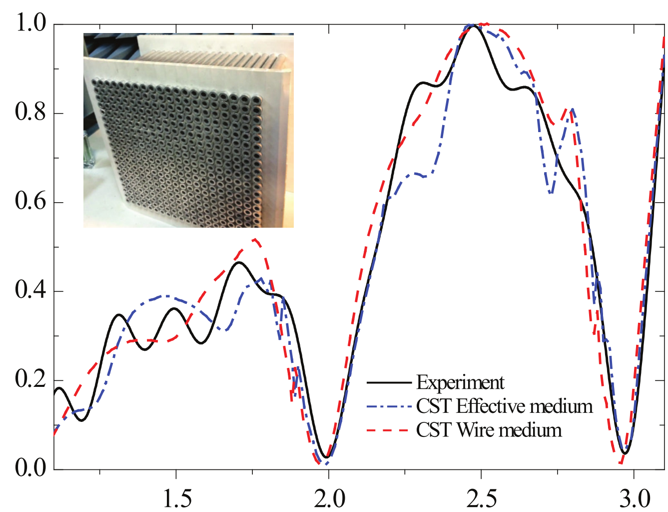

FIG. 4. (Color online) Scattering cross section of the wire metamaterial block. The inset shows a photograph of the structure. The experimentally determined scattering cross section is compared with numerical calculations performed in CST. The ripples in the experiment are explained in the main text.

where subscripts $\parallel$ and $\perp$ correspond to directions parallel and perpendicular to the wire axis, respectively. The scattering cross sections were normalized to unity. The above values for dielectric permittivities and magnetic permeabilities have been derived with the expressions in Eqs. (3) and (4). Finally, in the third model, we have calculated the scattering on a metamaterial block made from the anisotropic uniform media with the effective transverse dielectric and magnetic permeabilities equal to unity. Notably, it is quite obvious that in the third model, the scattering cross section should be vanishingly small for the case of normal incidence, since both the effective impedance and refractive index of this structure are equal to unity. The spectra of the forward scattering efficiency for the first two cases are shown in Fig. 4.

The observed oscillations in the experimental scattering spectra are due to the artificial Fabry-Pérot resonances between the metamaterial block interface and the source and receiving horn antennas. We observe a good correspondence between the experimental cross-section spectra and numerical results, namely, in the positions of the cross-section dips and in the cross-section modulation contrast. An even better correspondence is observed between the two numerical simulations. We note that such a correspondence could not be achieved in the approximation of effectively nonmagnetic media, since, as was discussed previously, such media do not scatter the normally incident electromagnetic radiation at all. The observed discrepancies between the experimental and numerical simulation results are mainly due to the wavelength-dependent wave-front distortions of the incident wave, produced by the horn antenna.

IV. CONCLUSION

We have shown both theoretically and experimentally that thick wire media exhibit a strong diamagnetic response, and that accounting for the effective magnetic susceptibility is crucial for obtaining adequate material parameters of these structures. The inclusion of the magnetic response of the wire medium is crucial for the design of wire-medium-based metadevices. Moreover, the strong diamagnetic response of a thick wire medium can be used for magnetic levitation [24]. Also, with low magnetic permeability in a wide frequency range, the thick wire medium can be considered as a base for so-called μ-near-zero (MNZ) [25] (permeability near zero) materials. We thus believe that our work can inspire research of the magnetic properties of wire metamaterials.

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[1] C. R. Simovski, P. A. Belov, A. V. Atrashchenko, and Y. S. Kivshar, Adv. Mater. **24**, 4229 (2012).
[2] P. A. Belov, Y. Zhao, S. Sudhakaran, A. Alomainy, and Y. Hao, Appl. Phys. Lett. **89**, 262109 (2006).
[3] A. Rahman, P. A. Belov, Y. Hao, and C. Parini, Opt. Lett. **35**, 142 (2010).
[4] G. Shvets, S. Trendafilov, J. B. Pendry, and A. Sarychev, Phys. Rev. Lett. **99**, 053903 (2007).
[5] S. Kawata, A. Ono, and P. Verma, Nat. Photonics **2**, 438 (2008).
[6] M. G. Silveirinha, P. A. Belov, and C. R. Simovski, Phys. Rev. B **75**, 035108 (2007).
[7] P. A. Belov, Y. Zhao, S. Tse, P. Ikonen, M. G. Silveirinha, C. R. Simovski, S. Tretyakov, Y. Hao, and C. Parini, Phys. Rev. B **77**, 193108 (2008).
[8] A. N. Poddubny, P. A. Belov, and Yu. S. Kivshar, Phys. Rev. B **87**, 035136 (2013).
[9] A. P. Slobozhanyuk, A. N. Poddubny, A. E. Krasnok, and P. A. Belov, Appl. Phys. Lett. **104**, 161105 (2014).
[10] P. A. Belov, R. Marques, S. I. Maslovski, I. S. Nefedov, M. Silveirinha, C. R. Simovski, and S. A. Tretyakov, Phys. Rev. B **67**, 113103 (2003).
[11] C. R. Simovski and P. A. Belov, Phys. Rev. E **70**, 046616 (2004).
[12] S. I. Maslovski and M. G. Silveirinha, Phys. Rev. B **80**, 245101 (2009).
[13] J. S. Tyo, IEEE Trans. Antennas Propag. **51**, 1093 (2003).
[14] E. Shamonina and L. Solymar, Eur. Phys. J. B **41**, 307 (2004).
[15] M. Lapine, A. K. Krylova, P. A. Belov, C. G. Poulton, R. C. McPhedran, and Yu. S. Kivshar, Phys. Rev. B **87**, 024408 (2013).
[16] L. Parke *et al.*, Appl. Phys. Lett. **106**, 101908 (2015).
[17] D. R. Smith, J. B. Pendry, J. Opt. Soc. Am. B **23**, 391, (2006).
[18] The numerical modeling was performed in the COMSOL MULTIPHYSICS package.
[19] A. M. Nicolson and G. F. Ross, IEEE Trans. Instrum. Meas. **19**, 377 (1970).
[20] W. B. Weir, Proc. IEEE **62**, 33 (1974).
[21] T. L. Blakney and W. B. Weir, Proc. IEEE **63**, 203 (1975).
[22] Simulations were performed in the CST Microwave Studio Package.
[23] www.trimcom.ru.
[24] Y. Urzhumov, W. Chen, C. Bingham, W. Padilla, and D. R. Smith, Phys. Rev. B **85**, 054430 (2012).
[25] N. Engheta, A. Alu, M. G. Silveirinha, A. Salandrino, and J. Li, in *Proceedings of the Mediterranean Electrotechnical Conference, 2006* (IEEE, New York, 2006), pp. 258–261.